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ERRATUM

SOME SPECULATIONS ON THE PATTERN OF QUARK AND LEPTON MASSES*

J. D. Bjorken
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

Conclusion (3) on page 8 is incorrect. It should be replaced by the following:

3) Our best guess (assuming SU(2) symmetry) for the masses of fourth generation quarks Q is > 150 GeV. Such quarks may be produced in pp or $p\bar{p}$ colliding-beam experiments. For example for $E_{cms} \gtrsim 2$ TeV one might expect a $Q\bar{Q}$ production cross section in the nanobarn range. Such a superheavy quark will decay into a light quark q and an intermediate boson W

$$Q \to q + W, \qquad W \to q + \bar{q}$$

leading to a final state typically containing six high-$p_\perp$ jets ($p_{\perp i} \sim 50$ GeV) of unmistakable signature.

* Work supported by the Department of Energy under Contract No. EY-76-C-03-0515.

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Stanford Linear Accelerator Center
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ABSTRACT

We conjecture that the logarithm of the mass of quark or lepton is a smooth function of its charge and generation. Together with an assumption of approximate mass-degeneracy for fourth-generation quarks, this leads to an estimate of $\sim$10 GeV for the mass of the fourth charged lepton, $\sim$27 GeV for the mass of the top quark, and $\sim$200 GeV for the (common) mass of the fourth-generation quarks.

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I. INTRODUCTION

The discovery and study of τ and Υ, as well as the progress in determining the couplings of quarks and leptons with respect to the weak SU(2) group,[1] has pointed fairly strongly toward a classification of fermions into families or, as Harari[2] puts it, generations, each isomorphic to the others in the absence of masses and Cabibbo-mixing. Thus the first-generation particles e, ν_e, d, u are isomorphic to the second generation particles μ, ν_μ, s, c, respectively. Furthermore it is rather widely believed[3] that the τ, ν_τ and b (the constituent of Υ) are members of a third generation, to be completed by a yet-to-be-discovered top quark t of charge 2/3 which is the weak-isospin partner of the b. If this is true--and in this note we shall assume it to be true--we will be confronted with a grand generalization of the old μ-e puzzle: not only are μ^- and τ^- copies of the e^-, the quarks also come in such copies. We do not now understand the reason for this repetitive pattern any better than we have ever understood the μ-e puzzle.[4] But certainly the pattern of masses and Cabibbo mixing, which after all is the feature distinguishing the generations from each other, is the main clue available to us at present. While in principle the Higgs structure of the spontaneously broken gauge theories should be an additional and powerful guide, the history of attempts to understand the pattern of mixings and mass-relations in that framework can only be called disappointing.[5] Nevertheless the amount of empirical information is not small, and there is perhaps something to be said for approaching the problem in a more phenomenological spirit. The point we wish to stress is that there at least appears to be a mass pattern: the mass-spectrum is not chaotic, despite the fact we cannot seem to comprehend it. In particular:

1) The generalized Cabibbo mixing angles (in the 6-quark Kobayashi-Maskawa[6] scheme) are <u>all</u> reasonably small,[3] so that it makes sense to assign quarks to generations with only perturbative mixing between generations. (We of course do not yet know the pattern of such mixing angles in detail; this requires a good deal of study of top and bottom production and decay properties.)

2) It is possible to assign the fermions to generations so that the masses increase as the generation increases; i.e.

$$\begin{aligned} &m_u < m_c < m_t \\ &m_d < m_s < m_b \\ &m_e < m_\mu < m_\tau \end{aligned} \tag{1}$$

While it is a matter of convention to assign leptons and, say $|Q|=1/3$ quarks in such a way, it is not a priori self-evident that the $|Q|=2/3$ quarks be ordered, as in Eq. (1). It could have been the case, for example, that the weak coupling of c was predominantly to d and u to s.

3) The mass-ratios of particles of given charge and of successive generations are <u>large</u>--at least an order of magnitude--and this is also often true when comparing masses within a generation (cf., Eq. (1)). (It is an attractive idea to try to connect these large mass ratios with the small mixing angles. A famous example is the relation[7]

$$\theta_c^2 \approx \frac{m_d}{m_s} \approx \frac{1}{20} \tag{2}$$

Another possibility is to connect them with $\alpha \sim 10^{-2}$ (or perhaps even to $\alpha_{strong} \sim 10^{-1}$?). However, we do not have anything more to say about that question and will not here approach the problem from that point of view.)

The above properties are not too much to build upon, but they are at least <u>something</u>. In order to continue, we shall make a strong assumption about the dynamics of mass generation, namely that for each given charge Q of fermion, we may write a mass formula with the general structure

$$\log\ m_{N,Q} = f(N,Q) \tag{3}$$

Here N denotes the generation, and the functions f(N,Q) are assumed to be smooth functions of N which may be approximated by low-order polynomials.

Why the property expressed above? Were it not for the logarithm, the formula would look like any old mass formula appropriate to broken internal symmetry or even to Regge trajectories. The logarithm is put in to take care of the quite remarkable empirical fact expressed above in item 3. And given the ubiquity of logarithms of masses in field-theory calculations, it may not be inconceivable that the logarithm could emerge someday from some fundamental framework. However, we do not have any specific suggestion to make.[8]

Equation (3) invites curve-fitting. In Fig. 1(a) is shown the primary data, with the solid lines an arbitrary interpolation to help guide the eye. For these purposes, our input data are the handbook lepton masses, plus the bare "current" masses for the quarks:

$$m_u \sim 4\ \text{MeV} \qquad m_s \sim 150\ \text{MeV}$$

$$m_d \sim 7\ \text{MeV} \qquad m_c \sim 1.2\ \text{GeV} \qquad (4)$$

$$m_b \sim 4.6\ \text{GeV}$$

The determination of m_u, m_d, and m_s is based on various electromagnetic and SU(3) mass splittings of mesons and baryons,[7] while $m_c \approx 1.2$ GeV is the the "current" mass determined from quantum chromodynamic charmonium sum rules.[9] The value of m_b is just a guess, but ought to be good to 10%. From Fig. 1(a), one notices the following features:

a) The logarithms of masses of charge 1/3 quarks are fitted rather well by a straight line, i.e.

$$\frac{m_d}{m_s} \approx \frac{m_s}{m_b} \approx .04 \pm .01 \qquad (5)$$

Extrapolation to N=4 would appear fairly reliable, leading to a conjectured mass for a fourth $|Q|=1/3$ quark h of $m_h \gtrsim 150$ GeV.

b) The corresponding curve for charged leptons is concave-down, since

$$\frac{m_e}{m_\mu} \sim 5 \times 10^{-3}$$

while (6)

$$\frac{m_\mu}{m_\tau} \sim 6 \times 10^{-2}$$

If there is a fourth-generation charged lepton λ , we thus expect

$$\frac{m_\tau}{m_\lambda} > 6 \times 10^{-2}, \qquad \text{or} \qquad m_\lambda < 30 \text{ GeV} \tag{7}$$

On the other hand, if one actually fits a second-order polynomial to the 3 data-points, and extrapolates to N=4, one gets

$$\frac{m_\tau}{m_\lambda} \approx \frac{m_\mu^3}{m_e m_\tau^2} \sim 0.7 \text{ !!!} \tag{8}$$

leading to the very low value

$$m_\lambda \sim 2.6 \text{ GeV !} \tag{9}$$

If we use the second-order polynomial to extrapolate to $N > 4$, the lepton masses become smaller than the τ mass. In fact N=4 is already beyond the maximum of the curve of mass _versus_ generation, which occurs at $N \sim 3.7$. We might speculate that once the monotonic nature of the mass formula no longer holds, there are no more generations present. Thus if we were to take the quadratic fit seriously, we could infer that the number of generations is three. Such an inference is in concord with the astrophysical evidence limiting the number of generations of neutrinos to a similar value.[10] However, there is no particular reason to insist on such a simple and definite formula. We therefore arrive at the rather feeble estimate that _if_ a fourth generation exists, the mass of the charged lepton λ is

$$3 \text{ GeV} < m_\lambda < 30 \text{ GeV} \tag{10}$$

c) The curve for charge 2/3 quarks, while containing only two data-points, appears more similar to the _lepton_ curve than to the curve connecting d, s, and b. In particular[11]

$$\frac{m_u}{m_e} \sim 8 \qquad \frac{m_c}{m_\mu} \sim 11 \tag{11}$$

Extrapolating this trend to the third generation gives

$$\frac{m_t}{m_\tau} \approx \left[\frac{m_c}{m_\mu}\right]^2 \cdot \left[\frac{m_e}{m_u}\right] \approx \left[\frac{11}{8}\right] \times 11 \approx 15 \tag{12}$$

leading to

$$m_t \sim 27 \text{ GeV} \tag{13}$$

Extrapolation to the fourth generation leads to an uncertainly in the $|Q|=2/3$ g-quark mass as large as for the fourth lepton and is not very helpful, other than restricting m_g to above 50 GeV. However inspection of Fig. 1(a) shows that the ratio of $|Q|=2/3$ to $|Q|=1/3$ quark mass first increases well beyond unity and then decreases with increasing generations. It is tempting to add one more speculation to this plethora of speculations, and to assume manifest SU(2) symmetry for the (final ?) fourth generation of quarks. Once we set $m_g \approx m_h$, then we may go backwards and guess the mass m_λ of the fourth lepton from m_g. If we fit a quadratic formula to the logarithms of m_d, m_s, and m_b, we get

$$\frac{m_b}{m_h} \approx \frac{m_s^3}{m_c m_b^2} \approx .023 \tag{14}$$

leading to

$$m_h \approx 200 \text{ GeV} \tag{15}$$

Extrapolating Eqs. (11) and (12) to the fourth generation particles gives

$$\rightarrow \frac{m_g}{m_\lambda} \approx \left[\frac{m_t}{m_\tau}\right]^2 \cdot \frac{m_\mu}{m_c} \approx \left[\frac{15}{11}\right] \cdot 15 \approx 20 \qquad (16)$$

Finally, upon assuming the fourth-generation SU(2) symmetry $m_g = m_h$, we get the estimate

$$m_\lambda \approx 10 \quad \text{GeV} \qquad (17)$$

d) We have ignored the masses, if any, of the neutrinos. Fig. 1(b) shows the present upper bounds, along with the astrophysical bound of $\lesssim 200$ eV on the sum of neutrino masses.[10] There is not much to say beyond the obvious statement that even on this logarithmic scale, neutrino masses are much smaller than the masses of the charged fermions.

Clearly all this is quite speculative guesswork and not to be trusted very much. In order of increasing improbability we note the following "conclusions":

1) If there is a fourth generation of fermions, the lightest member (other than the neutrino ν_λ) is most likely the charged lepton λ . Its mass should be negligible in comparison with fourth generation quarks and should probably be low enough to be found at PETRA and PEP. The best guess for the mass is $m_\lambda \sim 10$ GeV.

2) The (third generation) top quark may be too heavy to be found at PEP and PETRA; our best guess is $m_t \sim 27$ GeV, implying a mass for "toponium" ~55-60 GeV. However the uncertainties are very large.

3) Our best guess (assuming SU(2) symmetry) for the masses of fourth generation quarks Q is > 150 GeV. Such quarks may be produced in pp or $p\bar{p}$ colliding-beam experiments. For example for $E_{cms} \gtrsim 2$ TeV one might expect

a $Q\bar{Q}$ production cross section in the nanobarn range. Such a superheavy quark will decay into a light quark q and an intermediate boson W

$$Q \rightarrow q + W, \qquad W \rightarrow q + \bar{q}$$

leading to a final state typically containing six high-$p_\perp$ jets ($p_{\perp i} \sim 50$ GeV) of unmistakable signature.

4) The intergeneration mass relations of $|Q|=2/3$ quarks and leptons are similar despite the ratio of ~ 10-20 of m_q / m_ℓ within a generation. The $|Q|=1/3$ quarks behave somewhat differently. Perhaps this is indicative of intergeneration mixing occurring predominantly in the $|Q| = 1/3$ sector, so that d mixes with s, s with b, and b with h, with the mixing angles approximately equal to each other and to the Cabibbo angle:

$$\theta_c^2 \sim \frac{m_d}{m_s} \sim \frac{m_s}{m_b} \sim \frac{m_b}{m_h} \tag{18}$$

In terms of Kobayashi-Maskawa phenomenology, this would imply [3,6]

$$\begin{aligned} |\theta_2 + \theta_3 e^{i\delta}| &\sim \theta_c \\ |\theta_3 + \theta_2 e^{i\delta}| &\sim \theta_c \end{aligned} \tag{19}$$

Were there a formula similar to Eq. (2) for Q=2/3 quarks it would imply negligible u-c mixing. The c-t mixing <u>could</u> in principle be present at the level of θ_c, inasmuch as our estimate for m_t implies

$$\theta_c^2 \sim \frac{m_c}{m_t} \tag{20}$$

But there is too much that is not understood to state such things with any certainty.

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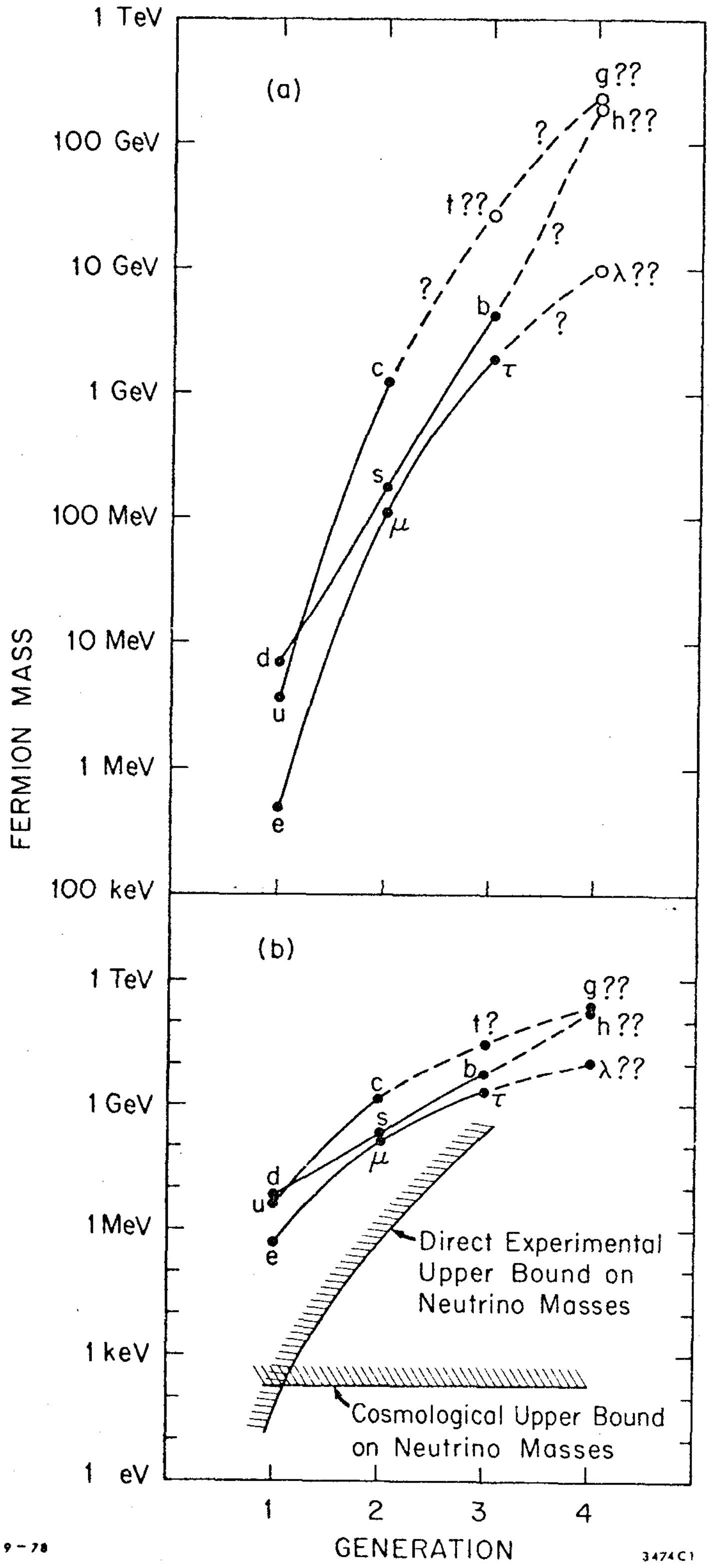

Fig. 1

Mass of basic fermions versus generation.