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MEASURING THE SPIN OF THE GLUON IN e^+e^- ANNIHILATION*

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ABSTRACT

We discuss the analysis of possible 3 jet events in e^+e^- annihilation and elsewhere. If one selects events with low thrust, the particles recoiling away from the thrust axis should appear as a pair of back-to-back jets in their centre-of-mass system. The angular distributions of such jets are very different in the case of vector gluon bremsstrahlung from what they would be for scalar gluons, or for the 3 gluon decay of a heavy quarkonium resonance. We outline a similar analysis of 3 jet events in photoproduction.

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1. Introduction

There has been much interest recently in the possibility of clean experimental tests of QCD in hadronic final states in e^+e^- annihilation and elsewhere. The basic idea is that the production of basic hadronic quanta (quarks, gluons) at large momentum transfers should be calculable in QCD perturbation theory, and reflected in the energy and angular distributions of final state hadrons. A prototype reaction is $e^+e^- \rightarrow q\bar{q} \rightarrow 2$ hadronic jets [1], which have been seen at SPEAR [2] with the angular distribution $\propto (1 + \cos^2\theta)$ expected [1] for spin 1/2 quarks, whereas spin 0 partons would have led to a $(1 - \cos^2\theta)$ distribution. Several authors have suggested that the higher order process $e^+e^- \rightarrow q\bar{q}$ + gluon may also be manifested in hadronic final states. In its most distinctive form this process may give 3 jet final states [3]. It should also lead to p_T broadening [4], characteristic patterns of hadronic energy deposition [5], angular correlations [6] and scaling violation [7] effects within 2 jet events. The purpose of this note is to discuss a method [8] of analyzing candidate 3 jet events so as to clarify their event structure. We propose that the angular distribution of the two less energetic jets in their centre-of-mass frame will provide a test of the spin of the gluon as sensitive as the test of quark spin provided by the $(1 + \cos^2\theta)$ angular distribution of 2 jet events. Our approach does nothing but reexpresses the known differential cross sections for $e^+e^- \rightarrow q\bar{q}$ + vector gluon (fig. 1(a)) [3], $e^+e^- \rightarrow q\bar{q}$ + scalar gluon (fig. 1(b)) [3], and quarkonium $\rightarrow$ 3 vector gluons (fig. 1(c)) [9] in a way which enables clear experimental discrimination between these physical processes. The same method of analysis could be applied equally well to electroproduction, neutrinoproduction [10,11] or photoproduction final states. We outline the application to photoproduction [12].

2. e^+e^- Annihilation

We start by clarifying and correcting a method of analysis suggested in ref. [8]. Define the thrust T of an e^+e^- final state by [13]

$$T \equiv \max_{\underline{n}} \frac{\sum_{\text{hadrons } h} |\underline{p}^h \cdot \underline{n}|}{\sum_{\text{hadrons } h} |\underline{p}^h|} \tag{1}$$

where $\underline{n}$ is a unit vector of arbitrary direction. The direction of $\underline{n}$ which maximizes T--the thrust axis--has a two-fold ambiguity which we resolve [8] by demanding that the sum $\sum_h |\underline{p}_T^h|$ of hadronic transverse momentum be smaller in the forward hemisphere about the thrust axis. If the event has a 2 jet structure, $T \underset{\sim}{<} 1$ with corrections due to nonperturbative effects which can only be unreliably estimated [8] at present. If the event has a 3 jet structure, $T < 1$ with the most energetic jet aligned along the forward thrust axis, and having energy $x_1 Q/2 \approx TQ/2$, where Q is the total e^+e^- centre-of-mass energy. The two less energetic jets will recoil into the backward hemisphere about the thrust axis. They should have a total energy

$$E = Q\left(1 - \frac{T}{2}\right) \tag{2}$$

and an invariant mass

$$M_x = Q\sqrt{1-T} \tag{3}$$

We propose [8] to analyze for 3 jet events by:

(A) Selecting events with low thrust, e.g., $T \underset{\sim}{<} 0.8$ at $Q \sim 15$ GeV, $T \underset{\sim}{<} 0.9$ at $Q \sim 22$ GeV, thus ensuring that $M_x \underset{\sim}{>} 7$ GeV, the centre-of-mass energy at which 2 jet events were most distinctive at SPEAR.

(B) Verifying that $\sum_h |\underline{p}_T^h|$ is small in the forward hemisphere about the thrust axis, as would be expected if a jet is headed in that direction.

(C) If so, discarding the hadrons in the forward hemisphere and Lorentz boosting (see fig. 2) the remaining hadrons by an amount ζ:

$$\mathrm{ch}\ \zeta = \frac{2-T}{2\sqrt{1-T}} \quad . \tag{4}$$

It can be seen from eqs. (2) and (3) that if the event does indeed have a 3 jet structure, the "jet boost" (4) should bring the 2 less energetic jets to their centre-of-mass frame where they should emerge back-to-back. Defining the angle which these jets make with the thrust axis to be $\tilde{\theta}$, we see that

$$\sin \tilde{\theta} = \frac{\pi\sqrt{S}}{4\sqrt{1-T}} \quad , \tag{5}$$

where S is the spherocity [14]

$$S \equiv \left(\frac{4}{\pi}\right)^2 \min \left(\frac{\sum_h |\underline{p}_T^h|}{\sum_h |\underline{p}_T^h|}\right)^2 \quad . \tag{6}$$

For three-jet events there is a kinematic bound

$$\frac{64}{\pi^2} \frac{(1-T)^2(2T-1)}{T^2} \leq S \leq \frac{16}{\pi^2} (1-T) \tag{7}$$

which can be re-expressed as a bound on $\tilde{\theta}$:

$$\frac{4(1-T)(2T-1)}{T^2} \leq \sin^2 \tilde{\theta} \leq 1 \quad . \tag{8}$$

As an example, if $T \gtrsim 0.8$ the bound (8) forces $\tilde{\theta}$ to lie between 90° and about 60°. One may therefore continue the analysis by :

(D) Verifying that the remaining hadrons do indeed form two jets in their centre-of-mass, with a $\tilde{\theta}$ angular distribution satisfying the constraints (8). Providing that the thrust selected is such that the dijet invariant mass M_x (3) is $\geq$ 7 GeV, it should be just as easy to distinguish jet structures from a phase space distribution and measure $d\sigma/d(\cos\tilde{\theta})$ as it was [2] when looking for 2 jet events at comparable centre-of-mass energies $Q \sim 7$ GeV.

It is easy to calculate the distributions in T and $\tilde{\theta}$ from the known cross sections for $e^+e^- \to q\bar{q}$ + vector gluon g [3], $e^+e^- \to q\bar{q}$ + scalar gluon s [3], and quarkonium $\to$ 3g [6]. We find that the angular distributions, normalized to 1 at $\cos\tilde{\theta} = 1$ are

$$N\,\frac{d\sigma(q\bar{q}g)}{d(\cos\tilde{\theta})} = \frac{1 + \dfrac{3(2-T)T^2\cos^2\tilde{\theta}}{4T^3 + (2-T)^3}}{1 - \cos^2\tilde{\theta}} \tag{9}$$

$$N\,\frac{d\sigma(q\bar{q}s)}{d(\cos\tilde{\theta})} = \frac{1 + \dfrac{T(3T-4)}{4-3T^2}\cos^2\tilde{\theta}}{1 - \cos^2\tilde{\theta}} \tag{10}$$

$$N\,\frac{d\sigma(ggg)}{d(\cos\tilde{\theta})} = \frac{1 + \dfrac{(6T^2-12T+4) + T^2\cos^2\tilde{\theta}}{8(1-T)^2 + (2-T)^2}\cos^2\tilde{\theta}}{\left(1 - \dfrac{T^2}{(2-T)^2}\cos^2\tilde{\theta}\right)^2} \tag{11}$$

The cross sections $\frac{1}{\sigma}\frac{d\sigma}{dT}$ and $\frac{1}{\Gamma}\frac{d\Gamma}{dT}$ have been plotted before [8]. In fig. 3 we plot the normalized angular distributions (9,10,11) for representative values of T = 0.8 and 0.9.

3. Results

We note the following serendipitons features of the distributions. For $e^+e^- \to q\bar{q}g$, the angular distributions are essentially independent of T, with the exception of the change in the kinematic bound (8). For

$\cos\tilde{\theta} \lesssim \frac{1}{2}$, the distributions are well approximated by

$$N \frac{d\sigma(q\bar{q}g)}{d(\cos\tilde{\theta})} \sim 1 + 2\cos^2\theta \qquad (12)$$

For $e^+e^- \to q\bar{q}s$, the angular distributions depend slightly on T, becoming flatter as T increases. However, a good approximation for $\cos\tilde{\theta} \lesssim \frac{1}{2}$ and $T \sim 0.8$ to 0.9 is

$$N \frac{d\sigma(q\bar{q}s)}{d(\cos\tilde{\theta})} \sim 1 + 0.2\cos^2\theta \quad . \qquad (13)$$

For quarkonium $\to$ ggg, the angular distributions are again essentially independent of T, and can be approximated, for $\cos\tilde{\theta} \lesssim \frac{1}{2}$, by

$$N \frac{d\Gamma(ggg)}{d(\cos\tilde{\theta})} \sim 1 - 0.1\cos^2\tilde{\theta} \quad . \qquad (14)$$

The approximate thrust independence of the angular distributions means that data with different values of T can easily be combined, as well as data from different centre-of-mass energies Q. The gross differences shown in fig. 3 between the angular distributions for $e^+e^- \to q\bar{q}g$ and $q\bar{q}s$, seen also in the approximate fits (12) and (13), mean that these angular distributions are sensitive probes of the spin 0 or 1 of the gluon. The only experimental tests of gluon spin to date seem to be the deep inelastic scaling violations, seen best in the neutrinoproduction analysis of the BEBC group [11]. However, that evidence for spin 1 gluons is rather abstract and formal, and we think the jet angular distributions (12) and (13) should be as striking as the $e^+e^- \to 2$ jets test of spin 1/2 for quark-partons. We also notice that the angular distributions on and off a quarkonium resonance (assuming the latter to be described by spin 1 gluons) are very different, which may be another good way of confirming the charmonium expectation that the dynamics of resonance decays are very different from those of the e^+e^- continuum.

4. Photoproduction

Finally we should note that analyses like the one discussed above, including angular distribution tests of the spin of the gluon, can be made equally well in electro-, muo-, and neutrinoproduction. Another process where the applicability of the QCD perturbation theory used here has recently been justified [12] is the production of large p_T jets in photoproduction [15]. In this case the relevant hard scattering subprocesses are those illustrated in fig. 4. One can proceed in a manner identical to that outlined earlier. Defining a thrust

$$T^* = \frac{\sum_h |p_{\parallel}^h|}{\sum_h |\underline{p}^h|} , \tag{15}$$

where all the momenta are measured in the photon-nucleon centre-of-mass, and triggering on events with low thrust. The process then involves scattering off a gluon or quark parton with longitudinal momentum fraction

$$x = 1 - T^* \tag{16}$$

at an effective

$$Q^2 = xs \tag{17}$$

where $\sqrt{s} \equiv \sqrt{2k^{\gamma}m_N + m_N^2}$ is the photoproduction centre-of-mass energy. Asymptotically free QCD perturbation theory can be applied to events with low T^*, which by the inequality (7) guarantees large spherocity (p_T). The jet boost (4) can again be applied to the forward-going particles to bring them to what should be a back-to-back two-jet configuration. The normalized angular distributions in the boosted frame for the different subprocesses indicated in fig. 4 are [12,16]

$$N \frac{d\sigma(\gamma+q \to q+g)}{d(\cos\tilde{\theta})} \quad \frac{4+(1+\cos\tilde{\theta})^2}{5(1+\cos\tilde{\theta})} \tag{18}$$

$$N \frac{d\sigma(\gamma+g \to q+\bar{q})}{d(\cos\tilde{\theta})} = \frac{-1+\dfrac{4-2\beta^2}{(1-\beta^2\cos^2\tilde{\theta})}-\dfrac{2(1-\beta^2)^2}{(1-\beta^2\cos^2\tilde{\theta})^2}}{1+2\beta^2-2\beta^4} \tag{19}$$

where

$$\beta = \sqrt{1-\frac{4m_q^2}{Q^2}}$$

In formula (19) we have retained the dependence on the quark mass: β is the quark velocity in the $\gamma+g$ centre-of-mass frame. The process $\gamma+g \to q+\bar{q}$ is clearly flavour-symmetric when $Q^2 = (1-T^*)s$ is large enough. Thus a low thrust trigger may be a good way to enhance the signal-to-background ratio when searching for heavy quark flavours in photoproduction. Verification of the angular distributions (18) and (19) would be good confirmation that QCD perturbation theory [11] is indeed applicable [12] to real photoproduction.

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References

[1] S. D. Drell, D. J. Levy and T.-M. Yan, Phys. Rev. 187 (1969) 2159; D 1 (1970) 1617; N. Cabibbo, G. Parisi and M. Testa, Lettere al Nuovo Cimento 4 (1970) 35; J. D. Bjorken and S. J. Brodsky, Phys. Rev. D 1 (1970) 1416.

[2] G. Hanson et al., Phys. Rev. Lett. 35 (1975) 1609.

[3] J. Ellis, M. K. Gaillard and G. G. Ross, Nucl. Phys. B111 (1976) 253.

[4] H. D. Politzer, Phys. Lett. 70B (1977) 430; G. Sterman and S. Weinberg, Phys. Rev. Lett. 39 (1977) 1436.

[5] C. L. Basham, L. S. Brown, S. D. Ellis and S. T. Love, Phys. Rev. D 17 (1978) 2298, and University of Washington preprint RLO-1388-759 (1978).

[6] S.-Y. Pi, R. L. Jaffe and F. E. Low, Phys. Rev. Lett. 41 (1978) 142; S.-Y. Pi and R. L. Jaffe, MIT preprint MIT-CTP-735 (1978).

[7] D. K. Choudhury and L.G.F. Vanryckeghem, Phys. Rev. D 17 (1978) 1766; J. F. Owens, Tallahassee preprint 78-02-06; H. Georgi and H. D. Politzer, Harvard preprint HUTP-77/A 071.

[8] A. DeRújula, J. Ellis, E. G. Floratos and M. K. Gaillard, Nucl. Phys. B138 (1978) 387.

[9] T. A. DeGrand, Y. J. Ng and S.-H.H. Tye, Phys. Rev. D 16 (1977) 3251.

[10] A. Mendez, Oxford University preprint, talk presented at the 1978 Topical Conference on Neutrino at Accelerators, Oxford, July 1978.

[11] D. Perkins, lecture at the Summer Institute on Particle Physics, Stanford Linear Accelerator Center, July 1978 (to be published).

[12] H. Fritzsch and P. Minkowski, Phys. Lett. 69B (1977) 316.

C. H. Llewellyn Smith, Oxford University preprint 56/78 (1978).

[13] E. Farhi, Phys. Rev. Lett. 39 (1977) 1587.

[14] H. Georgi and M. E. Machacek, Phys. Rev. Lett. 39 (1977) 1237.

[15] A. Donnachie, private communication.

[16] C. E. Carlson and R. Suaya, Stanford Linear Accelerator Center preprint SLAC-PUB-2076 (1977); L. M. Jones and H. W. Wyld, Phys. Rev. D 17 (1978) 1782; H. Georgi, S. L. Glashow, M. E. Machacek and D. V. Nanopoulos, Harvard University preprint HUTP-78/A 008 (1978); K. Hagiwara and T. Yoshino, Tokyo Metropolitan University preprint TMUP-HEL-803 (1978).

Figure Captions

1. Final states with three basic hadronic quanta: (a) $q+\bar{q}$ + vector gluon g, (b) $q+\bar{q}$ + scalar gluon s, (c) 3 vector gluons g.

2. The jet boost J brings the two secondary jets to their centre-of-mass frame.

3. The normalized angular distributions N $d\sigma/d(\cos\tilde{\theta})$ for A: $q\bar{q}g$, B: $q\bar{q}s$ and C: ggg plotted for T = 0.8 and 0.9. The vertical lines are the kinematic cutoffs given by eq. (8).

4. Feynman diagrams for low thrust T* events in photoproduction: (a) gluon bremsstrahlung [11], (b) $q\bar{q}$ pair production.

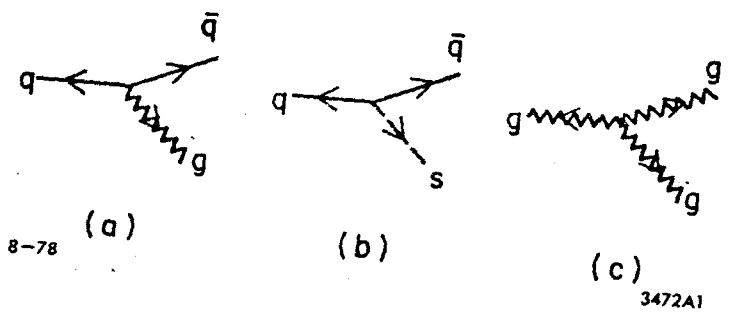


Fig. 1

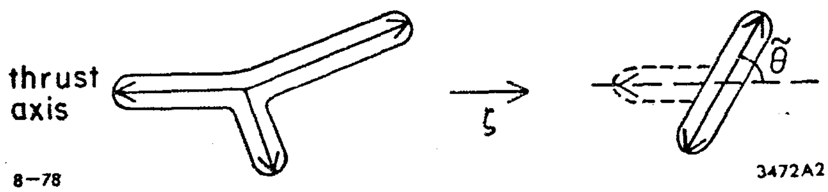
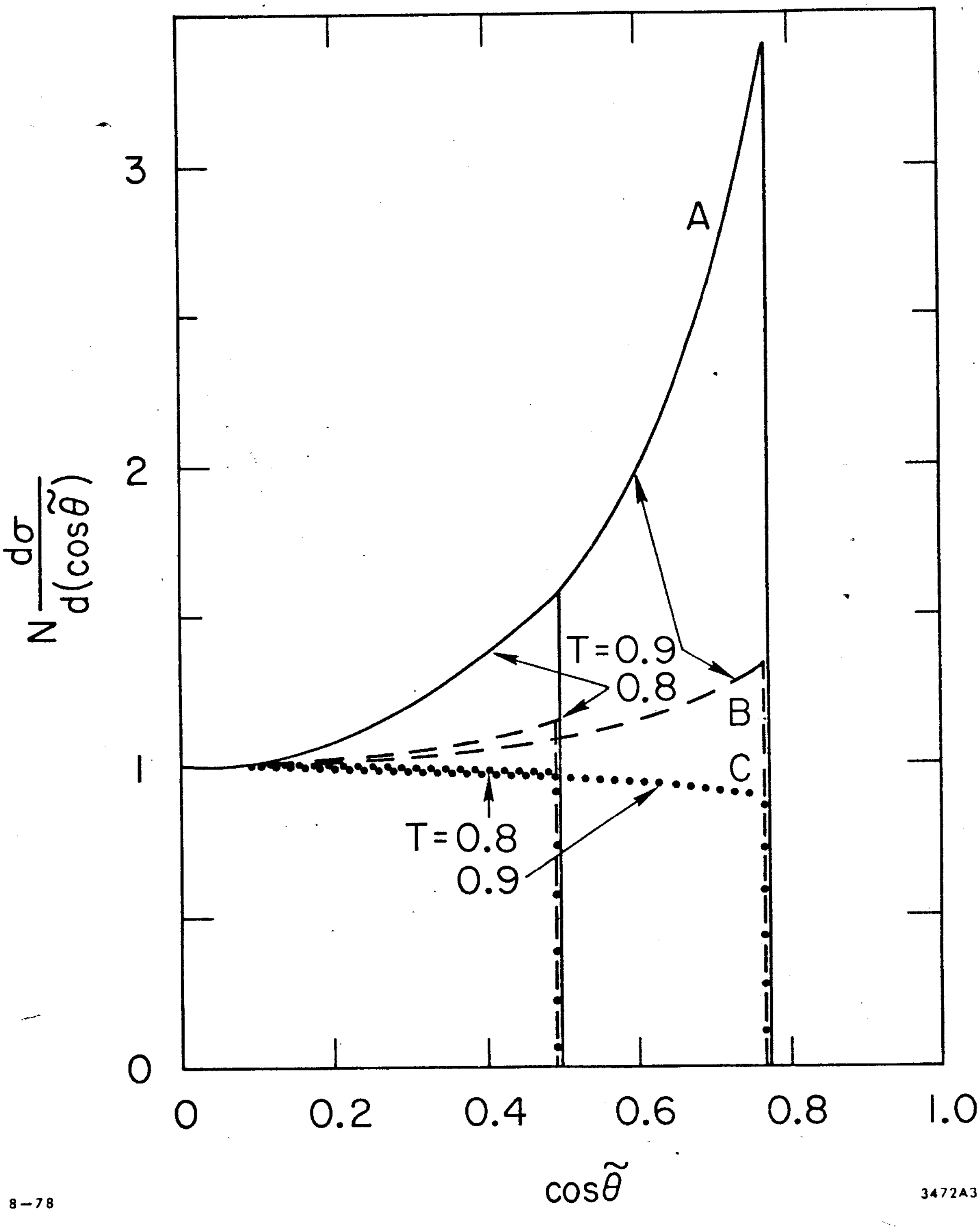
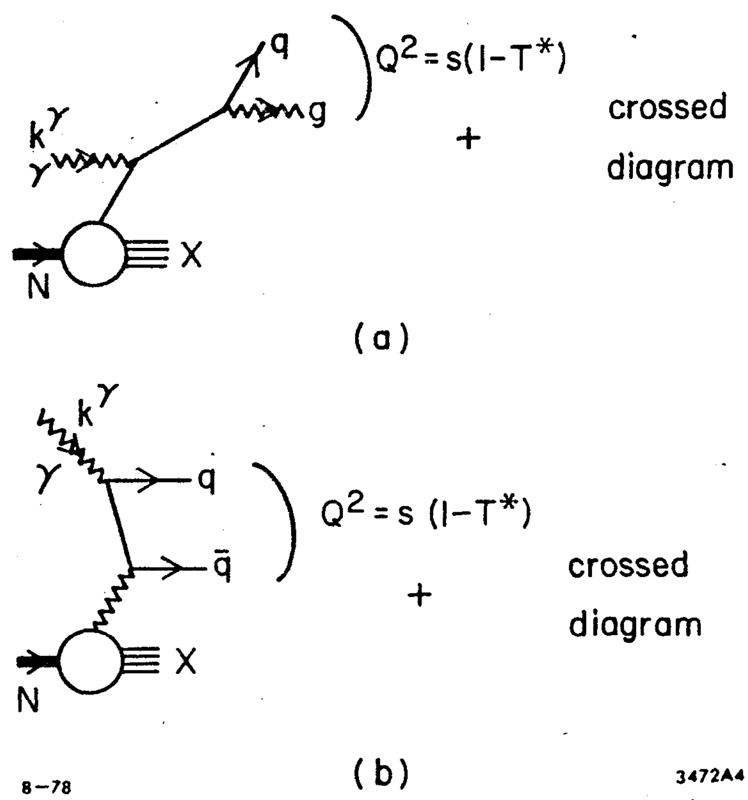


Fig. 2



3
2
1
0
$N \frac{d\sigma}{d(\cos\tilde{\theta})}$
A
T=0.9
0.8
B
C
T=0.8
0.9
0
0.2
0.4
0.6
0.8
1.0
$\cos\tilde{\theta}$
8-78
3472A3

Fig. 3



q
Q2 = s(I-T*)
k γ
γ
g
+
crossed
diagram
N
X
(a)
k γ
γ
q
q̄
Q2 = s (I-T*)
+
crossed
diagram
N
X
8-78
(b)
3472A4

Fig. 4