SLAC-PUB-2188
August 1978
(T/E)

TRANSVERSE MOMENTUM FLUCTUATIONS AND HIGH $p_\perp$ PROCESSES IN QUANTUM CHROMODYNAMICS*

R. R. Horgan†** and P. N. Scharbach†***
Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94305, U.S.A.

ABSTRACT

We discuss the introduction of constituent transverse momentum fluctuations in the parton model and examine their effects on the transverse momentum spectra of pions, jets and dilepton pairs in proton-proton collisions. We show that, when exact constituent kinematics are respected, the quark and gluon subprocesses of quantum chromodynamics do not suffice to explain the data in the present $p_\perp$ region. A hard scattering expansion, including constituent interchange model contributions, gives satisfactory agreement with the data.

(Submitted to Physics Letters)

---

* Work supported in part by the Department of Energy.
† Lindemann Trust Fellow
** Address after 1 Oct. 1978: CERN, Geneva, Switzerland.
*** Address after 1 Oct. 1978: Rutherford Laboratory, Chilton, Didcot, Oxon., U.K.

It has been experimentally established[1-4] that the invariant cross section for inclusive production of pions at high transverse momentum $p_\perp$ behaves as

$$E \frac{d\sigma}{d^3p}\bigg|_{\Theta_{CM}=90^o} = A \frac{f(x_\perp)}{p_\perp^N} \qquad (1)$$

where $x_\perp = 2p_\perp/\sqrt{s}$ and $N \simeq 8$ for $p_\perp$ in the range $1 \lesssim p_\perp \lesssim 8$ GeV/c. This behavior is contrary to that expected in a scale-invariant parton model for which, on dimensional grounds, N=4. Small deviations from this prediction are expected in any realistic quasi-scale-invariant theory such as Quantum Chromodynamics (QCD), but the predicted effects are insufficient to accommodate those observed experimentally. Dimensionally, a new mass scale must be introduced into the problem to achieve $N \neq 4$ in (1). This has led to the postulate[5,6] that partons possess non-negligible intrinsic transverse momentum $k_\perp$; the width of the $k_\perp$ distribution then provides the required mass scale. On the basis of the observed transverse momentum spectrum[7,8] of massive dilepton pairs produced in pp collisions, Feynman, Field and Fox have proposed[6] a Gaussian parton $k_\perp$ distribution with $\langle k_\perp \rangle \sim 850$ MeV/c. These and other authors[5] find agreement with the present single particle pp data when this "primordial" transverse momentum smearing is added to the scale violation effects inherent in asymptotically free QCD. Parton $k_\perp$ smearing has also been invoked[9] to improve agreement of QCD predictions with the observed Drell-Yan muon pair transverse momentum distribution at large $p_\perp$. However, as discussed in Ref. 10, there are serious theoretical difficulties in grafting large constituent $k_\perp$ fluctuations onto the parton model. The assumption is commonly made that transverse momentum fluctuations may be taken into account in a scattering process by convoluting $k_\perp$-dependent parton distributions with a subprocess cross section evaluated with partons on the mass shell. In fact, an interacting constituent a of the subprocess $a+b \to c+d$ with transverse momentum $k_{a\perp}$ and light-cone momentum fraction x_a may be forced far off-shell when $k_{a\perp}$ is large and/or $x_a \to 1$:

$$k_a^2 = -\left(\frac{k_{a\perp}^2 + m^2(x_a)}{1-x_a}\right) \tag{2}$$

This use of approximate rather than exact subprocess kinematics violates energy-momentum conservation and may lead to divergent contributions from kinematically forbidden regions of phase space. For example, in the case of the single-gluon exchange contribution to pp scattering, momentum smearing with on-shell kinematics allows a contribution from the $\hat{t}=0$ gluon pole which must be arbitrarily regulated,[6] leading to cut-off dependent results. With exact kinematics, the gluon pole lies outside the allowed phase space boundary. Furthermore, large $k_\perp$ fluctuations blur the identity of subprocesses and lead to double-counting problems in the enumeration of the possible subprocesses contributing to a given reaction. As an example, consider the quark-quark scattering contribution to high $p_\perp$ quark-jet production in pp scattering, where gluon bremstrahlung from an initial quark may induce quark $k_\perp$ fluctuations. When these fluctuations are large (of order $p_\perp$) the subprocess is better described as a large $p_\perp$ scatter of small $k_\perp$ quark and gluon constituents.

In an earlier paper[10] by Caswell, Horgan and Brodsky, a systematic study was made of various common approximations to the kinematics of high $p_\perp$ scattering with constitutent fluctuations. In ϕ^3 theory it is possible to compare the results of such treatments with the exact Feynman graph calculation and exhibit the effects of the singularities which arise when large constituent $k_\perp$ fluctuations are introduced without respecting the exact off-shell kinematics. The exact calculation for the large $p_\perp$ process $A+B \to C+X$ was shown to be reproduced to within 10 percent by a subprocess hard scattering expansion (HSE) in inverse powers of $p_\perp^2$ (at fixed $x_\perp$) determined by dimensional counting rules.[11] The HSE, identifying the origin of high $p_\perp$ exchanges in a series of hard scattering subprocesses involving on-shell, low $k_\perp$ constituents was suggested as an approach to high $p_\perp$ phenomena consistent with the basic conditions necessary for the applicability of the parton model. It is our purpose here to illustrate these ideas for the processes $pp \to \mu^+\mu^-X$, $pp \to \pi X$, and $pp \to$ jets. We shall show that the $p_\perp$ behavior of the present data is not reproduced by transverse momentum-smeared quark and gluon subprocesses if the correct off-shell kinematics are used, and that in the present moderate $p_\perp$ region, consistent interchange model (CIM) subprocesses[11] dominate the HSE and may not be neglected.[1] Indeed, these contributions (higher twist terms, in more formal language) must be present in subasymptotic QCD.

We begin with the process $pp \to \pi X$, for which the leading ($p_\perp^{-4}$) terms in the HSE are quark-quark, quark-gluon and gluon-gluon scattering. The contribution of these subprocesses to the invariant cross section at $\sqrt{s}$=27.4 GeV and $\sqrt{s}$=53 GeV is shown in Fig. 1. The dotted line shows the result of a calculation with scale-invariant coupling constant, parton distribution and fragmentation functions [2] with the strong interaction coupling constant normalized to the top end of the data at $\sqrt{s}$=53 GeV: α_s=0.14. Here the cross section scales as $p_\perp^{-4}$; the deviation from the experimental $p_\perp^{-8}$ behavior at intermediate $p_\perp$ is evident. The solid curves show the effects of scale violation in both α_s and the parton momentum and fragmentation distributions.[3] For each value of $\sqrt{s}$, the lower solid curve is calculated with no transverse momentum smearing of the initial constituents. The upper solid curve shows the result of allowing constituent fluctuations with a distribution $e^{-1.2k_\perp^2}$ corresponding to $\langle k_\perp \rangle$ = 810 MeV, calculated with exact off-shell kinematics. The effect of smearing is to change the magnitude of the cross section by a factor less than two and is certainly insufficient to account for the magnitude and $p_\perp$ dependence of the intermediate $p_\perp$ region. This is also shown by Fig. 2, in which $p_\perp^8 \, E \frac{d\sigma}{d^3p}$ is plotted against $p_\perp$. At no point does smearing with the correct kinematics flatten the curves into agreement with the data. We should not be surprised at the inability of the quark and gluon subprocesses to reproduce the moderate $p_\perp$ data. In this region the $p_\perp^{-8}$ quark-meson $\longrightarrow$ quark-meson scattering contribution to the HSE may be expected to dominate, since in this subprocess the observed π is produced directly, so that the cross section does not suffer fragmentation suppression (trigger bias effect). Indeed, it is possible to show[12] that the sum of the $p_\perp^{-8}$ Constituent Interchange Model and $p_\perp^{-4}$ subprocess contributions accounts well for the available $pp \to \pi X$ and $pp \to \bar{p}X$ data. Finally, Fig. 3 shows the cross section for jet production, with and without transverse momentum smearing. Again, the effect of smearing is small.

We finally turn to the $p_\perp$ behavior of the Drell-Yan process $pp \to \mu^+\mu^- X$ for which the dominant quark and gluon subprocesses are $qg \to q\gamma^*$ and $q\bar{q} \to g\gamma^*$. Figure 5 shows the data of Ref. 8 at rapidity $y=0$ and muon pair invariant masses $M=5.5$ and 9.5 GeV/c. Also shown are the results of our calculations in three cases. The short-dashed curves were calculated using scale invariant parton distribution functions and

α_s=0.14. They lie a factor of 4 below the data: their normalization cannot reach that of the data unless an unacceptably large value of α_s is adopted. The solid curves incorporate QCD scale violation in α_s and the parton distributions and again show the small effect of $k_\perp$ smearing with exact kinematics. As in the case of pp scattering, it is not clear at this level what is the appropriate parameter governing QCD scale violation; to show an extreme possibility we have chosen it to be $p_\perp^2 + M^2$, so that the violation is always sensitive to the largest of the two kinematical variables M^2, $p_\perp^2$. The sum of the short-dashed curve and the contribution of the CIM subprocess[16] $Mq \to q\gamma^*$, where M is a meson constituent of the proton, is shown by the long-dashed line. It should be noted that the normalization of the CIM contribution is not arbitrary and has been fixed[17] consistently with other branches of hadronic physics.

In conclusion, we have outlined the difficulties and uncertainties in principle and practice arising from the attempt to smear the transverse momentum distribution of on-shell partons. We have shown that, when subprocess kinematics are properly taken into account, the effects of ad hoc smearing are small and subprocesses involving only quarks and gluons are unable to account for present data for the production of high $p_\perp$ pions and $\mu^+\mu^-$ pairs in pp scattering. Satisfactory agreement is obtained with the inclusion of the CIM contributions to the subprocess hard scattering expansion. Charge correlation tests to discriminate between the quark-gluon and CIM contributions are discussed in Ref. 6, 12.

ACKNOWLEDGMENTS

We thank S. Brodsky, T. DeGrand, R. Field, D. Jones, J. Owens, and D. Silverman for interesting and useful discussions. Our numerical calculations were performed using VEGAS, a Monte Carlo multidimensional integration program developed by G. P. Lepage.[19] This work was supported by the Lindemann Trust and the Department of Energy.

REFERENCES

1. D. Antreasyan et al., Phys. Rev. Lett. 38, 115 (1977).
2. K. Eggert et al., Nucl. Phys. B98, 49 (1975).
3. F. Büsser et al., Nucl. Phys. B106, 1 (1976).
4. A. G. Clark et al., Phys. Lett. 74B, 267 (1978).
5. See for example, M. Della Negra et al., Nucl. Phys. B127, 1 (1977); A. P. Contogouris, R. Gaskell and S. Papadopoulos, Phys. Rev. D17, 2314 (1977) and R. D. Field, Phys. Rev. Lett, 40. 997 (1977).
6. R. P. Feynman, R. D. Field and G. C. Fox, Caltech preprint CALT-68-651 (1978).
7. D. C. Hom et al., Phys. Rev. Lett. 36, 1239 (1976); S. W. Herb et al., Phys. Rev. Lett. 39, 252 (1977); W. R. Innes et al., Phys. Rev. Lett. 39, 1240 (1977).
8. D. M. Kaplan et al., Phys. Rev. Lett. 40, 935 (1978).
9. K. Kajantie, J. Lindfors and R. Raitio, Phys. Lett. 74B, 384 (1978); G. Altarelli, G. Parisi and R. Petronzio, CERN preprint TH 2450 (1978).
10. W. E. Caswell, R. R. Horgan and S. J. Brodsky, SLAC preprint SLAC-PUB-2106 (1978) to be published in Phys. Rev. D.
11. D. Sivers, R. Blankenbecler and S. J. Brodsky, Phys. Rep. 23, 1 (1976).
12. J. Gunion and D. Jones, SLAC preprint SLAC-PUB 2157 (1978).
13. G. Chu and J. Gunion, Phys. Rev. D10, 3672 (1974).
14. J. F. Owens and E. Reya, Phys. Rev. D17, 3003 (1978); A. J. Buras and K. Gaemers, Nucl. Phys. B132, 249 (1978).
15. D. J. Gross, Phys. Rev. Lett. 32, 1071 (1974).

16. M. Duong-van, K. V. Vasavada and R. Blankenbecler, Phys. Rev. D16, 1389 (1977); C. Debeau and D. Silverman, SLAC preprint SLAC-PUB-2187 (1978).

17. R. Blankenbecler, S. J. Brodsky and J. F. Gunion, SLAC-PUB-2057 (1977), to be published in Phys. Rev. D.

18. The effect of $k_{\perp}$ fluctuations using off-shell kinematics has also been considered by M. Chase, Cambridge preprint DAMTP 77/129 (1977) and K. Kinoshita and Y. Kinoshita, Kyushu University preprint KYUSHU-78-HE-6 (1978).

19. G. P. Lepage, J. Comp. Phys. 27, 192 (1978).

FOOTNOTES

1. Estimates for the cross-over point in $p_\perp$ between the CIM and scale invariant $p_\perp^{-4}$ subprocesses are given in Ref. 12.

2. The parton momentum distributions were obtained from Ref. 13, the gluon distribution was taken as $xG(x)=2.5\ (1-x)$. The fragmentation functions were taken from Ref. 12.

3. For the scale-violating parton distributions used in this paper we used the counting-rule parametrization of Owens and Reya (Ref. 14). The earlier parametrixation of Buros and Gaemers[14] overestimates the QCD suppression of the sea and gluon distributions. The fragmentation functions are from Ref. 15. To lowest order in perturbation theory it is uncertain which kinematical variable governs the scale violation; we have chosen $p_\perp^2$. Other choices change the cross section normalization by at most 30 percent at $\theta_{CM}=90^o$, where the invariants $\hat{s}$, $\hat{t}$, $\hat{u}$ are all proportional to $p_\perp^2$.

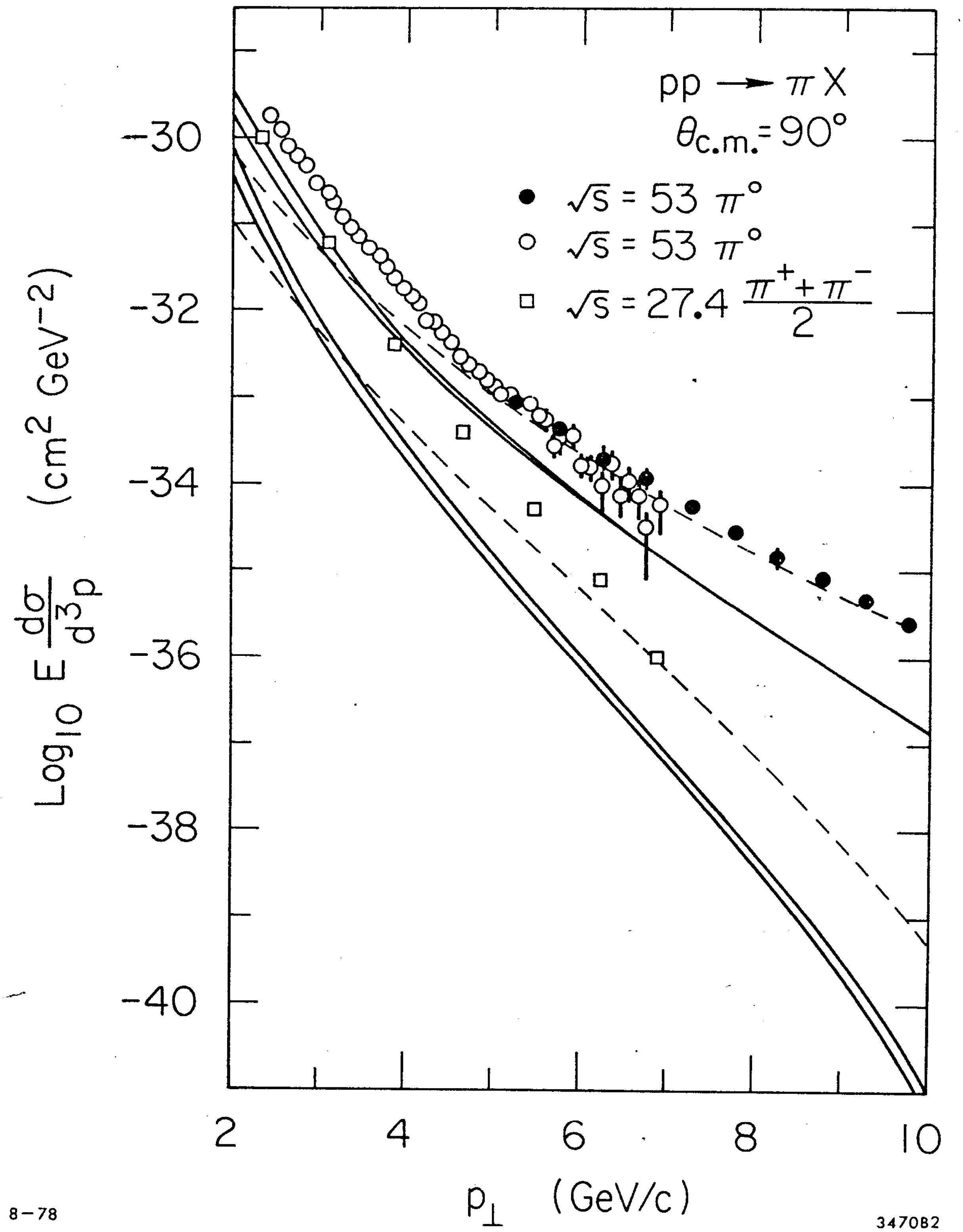
FIGURE CAPTIONS

1. Data and our results for $E\frac{d\sigma}{d^3p}$ $(pp \to \pi X)$ at $\theta_{CM}=90^{\circ}$. Solid dots ($\sqrt{s}$=53 GeV) are from Ref. 4, open dots ($\sqrt{s}$=53 GeV) from Ref. 3 and open squares ($\sqrt{s}$=27.4 GeV) from Ref. 1. The dotted line represents our results in the absence of scale violation and no $k_{\perp}$ fluctuations. The lower solid curves include scale violation; the upper solid curves include scale violation and $k_{\perp}$ fluctuations with off-shell constitutent kinematics.

2. Data and our results for $p_{\perp}^{8}\, E\frac{d\sigma}{d^3p}$ at fixed $x_{\perp}$=0.2 and 0.5 for $pp \to \pi X$ at $\theta_{CM}=90^{\circ}$. Data for $x_{\perp}$=0.2 is from Ref. 2, for $x_{\perp}$=0.5 from Ref. 1. The dashed curves include scale violation; the solid curves show the effect of $k_{\perp}$ fluctuations with off-shell constituent kinematics.

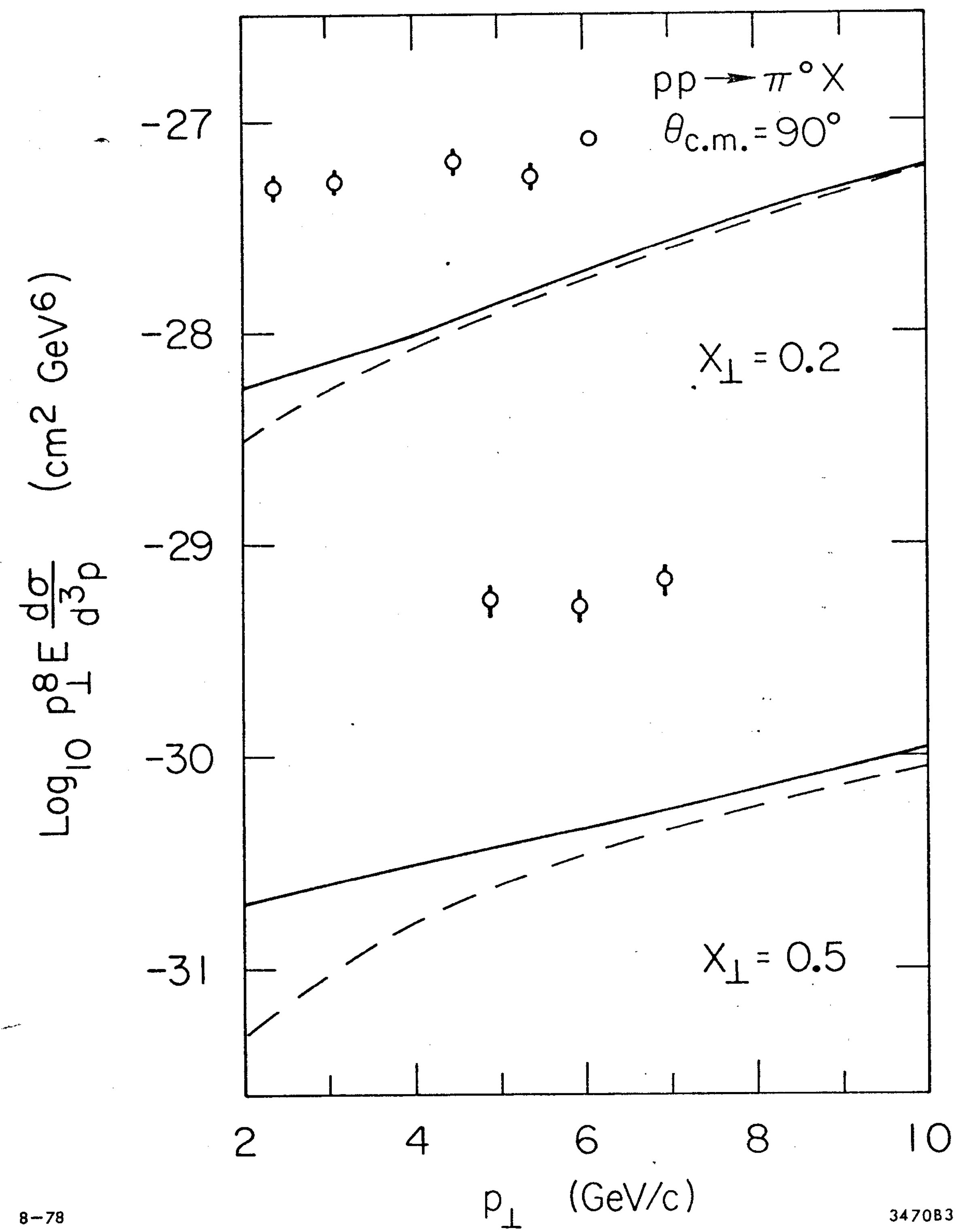
3. As in Fig. 1 for $pp \to$ jet + X at $\theta_{cm}=90^{\circ}$ for $\sqrt{s}$=27.4 and 53 GeV.

4. Data (from Ref. 8) for $pp \to \mu^+\mu^- X$ at y=0 $\sqrt{s}$=27.4 GeV and dimuon mass M=5.5, 9.5 GeV/c. The short-dashed curve incorporates no scale violation; lower/upper solid curves include scale violation and no-off-shell $k_{\perp}$ fluctuations. The long-dashed curve is the sum of the short-dashed and CIM contributions.



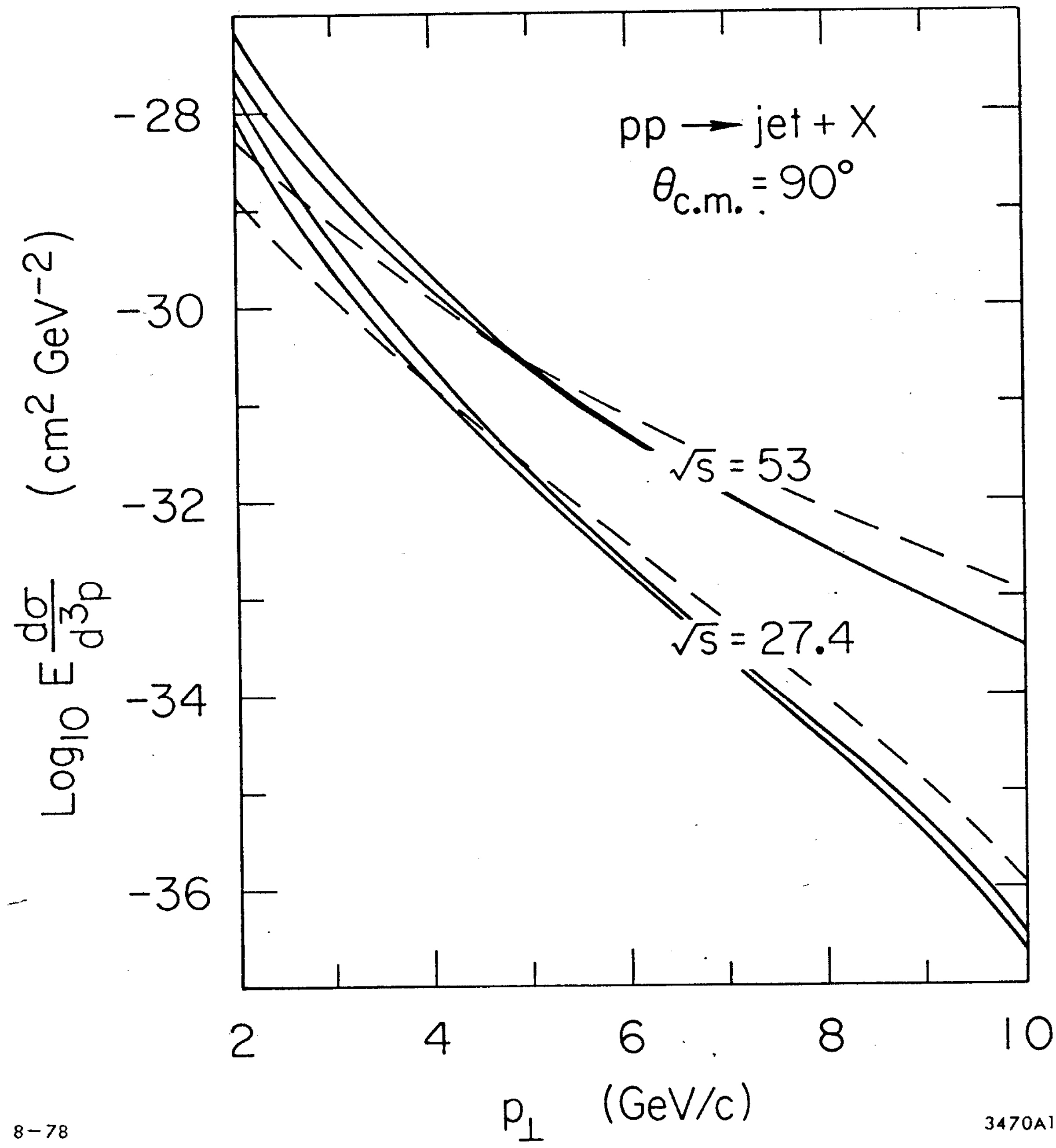
pp → π X
$\theta_{c.m.}$ = 90°
● $\sqrt{s}$ = 53 π°
○ $\sqrt{s}$ = 53 π°
□ $\sqrt{s}$ = 27.4 $\frac{\pi^+ + \pi^-}{2}$
-30
-32
-34
-36
-38
-40
$\text{Log}_{10}\ E \frac{d\sigma}{d^3p}$ $(\text{cm}^2\ \text{GeV}^{-2})$
2
4
6
8
10
$p_\perp$ (GeV/c)
8-78
3470B2

Fig. 1



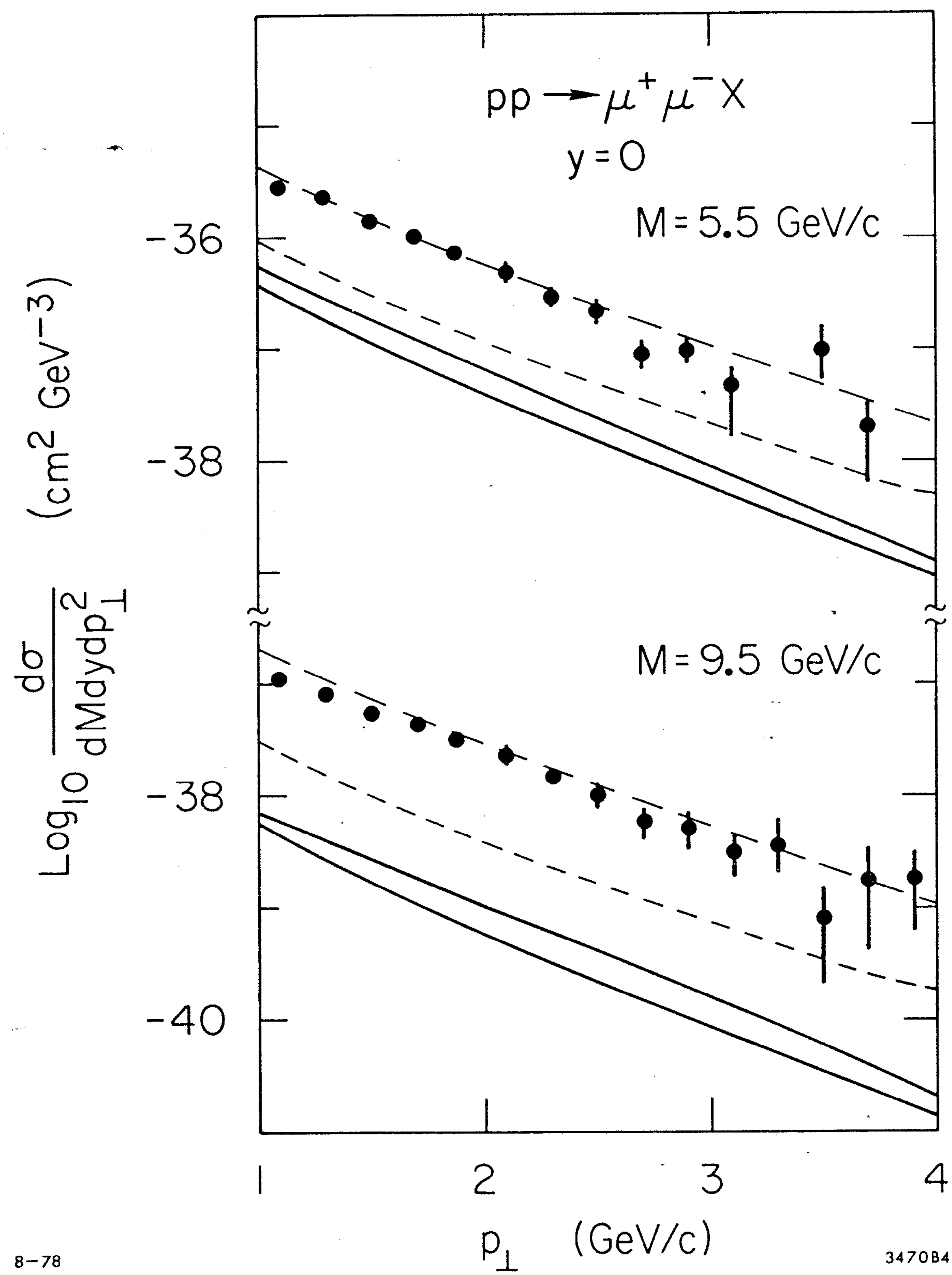
pp → π° X
θc.m. = 90°
-27
-28
-29
-30
-31
X⊥ = 0.2
X⊥ = 0.5
Log10 p⊥8 E dσ/d3p (cm2 GeV6)
2
4
6
8
10
p⊥ (GeV/c)
8-78
3470B3

Fig. 2



pp → jet + X
$\theta_{c.m.} = 90°$
$\sqrt{s} = 53$
$\sqrt{s} = 27.4$
−28
−30
−32
−34
−36
$Log_{10}\, E \frac{d\sigma}{d^3p}$ (cm^2 GeV^{-2})
2
4
6
8
10
$p_\perp$ (GeV/c)
8−78
3470A1

Fig. 3



pp → $\mu^+\mu^-$X
y = 0
M = 5.5 GeV/c
M = 9.5 GeV/c
-36
-38
-38
-40
$\mathrm{Log}_{10}\frac{d\sigma}{dMdydp_\perp^2}$ (cm^2 GeV^{-3})
1
2
3
4
$p_\perp$ (GeV/c)
8-78
3470B4

Fig. 4