SLAC-PUB-2185
August 1978
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NON-PERTURBATIVE EFFECTS IN DEEP INELASTIC SCATTERING*

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ABSTRACT

We study the implications of non-perturbative instanton field configurations for QCD predictions in deep inelastic scattering. We find that large-scale configurations cannot be neglected when $Q^2 \to \infty$, and renormalize the canonical Bjorken scaling functions. Studies of perturbative quantum fluctuations in the presence of an instanton indicate that the leading logarithms are identical to those occurring in conventional perturbation theory with zero background field. Thus the usual leading order QCD perturbative predictions for deep inelastic scaling violations are not altered by instanton effects.

(Submitted to Phys. Lett.)

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* Supported in part by the Department of Energy.
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At the moment most applications and tests of QCD[1] are made at large momentum transfers, and employ perturbation theory with the renormalization group underwriting asymptotic freedom.[2] It is of fundamental importance to justify this procedure in the presence of the non-perturbative effects[3] which are believed to be crucial in the infrared regions of small momentum transfers.[4] Such a justification is particularly necessary now that experiment[5] is supporting the perturbative predictions for the deviations from Bjorken scaling coming from the anomalous dimensions of QCD.[6] A start has been made on studying the magnitude of nonperturbative effects at large momentum transfers, by looking at the simplest "hard" process ($e^+e^- \to$ hadrons) and evaluating (see Fig. 1) the simplest diagram (a single fermion loop) in the presence of the simplest non-perturbative field configuration (an instanton).[7,8,9] It was found that this non-perturbative effect vanished very rapidly at large momenta Q,[8,9] supporting the "non-perturbative perturbation theory" prescription of ignoring more complicated field configurations at large Q^2. In this paper we extend our[8] previous studies by examining deep inelastic structure functions.

First we give general arguments, supported by a simple toy calculation, that if quantum fluctuations are ignored so that one is basically doing free field theory in a background instanton field, then Bjorken[10] scaling remains valid. However the structure functions themselves, which are related to operator matrix elements and hence[1,2] sensitive to the infrared behavior of the theory, are renormalized by non-perturbative effects due to instantons which are of arbitrarily large size, even at large Q^2. We then study quantum fluctuations using the development[11] of QCD perturbation theory in the presence of an instanton. Inspection of the effective QCD Lagrangian shows that the leading logarithms of perturbation theory are unchanged when an instanton is present. Hence in leading order asymptotic freedom[2] and the usual QCD anomalous dimensions[6] are not affected by instantons. Thus the leading order perturbative QCD predictions for deviations from Bjorken scaling in the moments of deep inelastic structure functions are not modified by the non-perturbative effects due to instantons. We point out, however, that there are problems in the use of factorization to make QCD perturbation theory predictions for other large momentum transfer processes.

It is well-known that the functional integral for QCD is a superposition of contributions from sectors with gauge field configurations having different Pontryagin numbers.[4] For a general vacuum expectation value $\langle 0|AB|0\rangle$

$$\langle 0|AB|0\rangle = \frac{\sum_{p=-\infty}^{+\infty} \langle 0|AB|0\rangle_p}{\sum_{p=-\infty}^{\infty} \langle 0|0\rangle_p} \tag{1}$$

where p is the Pontryagin number, and we have exhibited explicitly the division by the vacuum-vacuum amplitude. Up to now, many non-perturbative calculations[4] have used a dilute gas approximation where the contribution from the sector is approximated by a sum over configurations with many instantons and anti-instantons with net Pontryagin number p, and quantum fluctuations about these configurations are computed perturbatively. If the instanton-antiinstanton interactions are discarded, the contribution of each configuration is weighted be a factor exp $\left(-\sum_i 8\pi^2/g_i^2\right)$. Since the coupling $g_i^2 \to 0$ for small instantons, the configurations with just one instanton or anti-instanton should be the most important when it can be shown that only small-scale non-perturbative fluctuations are relevant, so that

$$\langle 0|AB|0\rangle \approx \langle 0|AB|0\rangle_0 + \left(\langle 0|AB|0\rangle_1 - \langle 0|AB|0\rangle_0 \langle 0|0\rangle_1\right) + \cdots \qquad (2)$$

One might expect this to be the case in processes involving large momentum transfers, and an example seems[7,8,9] to be the e^+e^- annihilation cross section at high Q^2. One might hope that the same would be true in deep inelastic scattering, but this is not obviously the case because other large scales of the order of the target hadron size are involved as well as the small distance $(x-y)^2 \sim 1/Q^2$ between the two virtual currents.

In the case of e^+e^- annihilation, the leading short distance singularity cancels between the two one-instanton terms in the dilute gas approximation (2) when one calculates the single fermion loop of Fig. 1. This is because the $(x-y)^2 \to 0$ behavior of the propagator in the presence of an instanton is the same as that of a fermion in zero field,[12] and the normalization of the fermion loop is fixed so that the one-instanton contribution to the vacuum polarization is subtracted[7,8]

$$\int d\rho\, d(\rho) \int d^4z \{T_r\left[-\gamma_\alpha S_1(x,y;z,\rho)\gamma_\beta S_1(y,x;z,\rho)\right] + \left[T_r\ \gamma_\alpha S_0(x,y)\gamma_\beta S_0(y,x)\right]\} \qquad (3)$$

and has no $1/(x-y)^6$ singularity.* In the case of deep inelastic scattering, the leading light-cone singularity in the presence of an instanton again has the same power of $1/(x-y)^2$ as that in the absence of an instanton. On

*$d(\rho)$ is the usual instanton density function,[4] where ρ is its size and z its position.

the other hand, the coefficient of this singularity may differ, because it is the matrix element of a bilocal operator between two external states, which may also be sensitive to the presence of an instanton.* Consider for example the artificially simple case of scalar currents bilinear in scalar quarks:

$$J(x) \equiv :\phi^+(x)\phi(x): \tag{4}$$

so that the connected piece of the time-ordered product of two such currents relevant to deep inelastic scattering is in free field theory just

$$T\left(J(x)J^+(y)\right)\Big|_0 = \Delta_0(x,y)\ :\phi^+(x)\phi(y):\Big|_0 + \text{(nonsingular terms)} \tag{5}$$

The matrix element of the bilocal operator between physical states of momentum p (see Fig. 2a) is just

$$\left\langle p\right|:\ \phi^+(x)\phi(y):\left|p\right\rangle_0 \equiv f_0\left((x-y)^2,(x-y)\cdot p\right) \tag{6}$$

In a semiclassical or dilute gas approximation to non-perturbative effects, one would first calculate the matrix element $\langle p|J(x)J^+(y)|p\rangle_1$ in the presence of an external (anti-)instanton field as in Fig. 2b, neglecting quantum fluctuations. Then the relevant operator product analogous to (4) can for an instanton of size ρ located at z_μ be represented by

$$T\left(J(x)J^+(y)\right)\Big|_1 = \Delta_1(x,y;z,\rho):\phi^+(x)\phi(y):\Big|_{c_1(z,\rho)} + \text{(nonsingular terms)} \tag{7}$$

*Here we have used the intuitively valid LSZ reduction formalism to extend the validity of (1) to a general matrix element involving strongly interacting particles on mass-shell. We hope to discuss this point further in a future publication.

The matrix element of the bilocal operator

$$\langle p|:\phi^{+}(x)\phi(y):|p\rangle_1 \equiv f_1\left((x-y)^2,(x-y)\cdot p;(x-y)\cdot z,z^2,z\cdot p\right) \tag{8}$$

is in general not the same as in the zero background field case (5). Therefore the delicate cancellation which eliminated the leading singularity in the vacuum polarization (3) will not in general apply to deep inelastic scattering. Hence the one-instanton correction term in (2) will have the same Bjorken[10] scaling behavior as the no-instanton term. Furthermore, we note that unlike the e^+e^- annihilation cross section, there is no kinematic reason why large-scale field configurations should be irrelevant at large Q^2.

To see these points explicitly, we consider the spin-zero analogue of deep-inelastic photon-photon scattering indicated in Fig. 3, with two currents (at x and y) supposed to be highly virtual, and two currents (at v and w) supposed essentially real. It is known[13] in QCD perturbation theory that, apart from a contact term which has no analogue in deep inelastic scattering off a hadron target, this process is dominated by the light-cone singularity in (x-y). If we calculate the zeroth order scalar box contribution to this process in the presence of an instanton we find (for convenience, we have used translation invariance to put the instanton at z=0):

$$\langle \gamma | T\left(J(x)J^{+}(y)\right) | \gamma \rangle_1 \rightarrow \langle 0 | T\left(J(x)J^{+}(y)J^{+}(v)J(w)\right) | 0 \rangle_1$$

$$= \frac{2}{(4\pi)^8}\left[\frac{1}{(x-y)^2}\,\frac{1}{(y-v)^2}\,\frac{1}{(v-w)^2}\,\frac{1}{(w-x)^2}\right] \times$$

$$\Bigg\{1 - \frac{\rho^2}{4}\left[\frac{1}{(x^2+\rho^2)(y^2+\rho^2)(v^2+\rho^2)(w^2+\rho^2)}\right]\Big\{\rho^4\ \left[(x-y)^2+(y-v)^2+(v-w)^2+(w-x)^2\right] +$$

$$+ \rho^2\left[(x-y)^2(v+w)^2+(x+y)^2(v-w)^2+(y-v)^2(x+w)^2+(y+v)^2(x-w)^2+ 2(x-v)^2(y-w)^2\right] +$$

$$+ 2\left[x^2y^2(v-w)^2+y^2v^2(w-x)^2+v^2w^2(x-y)^2+x^2w^2(y-v)^2\right]\Big\}\Bigg\}$$

$$+ \text{(crossed terms)} \tag{9}$$

There is an amusing technical shortcut which reduces the calculation of loops of propagators to the computation of Dirac γ-traces. One needs to evaluate the traces of γ-matrices $\sigma^{\mu} \equiv (i,\underline{\sigma})$, $\sigma^{+\mu} \equiv (-i,\underline{\sigma})$, which always occur in pairs. Furthermore the sum over instanton and anti-instanton configurations is hermitean:

$$\mathrm{Tr}\left[(\sigma^{\alpha}\sigma^{+\beta}\sigma^{\gamma}\sigma^{+\delta}\ldots\ \sigma^{\mu}\sigma^{+\nu}) + (\sigma^{+\alpha}\sigma^{\beta}\sigma^{+\gamma}\ldots\sigma^{\nu})\right] \tag{10}$$

But Dirac γ-matrices can be represented as

$$\gamma^{\mu} = \left(\begin{array}{c|c} 0 & \sigma^{\mu} \\ \hline \sigma^{+\mu} & 0 \end{array}\right)$$

so that

$$\gamma^{\alpha}\gamma^{\beta} = \left(\begin{array}{c|c} \sigma^{\alpha}\sigma^{+\beta} & \\ \hline 0 & \sigma^{+\alpha}\sigma^{\beta} \end{array}\right)$$

The combination (10) can therefore be written as

$$\mathrm{Tr}(\gamma^{\alpha}\gamma^{\beta}\gamma^{\gamma} \;\ldots\; \gamma^{\mu}\gamma^{\nu}) \tag{11}$$

It is evident that the expression (9) <u>does</u> reduce to the free field expression

$$\langle 0 | T\left(J(x)J^{+}(y)J^{+}(v)J(w)\right) | 0 \rangle_{0} = \frac{2}{(4\pi)^{8}} \left[\frac{1}{(x-y)^{2}}\, \frac{1}{(y-v)^{2}}\, \frac{1}{(v-w)^{2}}\, \frac{1}{(w-x)^{2}}\right] \tag{12}$$

in the <u>multiple</u> short distance limit $x\sim y\sim v\sim w$. Hence a cancellation analogous to that in (3) removes the leading non-perturbative renormalization of the contact term. However, Eq. (9) does <u>not</u> reduce to Eq.(12) when we go to the <u>single</u> light-cone or short-distance limit $x\sim y \neq v,w$,* which is the direct analogue of the kinematic region relevant to deep inelastic scattering from a hadron target.**

It therefore seems that instantons which are arbitrarily large renormalize the coefficient of the leading light-cone singularity in deep inelastic scattering—in a more picturesque language, the parton distribution can be distorted by fluctuations in the topological charge structure.

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* We remark in passing that there is a logarithmic divergence when Eq. (8) is integrated over instanton position. An analogous divergence occurs in the scalar loop contribution to the scalar vacuum polarization $\langle 0|T\left(J(x)J^{+}(y)\right)|0\rangle$. In both cases the absorptive part related to the cross section is convergent when the final state invariant mass is non-zero.

** For simplicity, we only discuss explicitly the short distance limit: the behavior near the light-cone is qualitatively similar.

This sensitivity to non-perturbative infrared effects should not surprise us, since we know[1,2] that in QCD perturbation theory the matrix elements of operators are infrared-sensitive and not reliably calculable. In QCD perturbation theory only the high Q^2 evolution of the moments of the structure functions (or equivalently the evolution[14] of the effective Q^2-dependent parton distributions) are reliably calculable. We should now ask whether these consequences of QCD perturbation theory are reliable in the presence of non-perturbative effects. The above analysis suggests that the dilute gas approximation[4] is not sufficient to answer this question definitively. This is because nonperturbative field configurations which have an arbitrarily large scale, and hence an arbitrarily small action $A \geq 8\pi^2/g^2$, are relevant in the large Q^2 limit. On the other hand it is presumably necessary for the validity of the conventional perturbative QCD analysis of deep inelastic scattering that quantum fluctuations in the presence of an instanton have logarithms of Q^2 which are the same as those found in the absence of a non-perturbative background field. We now study this question using the recent formulation[11] of QCD perturbation theory in the presence of an instanton.

Amati and Rouet[11] show that in the presence of an instanton the QCD Lagrangian can be written as

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}(A^{cl}+A^{qu})F^{\mu\nu}(A^{cl}+A^{qu}) - \frac{1}{2\alpha}(D_\mu A^{qu}_\mu)^2$$

$$-\frac{1}{2\beta\rho^2}\left(\int d^4x\, F^{cl}_{\mu\nu}A^{qu}_\mu\right)^2 - \frac{1}{2\gamma\rho^4}\left(\int d^4x\, x_\nu F^{cl}_{\mu\nu}A^{qu}_\mu\right)^2 \tag{13}$$

$$+\, c\, D_\mu U_\mu + (\psi_\nu + x_\nu\psi)F^{cl}_{\mu\nu}U_\mu$$

In Eq. (13) A^{cl} is the background instanton field, A^{qu} is the quantum fluctuation, α, β, and γ are gauge-fixing parameters, c the usual Faddeev-Popov ghost,

$$U_\mu \equiv D_\mu \bar{c} + \left(A_\mu^{qu} \times c\right) + (\bar{\psi}_\nu + x_\nu \bar{\psi})(D_\nu A_\mu^{qu} - F_{\mu\nu}^{cl}) + A_\mu^{qu}\, \bar{\psi} \tag{14}$$

D_μ is a gauge-covariant derivative, and ψ and ψ_ν are ghost fields corresponding to the translation and dilation zero modes of the instanton, just as c corresponds to the gauge zero modes. To study the leading logarithms of the perturbation theory developed using the Lagrangian (13), we need only look at the pieces in it which have operator dimension 4. Inspection reveals that most of the extra terms in (13) compared with the normal QCD Lagrangian are of dimension less than 4, and so do not affect the leading behavior as $Q^2 \to \infty$. The only new terms of dimension 4 are

$$\Delta \mathscr{L}_4 \equiv c D_\mu \left[(\bar{\psi}_\nu + x_\nu \bar{\psi}) D_\nu A_\mu^{qu} + A_\mu^{qu}\, \bar{\psi}\right] \tag{15}$$

Since there are no kinetic terms in the ψ and $\bar{\psi}$ "propagators", they are proportional to $\delta^4(k)$. Hence the contributions to the Callan-Symanzik β function which involve ψ and ψ_ν have no logarithm at lowest order in α_s. Therefore the coupling constant $\alpha_s(Q^2) \sim 1/\ln Q^2$ as $Q^2 \to \infty$ in the presence of an instanton, just as in the absence of a background field.[2] Similarly, the new ghosts ψ and ψ_ν clearly do not contribute to the $O(\alpha_s)$ calculation of the anomalous dimensions of leading twist operators. These two observations imply that the standard QCD perturbation theory predictions for the powers of $\ln Q^2$ in the deviations from Bjorken scaling of moments of the deep inelastic structure functions <u>also apply</u> to contributions from one-instanton background fields. As emphasized above this result is necessary, though not sufficient,* for the usual QCD perturbation theory results to survive non-perturbative phenomena.

We recall that the identity between (8) and the free field box in the multiple short distance limit $x\sim y\sim v\sim w$ means that subtraction (2) removes the leading singularity from the non-perturbative contribution to the contact term. This implies that the leading order QCD prediction[13] for deep inelastic scattering off a photon is also unaffected by non-perturbative effects.

Given that the leading Q^2 behavior of non-perturbative instanton configurations to deep inelastic scattering is identical to, and hence indistinguishable from, contributions with no background field, how suppressed are subasymptotic corrections in deep inelastic scattering? A recent paper[15] argues formally that all non-leading logarithms in the presence of an instanton are also identical with those in zero background field. On the other hand, it is clear from the analysis of the first part of this paper, and in particular from Eq. (8), that there will in general be non-perturbative corrections to deep inelastic scattering which are $O(1/Q^2)^n$ times some power of $\log Q^2$. They arise, for example, from instantons of size $\rho >> 1/Q$ which are located a long distance $(x-z) >> (x-y) \sim 1/Q$ from the deep inelastic currents. Hence their coefficients are not reliably calculable with our present understanding of nonpertubative

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*We are encouraged that it is obviously possible to generalize this result to multi-instanton configurations.

QCD.* They will correspond to the lower twist operator contributions of normal QCD perturbation theory. There will also be contributions from ρ, $x-z \approx O(1/Q)$ analogous to those we found[8] in the $e^+e^- \to$ hadrons total cross section which will show up in the coefficient functions of the operator product expansion. However they will be $O(Q^{-12})$ and hence not the dominant non-perturbative effects in deep inelastic scattering.

What of other hard scattering processes[1] and the applicability of QCD perturbation theory? The perturbative analyses[1-7] indicate that the leading logarithms in the cross sections for such processes can be factorized in terms of universal Q^2 evolution equations for parton distributions[14] interacting through QCD Born graphs written in terms of $\alpha_s(Q^2)$. The identity of leading logarithms in the presence of an instanton means that the usual Altarelli-Parisi[14] Q^2 evolution equations still apply to the parton distributions measured in deep inelastic scattering.

As usual, we introduce effective q^2-dependent parton distributions $q_i(x,q^2)$, $g(x,q^2)$. Their moments

$$q_i^n(q^2) \equiv \int_0^1 dx\, x^{n-1} q_i(x,q^2) \quad . \tag{16}$$

(and similarly for gluons) obey the evolution equations[18]

$$q^2 \frac{\partial}{\partial q^2} q_i^n(q^2) = \gamma_{ij}^n\left(\bar{g}^2\left(\frac{q^2}{\mu^2}\right)\right) q_j^n(q^2) \tag{17}$$

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*These effects will complicate analyses[16] of subasymptotic quark mass corrections to Bjorken scaling in the case of very light quarks with masses $m \lesssim O(\Lambda)$, whose Λ is a typical strong interaction scale of a few hundred MeV. On the other hand, perturbative corrections for very heavy mass quarks are probably still reliable.

As we saw above, at first order in $\bar{g}\,(q^2/\mu^2)$, γ^n_{ij} is insensitive to the vacuum structure of the theory. Therefore to this order the evolution equations are unchanged when we take into account non-perturbative effects, at least in a dilute gas approximation—only the infrared sensitive initial boundary conditions $q^n_j(q_0^2), q^n(q_0^2)$ are affected. Presumably the logarithms for other hard scattering processes calculated in background instanton fields will also be identical with those found[17] in QCD perturbation theory. In this case the same Q^2 evolution equations would apply to the parton distributions "measured" in these hard scattering processes. But it is not obvious that the infrared-sensitive initial conditions will factorize in the manner necessary for the normal ansatz of universal effective parton distributions to be applicable.* Stated in another way: an instanton or other nonperturbative field configuration may affect simultaneously the parton distributions in two colliding hadrons as in Fig. 4, so that they are not independent of each other, and the cross sections do not factorize. Schematically, in deep inelastic scattering one measures

$$q^{(o)}(x,q^2) + \left[q^{(1)}(x,q^2) - q^{(o)}(x,q^2)\right] \exp\left(\frac{-8\pi^2}{g^2}\right) + \ldots \tag{18}$$

whereas in (for example) a Drell-Yan collision the cross section is proportional to

$$q^{(o)}(x,q^2)\bar{q}^{(o)}(\bar{x},q^2) + \left[q^{(1)}(x,q^2)\bar{q}^{(1)}(\bar{x},q^2) - q^{(o)}(x,q^2)\bar{q}^{(o)}(\bar{x},q^2)\right] \exp\left(\frac{-8\pi^2}{g^2}\right) + \ldots \tag{19}$$

which contains no term of the type

$$q^{(o)}(x,q^2)q^{(1)}(\bar{x},q^2)\exp\left(\frac{-8\pi^2}{g^2}\right) \tag{20}$$

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*We note that this has not been demonstrated in the usual perturbative analyses,[17] either.

as would be required if the cross section were to factorize into two terms of the form (18). It is perhaps possible that more sophisticated non-perturbative effects may restore factorization, but this is a more complicated question requiring further study.

We should re-emphasize that our results were obtained in the context of a dilute gas approximation, and that the persistent relevance of large instantons at high Q^2 indicates that this approximation is invalid. Therefore our findings that QCD perturbation theory results survive non-perturbative effects are at best indicative and partial, and the problem cannot be fully resolved until more sophisticated non-perturbative calculational schemes are developed.

ACKNOWLEDGEMENTS

We would like to thank U. T. Cobley, S. Coleman, R. J. Crewther, A. Din, J. Finjord, L. Maiani and G. Parisi for useful discussions. One of us (L.B.) would like to thank the CERN theory division for its hospitality, while two others (J.E. and M.K.G.) would like to thank the FNAL and SLAC theory groups for their hospitality.

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FIGURE CAPTIONS

1. The single fermion loop contribution to the hadronic vacuum polarization calculated in the presence of an instanton.
2. The "handbag diagram" contribution to deep inelastic scattering calculated (a) without, and (b) with an instanton present.
3. The box diagram for photon-photon scattering in the presence of an instanton.
4. A hadron-hadron hand scattering process in the presence of an instanton.

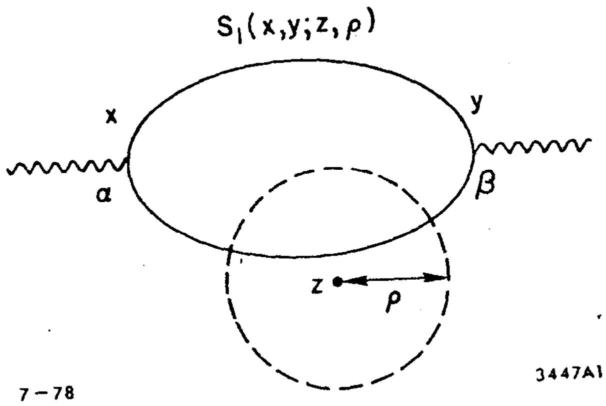


Fig 1

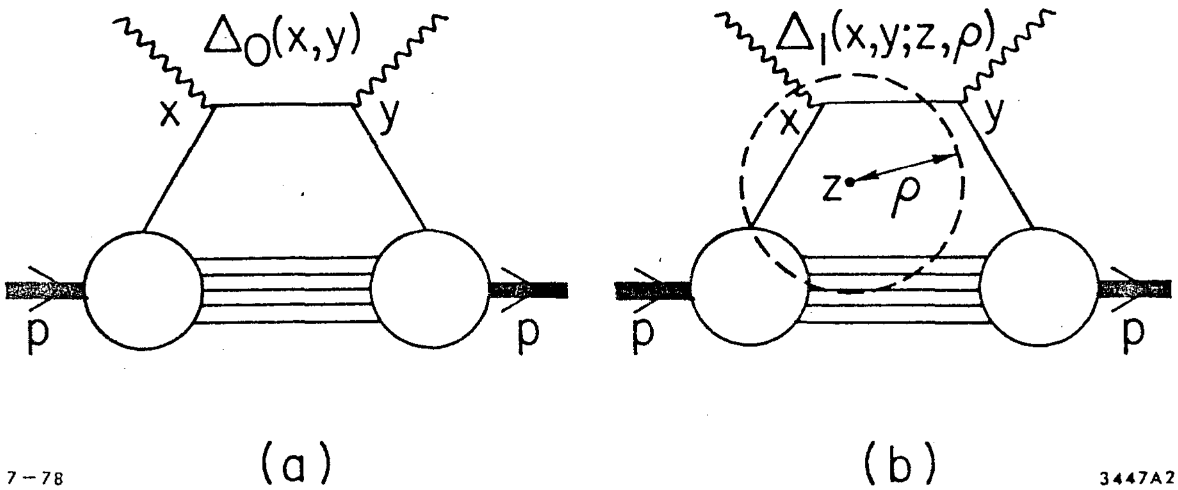
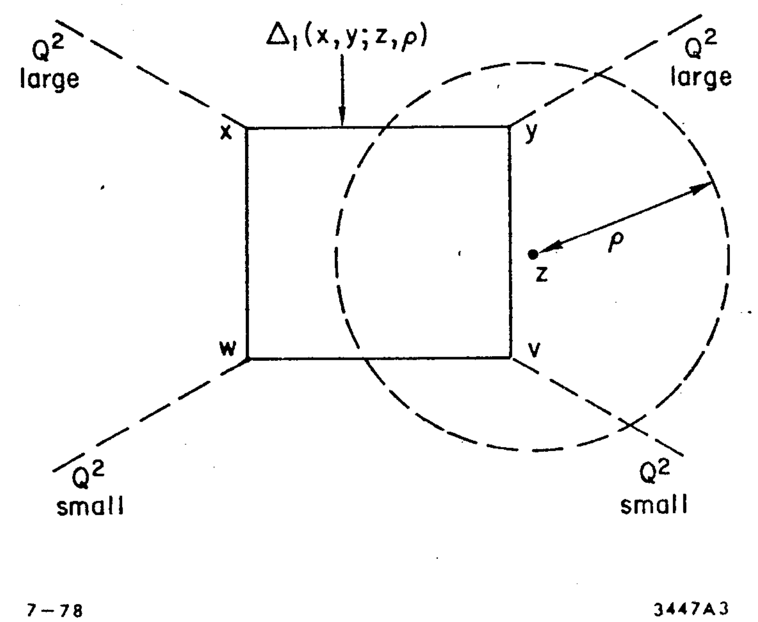


Fig. 2



$\Delta_1(x,y;z,\rho)$
Q^2
large
Q^2
large
x
y
w
v
ρ
z
Q^2
small
Q^2
small
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Fig. 3

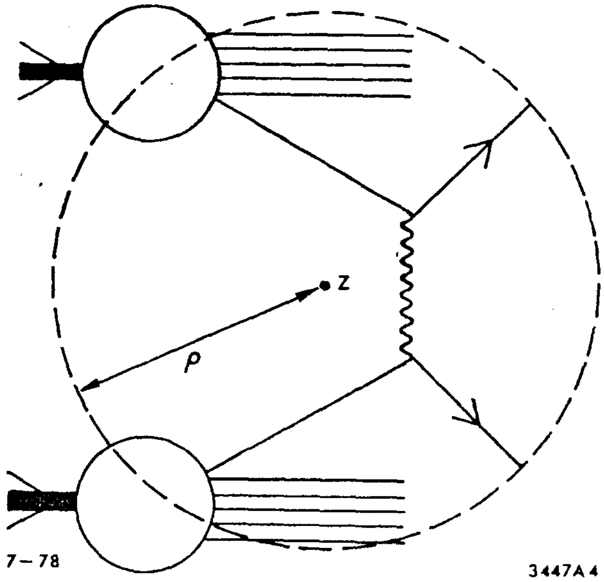

Fig. 4