PRINCIPLES OF AN OBJECTIVE QUANTUM THEORY OF WAVE PACKETS[*]

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CONTENT

ABSTRACT

In an attempt to overcome the conceptual difficulties of conventional quantum mechanics, and to find a new way for attacking the problems of relativistic quantum theory, a consistent conceptual framework is presented in which the basic constitutents are taken to be wave packets endowed with special properties. In particular, the wave packets must have a spatial extension larger than half their Compton length, and they carry an integer number of action quanta of size h^3. The mathematical formalism of non-relativistic quantum mechanics is given a new interpretation, and novel viewpoints concerning the relativistic domain are presented.

The theory is objective, i.e. it is formulated throughout in the language of philosophical realism: real properties, not merely observables, are ascribed to the wave packets. Exact values of position and momentum cannot be counted among these properties; only ranges of these quantities can. Heisenberg's relation and any non-commutativity of operators express the existence of correlations between properties of wave packets.

The conceptual problems concerning the measurement process, Schrödinger's cat, Wigner's friend, the Einstein-Podolsky-Rosen example, the spreading of macroscopic bodies and atomic screening are avoided within the proposed conceptual framework.

I. INTRODUCTION

"Say what you choose, so long as it does not prevent you from seeing the facts. (And when you see them there is a good deal that you will not say.)"[1]

Elementary particles like electrons, protons or pions that are the subject of quantum mechanics are unlike classical particles like grains, bullets or stars which are considered in classical mechanics. The essential point of difference is exhibited in the well known double-hole experiment[2] in which a beam of elementary particles approaches a wall with two holes in it; the distribution of the particles on some absorbing layer behind the wall depends on whether the two holes have been opened simultaneously or successively, even when the intensity of the incoming beam is so low as to let only one particle at a time pass throught the wall.

The distribution of the particles behind the wall is correctly given by the absolute square of the wave function, as calculated for classical waves with interference terms according to Huygens principle. These interference terms prevent one from saying that the particle has gone either through hole 1 or through hole 2. So this "particle" or "micro-object" is not a classical particle, i.e. the logical structure represented by the set of propositions of our language containing the word "particle" cannot be carried over to these micro-objects. Calling these objects "particles" is misleading and must be remedied by additional prescriptions accounting for the fact that their correct distribution behind the wall is given by superposition and interference of waves.

This wave-particle dilemma has been interpreted by Born and the Copenhagen school in terms of the probabilistic buckshot picture: the absolute square $|\psi(\vec{x},t)|^2$ ($|\psi(\vec{x},t)|^2 d^3x$) of the wave function is proportional to the probability that at time t the particle is observed at the location $\vec{x}$ (in $\vec{x}...\vec{x}+d\vec{x}$). Thus, in this interpretation, the particle does not have an exact location as long as it is not observed, but it does have a location, when it is observed.[3]

One may say, this does not bother us, for the formal rules of quantum theory contain both what may be called the wave aspect and what may be called the particle aspect. These formal rules give us the correct prescriptions for calculating the outcome of our experiments and that is all we want. Any pictorial interpretation which goes beyond is unnecessary and a matter of private preference. Right, in principle. However, one always notices that in the course of historical development as well as in the mind of any particular scientist, the final mathematical formalism describing a set of physical phenomena emerges from a more or less pictorial view, conception or model. A good picture is very helpful as it has the same logical structure as the region of reality which it represents, and it leads to a correct mathematical formulation of this reality. As a familiar example take Faraday's intuitive picture of lines of force and their mathematical formulation by Maxwell.[4] A bad picture leads to no or only a partially correct formalism. In this latter case it may happen that the emerging formalism in its initial stage describes the known phenomena correctly, but when it is developed further to include more and more experimental facts it sooner or later loses the track. This is what we think has happened to quantum theory.

It is well known that difficulties arise when quantum mechanics is extended to a relativistic quantum field theory: there are divergent integrals in perturbation expansions, and even if these are made finite in a systematic way by the renormalization procedures, nobody knows whether these expansions converge, and nobody has been able to give an exact solution of the field equations including interactions in the real space-time world of 4 dimensions. In view of the enormous efforts made in the last 50 years one might suspect that one is asking the wrong questions.[5]

We think that the particle picture is a bad picture in quantum theory, and we propose to replace it by another picture, the picture of a wave packet.

Imagine the track left by a "particle" in a bubble chamber. It consists of a chain of bubbles. In the particle picture one says that there is a wave packet traveling through the chamber, and the particle whose motion is governed by this wave packet has successively interacted with the atoms of the liquid at the positions where the bubbles have developed. In the wave-packet picture we shall say that there is a wave packet moving through the chamber and that it has undergone successive interactions with the atoms of the liquid at the positions where the bubbles have developed. The wave packet is like a cloud moving along while giving rise to thunder and lightning here and there. In this picture one is not tempted to ask about any path a point particle might have traveled. The probability for a flash of lightning to strike within d^3x around $\vec{x}$ and within the time interval dt around t is proportional (not necessarily equal) to $|\psi(\vec{x},t)|^2 d^3x \, dt$. This is the same expression,

except for the factor dt, which in the particle picture is interpreted as the probability for observing the particle at position $\vec{x}$. Quantum numbers like charge, spin etc., which in the particle picture are ascribed to particles are in our picture to be ascribed to wave packets. There are photon wave packets, electron wave packets, proton wave packets, and so on.

It is true that the idea of identifying particles with wave packets has already been tried in the early days of quantum mechanics. The idea was quickly abandoned primarily because the wave packets in general spread out rapidly, and other objections have also been raised. We shall deal with these objections in a later section of the paper. It will be shown that the wave-packet idea has been dropped too quickly and that, actually, a consistent interpretation can be built up with it. Indeed, Schrödinger has never dropped this picture, and our ideas, though incorporating new features, have much in common with his.

One fundamental point where our interpretation differs from that of traditional quantum mechanics is that we formulate the whole theory in the language of philosophical or epistemological realism. The traditional Copenhagen interpretation (in all its variants) forbids one to say that objects in the microscopic domain have any definite real properties when there is no observer to observe them; one may only speak of outcomes of observations. This interpretation is really baffling, both to students as well as to learned philosophers, and discussions on this point have never ceased. As Schrödinger[7] puts it: it is "a philosophical extravagance born out of despair."

On the other hand, to speak in the language of realism means to say that the micro-objects, viz. our wave packets, are real things and that they have real properties, not merely observables. They have their properties, whichever these may be, even when one does not observe them. The concept of the observer is unnecessary. The micro-objects are part of a reality which is the object of our study. This language of realism is used in all the other sectors of physics, in all the other disciplines of science as well as in everyday life. The Copenhagen language, we think, cannot consistently be extended to deal with the whole of physics not to speak of the whole of our experience.[8,9] Notice that the question of reality is not a matter to be proved or disproved. Realism is a way of speaking, or, rather, an attempt to characterize how our language works in normal (not philosophical) use.

There are notable rebels against the edict of Copenhagen. Einstein, for one, has repeatedly stressed that quantum mechanics should be formulated in the language of realism.[8,10,12,13] In addition, nowadays there are many physicists which feel the need for such an objective formulation. Especially noteworthy are the writings of Bunge.[15-18,20] Bell[21] postulates "beables" to replace the "observables". See also Jordan,[23] Schrödinger,[7] Popper,[24] Jauch,[25] Ludwig,[27] Haag,[28] Lévy-Leblond,[29] Stapp,[30] and Roberts.[31]

We shall show in this paper that it was a wrong idea of the nature of the microscopic objects that has lead to the refusal of realism. The interpretational difficulties met so far will, of course, imply that in a realistic description of micro-objects, these supposed real objects must be endowed with unfamiliar properties. I think, nevertheless, that a realistic description is the indispensable requirement for overcoming

the difficulties. I am also pretty sure that wave packets reflect essential properties of micro-objects. Perhaps not all the properties I have ascribed to wave packets will turn out to be the right ones. Here, I stick my neck out, for I think that truth emerges faster from error than from confusion.

This paper is meant to be preparatory. It is intended to remove stumbling blocks on the way to a new mathematical formulation of relativistic quantum theory, or, rather to propose another way. As a first check, in this paper we want to show that our general concept does not lead to any fundamental contradictions in interpreting those phenomena that are considered in non-relativistic quantum mechanics. The mathematical formalism of non-relativistic quantum mechanics is left untouched. What is changed is its interpretation. However, at many places we deliberately consider phenomena going beyond that scope in order to ensure that we use only such concepts that can describe the relativistic domain as well.

When surveying the literature on the conceptual problems of quantum mechanics one observes that almost all authors confine themselves to non-relativistic theory. I think that this is the reason for the general sterility of the discussions. It is my opinion that the solution to the conceptual problems of quantum mechanics will only become possible when the achievements of relativistic quantum theory and high-energy physics are taken into consideration. Indeed, led by a vital instinct, physicists have left quantum mechanics with its problems and have proceeded to explore new domains of reality, thereby tacitly abandoning many of the concepts of orthodox quantum mechanics. Quantum mechanics can learn

much from the achievements of high-energy physics and relativistic quantum theory. Likewise, high-energy physics can learn from quantum mechanics when the two are interpreted using the same concepts. We hope that once the basic conceptual features are clarified, further development and mathematical formulation of the new concepts will not meet unsurmountable difficulties, i.e. will not take another 50 years.

II. WAVE PACKETS AND COMPTON LENGTH

We begin by summarizing some basic properties of customary wave packets, i.e. linear superpositions of plane waves

$$\psi(\vec{x},t) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} \tilde{\psi}(\vec{k}) \exp\left[i(\vec{k}\vec{x}-\omega t)\right] d^3k \quad , \qquad (2.1)$$

where $\tilde{\psi}(\vec{k})$ is normalized

$$\int_{-\infty}^{+\infty} |\tilde{\psi}(\vec{k})|^2 \, d^3k = A < \infty \qquad (2.2)$$

and $\omega=\omega(\vec{k})$. When $\tilde{\psi}(\vec{k})$ is given, the behavior of the wave packet (2.1) is determined by the dispersion law $\omega=\omega(\vec{k})$. The free Schrödinger equation gives

$$\omega = \frac{\hbar}{2m}|\vec{k}|^2 \equiv \frac{\hbar}{2m} k^2 \qquad (2.3)$$

whereas the Klein-Gordon equation or, in fact, any relativistic wave equation gives

$$\omega = c\sqrt{k^2+(1/\lambda_C)^2} \qquad (2.4)$$

where

$$\lambda_C = \frac{\hbar}{mc} \qquad (2.5)$$

is the Compton wavelength belonging to the (rest mass) parameter m.

We consider Gaussian wave packets

$$\tilde{\psi}(\vec{k}) = \sqrt{\frac{Ab_o}{\pi^{3/2}}} \ \exp\left[-\frac{b_o^2}{2}\ (\vec{k}-\vec{k}_o)^2\right] \quad . \tag{2.6}$$

The distribution

$$|\tilde{\psi}(\vec{k})|^2 = \frac{Ab_o}{\pi^{3/2}} \ \exp\left[-b_o^2(\vec{k}-\vec{k}_o)^2\right] \tag{2.7}$$

then has a width

$$\Delta k = 1/(\sqrt{2}\ b_o) \quad . \tag{2.8}$$

We use the general relativistic dispersion law (2.4). In order to be able to evaluate the integral (2.1) analytically we consider the case

$$k_o \gg 1/b_o \quad , \tag{2.9}$$

i.e. narrow wave packets that are concentrated around $\vec{k}_o$ in k-space and have negligible contributions from small and negative $\vec{k}$ values.[32] We can then expand $\omega(\vec{k})$ into a three-dimensional Taylor series around $\vec{k}_o$ and break off the series after the quadratic terms. We choose the coordinate frame so that $\vec{k}_o = (k_o,\ o,\ o,)$, i.e. the x-axis points in the direction of $\vec{k}_o$, and the y- and z-axes are in the plane normal to $\vec{k}_o$. The result then is

$$|\psi(\vec{x},t)|^2 = \frac{A}{(2\pi)^{3/2}} \ \frac{1}{\Delta x_{||}(t)\left[\Delta x_{\perp}(t)^2\right]} \ \times$$

$$\times \ \exp\left[-\frac{(x-(k_o c^2/\omega_o)t)^2}{2\left[\Delta x_{||}(t)\right]^2} - \frac{y^2+z^2}{2\left[\Delta x_{\perp}(t)^2\right]}\right] \tag{2.10}$$

where

$$\Delta x_{||}(t) = \frac{b_o}{\sqrt{2}}\left(1 + \frac{(1/\lambda_c)^4 c^8}{\omega_o^6\ b_o^4}\ t^2\right)^{\frac{1}{2}} \tag{2.11}$$

$$\Delta x_\perp(t) = \frac{b_o}{\sqrt{2}} \left(1 + \frac{c^4}{\omega_o^2 b_o^4} t^2\right)^{\frac{1}{2}} \qquad (2.12)$$

and $\omega_o := c\left(k_o^2 + (1/\lambda_c)^2\right)^{\frac{1}{2}}$ according to Eq. (2.4).

The packet in coordinate space again has Gaussian shape, and widths $\Delta x_{\parallel}$ and $\Delta x_\perp$ in longitudinal and transverse directions, respectively. Its center moves in x-direction with velocity

$$v_o = k_o c^2/\omega_o \qquad . \qquad (2.13)$$

The widths (2.11), (2.12) increase with time, the packet spreads out. The minimum extension occurs at t=0. This point is arbitrary, for we can replace t by $t-t_o$ without changing the wave equations. With this replacement the minimum would occur at $t=t_o$. At times $t < t_o$ the packet has contracted.

With the de Broglie relation $\lambda_B = \hbar/p$ and the Einstein-Planck relation $E = \hbar\omega$ it can be seen that the dispersion law (2.3) which follows from the Schrödinger equation corresponds to the non-relativistic relation $E = p^2/(2m)$ between energy and momentum of a macro-particle, whereas the law (2.4) corresponds to the general relativistic relation $E = \left(p^2 c^2 + (mc^2)^2\right)^{\frac{1}{2}}$. The features of wave packets considered here are essentially determined by the dispersion law $\omega(\vec{k})$, and this law in turn essentially reflects the relation $E(\vec{p})$ and not the specific properties of any wave equation.

From (2.8), (2.11) and $p = \hbar k$ we have

$$\Delta p \cdot \Delta x_{\parallel}(t) = \hbar \Delta k \cdot \Delta x_{\parallel}(t) =$$

$$= \frac{\hbar}{2}\left(1 + \frac{(1/\lambda_c)^4 c^8}{\omega_o^6 b_o^4} t^2\right)^{\frac{1}{2}} \geq \frac{\hbar}{2} \qquad . \qquad (2.14)$$

This is Heisenberg's "uncertainty" relation. The same holds when using $\Delta x_\perp$ instead of $\Delta x_\parallel$. For later reference we note that Heisenberg's relation has been derived here by considering only properties of wave packets.

The existence of the minimum value and the general form of the time dependence does not rest on the specific Gaussian shape of the packet but is a general property of wave packets. Only the equality sign requires a Gaussian shape.[33] Also, the hyperbolic time dependence of (2.11) and (2.12) is a general feature,[35] as is the asymptotic form of the time dependence of (2.10). The spatial maximum of $|\psi(\vec{x},t)|$ decreases like $t^{-3/2}$ when $t \rightarrow \infty$. This agrees with a lemma of Ruelle[36] stating that this form of decrease is valid for a very general class of superpositions of solutions of the Klein-Gordon equation with $m > 0$, including the Gaussian superposition.

Let us now more closely consider the spreading of the wave packet (2.10). The spreading velocities in longitudinal and transverse directions increase with time and tend toward finite values as $t \rightarrow \infty$

$$\dot{\Delta}_\parallel := \frac{d}{dt} \Delta x_\parallel(t) \longrightarrow \frac{(1/\lambda_c)^2 c^4}{\sqrt{2}\, b_o \omega_o^3} =: \dot{\Delta}_{\parallel \infty} \tag{2.15}$$

$$\dot{\Delta}_\perp := \frac{d}{dt} \Delta x_\perp(t) \longrightarrow \frac{c^2}{\sqrt{2}\, b_o \omega_o} =: \dot{\Delta}_{\perp \infty} \tag{2.16}$$

It is easy to verify that in both cases 90% of the asymptotic spreading velocity is reached when the packet has spread out to roughly double its initial ($t=0$) extension.

We postulate that the asymptotic spreading velocity be smaller than or equal to the velocity of light

$$\dot{\Delta}_{\parallel \infty} \ , \quad \dot{\Delta}_{\perp \infty} \leq c \quad . \tag{2.17}$$

Then it follows from (2.16) that

$$b_o \geq \frac{c}{\sqrt{2}\ \omega_o} = \frac{1}{\sqrt{2}} \left(k_o^2 + (1/\lambda_C)^2 \right)^{-\frac{1}{2}} \quad . \tag{2.18}$$

In the non-relativistic limit, $k_o \ll 1/\lambda_C$, this reads $b_o \geq \lambda_C/\sqrt{2}$ or

$$\Delta x(t=0) \geq \frac{\lambda_C}{2} \quad . \tag{2.19}$$

That is, unless the wave packet spreads out with over-light velocity it can never have an extension smaller than half a Compton length. Notice that this is a general result, not restricted to non-relativistic situations (except when m=0, see below); the non-relativistic limit has only been used to reveal the general restriction (2.19) on the constant b_o appearing in (2.6). The minimum extension refers to the longitudinal as well as to the transverse direction, for (2.19) can as well be derived by using (2.15) instead of (2.16). Also, starting with a wave packet that has different extensions in different directions in k-space, i.e. replacing (2.6) by the more general distribution

$$\tilde{\psi}(\vec{k}) = N \exp \left[- \frac{b_{ox}^2}{2} (k_x - k_{ox})^2 - \frac{b_{oy}^2}{2} k_y^2 - \frac{b_{oz}^2}{2} k_z^2 \right] \tag{2.20}$$

where N is some normalization constant, would not alter the result. And finally, wave packets different from Gaussian packets would only result in a larger minimum extension. However, closer inspection reveals a possible objection, namely, that the result (2.19) lies just outside the domain where the conditions for its derivation are justified; the non-relativistic limit means using the non-relativistic dispersion law (2.3)

in the integral (2.1). Therefore the integrand must only give appreciable contributions from k-regions where (2.3) is justified, i.e. where $k \ll 1/\lambda_C$. This is the case when $k_o + 1/b_o \ll 1/\lambda_C$. With the minimum value $b_o = \lambda_C/\sqrt{2}$ this condition reads $k_o + \sqrt{2}/\lambda_C \ll 1/\lambda_C$ which cannot be satisfied by any (positive) value of k_o. Nevertheless, Eq. (2.19) does hold, as can be seen by subdividing the integral (2.1) into three integrals over the three regions $-\infty \dots -q/\lambda_C$, $-q/\lambda_C \dots +q/\lambda_C$ and $+q/\lambda_C \dots \infty$. In the integral over the second region the dispersion law can legitimately be approximated by the non-relativistic formula (2.3). The number q sets the error level: the smaller the q is, the smaller the error. Numerically, even for q=1 ($|\vec{k}| \leq 1/\lambda_C$) Eq. (2.3) differs from (2.4) by less than 6%. Evaluation of the integral in turn results in a sum of terms one of which shows the same behavior as (2.10), i.e. spreads out with over-light velocity if $b_o < \lambda_C/\sqrt{2}$. As the other terms are different and harmless, as are the integrals over the other regions, the spreading of this sub-integral suffices for the whole integral (2.1) and $|\psi(\vec{x},t)|^2$ to eventually spread out with over-light velocity unless $b_o \leq \lambda_C/\sqrt{2}$.

For particles of zero rest mass there is no finite Compton wavelength and no non-relativistic limit. In the m=0 case Eq. (2.18) reads $b_o \geq 1/(\sqrt{2}\,k_o) = \lambda_{B_o}/\sqrt{2}$ or

$$\Delta x_i(t=0) \geq \lambda_{B_o}/2 \quad . \tag{2.21}$$

The de Broglie wavelength of the center of the wave packet now plays a role similar to the Compton wavelength in the m>0 case.[37] Again, the result $b_o \geq 1/(\sqrt{2}\,k_o)$ violates a supposition, namely condition (2.9), $k_o \gg 1/b_o$. But again Eq. (2.21) can be maintained by a calculation not

using condition (2.9): the integral (2.1) is written as a sum of different terms one of which can be shown to have a behavior that leads to (2.21). This term comes from the integral

$$\int\int_{-qk_o}^{+qk_o} dk_y\, dk_z \left(\int_{k_o/q}^{\infty} f_1 dk_x + \int_{-\infty}^{-k_o/q} f_2\, dk_x \right) \qquad (2.22)$$

where q, as above, sets the error level.

One might object that in relativistic quantum theories the quantity $|\psi|^2$ considered so far is not the relevant one. For example, when we replace the Schrödinger equation by the Klein-Gordon equation the role of $|\psi|^2$ is taken over by the expression

$$\rho = \frac{i\hbar}{2mc^2} \left(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t} \right) = i\, \frac{\lambda_C}{2} \left(\psi^* \frac{\partial \psi}{\partial(ct)} - \psi \frac{\partial \psi^*}{\partial(ct)} \right) \qquad (2.23)$$

which means that our wave packets are to be constructed in a somewhat more complicated manner. However, with narrow wave packets in k-space (2.6), (2.9) the expression

$$\frac{\partial}{\partial t} \psi(\vec{x},t) = N_1 \int \omega(k) d^3k \, \exp\left[-\frac{b_o^2}{2} (\vec{k}-\vec{k}_o)^2 + i\left(\vec{k}\vec{x} - \omega(k)t\right) \right] \qquad (2.24)$$

can be written as

$$\frac{\partial}{\partial t} \psi(\vec{x},t) = N_2 \omega_o \psi(\vec{x},t) \qquad (2.25)$$

since in the k-region from which the integral gets its main contribution, $\omega(k)$ is a slowly varying function of k compared to the exponential factor, and can thus be extracted out of the integral. Hence ρ becomes proportional to $|\psi|^2$ and our conclusions also hold in this case. Also, ρ is

(positive or negative) definite. It is perhaps not well known but true nevertheless that this is not the case for any superposition of solutions of the free Klein-Gordon equation, even if one restricts oneself to positive-frequency solutions and to normalizable superpositions. Gerlach et al.[38] have given a counter example. The nature of this example, however, lets one suspect that definiteness is preserved for wave packets that are broader than half a Compton wavelength.

Would the over-light spreading mean that signals can be transmitted with over-light velocity? Yes. If one could produce such packets, a distant observer could cause them to interact. We do not know whether the existence of a minimal extension of wave packets due to the postulate (2.17) has been noticed before; in any case it gains a new significance from our point of view: there can be no "particle" with a dimension smaller than half its Compton wavelength. This observation gives the Compton length its real significance.

From $\Delta x \Delta p \geq \hbar/2$ we have

$$\Delta p \geq \frac{\hbar}{2\Delta x} \quad . \tag{2.26}$$

Consider the case where Δx takes on its minimum value $\Delta x_{min} = \hbar/(2\,mc)$; then

$$\Delta p \geq mc \quad . \tag{2.27}$$

This large range in momentum and hence in energy suggests that the case when $\Delta x = \Delta x_{min}$ may have something to do with production of new particles, i.e. new wave packets. This might provide a "physical" Lorentz-invariance of the Compton length. When seen from a moving frame, a wave packet appears contracted, and may even "show" a less-than-minimum extension.

However, this statement must be given a physical meaning, i.e. the wave packet must be made to interact with wave packets at rest in the moving system. In these interactions the smaller-than-minimum wave packet, according to (2.27), will be converted into a number of larger-than-minimum wave packets, i.e. will give rise to packet (particle) production.

It seems relevant here to point out that, in spite of many efforts, it has not been possible to define relativistic particle position operators with eigenfunctions that are both orthogonal and transform properly under Lorentz transformations.[39] Postulating orthogonality means postulating exact localizability. The wave packet picture suggests that this postulate be given up. From the wave packet view, the unceasing efforts to construct orthogonal localized states then remind one of the efforts of the fly to go through the window pane. In fact, when the orthogonality postulate is given up it is possible to construct position operators with eigenfunctions that transform properly under Lorentz transformations. The scalar product of two of these eigenfunctions is not a δ-distribution but decreases like $\exp\left[-|\vec{x}'-\vec{x}|/\lambda_c\right]$. This will be further discussed when we consider the space-time position operators in Section XX.

We think that these remarks also shed new light on the observation made in relativistic quantum field theory, namely that the field quanta cannot be localized within regions that are smaller than their Compton wavelength.[40,41] This is a typically relativistic effect; in non-relativistic quantum field theory the quanta can be localized within arbitrarily small regions. Notice that condition (2.17) contains the velocity of light.

The Compton wavelength playing the role of a fundamental length for particles (wave packets) of mass m gives support to those papers which deal with the problem of introducing a fundamental length and which take the Compton wavelength for that length.[42] However, the real problem of the fundamental length is to find a fundamental length that is universal, i.e. valid for particles of any mass. This is equivalent to finding a universal mass, and this question is outside the scope of our considerations.

III. EVENTS AND BASIC INTERACTIONS

The wave packets interact with one another; they coalesce, split and build up stationary aggregates, as will be described in the subsequent sections. If one knew all wave packets and their developments one would know everything. However, the details of these processes are beyond the possibility of direct observation since the observers and any measuring apparatus in turn are composed of wave packets. Since for any wave packet there exists a minimum extension equal to half the Compton length the concept of exact space-time localization does not seem to be appropriate here. Nevertheless, we want to get in touch with the ideas of conventional non-relativistic quantum mechanics where such a concept exists and where the expression $|\psi(\vec{x},t)|^2 d^3x$ gives the probability for finding a particle at time t in d^3x. In the wave-packet picture the expression $|\psi(\vec{x},t)|^2 d^3x\, dt$ with the same $\psi(\vec{x},t)$ is interpreted as being proportional to the probability that the wave packet interacts with another one within d^3x and within dt; space and time are thus treated in a symmetrical way. Such an interaction when it is regarded only under the aspect of the space and time of its occurence is called an event.

The localization of any event can only be defined within the spatial extension of the smallest wave packet participating in the interaction. Notice that there are wave packets as long as 10^7 m (coherence length of photon beams[43]). For these the name micro-objects then seems no longer quite appropriate. Actually, the best possible physical localization is even more inaccurate. The wave packet must interact with another one which forms part of a detecting device. The wave packet of the detecting device must, through subsequent interactions with other wave packets of the device, initiate a cascading process which must lead to a macroscopic effect, say, a black spot in a photographic emulsion. This black spot then defines as accurately as it can the position of the initiating event. Details of this process will be discussed in Sections VIII-X.

The quantity $|\psi|^2 d^3x$ dt is conceptually more closely related to a cross section than to the position probability of a particle in quantum mechanics. Consider the interference fringes on the absorbing layer behind the wall with the two holes in it as mentioned in the introduction. The fringe intensity at some position $\vec{x}_o$ can be calculated from the probability for a wave packet to induce an event at $\vec{x}_o$, i.e. $|\psi(\vec{x}_o)|^2$, when the time of the event is left out of account. On the other hand, one could calculate the fringe intensity from the cross section σ for the incoming wave packet $\psi_i(\vec{x})$ to excite an atom at $\vec{x}_o$ in the absorbing layer from its ground state $\phi_o(\vec{x})$ to some state $\phi_k(\vec{x})$. This atom then can give rise to an observable event at $\vec{x}_o$. Let the outgoing wave packet be $\psi_f(\vec{x})$, then

$$\sigma \propto \left| \int \phi_k(\vec{x}) \psi_f(\vec{x}) \; 0 \; \phi_o(\vec{x}) \psi_i(\vec{x}) d^3x \right|^2 \quad . \tag{3.1}$$

Since the atom serves to localize the event we can assume that its extension is small compared with the extension of the incoming wave packet. Hence, we can extract $\psi_i(\vec{x}_o)$ out of the integral leaving

$$\sigma \propto |\psi_i(\vec{x}_o)|^2 \quad . \tag{3.2}$$

The main difference between a cross section and the quantity $|\psi|^2 \, d^3x \, dt$ is that a cross section is defined for stationary conditions whereas $|\psi|^2 \, d^3x \, dt$ pertains to a definite space as well as to a definite time interval.

In contrast to ordinary quantum mechanics, the observation of an interaction of a wave packet via, say, a black spot in a photographic emulsion is not a position measurement of the wave packet under consideration; there is no single sharp value of position for a wave packet. In order to compare this situation with ordinary quantum mechanics let us assume that the wave packet has a well defined maximum (e.g. a Gaussian packet), and let us define a position measurement as a determination of the wave packet's maximum. If the packet induces an event at a certain position we can conclude that its maximum is somewhere around. We even can—supposed we knew the form of the packet—calculate by maximum likelihood methods the probability that the region of the observed black spot includes the maximum of the wave packet.[44] In general, this probability will be smaller than 1. Only in the case when the wave packet is narrower than the black spot can we get a probability of 1. In ordinary quantum mechanics it is assumed that a position measurement throws the system into an eigenstate of the position operator, i.e. into a δ-distribution. So, only after the position measurement is there a probability of 1 that the wave packet is at a particular position. In our conception there is

no need for such an assumption (see Sections X, XI), and we do not assume

that any position measurement will have the drastic effect of reducing

any, however broad, wave packet to a δ-distribution. Of course, there is

some alteration of the wave packet when it induces an event that leads to

an observable effect. However, this is a quite normal alteration occurring

in every event, and it might be very small and negligible under the con-

sidered circumstances. If a wave packet was broad before the observation

it can remain broad after it.

Of course, a reduction to a spatial δ-distribution is also forbidden

by the limiting spreading velocity discussed in the preceding section;

however, the above argument is still valid even if there were no such

additional restriction. For momentum measurements quite analogous

considerations apply, and there is no lower limit on the width of a wave

packet in momentum space.

For the purpose of coherent presentation we here describe some

further conceptions concerning wave packets and their interactions which

will be more fully substantiated only in later sections (in particular

Sections VII, IX, XI). Basically a wave packet is not modified continuously

but discontinuously, by a series of distinct interactions with other

wave packets. Such a distinct basic interaction need not occur instan-

taneously; it may take some time but it is conceptually differentiated

from any other interaction. The wave packets emerging from a basic inter-

action are completely independent of each other.

One may qualitatively subdivide the interactions into mild and violent

ones with respect to a particular wave packet. Mild interactions cause

only a very small modification of the wave packet under consideration.

Modifications of a wave packet which are described by a Schrödinger-type

equation containing an external field or potential are not due to one

of the basic interactions considered here. Rather, those modifications

are the result of a series of mild interactions. Consider a charged

particle (wave packet) moving through a constant magnetic field. The

curvature of its trajectory in our conception is due to a series of mild

interactions. One interaction may result in emission of a soft bremsstrah-

lung photon (wave packet). As soon as particles that move at relativistic

velocities are considered, these interactions become violent; they form

the synchrotron radiation and cause an appreciable energy loss of the

charged particle. The particle motion then cannot any longer be treated

simply by a Schrödinger equation with potential.

The violent interactions cause a strong modification of the wave

packet under consideration. Examples are: particle decay, or any inter-

action typically considered in high-energy physics. These processes are

essentially processes involving many particles, i.e. wave packets in the

final state, and they are well known to be beyond the reach of any

Schrödinger-type equation. It is seen that this view is closer in spirit

to the picture of quantum field theory when applied to high-energy physics

where the interactions between particles are ascribed to exchange of

(virtual) quanta, and where the particles before and after the interaction

are conceived to be in definite states and independent of each other.

Also, this view brings to mind the derivations of the Schrödinger equation

with potential from some kind of Bethe-Salpeter equation.[41]

The detailed mathematical description of these basic interactions of

wave packets with each other falls outside the scope of traditional non-

relativistic quantum mechanics. With regard to these interactions quantum

mechanics is in a position analogous to that of thermodynamics with regard to the interactions between molecules described in kinetic theory or statistical mechanics. The detailed laws governing the basic interactions, according to Wigner, must be expected to be non-linear (see Section VII and also Einstein [45]). Their formulation is the most difficult task since obviously it would be the solution to the basic problems concerning elementary particles. We do not have this solution. We only mark the place in a general conceptual framework where the problem lies and where the solution should be sought, and we try to formulate some general principles to be incorporated into any mathematical description of the basic interactions.

In trying to find out basic features it may also be useful to have a look at existing models. The high-energy reader might already have asked whether there is a connection between the idea of a wave packet in coordinate as well as momentum space, and the quark-parton model.[46] If so, certainly some novel viewpoints on structure functions, quark confinement, gluons, etc. are to be expected. A soliton physicist, trained in regarding particles as extended objects, might also ask about the connection of wave packets with soliton solutions.[47] Also, there are the bag models,[48] and there are form factors. Surely, here are questions that appear worthwhile to investigate.

IV. NORMALIZATION AND GAUGE INVARIANCE

The wave packet (2.1) is normalized to A

$$\int_{-\infty}^{+\infty} |\psi(\vec{x},t)|^2 \, d^3x = A \qquad (4.1)$$

provided $\tilde{\psi}(\vec{k})$ is normalized to A. In particular, the normalization is time independent, that is, the left-hand side of (4.1) denotes a conserved quantity. In the partcle interpretation one wave packet (pure state) corresponds to one particle, and (4.1) is the probability for observing the particle anywhere in space. Since in non-relativistic quantum mechanics the particle can neither disappear nor multiply the interpretation of (4.1) as conservation of probability appears natural and it is convenient to put A=1. In our picture the wave packet does not correspond to one particle, so what is conserved then?

This question cannot be answered without considering further physical situations. From considering identical wave packets in Section V we arrive at the following interpretation: with k=p/h (2.1) becomes $\psi(\vec{x},t) = (\sqrt{2\pi}\,\hbar)^{-3} \int \tilde{\psi}(\vec{p}/\hbar)\, \exp\left[i(\vec{p}\vec{x}-\hbar\omega t)/\hbar\right] d^3p$ and we can write (4.1) in the form of an integral over phase space

$$(2\pi)^{3/2} \iint \psi^*(\vec{x},t)\tilde{\psi}(\vec{p}/\hbar)\ \exp\left[i(\vec{p}\vec{x}-\hbar_\omega t)/\hbar\right]\cdot \frac{d^3x\ d^3p}{h^3} = A = \text{const.} \qquad (4.2)$$

The quantity $\kappa\psi^*\psi\, d^3x\, dt$ is interpreted as the probability for an event to occur within d^3x and within dt; therefore it should be physically dimensionless. Then, when the proportionality factor κ is given the dimension $(\text{time})^{-1}$ it follows that the integral (4.2) and the quantity A are dimensionless. Following a suggestion by Jaynes[49] we call them the number of spatial _action_ quanta. By a spatial or three-dimensional action quantum we mean the quantity h^3. A wave packet then contains "spatial" action; that is $(\text{action})^3$ in units of spatial action quanta h^3. The feature of a proportionality factor with the physical dimension of $(\text{time})^{-1}$ stems from the feature that the above probability refers not only to d^3x

but to the temporal interval dt as well. This equal treatment of space

and time intervals is essential for a covariant formulation, as will be-

come clear in Section XX.

Any factor exp(iα) of modulus 1 applied to a wave packet $\psi(\vec{x})$ is

irrelevant, i.e. has no observable consequences. Interference between

different wave packets never occurs; a wave packet can interfere only with

itself. This is the generalization of Dirac's statement[50] that each

photon interferes only with itself. Interference between two laser beams

from different lasers has actually been observed.[51] This is interpreted

by saying that the two photon wave packets first coalesce into one single

packet which then interferes with itself. This view has already been

expressed by the experimenters themselves (Ref. 51 first paper, p. 282). It

will be built into a general scheme when we discuss the mechanism of

coalescence of identical wave packets in Section V and XVI.

While a factor $e^{i\alpha}$ of modulus 1 applied to $\psi(\vec{x})$ does not affect the

probability for an interaction to take place at $\vec{x}$, a constant real factor

$\neq 1$ does. In our interpretation $|\psi(\vec{x})|^2$ no longer determines the probability

for observing a particle, so there is no longer any motivation for putting

A=1 in the normalization (5.1). The probability for interaction at $\vec{x}$

then is proportional to the number A of spatial action quanta h^3.

The content of action of a free wave packet is constant. It is also

constant if there are external fields in the Schrödinger equation. Not

all interactions can be accounted for by external fields;[52] only the

"mild" interactions can (Section III). In these the change in the content

of action of the wave packet can be neglected; one may regard this as

some kind of approximate adiabatic invariance of action.[53] In the

"violent" interactions the wave packets may exchange non-negligible amounts of action with one another. Of course, it is conceived that action is only exchanged in units of h (or equivalently, action^3 in units of h^3).

Whether the combined amount of action of the wave packets is conserved in any interaction is another question, which goes beyond the scope of non-relativistic quantum mechanics. Let us explain why we think that action is indeed always conserved. The time independence of $\int |\psi(\vec{x},t)|^2 d^3x$ i.e. of action or "probability" due to Noether's theorem results from the fact that the Lagrange density $\mathscr{L}$ from which the Schrödinger equation follows is invariant under global gauge transformations $\psi(\vec{x}) \rightarrow \psi'(\vec{x}) = e^{i\alpha} \psi(\vec{x})$. When Lagrangians are constructed with interacting fields and when the interaction term $\mathscr{L}_{int}$ in these Lagrangians is also gauge invariant, this, according to the common interpretation, leads to another conserved additive quantity.[55] This quantity is electric charge when the gauge invariant $\mathscr{L}_{int}$ contains electrically charged particle fields, it is baryonic charge if $\mathscr{L}_{int}$ contains baryon fields, and so on. Every new type of field functions appearing in $\mathscr{L}_{int}$ in this way means a new type of conserved charge; thus, one can produce infinitely many conserved quantities from one principle. That is black magic. Therefore, we interpret the situation differently: there is only one conserved quantity, namely action. The gauge invariance of a specific interaction Lagrangian means that action is conserved in that specific type of interaction. Gauge invariance of the free Lagrangian means the same as gauge invariance of the interaction Lagrangian: conservation of action. Thus action gets its own general conservation law, which it does not have in classical (not

quantum) physics. The great success of gauge invariance[56] is the basis

for our belief in that conservation law. Thus the basic feature[57] of

quantum theory, namely that it operates largely with probability ampli-

tudes and only in the very end of any calculation goes over to bilinear

forms of the type $|\psi|^2$, means that there is some kind of gauge invariance.

This gauge invariance, when consistently built into the theory, then

means total conservation of action, a quantity that is quantized in units

of h, the fundamental constant in quantum theory. More precisely, it is

the principle of local gauge invariance ($\alpha \to \alpha(\vec{x})$) that is so fruitful.

In our interpretation, the fact that the gauge parameters ($\alpha(\vec{x})$) can vary

from one space point to another means that they can vary from one wave

packet to another. This is in line with the idea that the extension of

the packets in some sense represents fundamental lengths, and that there

are no phase relations between different wave packets: a wave packet can

only interfere with itself.

V. IDENTICAL PARTICLES AND COALESCENCE

When one has a system of several identical particles one requires

that the Hamiltonian of the system be invariant under the permutation of

the particle labels, i.e. the Hamiltonian H must commute with the permu-

tation operator P, $[H,P]=0$. Then $P\psi$ is a solution of the Schrödinger

equation, provided ψ is a solution. This does not imply that ψ is an

eigenfunction of the permutation operator, e.g. that ψ is a symmetric or

an antisymmetric function of the particle labels. It is an additional

postulate that the wave functions describing identical elementary particles

are either symmetric or antisymmetric. It has turned out that this con-

jecture is verified by Nature: there are only bosons and fermions.

The postulate that the state functions of identical particles be either symmetric or antisymmetric is a sensible quantum mechanical statement only when the particles interact. When they do not interact the postulate does not lead to any observable effects that go beyond those of classical physics.[58] This fact is easily checked, e.g., for the hydrogen molecule in the treatment of Heitler and London,[59] or for para and ortho helium. The exchange integrals describing the contribution from the effect of (anti)symmetrization always appear multiplied by the coupling constant e^2.

Thus, our interpretation of (anti)symmetrization is this: in the interaction between identical wave packets there is an intermediate coalesced state. In this state the single incoming wave packets completely lose their individuality. For example, when two electrons have coalesced it is only by the total charge of $2e$ that one can conclude that the state was built up by two electrons. The coalesced state is really a function of only one set of variables pertaining to only one system. For example, there is only one space coordinate vector $\vec{x}$. Writing the coalesced state as a function of several sets of variables is not really justified physically. Rather this procedure has to be remedied and the remedy is (anti)symmetrization with respect to the labels pertaining to the single systems before (or after) the coalescence. Secondly, the difference between symmetrization and antisymmetrization physically means that the coalesced state resulting from coalescence of identical bosons can have essentially (up to normalization and spin) the same properties as those of an initial boson; in particular it belongs to the same spin class. The coalesced state resulting from coalescence of identical fermions cannot have essentially the same properties as those of an initial fermion; in particular it does not belong to the same spin class. This latter statement is Pauli's exclusion principle in our version.

Identical wave packets can coalesce irrespective of the direction of their spin: electrons can coalesce irrespective of whether their spins are parallel or antiparallel. The coalesced states in the two respective cases are, however, different from each other (total spin 1 and 0, respectively), and there are no transitions between them under interactions that do not change the spin direction. When observing this one can easily obtain Feynman's prescription for transition amplitudes, which does not use the (anti)symmetrization postulate, from the usual prescription, which does use the postulate.[60]

Speaking of coalesced packets may be regarded as just another formulation of the usual statement that the observer is unable, in principle, to distinguish the particles. The coalesced states make the loss of individuality of the original particles a physical process. This formulation is closer to the attitude of realism in which there is no need for the mentioning of an observer. (Anti)symmetrization is then a mathematical method of dealing with the existence of coalesced states just as the superposition and interference of plane waves is a method of dealing with the extended structure of micro-objects. The different superposed waves with their definite frequencies and wave lengths are fictitious in the same way as are the individual particles in the coalesced packet. Notice that in (anti)symmetrization we speak of the mathematical operation of exchanging particle labels, not of the physical process of transporting real particles from one place to the other.

Consider the H_2 molecule. In the calculation of its states starting from those of two far separated H atoms, antisymmetrization is introduced from the beginning. The H_2 molecule is a stationary aggregate of wave packets (see Section IX), and the electron packets interact with each other, perhaps mediated by some photon packets going to and fro between

them, and in addition they coalesce and split. When they are far apart

there is no coalescence, and antisymmetrization is merely a formal pre-

scription. It begins to acquire physical significance when the electron

packets come close to each other and begin to overlap, in which case the

fraction of time they are coalesced increases. The same applies to (anti)

symmetrization of transition amplitudes. When the scattering angle is

small the exchange terms can be neglected. When the scattering angle is

large the packets come close to each other, coalescence sets in, and the

exchange terms become appreciable.

When two identical bose wave packets coalesce into one single wave

packet this means that the amount of action of the new wave packet is the

sum of the action of the original wave packets. This is expressed by the

normalization. The norm of the new wave packet is the sum of the norms

of the original wave packets. Hence, the probability that the new wave

packet undergoes an interaction is proportional to the sum of the corre-

sponding probabilities of the original wave packets with the same propor-

tionality factor. Thus a bose wave packet which is the result of coales-

cence of n wave packets with 1 quantum of action each, contains n quanta

of action. The amount of action is conserved whether the wave packets

are well separated or have completely coalesced.

With these principles we can derive the Planck law by the standard

Bose-Einstein method.[61] We consider equilibrium radiation in a black-

body cavity and ask for the most probable number of photons $dn/(d^3x \, d\nu) \cdot d\nu$

per unit volume in the small frequency interval $d\nu$. The standard pro-

cedure is to subdivide the whole frequency range into discrete intervals

numbered by i=1,2,3... Since to the frequency ν_i there corresponds an energy $\varepsilon_i = h\nu_i$ we may regard this also as a subdivision into discrete energy intervals. To the frequency interval $d\nu_i$ there also corresponds a momentum interval $(c/h)dp_i$, and one starts by considering the phase space region $4\pi p^2 dp d^3x$ corresponding to this interval, and by calculating the most probable number of photons in this region. This is done by counting the number of elementary phase-space cells of magnitude h^3 contained in this region

$$g_i = 4\pi p^2 dp \ d^3x/h^3 \tag{5.1}$$

and by evaluating the number of possible ways $W_i = (g_i + n_i - 1)!/$ $\left[n_i! \ (g_i-1)!\right]$ in which n_i photons can be distributed over these elementary cells, where the interchange of photons within one elementary cell does not count as another way. Then the logarithm of the product ΠW_i is maximized by varying the n_i subject to the constraint that the total energy is constant

$$\sum_i n_i h\nu_i = \text{const.} \tag{5.2}$$

The n_i for which the maximum is achieved are then the required most probable number of photons in the considered phase-space region:

$$\bar{n}_i = g_i (\exp\left[\beta h\nu_i\right]-1)^{-1} \quad , \tag{5.3}$$

where β is the Lagrange multiplier in the maximization procedure. By comparison with thermodynamic relations one sets $\beta=1/kT$ and by using $p=h\nu/c$ and dropping the indices i one finally obtains[63]

$$\frac{dn}{d^3x \ d\nu} = \frac{4\pi\nu^2}{c^3} \frac{1}{\exp\left[h\nu/kT\right]-1} \quad . \tag{5.4}$$

In our interpretation there is no distribution of photons over phase-space cells but rather there is distribution of action quanta h^3 over photon wave packets. The number g_i in (5.1) so far taken as the number of elementary cells contained in a given phase-space region now is taken to be the maximum possible number of independent wave packets in that region, since each wave packet must contain at least one action quantum h^3. This is what is behind the well-known statement that one discrete state is equivalent to one elementary phase-space cell h^3(Ref. 50, p. 238). If one distributes n_i action quanta over g_i packets and if $n_i > g_i$ then some packets must contain more than one action quantum; these are regarded as coalesced packets. The several action quanta then cannot be discerned because of the above described properties of such packets. To speak of exchange of action quanta within one (coalesced) wave packet does not make sense and this exchange must not appear in the calculation. The coalescence is Einstein's "mutual interaction of the molecules whose nature is at present completely mysterious" or Schrödinger's "strange sort of molecular interaction".[64] Exchange of action quanta from different wave packets does make sense. Attributing an action quantum h^3 to a wave packet of average frequency ν_i means attributing to it an average energy $h\nu_i$, and this is why energy conservation play its role, via the constraint (5.2).

In the photon case there is no constant total number of "particles",

$$\sum_i n_i = \text{const} \quad , \tag{5.5}$$

which in our interpretation would mean a constant total number of action quanta. This is due to the zero rest mass of photons which makes it possible to have action quanta carried by wave packets of very low frequency and hence almost no energy. The lowest possible frequency is determined

by the longest possible wave length $\lambda=c/\nu$, and this in turn is determined by the dimensions of the cavity. A 3 cm cavity would correspond to a radio frequency and an energy as low as 4×10^{-5} eV. On the other hand, for electrons in a metal at room temperature there should be conservation of the number of action quanta (and hence a chemical potential $\neq 0$), because the energy ε_i in the formula $\varepsilon_i = h\nu_i$ means total energy and is never smaller than the rest energy. The calculation of the fermi distribution of electrons cannot be interpreted in the same neat way as the bose distribution. This difference is connected with the way we describe things, namely in terms of non-coalesced states while the fermi coalesced states are essentially different from these.

The above arguments get further support by the following consideration. The bose (fermi) distribution of particles of mass m confined within some box is known to deviate from the classical Boltzmann distribution when the "thermal de Broglie wavelength"

$$\lambda_{th} = h/\sqrt{2\pi \, m \, kT} \tag{5.6}$$

is greater than the average distance between two particles.[65] The name is chosen because λ_{th} is of the order of the de Broglie wavelength of a particle of mass m with the kinetic energy kT, and one regards λ_{th} somehow as the spatial extension of the particle. We can make this more precise by observing that the thermal de Broglie wavelength gives a lower bound for the spatial extension Δx of a wave packet. The range Δp of momenta within one wave packet evidently cannot be broader than the range Δp_{obs} observed experimentally. The observed range is the superposition of two ranges; first, there is the range Δp_1 due to the presence of many wave packets each with a somewhat different velocity and average momentum,

and second there is the range Δp in the momenta of each individual wave packet. The observed range is obtained from the classical Boltzmann distribution

$$\Delta p_{obs} = \left(\overline{(\Delta p)^2}\right)^{1/2} = \sqrt{3 \ kT \ m} \tag{5.7}$$

Hence $\Delta p \leq \sqrt{3 \ kT \ m}$ and $\Delta x \geq h/(2\Delta p) \geq h/(2 \ \sqrt{3 \ kT \ m})$, i.e.

$$\Delta x \geq \lambda_{th}/\sqrt{24\pi} \approx 0.12 \ \lambda_{th} \ . \tag{5.8}$$

Thus, when the de Broglie wavelength is of the order of the mean separation of the particles, the wave packets begin to overlap and to coalesce.

The genuine quanta thus are not energy quanta but action quanta. We suspect that the distinguished role ascribed to the harmonic oscillator in the quantum theory of radiation stems only from thinking in terms of energy quanta, for the harmonic oscillator is the only quantum mechanical system whose discrete energy eigenvalues are equidistant. So, one has conceived the idea that oscillators are in the walls of the black-body cavity where everybody would expect to be atoms. But atoms have no equidistant energy eigenvalues.

Also, the suspicion arises that thinking of the radiation field as equivalent to a system of harmonic oscillators in quantum field theory is not really justified. Remember that in this way the infinite zero-point energy comes in, which then, in turn, must be eliminated by normal ordering. Let us have a look at the formalism of field quantization (second quantization) for boson fields. Consider the creation operator $a^{+}(\vec{p})$. When it operates on the vacuum state it generates a state where one boson with momentum $\vec{p}$ is present. When it operates again on the one-boson state it

generates a two-boson state, and so on. The point is that nothing, except the normalization factor, indicates that this state is an n-particle state: there is only one phase and only one momentum $\vec{p}$. Hence, it is very much like our coalesced states and may consequently be interpreted as meaning not n particles but rather n quanta of action. This suggests that one should regard field quantization as a formalism that deals with quantization of action.[66]

VI. THE MEASUREMENT PROBLEM IN QUANTUM MECHANICS

In conventional quantum mechanics observers and measurements are necessary elements in defining the theory. There are, nevertheless, serious conceptual difficulties with the measurement process and we are now going to discuss these. A very clear exposition has been given by Wigner;[68] we shall use here his language and notation. Actually, there are several conceptions of the quantum mechanical measurement process, none of which satisfies everybody. We do not give here a proper account of these;[26] we shall only mention certain features that will have some bearing on our proposed view.

Usually, it is assumed that a type of measurement exists where the measuring apparatus A does not change the measured object σ, though of course the object influences the apparatus. This feature is taken over from the features of a macroscopic measurement. In the microscopic realm it may appear to be a somewhat unrealistic assumption in view of the general principle that there is no action without reaction. Let us assume however that it represents an allowed approximation. Further, let us assume that both apparatus and objects are described by one pure state vector each, and the measurement is taken as an interaction between

these two. This case, then, may serve to outline the characteristic difficulties. Let A_o denote the state vector of the apparatus before measurement. The apparatus is to measure the quantity Q on the object. Let σ_μ denote the state vector of the object. σ_μ is one of the eigenstates of the operator belonging to Q, and q_μ is the corresponding eigenvalue, which represents the outcome of the measurement. It is further assumed that the Hilbert space of apparatus plus object is the direct product of the Hilbert space of the apparatus and that of the object. The considered type of measurement can then be symbolized as

$$A_o \times \sigma_\mu \rightarrow A_\mu \times \sigma_\mu \quad . \tag{6.1}$$

When, more generally, the object before measurement was not in the state σ_μ but rather in a superposition

$$\sigma = \sum_\mu c_\mu \sigma_\mu$$

then, because the arrow in (6.1) represents a linear (unitary) operator, the measurement is described by

$$A_o \times \sigma = \sum_\mu c_\mu (A_o \times \sigma_\mu)$$
$$\rightarrow \sum_\mu c_\mu (A_\mu \times \sigma_\mu) \quad . \tag{6.2}$$

Would the sum (6.2) consist of a single term only it would describe a system where the apparatus is in the state A_μ and simultaneously the object in the state σ_μ, and hence the measurement could be interpreted as giving the result q_μ. However, this is not the case; (6.2) rather consists of a linear superposition of such terms. On the other hand, if we are to be able to call this interaction an observation at all, it has to lead to one of the pure states $(A_\mu \times \sigma_\mu)$, i.e. the sum has to

be reduced to one of its terms. This reduction is the postulate of von Neumann.[69]

There are then supposed to exist two kinds of changes of the state vector of a quantum mechanical system in time, first according to the Schrödinger equation and second according to v. Neumann's reduction or projection postulate in a measurement. The change due to the Schrödinger equation is completely determined and calculable; the change due to the reduction process is not completely determined, and only probabilities can be calculated. This situation has been felt to be unsatisfactory by many, and attempts have been made to specify the supposed extra mechanism by which only a single pure state rather than a superposition of many states appears as the result of a measurement. Is it only in a measurement interaction that the reduction occurs? If so, what specific features of a measurement interaction bring about this reduction? Is it the observation by a human being, or is it other features independent of the presence of a human observer? In particular, several ways have been tried to find out how the superposition could be resolved into the corresponding mixture, i.e. how the interference terms could be made to vanish. The mixture then could be treated using concepts familiar from statistical mechanics.

When averages or eigenvalues on the system (6.2) are calculated pertaining to only one system, either object or measurement apparatus, the considered subsystem behaves mathematically like a mixture.[70] This has been regarded, e.g. by Margenau,[71] as already being sufficient to settle the problem of measurement. Zeh[72] has made the idea very plausible that the larger the apparatus (or the system: apparatus plus object) the easier

it will be for it to interact with its surroundings and eventually with the whole universe. Therefore, the apparatus is a subsystem of a larger system and hence describable as a mixture. On the other hand, it might be asked how the separation of object and apparatus, or apparatus and the rest of the universe then is achieved, or defined physically (see Section XII on the Einstein-Podolsky-Rosen problem).

One may try to perform a second measurement, now regarding the first apparatus A as the new object. If, by this measurement one observed, say, the state A_5 of the apparatus A one would know that the original object is in the state σ_5 and would yield the result q_5. One may try to measure a quantity R whose eigenvectors are just the states A_μ. Let the state vector of the second apparatus before the measurement be B_o. According to the above procedure this measurement could then be denoted by $B_o \times A_\mu \to B_\mu \times A_\mu$ and

$$B_o \times \sum_\mu c_\mu (A_\mu \times \sigma_\mu) \to \sum_\mu c_\mu (B_\mu \times A_\mu \times \sigma_\mu). \tag{6.3}$$

One sees that nothing new is accomplished. The new apparatus B and the new object A are interrelated in the same way as were the original apparatus A and the original object σ. Moreover, both systems A and B are now correlated with the object σ. One can extend this chain to a third apparatus C and, indeed, as far as one wants without ever getting a definite result:

$$\sum_\mu c_\mu (...Z_\mu \times Y_\mu \times ... \times C_\mu \times B_\mu \times A_\mu \times \sigma_\mu) \quad . \tag{6.4}$$

It does not matter whether the apparatuses are microscopic or macroscopic so long as they are described by one state vector each. Also, evidently, it does not help to conceive an apparatus (more realistically, one may

argue) as a mixture. This would only mean that we do not know the state vector of the apparatus but only a set of possible states with their respective probabilities. Some other physicist may well know the actual pure state in which the apparatus really is.[73]

It has also been argued that the chain (6.4) can be terminated when it is so long that the terms in the sum, by means of the last apparatuses Y, Z etc., are well separated dynamically; that means when all transition matrix elements between any two terms with different μ vanish, and when they do so for any physical interaction (transition) operator.[74,75] In this case the sum (6.4) is mathematically equivalent to a mixture of the different terms. However, though mathematically the sum may behave as a mixture, conceptually it is not a mixture, for there is nobody who could, in principle, know in which definite state the system is. Also, the separated terms could, in principle, be brought together and be superposed with coherent interference; this is not possible with the components of a mixture.[74,76]

VII. SCHRÖDINGER'S CAT AND WIGNER'S FRIEND

The problem of superposition is presented very drastically in the example of Schrödinger's cat.[77] Schrödinger considers a cat in a closed box. Also present within the box are a certain amount of a radioactive substance, a Geiger counter and a cat-killing device, all protected against the cat. Circumstances are arranged so that the probability for the Geiger counter to count the decay of at least one nucleus within one hour is just 1/2. In the case where decay occurs, the counter discharge triggers the cat-killing device which consists of a hammer and a flask of prussic acid. The flask then is smashed, the acid penetrates into the box, and the cat is poisoned. The probability that after one hour

the cat is dead is 1/2. Since the box is closed we cannot know after
an hour whether the cat is dead or alive, unless we open the box (using
the necessary precaution) and look into it. In orthodox quantum mechanics
an allowed abstraction is to describe the living cat by one wave function
ψ_1 and the dead cat by another wave function ψ_2 orthogonal to ψ_1. Further-
more, according to the principle of superposition the cat in the closed
box after one hour is in the state $(\psi_1 + \psi_2)/\sqrt{2}$, i.e. there is neither a
dead cat nor a living cat but a superposition of both. This is independ-
ent of whether ψ_1 and ψ_2 are orthogonal to each other or even whether
they are dynamically separated or not. Indeed, ψ_1 and ψ_2 are very well
dynamically separated, there is no finite transition probability from
the state of the dead cat to that of the living cat. We may consider
the cat to be one of the last apparatuses in the chain (6.4). The micro-
scopic object is the nucleus and the two possible experimental outcomes
($\mu=1,2$) are whether it decays within one hour or not. Both outcomes are
coupled by the chain (6.4) to macroscopically different situations, namely
the dead and living cat, respectively.

In an attempt to solve the discussed problem Wigner[76,78] has specu-
lated that the chain (6.4) might terminate and the sum reduce to one of
its terms when one of the successive apparatuses is identical with my
consciousness. This proposal can be interpreted in two different ways
which are often confused. First, in von Neumann's sense of psycho-
physical parallelism (Ref. 69, Section VI,1), that is, in some philosophical
way, by saying: I can perceive that my state is A_5, hence I am in state
A_5. This is tautogological in the same way as: cogito ergo sum.[79]
Second, it can be interpreted in the sense of realism meaning that the

process of becoming aware of something is just the extra physical mechanism

connected with von Neumann's postulate. Then the following difficulty

arises. The role of some apparatus in the chain (6.4) ending in my brain

can as well be played by another person—Wigner's friend in[78]. Then the

reduction of the sum already takes place in his brain. Now imagine that

the task of the apparatus or respective person is really very primitive.

Then the person can be very stupid, with little consciousness, and one

may even replace the person by an ape, a bird, a worm etc. The problem

then is to which being in this declining line does one still ascribe

consciousness and to which does one no longer ascribe it. That means one

has to define consciousness in physical terms.

In the same papers Wigner actually makes a proposal in that direc-

tion. He points out that the equations of motion in quantum mechanics

must cease to be linear if conscious beings enter the picture. It will

be seen that, though we dismiss the conscious observer as an essential

element, we need interactions with properties that cannot be described

by a Schrödinger equation; these interactions are our basic interactions

introduced in Section III. They thus represent Wigner's non-linear inter-

action in our conceptual framework. The difference is that our basic

interactions do not act only after the interaction described by the

Schrödinger equation, but rather the Schrödinger interaction itself is

a phenomenological account of a series of basic interactions.

Schrödinger's cat and Wigner's friend point at two basic problems

in the usual conception of the measurement process: determinism and

realism. The problem of determinism comes in when one tries to explain

v. Neumann's postulate by some extra mechanism. Indeed, any attempt

to specify a physical mechanism which turns the superposition (6.2) or (6.4) into a mixture basically means a refusal of the Copenhagen interpretation of quantum mechanics. The Copenhagen interpretation asserts that the probabilistic appearance of quantum effects is irreducible to any more basic process and has to be accepted as such. To try to find a mechanism which turns the superposition into a mixture means to try to find a mechanism which in any single measurement gives a definite result, and which in repetitions of the measurement gives the different results with the frequencies that are observed and predicted by quantum mechanics. The individual result is determined; it is only that we are unable to predict it with certainty. Then, provided we knew the deterministic laws of the extra mechanism, we must conclude that the fact that we are still unable to predict the individual result is because we are unable to get the initial conditions completely under control. This is a situation that reminds us of statistical mechanics. We discuss it in more detail in Section XIX.

The problem of realism in the usual conception of the measurement process is this: it is characteristic of traditional quantum mechanics that it requires the existence of external equipment adapted to the observable in question. The world is divided into two parts, a part which is observed and a part which does the observing.[80,81] The borderline between these two parts must not be confused with von Neumann's boundary (Ref. 69, Section VI,1) which is the boundary between the real world and the philosophical or grammatical ego which is not part of the real world.[79] The borderline discussed here is between two parts of the real world, namely between the domain of the micro-objects and the domain of

the micro-objects and the domain of macroscopic external equipment. In conventional quantum mechanics the borderline between these two parts becomes, however, at the same time the borderline between the domains of two different philosophical or epistemological attitudes. In the observing part the language of philosophical realism is spoken whereas in the observed part this language is prohibited by the edict of Copenhagen. Schrödinger's cat is a macroscopic object and falls into the domain of realism. One is allowed to say that the cat is either dead or alive even when one did not yet look into the box. The radioactive nucleus, on the other hand, is a microscopic object and falls outside the domain of realism. The chain (6.4) then begins (at the right end) with a microscopic but ends with a macroscopic object, and somewhere the borderline into the domain of realism must be crossed. As both the microscopic and the macroscopic objects are described by wave functions, the interpretation of the ψ-function must change radically when that crossing is done. This is the real difficulty.

The refusal of realism sometimes is driven so far as to assert that the ψ-function only represents one's knowledge (whatever this may be physically). There are evidently cases within the framework of the usual particle interpretation where this cannot be maintained, for example in the double-hole experiment discussed in the introduction. There, one cannot say: "the particle has gone either through hole 1 or through hole 2, only we do not know through which."

VIII. MACROSCOPIC RECORD

Before we enter the more detailed discussion of our own conception
of measurements we want to emphasize one general point which plays an
essential role: the outcome of any measurement must have a macroscopic
fixation, representation, or record. Conversely, the measurement is
completed when a macroscopic record has been achieved. The macroscopic
record can be achieved in a variety of ways. Suppose we want to measure
the path of an electron. When the electron (wave packet) moves through a
photographic emulsion it induces a series of events that eventually lead
to a track of blackened grains. These grains are the macroscopic record
of the induced events from which the path of the wave packet is deducted.
Or consider a Wilson cloud chamber where, after a particle has gone by,
a track of drops appears and then after a few seconds disappears. After
the disappearance of the track no macroscopic record seems to remain.
However, we may take a photograph of the track while it is visible. Then
the picture of the track on the photograph (the respective grains) is the
macroscopic record of the original track events. The record by means of
the grains of this photograph differs from the record by means of the
grains of the above photographic emulsion where the particle (wave packet)
has directly moved through. Of course, there may be many records of one
and the same event. Or we may simply look at the track in the Wilson
chamber seeing it appearing and disappearing; this, then, is also a
macroscopic record, namely some structure built up by the atoms which
form the memory part of our brain.

Let us have a closer look at the physical realization of the macro-
scopic record of a measured event. This record can be called a memory
of the event, and therefore the physical object acting as this memory

must have a certain degree of stability; it must not change with time
to any great extent, even when subjected to disturbances from outside.
Certainly microscopic objects (like electron wave packets) do not meet
this requirement. The memory therefore must be a macroscopic object,
and more specifically, a stable macroscopic object. Whether a macro-
scopic object is stable or not depends upon the environment in which it
is placed. Under favourable circumstances a body of 10^{-12} g, consisting
of 6×10^{11} nucleons (Millikan drop), may be considered a stable macro-
scopic object. However, when this body is immerged into a fluid the
situation is less favourable since the body is then subjected to irregu-
lar Brownian motion. Or consider a book in a library: one would call this
a stable macroscopic object, well suited to fulfill the requirements of
a memory. However, if it were on the sun, it would soon be burned up.
(By the way, this will inevitably occur to all libraries on earth when
the sun becomes a red giant star, in about 10^9 years.) These remarks
should make it clear that the choice of a stable macroscopic object as
a record of an event is arbitrary to some extent and depends on the scale
considered. The basic measure of this scale for us is given by the struc-
ture of the human sense organs and the brain. This arbitrariness does
not, however, present any difficulty. By the very fact that we speak of
an event the event has a macroscopic record, i.e. is engraved in a memory.

Of course, the macroscopic record of one result (event) must be dif-
ferent from that of a different result (event), and the two macroscopic
records must be different by macroscopic standards. We then say they
are macroscopically different. Think of two well distinguished positions
of a pointer or two different figures on a digital display of an ammeter.

One may also speak of different macroscopic states of a macroscopic device. More specifically, the macroscopic body which serves as the macroscopic record of an event is conceived to be an aggregate which we shall explain in the next section.

Our requirement of a macroscopic record of the outcome of a measurement may bring to mind the feature of conventional quantum mechanics requiring the existence of external equipment and observers in order to preserve a realm of realism. This comparison is superficial and misleading. Our microscopic wave packets with all their properties are just as real as any macroscopic body, and there is no change in the interpretation of the ψ function when we go from microscopic to macroscopic objects.

IX. AGGREGATES

We do not interpret a macroscopic body to be a single wave packet. As a more realistic model of a macroscopic body we conceive it to be an aggregate. By this we mean a set of independent wave packets which incessantly undergo interactions with one another. For this conception we need our basic interactions leading to completely independent wave packets when the interaction is over. This cannot be achieved by interactions described by a Schrödinger equation as discussed in Section VI on the measurement problem in traditional quantum mechanics. An aggregate is also conceptually different from a mixture since a mixture would again mean that any individual macroscopic body is described by one wave function (of many variables), it is only the investigator that does not know exactly which one out of a given set it is. The concept of an aggregate is essential to solving the problems of measurement (Sections X, XI), of screening in atoms (Section XVII) and of spreading.[82]

Recall the spreading of the wave packets as considered in Section II.
It can be easily calculated in the non-relativistic limit from formula
(2.12) that a macroscopic body with the dimensions of Millikan's drops
(radius 5×10^{-5} cm, mass 4.7×10^{-13} gr) when described by a Gaussian wave
packet spreads out to double its initial extension in 23 days! Einstein[86]
has regarded situations like this and their interpretation in quantum
mechanics as an unacceptable feature. This spreading is hindered in our
conception of a macrosocpic body by the permanently occurring interactions
of the constituent wave packets. It is understood that these interactions
also lead to some stationary size of the wave packets. This view agrees
with the fact that stationary wave packets exist when considered in some
external potential, such as the harmonic oscillator and the Coulomb
potential. As explained in Section III, any calculation with the help
of a potential is only a phenomenological method to account for a series
of mild events, i.e. basic interactions between wave packets.

The number of wave packets that constitute a macroscopic body must
be large. A macroscopic body is an aggregate, but not every aggregate
is a macroscopic body. There are also stationary aggregates which con-
sist of only a few interacting wave packets. Among these we count the
molecules, the atoms, the nuclei and any system with a rest mass that is
smaller than the sum of the rest masses of its constituents. With this
last condition we want to exclude the elementary-particle resonances;
and we also want to exclude the view that all the elementary particles
are bound states of a number of quarks and antiquarks. Thus, our wave
packets are only meant to describe the elementary particles as listed in

the Rosenfeld tables.[87] Consider, for instance, the proton-antiproton
system $p\bar{p}$. There are discrete states with energies both above and below
the sum of the rest energies of the constituents, $2m_pc^2$.[88] The lower
states are to be considered as aggregates whereas the upper states are
not.

Evidently, the concept of an aggregate as used to describe stable
bound states and macroscopic bodies leads to another restriction of the
principle of superposition, in addition to the superselection-rule re-
striction. In fact, it is a rather drastic restriction which means that
not every system can be described by one pure coherent state vector (wave
packet). Only the elementary particles mentioned above can be so described.
For example, a freely moving atom cannot be described by a free wave
packet; a free wave packet will spread out, but an atom will not; its
size in the ground state will always be determined by the Bohr radius.
In conventional quantum mechanics both an electron and an atom may be
described by free wave packets. This procedure is in accordance with
the idea that the wave function governs the behavior of some particle,
and it does not seem to make much difference whether this particle is an
electron, an atom, or even a macroscopic body. The situation is different
when the wave packets are not to direct particles but are themselves the
fundamental constituents. Our picture therefore suggests that the whole
formalism of Hilbert-space vectors and interfering wave functions only
applies to elementary particles. Hence, nuclei, atoms, molecules and
macroscopic bodies, strictly speaking, are not the proper objects of
quantum mechanics. Quantum mechanics applies to these objects only inso-
far as they are built up of elementary wave packets and it is these
elementary wave packets that are the proper objects of quantum mechanics.

One must, however, be prepared that coalescence may occur in aggregates that consist of identical wave packets. We mean the explanation of superfluidity of He^4-atoms by means of condensation of bose particles into one coherent state. One might have expected that only coalescence of elementary identical wave packets is an easy process, not of aggregates which themselves represent systems of interacting elementary wave packets. One must conclude that under certain circumstances aggregates are also capable of building up large-scale coherent states. The specific aggregate of identical wave packets may then be something in between a set of completely independent wave packets and a single coalesced packet. The interactions occurring between the identical constituents establish momentary correlations (coalescence) between them. Since extent and frequency of occurrence of these correlations may vary to a large extent, a continuous variety of intermediate aggregates between the above mentioned limiting cases are conceivable. We note that there must be quite specific circumstances in order for superfluidity or superconductivity to occur, and even in the current theories it is by no means a simple matter to account quantitatively for the observed effects.

There are, as far as I can see, only two physical situations where a macroscopic body is actually described by one coherent state vector when the intrinsic properties of this body are being calculated: in quantum statistical mechanics[89] and in the measurement process of traditional quantum mechanics. In quantum statistical mechanics, the fact that the system is in one pure state (though as a member of an ensemble) does not, however, influence any observable result since interference effects between parts that in any sense may be ascribed to single constituents do not influence any observable result. One might regard the bose

or fermi degeneracy as an effect of (anti)symmetrization of the one total wave function of the system. We think, however, that we have given a satisfactory physical explanation in terms of there being many wave packets,in Section V. Notice also that the wave packets that constitute the macroscopic rigid body appear in the actual calculations only phenomenologically by determining the form of some external potential used in the Schrödinger equation for calculating the behavior of some extra wave packet. Cf. the calculations of energy bands: an electron moves in a periodic potential.

The discussions of the measurement process in conventional quantum mechanics are phony; no experimentalist can learn anything from them for the construction of his or her measurement apparatus. In our conception of the measurement process we do not describe the measurement apparatus by one pure state vector or wave function. Instead, in our conception the measurement apparatus is a macroscopic aggregate, and the macroscopic states mentioned in the preceding section comprise a great variety of arrangements and mutual interactions of the individual constituent wave packets. These macroscopic states can then be compared with the macrostates in the kinetic theory of gases which are characterized by macroscopic parameters such as pressure, temperature and volume, and which are regarded as comprising a great variety of micro-states characterized by the positions and momenta of the individual constituent molecules.

X. MEASUREMENTS OF POSITION AND MOMENTUM

Although no observer is needed to define our theory measurements
are needed to test it. In our conception there is not one single type
of measurement process, rather there are different types which require
special treatment each. As representatives for the basic types we treat
in this section the measurement of the position of an event and the
measurement of the average momentum of a wave packet, and in the next
section the operation of a Stern-Gerlach-type apparatus which can be
used for a spin-component measurement.

We begin with a measurement of the position of an event. For
definiteness we consider a wave packet that moves in y-direction towards
a thin layer of photographic emulsion and penetrates it. The layer is in
the x-z plane. We want to measure the point (x,z) where the wave packet
has induced an event. Notice that it is not the "position" of the wave
packet that we are talking about. The position of the event gives only
some probability that the center of the packet has moved through the
region of the event, as has already been explained in Section III. One
can determine the center of the packet if one can determine its shape
and that can only be done approximately, by measuring the position of
many events under the assumption that the shape is not altered signifi-
cantly by the interaction events. Only in the case where the width of
the packet is smaller than the (physical, i.e. extended) point (x,z) can
one say with a probability of 1 that the whole packet has passed through
this point. The wave packet undergoes an interaction with one of the

emulsion packets and then leaves the emulsion. After that it is completely separated from every emulsion packet, since every interaction in our concept is effected by the basic interactions which lead to complete separation.

The wave packet is modified to some extent by the interaction with an emulsion packet. This modification may be mild or strong. When one considers an electron packet moving through the emulsion and leaving a track of blackened spots behind, the modification, evidently, must be relatively mild. It might even be neglected completely in special situations or considerations. On the other hand, one may consider an incoming positron wave packet which then is annihilated. This, of course, means a strong modification.

According to what has been said in the preceding section the emulsion layer is a macroscopic aggregate. After the wave packet has passed by, the layer is developed and a black spot appears at the interaction point. This particular state of the emulsion layer is a macroscopic state in the thermodynamic sense of Section IX. It is macroscopically different from a state describing an emulsion layer with a black spot at a different position or one with no black spot at all. We need quite specific macroscopic bodies which are in some meta-stable state. Then the incoming wave packet can act like a snow ball triggering an avalanche—a general feature of macroscopic apparatuses for measurements of events induced by micro-objects.[90]

The details of the avalanching process and the microscopic details of the final macroscopic state of the emulsion layer are unimportant; what counts is the resulting black spot at the position (x,z). Of course, the problem of characterizing those macroscopic systems which are capable of meta-stable states and transitions to other states remains. This, we think, is not a proper problem of quantum theory but rather of statistical mechanics, though quantum theory might contribute. We note that in the attempts of Prigogine and his school to solve this problem the quantum superposition principle has also been found not to be applicable to macroscopic states.[91]

The measurement of the position of an event is a basic element in the measurement of every physical quantity. Generally the measurement of any quantity can be transposed into a (series of) position measurement(s); pointers can be made to answer every question. This is done by using physical laws according to which different values of the quantity in question lead to different space-time positions of the considered object and hence to different positions of the observed events. From any description, e.g.[92], of how real measurements in the realm of elementary particles are performed one sees, for example, that ample use is made of conservation laws.

We consider the measurement of the momentum of a particle (wave packet) by deflection in a constant magnetic field. According to what has been said in Section III, the deflection is interpreted to be the result of a series of mild interactions with the wave packets that in some way constitute the magnetic field. We notice that the device is in no way, in principle, different from one that would be used for

measuring the momentum of a macroscopic charged body. A physical law (Newton's law with Lorentz force) connects momentum with the curvature of the trajectory, and from measuring the curvature by means of position measurements the value of the momentum is derived. Only in the details of the position measurements do quantum and classical mechanics differ from each other. The Ehrenfest theorems then tell us that what is measured here is the average of the absolute value of the wave packet's momentum.

We do not assume that the production of the black spots in a photographic emulsion or of the bubbles in a bubble chamber has a fundamental influence on the wave packet in the sense that every bubble would mean a particle-position measurement, as in conventional quantum mechanics, and hence would reduce the packet in size to at least the size of the bubble. These interactions may or may not have an effect of this kind but this does not play any role in our argument.

XI. SPIN OPERATIONS

Finally we consider the passage of a spin-$\hbar/2$ atom through a Stern-Gerlach magnet. It is this situation, we think, that has kept v. Neumann's reduction postulate alive because it seems most difficult to dispense with it here. Nevertheless, we do dispense with that postulate, but we shall also show that this can only be done with the help of the basic interactions (discussed in Sections III and VII) which go beyond the framework of traditional quantum mechanics.

We follow the description given in the Feynmann lectures because it exhibits the essential points in a simple way. Actually, the described operations of a Stern-Gerlach magnet on an atom are not equally valid

for an electron.[93] We do not think that this difference between an atom and an electron is unimportant in every respect; after all, the atom is an aggregate of wave packets whereas the electron is a single packet. We think, however, that with respect to the fundamental considerations to follow, this difference is not important. We could replace the atoms everywhere in the text by electrons or protons if at the same time we replaced the Stern-Gerlach magnet by some more complicated (scattering) device as described, e.g., by Mott and Massey[93] and with experimental details in[94]. Let the magnetic field and its gradient both be in the z-direction, and let the atom enter the Stern-Gerlach magnet in the y-direction. Theory says that if the atom is deflected upward the z-component of its spin is $+\hbar/2$ and that if it is deflected downward the z-component is $-\hbar/2$. In general the single atom goes both up and down because its wave function can be written as a coherent superposition of a spin-up and a spin-down wave function

$$\psi = a_1 \psi^{up} + a_2 \psi^{down} \quad , \qquad (11.1)$$

the coefficients a_1 and a_2 being determined by the initial spin direction of the atom.[95] It is only in the case where either a_1 or a_2 is zero, i.e. when the initial spin was either down or up, that the atom takes only one way, either the lower or the upper one. In the general case the atom is pushed into a definite path only by a further position measurement. This has also the following consequence. Let an atom with spin in x-direction enter the Stern-Gerlach magnet. It then covers the upper and lower path with equal probabilities. Put additional magnets behind the first one so that in the end the different paths are being

brought together again, as described in the Feynman lectures or by
Bohm.[74] Ordinary quantum mechanics says that if no position measure-
ment is done the atom still has its spin in x-direction when it leaves
the whole arrangement. So, only when the atom is observed in the upper
path, it suddenly retires from the lower path and becomes a pure upper-
path atom, and its wave function becomes an eigenfunction of the operator
of the spin component in z-direction. This is v. Neumann's postulate
in this particular situation.

In our view observations do not have this special effect. In
accordance with the attitude of realism, observations observe what
already is, notwithstanding the fact that there might only be a finite
accuracy obtainable and that there might be some influence on the
measured reality. The fact that a wave packet has induced an event
which is localized in the spin-up path of some Stern-Gerlach-type appa-
ratus in our view does not imply that the wave packet becomes strongly
concentrated around the location of that event, as we already explained
in Section III. Nevertheless, we do assume that the wave packet (atom)
is pushed into a definite eigenstate of the operator of the spin compo-
nent. However, this is effected by normal physical interactions which
act independently of whether there is a position measurement or not.
These interactions are the basic interactions of our considered wave
packet with the wave packets that in some way constitute the magnetic
field of the Stern-Gerlach magnet. Here, also our concept of a macro-
scopic apparatus as an aggregate enters. As we already explained in
Section III, VII and IX, the basic interactions cannot be described by
a Schrödinger equation. Only their combined action is described by a

Schrödinger equation with external potential. The basic interactions
lead to complete separation of the outgoing wave packets from each other,
and here this leads to complete separation of the outgoing wave packet
from the packets of the magnetic field, i.e. the Stern-Gerlach apparatus.
It is interesting to note that it was in the same year (1922),when Stern
and Gerlach published their results,that Einstein and Ehrenfest revealed
serious difficulties in explaining how the atoms take up their orienta-
tions in a Stern-Gerlach magnet, when only those physical (radiation)
processes that are usually considered to be responsible for this are
taken into account.[96] The basic interactions have already acted when the
measurement of the path by means of a position measurement is performed.
Thus, an atom that enters the above described combination of normal and
reversed Stern-Gerlach magnets in our view no longer leaves the whole
arrangement with its spin in x-direction, but rather in either +z or
-z direction. This might in principle, though perhaps not in practice,[74]
be decided by experiment. Actually, that particular assertion of quan-
tum mechanics has never been verified experimentally. By the way, our
assumption is sloppily made in many articles on quantum mechanics. The
priests of orthodox quantum mechanics can, however, only raise their
hands and warn us that we are in contrast to quantum mechanics, but they
cannot put their hands on facts that would be in contrast to our assump-
tion.

It is true that the whole process is no longer time-reversal
invariant: in traditional quantum mechanics the atom enters and leaves
the arrangement with spin in x direction, and it even does so if one
lets it go through the arrangement in reversed direction. In our

conception the atom enters the arrangement with spin in x direction and leaves it with spin in ± z direction. If we let the outgoing atom with +z spin go through the arrangement in reversed direction it does, however, not come out with spin in the initial x direction but in +z direction. This confirms the consistency of our view where the whole process is not one interaction but rather a result of many basic interactions with wave packets that are to some degree distributed randomly. With that assumption all further implications of our conception are the same as those of conventional quantum mechanics with a position measure-having been performed. The difference is that we assume these implications to hold even when the electron has left the Stern-Gerlach type apparatus without a position measurement having been performed.

It also fits with this view that the atom must spend some minimum time in the inhomogenous magnetic field so that the apparatus can accomplish the task for which it was constructed.[97] Also, the Stern-Gerlach magnet is affected by the atom. The energy of the atom is changed by $\pm \vec{\mu}\vec{H}$, where $\vec{\mu}$ is the atom's magnetic moment and $\vec{H}$ is the magnetic field in the apparatus. This energy must be exchanged with the magnet. The magnet acts as a polarizer, and even a transfer of angular momentum between the atom and the apparatus can be measured. This transfer has been demonstrated in the analogous case of polarization experiments with photon wave packets.[98]

We thus dispense with v. Neumann's reduction postulate, but we do not dispense with the reduction process altogether. The reduction process, however, is a physical process; it is the apparatus, not the observation, that forces the wave packet to become a particular eigenfunction of the corresponding operator. Also, the reduction process

does not occur in a proper measurement. One can of course expand any wave function into a series of eigenfunctions of some Hermitian operator, this is pure mathematics. However, whether the eigenfunctions in the expansion acquire a physical meaning, i.e. whether the system actually is capable of developing into one of the eigenstates depends on the particular physical situation. In conceiving the appropriate physical situations in quantum mechanics ample use is made of classical analogues. Accordingly, we assume that the Stern-Gerlach apparatus indeed gives the described physical meaning to the expansion (11.1). The eigenstates of the spin-component operator are regarded as stationary states of the atom in the magnetic field. It is perhaps worth remembering here the general fact that if both $\psi^*\psi$ and the current $\psi^*\partial\psi/\partial t-\psi\partial\psi^*/\partial t$ are independent of time, the Schrödinger equation is equivalent to an eigenvalue equation (Ref. 34 p. 40/41). This fact suggests that a large class of eigenvalue problems are to be regarded as being similar to the described spin operation, and this in turn is like the formation of standing sound waves. The latter, in principle, is derivable from the intermolecular interaction laws. Consider, for example, the process in which a hydrogen atom is formed by the capture of an electron in the Coulomb field of a proton. The electron will settle down in one definite state out of the set of discrete eigen-states offered by the Schrödinger equation. The process of the settling down of the electron as well as its resulting stationary state are due to basic interactions between the electron and proton wave packets. The settling down is accompanied by the emission of photons which is clearly beyond the reach of the Schrödinger equation with its conservation of particle number. Cf. the assumption of random phases between the different

states of an atom when radiative transitions are treated.[99] This assumption is equivalent to describing the atom by a mixture, i.e. as really being in some definite state though one does not exactly know in which state it is.[23] Indeed, the whole concept of transition amplitudes between stationary states is based on this view. The concept of transition amplitudes has been extended to include transitions between scattering states, this is the S-matrix concept. In our view this extension means something different, namely that there are definite and independent states of the individual wave packets both before and after the interaction, whether one measures them or not. This is one of the places, alluded to in the introduction, where physicists have tacitly abandoned the concepts of orthodox quantum mechanics. And this is just the feature we ascribe to our basic interactions.

The discrete eigenvalues of operators then are no more genuine quantum effects than the discrete frequency values of sound waves. The real quantum effects are represented by the quantization of action in units of h as discussed in Section V.

We may tentatively assume that those apparatuses that can be associated with operators with a discrete eigenvalue spectrum can force the wave packet into an eigenstate. This is the general operation of such an operator. However, we do not say that every apparatus whose operator has a discrete spectrum forces the wave packet into an eigenstate. There are charge measurements, for example, and we may associate with them a charge operator, but we would regard this as a purely formal operation.

The operation of the Stern-Gerlach magnet in the described way is often called a measurement of a spin component.[100] We have carefully avoided doing so. Sure, a position measurement on the atom when it leaves

the apparatus measures its spin component, but this in general is not the
original spin component of the atom before it entered the apparatus.
Calling the above operation a measurement would be similar to calling
the process where a ball runs down a Galton pin board a measurement of
the ball's initial position. We suspect that the reason why the above
operation has been called a measurement is that this situation calls so
much for a "reduction". Since Schrödinger's equation, on closer inspec-
tion, is not able to give this reduction it was only v. Neumann's axiom
of measurement that provided it. In our view v. Neumann's postulate is
a misleading attempt to force the description of essentially different
situations into one single scheme.

A proper measurement of the spin direction (in the quantum mechanical
sense) of the incoming atom can indeed be brought about with a Stern-
Gerlach magnet, but not by the operation described so far. A proper spin
measurement cannot be made on a single atom, rather a large number of
equal atoms, i.e. an ensemble must be given. Then the direction of the
spin-reference axis of the Stern-Gerlach apparatus must be varied until
a position is reached in which all incoming atoms are deflected, say, up-
ward. This direction then is the spin direction of the incoming atoms.
We note that the necessary final position measurement influences the wave
function of the atom, but this influence is not longer relevant.

Also, the Stern-Gerlach apparatus can be used for what is called a
preparation. If, say, the lower of the two possible paths is blocked
off, the apparatus prepares spin-up atoms irrespective of the spin orien-
tation of the incoming atoms (except in the degenerate case where the
incoming atoms all have spin down no atom would leave the apparatus.)

Notice that in such a preparation the path of the atom actually is no longer measured by position measurements. Rather, earlier measurements have shown that the apparatus works in the intended way. This preparation is sometimes called a filtering.[101] This is a misleading denomination from our point of view since it suggests that the measuring apparatus would do nothing more than let pass atoms having certain properties they already had before entering the apparatus.

XII. THE EINSTEIN-PODOLSKY-ROSEN (EPR) PROBLEM

We now want to discuss the Einstein-Podolsky-Rosen (EPR) problem, or paradox, as it is sometimes called. This will add new features to the general conceptual framework. In their paper "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?"[102] EPR want to demonstrate that the question has to be denied. Reality is defined by the following (sufficient) criterion: "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity". For a physical theory to be complete it is necessary that "every element of the physical reality must have a counterpart in the physical theory." In order to prove their proposition they consider two systems, I and II (imagine two particles), which have interacted from time $t=0$ to $t=T$, after which time there is no longer any interaction between the two systems. Let the (calculable) wave function of the combined system I + II after the time T be $\Psi(x_1, x_2)$. x_1 stands for the variables used to describe the first system and x_2 for the second system. However, the state in which either of the two subsystems I or II is left after the interaction cannot be

calculated. According to v. Neumann's postulate of conventional quantum mechanics,[69] that state can only be obtained with the help of a further measurement in which the reduction of the wave packet must take place: let μ_1, μ_2, μ_3 ... be the eigenvalues of some physical quantity M pertaining to system I and $u_{\mu_1}(x_1)$, $u_{\mu_2}(x_1)$, $u_{\mu_3}(x_1)$ ... the corresponding orthonormal eigenfunctions. Then $\Psi(x_1, x_2)$ can be expanded into a series of these eigenfunctions with coefficients that are functions of x_2

$$\Psi(x_1, x_2) = \sum_k \zeta_{\mu_k}(x_2)\, u_{\mu_k}(x_1) \quad . \tag{12.1}$$

From (12.1) it follows that the system I + II is in the definite state $\Psi(x_1, x_2)$ but there is no longer any independent state of system I or of system II although the two have become correlated in a one-to-one manner. The situation is quite the same as that considered in the measurement problem in Section VI. Indeed the sum (12.1) is only a different notation of the sum (6.2). The coefficient functions $\zeta_{\mu_k}(x_2)$ in general are neither normalized nor orthogonal to each other. Suppose that the quantity M is measured on system I and that the value μ' is found. It is then concluded that, (immediately) after the measurement, the first system is left in the state with wave function $u_{\mu'}(x_1)$, and from (12.1) it then follows that the second system is left in the state with the wave function given—apart from normalization—by $\zeta_{\mu'}(x_2)$. This is the reduction of the wave packet: the wave packet given by the sum (12.1) is reduced to a single term $\zeta_{\mu'}(x_2)\, u_{\mu'}(x_1)$. The set of functions $u_\mu(x_1)$ is determined by the choice of the physical quantity M. If another quantity, say S, is chosen, with eigenvalues σ_1, σ_2, σ_3 ... and orthonormal eigenfunctions $v_{\sigma_1}(x_1)$, $v_{\sigma_2}(x_1)$, $v_{\sigma_3}(x_1)$ ... instead of (12.1) another expansion is obtained

$$\Psi(x_1,x_2) = \sum_k \eta_{\sigma_k}(x_2) \, v_{\sigma_k}(x_1) \tag{12.2}$$

where the $\eta_{\sigma_k}(x_2)$ are the new coefficient functions. If now the quantity

S is measured on system I and the value σ' is found, then system I is left

in the state $v_{\sigma'}(x_1)$ and system II in the state given—apart from normali-

zation—by $\eta_{\sigma'}(x_2)$.

Therefore, as a consequence of two different measurements performed

upon the first system, the second system may be left in states with two

different wave functions (of course different by more than a proportion-

ality factor). On the other hand, at the time of measurement, the two

systems no longer interact, i.e. by definition no real change can take

place in the second system as a result of anything that may be done to

the first system. Thus it is possible to assign two different wave func-

tions to the same reality—system II after the interaction—a rather para-

doxical situation.

The paradox is sharpened in an example in which the two sets of coef-

ficient functions $\zeta_{\mu_k}(x_2)$ and $\eta_{\sigma_k}(x_2)$ are eigenfunctions of two non-commuting

operators corresponding, say, to the physical quantities of momentum P and

position Q, respectively. Since in our view no sharp position does exist

this EPR example using Q (and P) is not convincing at all. We may, how-

ever, replace Q and P by two different spin component operators; then

the EPR conclusions remain valid. EPR then show that by measuring either

the momentum or the coordinate of the first system, one is in a position

to predict with certainty, and without in any way disturbing the second

system, either the value of the momentum or the value of the position

of the second system. According to the above criterion of reality, both

the momentum and the position of system II are elements of physical reality.
Therefore, the values of both of them must enter into the complete descrip-
tion of reality. On the other hand, in conventional quantum mechanics
no wave function can contain both an eigenvalue of some operator M and
an eigenvalue of an operator N which does not commute with M. Therefore
EPR conclude, that the quantum-mechanical description of reality by the
wave function is not complete. - As EPR remark, one would not arrive at
this conclusion if one regarded the given criterion of reality as not
sufficiently restrictive. One might insist "that two or more physical
quantities can be regarded as simultaneous elements of reality only when
they can be simultaneously measured or predicted. On this point of view,
since either one or the other, but not both simultaneously, of the quanti-
ties P and Q can be predicted, they are not simultaneously real. This
makes the reality of P and Q depend upon the process of measurement
carried out on the first system, which does not disturb the second system
in any way. No reasonable definition of reality could be expected to
permit this." See our example of a solvable and burnable sugar cube in
Section XVIII.

XIII. BOHR'S VIEW ON THE EPR PROBLEM

The EPR paper was immediately answered by Bohr,[103] Schrödinger[77]
and Furry.[104] Schrödinger was on Einstein's side, stressing the con-
ceptual inconsistencies of quantum mechanics, whereas Bohr and Furry
rejected the conclusion of EPR. The arguments of Bohr and those of
Furry are of quite different natures. In this section we sketch Bohr's
view, and in the next section we present the arguments of Furry, which
are those of orthodox quantum mechanics.

The essential point of Bohr's argument on the EPR paper, we think,
lies in his statement[105] that "no sharp separation can be made between an
independent behaviour of the objects and their interaction with the meas-
uring instruments which define the reference frame." In more detail this,
we think, can be expressed in the following way. There are properties
of particles that can only be defined relative to some coordinate system,
for example position and momentum. Only such properties are considered.
The coordinate system must be the same for all particles, and it has to
be defined experimentally with the help of some rigid body. Now a
measurement on particle I implies an interaction between it and this
body, possibly mediated by some instrument, and this will influence the
body to some degree. That this influence, according to Bohr, is not
completely controllable is the decisive point. This influence will then
implicitly enter into the future measurements on particle II. In fact,
it will even enter into the definition of the properties of particle II
since these properties are only defined relative to the coordinate system,
i.e. the rigid body. Hence, due to these uncontrollable modifications
of the common coordinate system there are—in the last resort— no two
particles or systems that are completely independent of each other even
if they are well separated in space. Thus, Bohr arrives at the same
conclusion as Furry and the other proponents of orthodox quantum mechanics
as will be seen in the next section. However, the nonseparability of
Bohr is not due to the superposition nature or coherence of the state
of the two particles after the interaction, but is, in a way, more
fundamental. In fact, Bohr's argument, when driven to its extreme, would
mean, for example, that the very concept of a spatial coordinate,

ascribed to any particular system, must be abandoned. However, even if there is a fundamental uncertainty on the observational level this does not necessarily imply an uncertainty on the level of the theoretical description. Although our wave packets and their Compton wavelengths set an upper limit on the accuracy of any physically possible localization, the wave packets themselves nevertheless are conceived to be given as a function of the space coordinates. In this way an accuracy is introduced that goes beyond the possibilities of direct experimental verification. This is done in order to be able to define and control the uncertainties on the observational level. It is analogous to the procedure in quantum mechanics where $\psi(\vec{x})$ is used, and to relativistic quantum field theory when the field operators are written as functions of the space coordinates and yet do not allow an exact localization of the field quanta.

Another point which we regard of fundamental importance is this: Bohr's consideration, in principle, applies to micro- as well as to macro-objects. The difference is that in the case of macro-objects the disturbances due to a measurement cause only a relatively small change in the considered properties, and one is contented with the obtainable accuracy. The relatively small change in turn is due to the fact that for measuring the properties of macro-objects, for example their coordinates, one mostly uses micro-objects, namely photons, when one looks at the objects. When measuring micro-objects one can only use micro-objects again, and the disturbances will be relatively large though absolutely they are of the same order as for the macro-objects. Imagine a world whose basic constituents are billiard balls. One wants to measure the path of a specific billiard ball. For this purpose one can only let the

ball collide with other balls and observe the paths of these other balls. However, one cannot measure their paths, say, by taking a film of the whole process and then measuring the tracks on the film. The film itself has to consist of billiard balls as well as the film camera, one's eyes etc. This drastic example shows that certain difficulties in measuring micro-objects arise already from the fact that all measuring apparatuses and the investigators themselves are composed of micro-objects. This, we think, may form the root for Bohr's assumption that the influence of a micro-object on the measuring instrument is not completely controllable. Is is good, however, to conceptually separate this kind of difficulty from those that arise from the specific nature of the micro-objects themselves.

XIV. REPLY OF QUANTUM MECHANICS TO EPR

The adherents of orthodox quantum mechanics point out that the considerations of EPR only demonstrate that two systems cannot be considered to be independent even when there are no longer any dynamical interactions between them. We have here a new kind of correlation between systems that have formerly interacted. Before the appearance of the EPR paper this correlation had not fully been noticed. Nowadays it is also called non-locality, quantum interconnectedness[106] or quantum nonseparability.[107] Since EPR do assume that the systems are completely independent of one another they are, on this point, not in accordance with the rules of quantum mechanics. Hence they cannot maintain that quantum mechanics gives only an incomplete description of reality.

The orthodox conception is that the two systems are not completely separated until a measurement has been performed. Here again the ghost of v. Neumann's postulate lurks. Since the time of this subsequent measurement is at the observer's disposal he or she may perform it an arbitrarily long time after the interaction, so that the wave functions

of the two systems can be taken to be well separated from one another. This non-separability would then seem to be a rather peculiar feature of conventional quantum mechanics.

Therefore, now another question replaces the original one: is quantum non-separability a feature of reality? This question can, in principle, be answered by experiment. The experimental tests are mostly based on physical situations such as those proposed by Bohm and Aharonov.[109] Imagine, for instance, the scattering of two protons through an intermediate state of zero angular momentum. When the protons are far apart from each other we may force, by devices acting like Stern-Gerlach magnets, the spin component of proton I into a given direction $\vec{a}$ ("measurement of spin component" as it is often wrongly called). The result can only be either $+\hbar/2$ or $-\hbar/2$. Let the result be $+\hbar/2$. Then, according to quantum mechanics, a Stern-Gerlach type apparatus with spin-reference axis $\vec{a}$ operating on proton II must give the result $-\hbar/2$. Thus we can know with certainty the spin component of proton II by previously, and at a very distant place, knowing the same spin component of proton I.

In order to contrast the separability hypothesis (ascribed to EPR), to quantum mechanics, a definite mathematical formulation must be given to this hypothesis. This can be done in several ways. We here discuss a representative formulation in which it is assumed[104] that either of the two systems in the interaction made a transition to a definite state in which it then is. In the proton-proton S-scattering case this then means that in any individual case the spin of each proton is definite in some direction while that of the other proton is opposite. In order to retain spherical symmetry in the statistical sense it is further supposed that in a large sample of similar cases there is the same

probability for any direction of the spin axis. Bohm and Aharonov[104]
write: "It is true that in any single case, the total angular momentum
will not be conserved... . However, thus far, there has not been given
an experimental demonstration of the detailed conservation of every
component of the angular momentum, for particles that are far apart and
not interacting. On the other hand, ... the uniform probability of all
directions will lead to the experimentally observed fact of conservation
on the average." It is clear that the separability hypothesis will give
predictions different from those of quantum mechanics when correlations
between the spin components of the two protons from the same scattering
interaction are considered.

In fact, such experiments have been performed, as well as analogous
experiments with photons. Though some discrepancies appear to remain in
the experimental results, on the whole, and especially so in recent
times, they confirm quantum non-separability.[94,110-112] So, we shall
base our argumentation on the assumption that in the experimentally ex-
amined situations quantum non-separability is a fact, although we shall
organize the facts using different concepts.

XV. NON-SEPARABILITY AND RELATIVISTIC CAUSALITY

Before we go on to explain our view on the observed quantum non-
separability let us analyse more closely one specific aspect of the
situation. Let us again consider proton-proton scattering through a $J=0$
intermediate state. In some distance from the place 0 of the interaction
there is an observer A, and in the opposite direction from 0 there is an
observer B. Both the distance OA and OB are as large as one wishes but
the distance OA is somewhat shorter than OB. The whole situation is

regarded in the center-of-momentum system. After the interaction, both
A and B operate with their respective Stern-Gerlach-type apparatus on
their respective proton, thereby pushing it either into the upper or into
the lower path, i.e. effecting it to become either a $+\hbar/2$ or a $-\hbar/2$
proton with respect to the spin-reference axis of the apparatus. Let
A have the reference axis of his apparatus in the direction $\vec{a}$. When
his operation is completed let B operate on the spin component of his
proton with respect to the same direction $\vec{a}$. If A has effected spin up
then B will effect spin down, because the intermediate state had zero
angular momentum, and angular momentum is conserved. One can always, in
principle, arrange the time and place of the operation of the two inde-
pendent spin apparatuses so that they are space-like to one another.
That would mean that the first operation could influence the result of
the second operation which takes place so early after the first one that
no light signal could have connected the two, and that there is some kind
of action-at-distance.

Does that mean that relativistic causality is violated, i.e. that
signals can be transmitted with overlight velocity? We want to demon-
strate that this is not so: one has to take the random nature of the
quantum effects into account. The point is that B cannot be sure that
his result is caused by A's result; it might as well be a random result.
Conversely, A cannot cause with certainty that B effects spin down since
A cannot cause with certainty a spin-up effect. The spin-up and spin-
down results of A occur randomly, the respective probabilities being
1/2.

One may try to compensate for the stochastic character of the single event by using a large number of the described scattering arrangements simultaneously. B may measure a large number of spin-up to spin-down ratios, and so with respect to every direction. Nevertheless, the statistics of B will always be the same no matter what A does with his proton. With any direction $\vec{a}$ of A's apparatus A produces an unpolarized ensemble ("beam") of protons, and an ensemble of protons consisting of an equal number of spin-up and spin-down protons with respect to $\vec{a}$ is equivalent, with regard to polarization (not with regard to any conceivable measurement) to an ensemble of unpolarized protons made up in any other way.[113] So, this method is also ineffective at transmitting any message with over-light velocity. What can be established are certain correlations between space-like events: A and B may register their respective up/down results along with the time of their occurance, and A and B may afterwards come together, compare their records and detect correlations. For example, in the case when A and B both had chosen the same spin-reference direction $\vec{a}$ they will notice that whenever A had obtained a spin-up proton B had obtained a corresponding spin-down proton. But eveidently these correlations are not sufficient to achieve signal transmission with over-light velocity.

One might wonder whether there should not exist any possibility for B to distinguish the case where A has done nothing, from the case where A has done a spin operation. In the first case B receives coherent superpositions of spin-up and spin-down states and these superpositions are equivalent to protons with spin axes oriented in all directions. In the second case B receives protons with spin axes oriented randomly

either up or down with respect to only one specific direction. However, here again the above-mentioned fact applies that all ways of constructing an unpolarized ensemble are equivalent.

Other devices might be invented with the aim of showing a possible violation of relativistic causality by means of the correlations between space-like events. We expect that, as in the case of the above protons, all these attempts will finally fail in that when one goes to work them out in detail some features will always turn up which prevent one from completing the demonstration.[106] If this is true, then there is no problem with the fact that two space-like events may be seen in reversed order of time by a moving observer, (Ref. 108, p. 86-91) since this does not mean a reversed order of causally connected predictable effects.

The above considerations may also have some bearing on the postulate of locality in relativistic quantum field theory[114] which essentially means that two fields must commute at space-like separations. The motivation for this axiom is to guarantee that relativistic causality is preserved. Our considerations suggest that relativistic causality may well be preserved without that axiom so it probably is unduly restrictive.

XVI. PROPOSED INTERPRETATION OF THE EPR PROBLEM

Our proposed interpretation of the experimentally observed quantum non-separability is this: first, it has nothing to do with the expansions (12.1) or (12.2) considered in the original discussions of the EPR problem or the quantum non-separability problem. After an interaction the two incoming wave packets are again completely separated from each other, as has been discussed in the Sections IX - XI on the measurement problem

and is accounted for by our basic interactions. How then, can quantum non-separability come about? Well, realize that all existing experimental confirmations of quantum non-separability are only concerned with the special case of identical particles, namely with photons and with protons, respectively. This gives the explanation: quantum non-separability is the effect of coalescence of identical wave packets. In the considered interaction the two packets coalesce into one single packet. After the interaction there is only this coalesced packet which is a function of really only one set of variables pertaining to one single system. It is only through a subsequent interaction with another wave packet that the coalesced packet splits into several others. This splitting interaction cannot be described by a Schrödinger-type equation, but again belongs to the category of the basic interactions. It need not be the very first interaction of the coalesced packet after its creation. The coalesced packet may undergo several interactions before it undergoes one that induces splitting. In order to be in accord with the experimental findings, the coalesced packet must be supposed to be split by a basic interaction that does not occur before the first Stern-Gerlach-type apparatus has attained the "reduction", i.e. the spin orientation of "its" proton. Actually, since we regard the two protons as really forming only one single wave packet it is this coalesced packet that is forced into some stationary state. Though the coalesced packet is in some definite state one cannot say this of each of the two proton packets separately since two separate protons do not even exist at this moment. This state then is such that when it is split by another interaction, perhaps during the necessary position (path) measurement, both

emerging proton wave packets have their respective spin components correlated in a definite way. This may not be the only way for the coalesced packet to become split. In fact, care must be taken in order to avoid unwanted splitting interactions, e.g. the apparatus must be evacuated.

The coalesced state is a coherent wave packet. It is not always necessary to imagine the packet as spatially connected. If there are spatially disconnected parts we would count them as one wave packet if they are coherent, i.e. if they can be brought to interference. This is in line with the view that a wave packet can only interfere with itself (Section IV). When the coalesced packet has split, the resulting wave packets are separated in every respect and coherence between them is lost. The idea of invoking coherence for the account of quantum non-separability has already been mentioned by Born (Ref. 11, letter no. 89); see also Schrödinger.[115] The splitting of wave packets is a necessary supplement to coalescence, because otherwise all identical wave packets could coalesce into one single wave packet in the course of the evolution of the universe.

In our conception quantum non-separability can only occur between formerly identical packets because only identical packets can coalesce. Hence, if the proton-proton scattering experiment[94] were to be repeated with the incoming proton replaced by an antiproton there should be no effect of any quantum non-separability. Here we have another prediction deviating from traditional quantum mechanics.

The fact of quantum non-separability can then be expressed as a general property of any wave packet, coalesced or not, namely that when it undergoes an interaction it is modified as a whole. It is not necessary that the modification be abrupt, i.e. take place within an infinitesimally small time interval; it is only necessary that any modification extends simultaneously over the whole wave packet. Thus, in this respect the wave packet behaves like the rigid body of classical (not relativistic) mechanics. The correlation over space-like distances is then expressed as the possibility of a wave packet to induce events at space-like separations. We note that the contrasting hypothesis, namely that a wave packet never induces events at space-like but only at time-like separations, means assuming that there is some causal propagation of signals within a wave packet. This would not fit with our idea that wave packets form the fundamental constituents, and that the extension of a wave packet represents some kind of fundamental length, thus making any localization impossible that is more accurate than its extension. Notice that any possible localization of the space-like or time-like interaction events of this wave packet is only given by the smaller extension of the other wave packets with which it interacts.

XVII. THE OLD OBJECTIONS AGAINST WAVE PACKETS

We now have available the prerequisites to deal with the old objections that have been raised against the identification of the micro-objects ("particles") with wave packets. These objections are often formulated rather sloppily, and most of them refer to a very naive wave-packet picture. One sometimes gets the impression that they merely represent rationalizations of a strong prejudice. Accordingly, they do

not form a heavy obstacle for a determined attack. As we mentioned in the introduction, Schrödinger has always stuck to the wave-packet idea, and our explanations might be regarded just as elaborations along the lines proposed by him.[116-118]

(i) The most popular objection is that wave packets spread out rapidly. - A corresponding difficulty already exists in ordinary quantum mechanics where the wave packet is to govern the behavior of a particle. Even here the spreading of the wave packet must be slow otherwise there would be no clear track but rather blobs or grains that are distributed erratically over larger and larger regions of space. One treats this difficulty in quantum mechanics by saying that the wave packet need not have a smaller extension than the observed width of the track. This is at least the dimension of a bubble (10^{-2} cm) or a grain (5×10^{-4} cm), and a wave packet with a minimum extension that is that large does not spread appreciably in normal experimental situations. Nothing prevents us from carrying over this argument when we consider the wave packet itself as the physical object. Difficulties would only arise if we identified an electron, say, with a wave packet of fixed size, and if this packet had the dimension of the "electron radius" $e^2/m_e c^2 = 2.8 \times 10^{-13}$ cm or the Compton wavelength of the electron $\lambda_C = \hbar/m_e c^2 = 7.7 \times 10^{-11}$ cm. However, the Compton length is only the minimum extension; in principle, a wave packet can have any size, and there is no compelling reason to give our wave packets a constant size and to give them smaller dimensions than the wave packets of ordinary quantum mechanics.

As to high-energy particles, formulae (2.11) and (2.12) or (2.15) and (2.16) show that the spreading ceases when the velocity of the packet approaches the velocity of light, viz. when the center energy $\hbar\omega_o$ becomes large compared to mc^2.

Furthermore, wave packets that incessantly undergo interactions with one another and form stationary aggregates do not spread out. Consider for example the hydrogen atom. The stationary electron wave packet in our conception is given by an eigenfunction of the Hamilton operator under the usual conditions. It is not given by a still narrower packet orbiting around the nucleus. Such a packet, at least for high quantum numbers, can be constructed by a superposition of the above atomic eigenfunctions, as has been shown by Schrödinger.[119] However, as Heisenberg pointed out,[120] this packet would inevitably spread out all around the nucleus though the higher the quantum numbers of the contributing states, the slower the spreading.

(ii) ψ can take on complex values.- This objection dissolves as soon as one realizes that a complex function is nothing but a couple of real functions united in a convenient way, and as soon as one remembers other situations in physics where complex quantities are used; e.g. the impedance of electric circuits $Z=R + iX$; see Bunge.[121]

(iii) ψ undergoes a discontinuous change during a process of measurement.- This is v. Neumann's axiom and we have dispensed with it in Section XI. Nevertheless, we do have a physical reduction process, as discussed for the Stern-Gerlach apparatus. In this process, however, the change of the incoming wave packet is considered to be the result of many interactions with the wave packets of the magnetic field and hence is also

not brought about instantaneously. We have not yet met with a compelling

reason for assuming that the basic interactions would have to occur

abruptly. Actually, we cannot regard this objection as a serious one.

While it might constitute an argument against a naive wave packet model,

our model is certainly not naive; among other things, the wave packet is

changed simultaneously over its whole extension in an interaction, and

it can induce events at space-like separations.

There might be another difficulty. If the transition of the wave

packet from a discrete state to another discrete state takes some time,

one might question whether the idea of transitions between quasi-stationary

discrete states can be retained at all. On the other hand, Einstein's

semi-classical derivation of Planck's radiation law[122] is based on that

idea. Setting aside the obvious possibility that the time for a transi-

tion may be very short compared with the time the packet spends in a

quasi-stationary discrete state, there still is our idea that each wave

packet represents $(action)^3$; that action is conserved and that it can

only be exchanged in units of h. This fact suffices to derive Planck's

law, as shown in Section V. Planck's law is essentially an effect of

quantization of action, not of the existence of stationary states, and

quantization of action is regarded as meaning field quantization, i.e.

second, not first, quantization. Schrödinger explicitly stresses the

paramount importance of field quantization in this connection.[116] The

idea that quantization of action and not the existence of stationary

states is the basis of Planck's law fits with our idea that (most)

operators with a discrete spectrum force the wave packet into one of

their eigenstates by means of physical interactions with the corresponding

apparatuses. There is no such apparatus in the case of blackbody radiation. Shape and material of the box do not influence the Planck distribution, and the volume can, at most, determine the value of the temperature.

(iv) A wave function in a space of many dimensions makes a simple interpretation impossible.- This objection can easily be encountered by noticing that the coalesced state resulting from an interaction of two incoming wave packets is considered to describe a single system, and that a many-particle system is considered to be an aggregate, not a pure state of many variables. Again, we cannot regard this objection as a serious one. In fact, the interpretation of our wave packet is not altogether simple; for example, correlated ranges in coordinate and momentum space are ascribed to it (Section XVIII below).

There is another related difficulty. Consider a many-electron alkali atom. This atom is an aggregate in our conception. When calculating the state of the outer electron the charge of the nucleus can be regarded as shielded or screened by the charge of the inner electrons. To regard the whole system of electrons as one wave packet would mean that some parts of the wave packet are screening or shielding other parts of the same packet from the nucleus. This screening should then also apply when the state of the hydrogen atom, with only one electron, is calculated. For hydrogen, however, no such shielding must be applied if one wants to get the correct state functions. Why shielding must not be applied in the case of the hydrogen atom is a different question. It arises, as we think, from interpreting the Schrödinger equation as describing point particles in a Coulomb field. In our view, the Coulomb law itself is the collective result of many basic interactions.

(v) Always "a whole particle" is observed, and never parts of it. Schrödinger[116] writes: "But I do not think they [the arguments] are [conclusive], provided only that one holds on to the wave aspect throughout the whole process... . One must regard the "observation of an electron" as an *event* that occurs within a train of de Broglie waves when a contraption is interposed in it which by its very nature cannot but answer by discrete response: a photographic emulsion, a luminescent screen, a Geiger counter." The fact that all interactions of some wave packet carry the same features, the existence of stationary states and the quantization of action we think is enough to substantiate Schrödinger's suggestion in any particular situation.

XVIII.   REALISM AND HEISENBERG RELATION

In this section we want to give an objective interpretation of the Heisenberg relation

$$\Delta p \times \Delta x \geq \hbar/2 \quad . \tag{18.1}$$

We presuppose that there is one reality, the same for all, existing independently of an observer, and that every real object has real properties.   That means we express things in the language of philosophical realism, i.e. in normal language.

When a wave packet has passed a Stern-Gerlach-type apparatus we say it has a definite spin component.   It cannot, however, be said that the wave packet has a definite position and momentum.   To ask for any definite value of position or momentum of a micro-object is to ask the wrong question; it is as if a child asked whether "the Spain" is in Madrid or in Barcelona.   It is in order to make these questions appear absurd that we call the micro-object a wave packet.   We think the basic reason

for abandoning realism has been the difficulties encountered when trying to attribute exact position and exact momentum to micro-objects. The abandonment of realism was really unjustified. The difficulties with position and momentum arise only from a misconception of the nature of the microscopic objects.

We have already said that only ranges of momentum and position can be ascribed to the micro-objects. With regard to micro-objects viz. wave packets to which spin and angular momentum (and magnetic moment) must also be ascribed, our view is that such a packet can be forced by suitable experimental devices to become a stationary state described by an eigenfunction of the operator of an angular momentum (spin) component with respect to one particular space direction. Only ranges of angular momentum components in the other directions can then be ascribed to this wave packet. This is regarded as a quite general feature, not restricted to angular momentum. It also applies to e.g., the discrete eigenvalues of the energy operator (bound state Hamiltonian). Excepted are those discrete values (quantum numbers) which distinguish between different super-selection subspaces.

What, then, is the meaning of the non-commutativity of two operators, as of momentum and position operators, for example? In commenting on the Einstein-Podolsky-Rosen paper, Bohr describes physical situations where the experimental arrangement suited for measuring the exact positions excludes the experimental arrangement suited for measuring the exact momenta of some particles, and he writes (Ref. 105, p. 233): "As repeatedly stressed, the principal point is here that such measurements demand mutually exclusive experimental arrangements." We do not think that

this is a principal point, characteristic of situations in quantum mechanics. Consider a sugar cube and the two properties of being soluble and of being burnable. Evidently, measurements of these properties demand mutually exclusive experimental arrangements. Yet nobody would find anything unusual in this situation, and everybody would say that the sugar cube is both burnable and soluble. The justification for calling the individual cube both burnable and soluble rests on the fact that one has at one's disposal many cubes which are presupposed to be identical with respect to the considered properties. The language of quantum mechanics would express this fact by saying that an ensemble of sugar cubes must be given. It is easy to imagine cubes made up of a mixture of several substances such that the cubes are easier to burn up but more difficult to dissolve. Moreover, it is easy to conceive of substances that can be mixed in different proportions so that the degrees of solubility and combustibility are in a certain complementary relation to each other.

According to the general relation[123]

$$\Delta A \times \Delta B \geq \frac{1}{2} |\langle [A,B] \rangle| \quad , \tag{18.2}$$

the non-commutativity of two operators then means that to the micro-object only a range of (discrete or continuous) eigenvalues of each of the respective operators can be attributed, and that these two ranges are interrelated in a definite way, given by the commutation relation between the operators. In the case of operators with discrete eigenvalue spectrum this is to be understood to mean that the micro-object cannot, in the given situation, be forced to become one definite state

out of the considered spectrum of stationary states. All these state-
ments are about how Nature acts and not about how we mutually disturb
ourselves in our respective measurements.

The interrelation of the ranges of two quantities may be called
complementarity. This could give Bohr's principle of complementarity a
sharp formulation. Such a complementarity is not, however, a character-
istic feature of the micro-objects, as the sugar-cube example shows.
Those properties that are complementary in macro-physics are different
from those properties that are complementary in quantum physics. In
macroscopic theories there is no complementarity in the above sense
between position and momentum, or between two components of angular
momentum. The complementarity between these particular quantites is the
characteristic and unexpected feature of the micro-objects considered in
quantum mechanics. Again, it is a matter of experimental verification
as to which properties ("flavors") can be attributed to the micro-objects
and which of them are complementary to which.

Accordingly, Heisenberg's relation (18.1) expresses neither more nor
less than a property of wave packets. It has nothing whatever to do with
any measurement. It obtains its central importance from the central
importance of the wave packets. Though one cannot ascribe the wave packet
a definite value of position and of momentum one can ascribe to it a
definite range of positions Δx and a definite range of momenta Δp,
and one can ascribe to it these two ranges simultaneously.[124] Thus, with
regard to the EPR problem one can say that the description of micro-
objects by means of wave packets is as complete as it can physically be.
Heisenberg's "uncertainty" relation actually does not refer to any

uncertainty. It would be more appropriate to call it a complementarity relation, namely a special case of the general complementarity relation (18.2).

The concept of wave packets as real physical objects with correlated ranges in coordinate and momentum space might suggest an explanation of some effects observed with laser beams, namely that a focused beam seems to have a broader spread in the absolute value of momentum than the unfocused beam.[125] This would be a third experimental test of our ideas, besides the tests mentioned in Sections XI and XVI.

XIX. DETERMINISM

Einstein's principal objections against quantum mechanics were of twofold nature. First, he held up the belief that real progress would only be possible when quantum mechanics is re-formulated in the language of realism.[8,10,12,13]

Second, Einstein refused to accept that the statistical description of physical processes is really fundamental and would never be superseded by a causal description.[126] In fact, to regard the world not as a three-dimensional spatial structure that develops in the course of time but rather "sub specie aeternitatis", as a four-dimensional space-time structure, means to regard it as existing in the time directions of both past and future, just as it exists in the six space directions. Efforts made towards developing a causal theory include the theories of hidden variables, which conceive the stochastic appearance of quantum phenomena to be really determined in a causal way by the values of some extra variables, either carried by the measured object itself or by the measuring device. These variables are called hidden because so far they

have escaped detection. Einstein, was not, however, satisfied with the proposed theories. We do not give here a proper account of the hidden-variables theories.[127] Rather we want to mention some other viewpoints that, to our knowledge, are not taken into account in the existing theories.

Of course there is no indeterminism in the sense that momentum or position are undetermined within some ranges. The indeterminism concerns the behavior of the wave packets as such, their interactions, their transmutations. Imagine that the hidden variables are to be found in the surroundings of the considered wave packet. Consider an individual electron wave packet entering a Stern-Gerlach-type apparatus. It is then forced into one of two possible stationary states. Which of the two states is chosen is thought to be pre-determined by some very mild interactions the wave packet has undergone on its way to the magnetic field or upon just entering it. We may say that the electron packet becomes unstable when placed in that apparatus. Actually, the problem concerns a wide range of phenomena including "spontaneous" symmetry breaking in various gauge field theories, "spontaneous" emission of radiation from atoms, as well as decay of nuclei and all the unstable particles. It is impossible to keep away all outside influences; it is hardly possible to keep away neutrinos and photons, and it is absolutely impossible to keep away the gravitational field.

Consider the unstable state of a pencil standing on its point. In classical mechanics it is assumed that the unstable state will decay sooner or later due to tiny disturbances: air movements, vibrations of the support etc. Or consider the disturbances that cause what is called

the statistical error in classical (non quantum) measurements. Often the measurement can be improved, with the result of smaller error bars. An essential part of this improvement consists in getting control over more and more disturbing effects so that what formerly had to be counted among the causes for the statistical errors now can be treated systematically, or can be kept away. Correspondingly, one may conceive the idea that one can get the outside disturbances that determine the behavior of an individual wave packet under control, and that this control in principle can be made complete. That this is not the case, not even in principle, has been pointed out by Janes.[129] Consider the space-time diagram of Fig. 1. An observer at O can at most have a knowledge of the events within his past light cone. He can predict anything occurring in his future light cone only on the basis of what he knows. However, the event at A obviously can be influenced by events lying outside his past cone, for instance at B. Hence, as a matter of principle, the future may be completely determined but still it is not completely predictable.

One might try to regain predictability be extending the past light cone far backward in time. If one assumes an appropriate cosmological model of the world the space-time diagram will be deformed as a result of non-zero space-time curvature and the past light cone will eventually include all of the world, and no events such as even B in Fig. 1 will be left outside it.[130] But even so one cannot obtain complete predictability for one has to observe that all things consist of wave packets. If one wants to fully describe one specific wave packet at one particular time (or time interval) one can do this only with the help of macroscopic

stable aggregates such as books, microfilms, magnetic tapes, etc. All
of these aggregates consist of a large number of wave packets. So, for
noticing and registering one wave packet many other wave packets are
needed. Only a part of the world can thus be "known", the other part
is used in representing the "knowledge". On the other hand, some wave
packet from the registering aggregate may well influence the registered
wave packet at some later time.

XX. CONVARIANT FORMULATION

Wigner writes:[52] "The greatest conceptual difficulty in the recon-
ciliation of the quantum mechanics with the general theory of relativity
is the basic difference in the observables they consider. This is the
coincidence of two particles in general relativity, that is an event,
whereas the quantum mechanical scattering and reaction theories do not
attribute a space-time point to such an event." Wigner also points out
that there are two types of position operators which have been discussed.
The one type refers to the position of a particle at a definite time.
These are the conventional position operators, determined by Newton,
Wigner, Wightman and Fleming. Wigner expresses reservations concerning
the basic nature of these. The other type of position operators refers
to a position in space-time. This localization refers not to a particle
but to an event. The search for the corresponding operators constitutes
a departure from the now accepted concepts of quantum mechanics. The
physical meaning of these operators, according to Wigner, was most clearly
explained by A. Broyles.[131] Nevertheless, Wigner concludes "A conceptual
problem remains to be solved here—apparently, the mathematical problem
is easier in this case than the interpretation of the formulae obtained."

We think that our events can be identified with the events dealt with in the formalism of Broyles. Therefore, we shall report here on some of the basic features of this work, using as far as possible the author's own words. Broyles only considers the case of zero spin. We think that the essential features are already revealed in this case.

A probability amplitude $\psi(x)$ for observing an event at a space-time point $x = (x^0, x^1, x^2, x^3)$ is considered. (In this section, x means a four vector, not the x-component of a three-vector). This is already an essential point. For the process of conventional quantum theory of looking at space at each instant of time and watching the development of systems with time puts time on a different basis than space. This lies already in the very kind of the questions that are asked and is further expressed by treating time as a parameter whereas for space-coordinates operators are required. Although relativistic quantum mechanics today is covariant, this distinction between time and space coordinates is contrary to the spirit of Einstein's theory of relativity.[132]

For this reason, an operator corresponding to the time coordinate is added to those for the space coordinates. A Hilbert space is defined in which the wave function $\psi(x)$ is the result of a scalar product $\psi(x) = \langle x|\psi\rangle$, with $|\psi\rangle$ = general state vector and $|x\rangle$ = simultaneous eigenvector of the four space-time position operators X^0, X^1, X^2, X^3. $[X^\mu, X^\nu] = 0$ is required, where $\mu, \nu = 0, 1, 2, 3$. These operators can be related to the displacement generators (18.2) below so that they transform like the components of a four vector. The space-time position vectors $|x\rangle$ are taken to be orthogonal and normalized so that $\langle x|x'\rangle = \delta(x-x')$. This orthonormality of the eigenfunctions of the

quartet of position operators already means that the position in question is not the position of a particle. If it were, and if one had observed the particle at position t, x, y, z, the probability of observing it at any point t', x', y', z' different from t, x, y, z would be zero. This is surely not the case: if one finds the particle somewhere at time t, it will be somewhere also at time $t' \neq t$.[52]

In the conventional particle picture it is meaningful to ask about the probability that a particle is found within d^3x at the time t. In the event picture the probability for an event to occur within d^3x at the time t is zero. Therefore one can only ask about the probability that an event occurs within d^3x and within the time interval dt.

One may define a quantity $\rho_s(x)$ that is formally equivalent to the conventional particle probability by

$$\rho_s(x) = \lim_{dx^0 \to 0} \psi^*(x)\psi(x)dx^0 \bigg/ \iint_{\vec{x}'} \psi^*\psi \, d^3x' \, dx^0 \quad , \qquad (20.1)$$

that is, by first counting the number of events occurring in a very small volume of size d^3x around the point $\vec{x}$ in the time interval of length dx^0 around x^0, then dividing this number by the total number of events occurring throughout the whole space in this time interval, and then letting $dx^0 \to 0$. The quantity $\rho(x) \equiv \psi^*(x)\,\psi(x)$ transforms as a Lorentz-scalar since the number of events in space-time volume d^4x is proportional to $\rho(x)d^4x$. This number will not change when viewed from a different inertial frame, and the volume size d^4x also is an invariant. The space probability distribution $\rho_s(x)$ can be shown to transform like the zeroth component of a four-vector, provided the space integral $\int \rho(x)d^3x = \int \psi^*\psi \, d^3x$ is constant in time.[133] This condition corresponds to setting the normalization integral (4.1) independent of time and equal

to one. This is self-evident in the conventional interpretation. Broyles considers the states in his extended Hilbert space as referring to one event. This would mean putting $\int \psi^* \psi \, d^4x = 1$, and this implies $\int \psi^* \psi \, d^3x = 0$. At this point we deviate from the interpretation of Broyles. In our conception, the vectors are for an indefinite number of events. Even if the vector is modified to some extent by inducing an event, we want to regard it as the same vector before and after that event, at least for mild events. We have instead motivated the time independence of the space integral (not the value 1) by conservation of action, see Section IV on normalization. In any case we have to put up with the inconvenience that the integral over space and time $\int \rho(x) \, d^4x$ diverges as long as $\psi(x)$ extends throughout all time, as is the case for any conserved quantity.

In the extended Hilbert space of Broyles, displacement generators can then be defined by

$$P_\mu |x\rangle = i \lim_{\ell \to 0} \ell^{-1} (|x + i_\mu \ell\rangle - |x\rangle) \tag{20.2}$$

where i_μ is the unit vector in the μth direction of space-time. One has then

$$\langle x|P_\mu|\psi\rangle = -i\partial/\partial x^\mu \psi(x) \tag{20.3}$$

and the P_μ are generators of the inhomogenous Lorentz or Poincaré group. The space components P^j (j=1,2,3) can be shown to be the familiar three-vector momentum operators and P^o the energy operator ($\neq$ Hamiltonian, however). $P_\mu P^\mu$ is interpreted as the rest-mass operator in Broyles' Hilbert space. It is an invariant operator of the Poincaré group. State vectors in Broyles' Hilbert space that are eigenvectors of the mass

operator produce position amplitudes satisfying the Klein-Gordon equation:

let $|\psi k\rangle$ be defined as an eigenvector of $P_\mu P^\mu$ so that $P_\mu P^\mu |\psi k\rangle = -k^2 |\psi k\rangle$.

Projecting this equation on to $\langle x|$ and using (20.3) gives

$$\partial_\mu \partial^\mu \langle x|\psi k\rangle = k^2 \langle x|\psi k\rangle \quad . \qquad (20.4)$$

This is the Klein-Gordon equation if one makes the identification

$k=mc/\hbar = 1/\lambda_C$, λ_C = Compton length.

The state vector $|\psi\rangle$ is specified when all of its projections

$\langle x|\psi\rangle$ on the $|x\rangle$ vectors are given. Thus a knowledge of $|\psi\rangle$ is equivalent

to the knowledge of the wave function $\psi(x)$ at all points of space-time.

However, the knowledge of the values $\langle x|\psi\rangle$ and its time derivative at

one time is equivalent to the knowledge of $\langle x|\psi\rangle$ at all space-time points,

provided $|\psi\rangle$ is known to be an eigenvector of the mass operator, i.e. to

lie on a mass shell.[134]

It can, however, be shown that the mass operator does not commute

with any of the space-time position operators. That is, the space-time

position eigenvectors $|x\rangle$ do not belong to a definite rest mass and hence

cannot be contained in a Hilbert space confined to solutions of a

Schrödinger type equation with a given rest-mass parameter. Such a

Hilbert space is, however, always considered in conventional quantum

mechanics, since quantum mechanics was conceived to describe particles

and particles are conceived to have a mass. Thus, in order to put

Broyles' formalism in contact with the more standard treatment, pro-

jection operators into a subspace belonging to a given non-negative real

mass, i.e. into the familiar Dirac Hilbert space, are introduced. With

the aid of the projector

$$Q_{kr} = \frac{1}{2} \int |p_{kr}\rangle\langle p_{kr}| \epsilon_k^{-1} \, d^3p \, (2\pi)^{-4} \qquad (20.5)$$

where $p^j_{kr} = p^j$, $p^o_{kr} = r\varepsilon_k$, $\varepsilon_k = (p_j p^j + k^2)^{\frac{1}{2}}$, $r = \pm 1$, $j=1, 2, 3$, $k=mc/\hbar$, the position vectors

$$Q_{kr}|x\rangle \qquad (20.6)$$

can be defined, being simultaneous eigenvectors of mass and position operators. These vectors transform properly under Lorentz transformations; they are, however, not orthogonal. Instead they give

$$\langle x'|Q_{k'r'} Q_{kr}|x\rangle = \frac{1}{2}\int \exp\left[ip_{kr}(x'-x)\right] \varepsilon_k^{-1} \, d^3p \, (4\pi)^{-4}$$

$$\times \delta(k'^2-k^2)\delta_{r,r'} \qquad (20.7)$$

$$= ir(2\pi)^{-1/2}\Delta^r(x'-x)\delta(k'^2-k^2)\delta_{r,r'}$$

where $\Delta^r=\Delta^{\pm}$ are singular functions well known in relativistic quantum field theory.[137] These functions contain a δ-function singularity on the light cone and decrease exponentially outside of it for spacelike separations $x'-x$. For example, for $t=0$ they behave as

$$\Delta^+(0,\vec{x}'-\vec{x}) = -\frac{im}{4\pi|\vec{x}'-\vec{x}|} K_1(|\vec{x}'-\vec{x}|/\lambda_C) \qquad (20.8)$$

where K_1 = modified Bessel function of order 1.[138] For large $|\vec{x}'-\vec{x}|/\lambda_C$ this can be approximated by

$$\Delta^+ = -\frac{im}{\pi|\vec{x}'-\vec{x}|}\left(\frac{\pi\lambda_C}{2|\vec{x}'-\vec{x}|}\right)^{\frac{1}{2}} \exp(-|\vec{x}'-\vec{x}|/\lambda_C) \qquad (20.9)$$

At $|\vec{x}'-\vec{x}|/\lambda_C \geq 2$ the error of formula (20.9) is already smaller than 15%. Hence, Δ^+ is different from zero practically only in a region $|\vec{x}'-\vec{x}|$ of the order of a Compton length λ_C. The vectors $Q_{kr}|x\rangle$ may be called "almost orthogonal" for space points at a distance greater than λ_C. In view of our considerations of the spreading of wave packets

in Section II this would seem an acceptable property of position eigen-vectors. In the non-relativistic limit $c \to \infty$ one has $\lambda_c \equiv \hbar/mc \to 0$, and the position vectors become strictly orthogonal. In fact, the whole formalism of Broyles reduces to the usual non-relativistic quantum theory, since the Klein-Gordon equation reduces to the Schrödinger equation in the limit as $c \to \infty$.

When we confine ourselves to a consideration of the mass-shell eigenvectors, i.e. the particle states considered in standard quantum mechanics, we would take the operators $Q_{kr}X^j$ as the most reasonable position operators for these states. They reduce to the position opera-tors in non-relativistic quantum mechanics as $c \to \infty$. Their eigenvectors are almost orthogonal and transform properly under Lorentz transforma-tions. On the other hand, the Newton-Wigner position operators do have exactly orthogonal eigenvectors, but these do not transform properly under Lorentz transformations. Also, the Newton-Wigner operators do not reduce to the expressions of non-relativistic quantum theory. All efforts to define position eigenstates in relativistic quantum mechanics that are (1) orthogonal, (2) possible particle states (on a mass shell), and (3) transform properly under Lorentz transformations have failed. It is necessary, therefore, to give up one of these conditions. Newton and Wigner gave up (3). In Broyles' Hilbert space it is possible to give up (2). For the conventional particle states we propose to give up (1).

We mention that the non-localizability of the field quanta, mentioned in Section II on wave packets, also follows naturally from Broyles' formalism.

The vectors $Q_{k1}|\psi\rangle$ belonging to positive-energy (r=1) mass shell are considered as the vectors most closely related to the usual kets in Dirac's Hilbert space. There is one inconvenience, namely the δ-distribution $\delta(k'^2-k^2)$ appearing in scalar products such as (20.7) or

$$\langle\psi'|Q_{k'r'}Q_{kr}|\psi\rangle = \frac{1}{2}\int\langle\psi'|p_{k'r'}\rangle\langle p_{kr}|\psi\rangle\varepsilon_k^{-1}d^3p\ (4\pi)^{-4}$$
$$\times\ \delta(k'^2-k^2)\delta_{r,r'}\quad.$$

$$(20.10)$$

However, this might be dealt with in the same way as e.g. the scalar product of conventional momentum eigenvectors, i.e. plane waves, in Dirac's Hilbert space. It might perhaps be worthwhile to consider the question of whether the states that conventionally are to describe a particle, are really states from Broyles' Hilbert space that belong to a definite mass value. Consider the elementary-particle resonances with their finite life time and a corresponding rest-mass distribution of width $\Gamma>0$. These resonances might perhaps be better described by a superposition of different mass eigenvectors from Broyles' Hilbert space, the stable particles then being the limiting cases $\Gamma\rightarrow0$. The mass parameter of a wave packet then would play a different role than, say, the charge parameter; the mass parameter seems more comparable with momentum. There are other hints, as we think, at the special role of the mass parameter, for example the photon-like behavior of the neutral vector mesons ρ, ω, ϕ ... as described in the vector-dominance model.[139]

Of course, Broyles' Hilbert space can only form a possible framework for an elaboration and covariant formulation of the wave-packet picture presented here. Nevertheless, it demonstrates that such a possibility exists and moreover that it exhibits, as we think, several very satisfactory features.

XXI. SUMMARY

A novel conceptual framework for quantum physics has been presented. The basic constituents are taken to be wave packets, not point particles. Matter is thus explained by a field concept. The wave packets in the first instance are the usual wave packets of non-relativistic or relativistic quantum mechanics. They are, however, interpreted differently, namely as real physical objects, and they are endowed with additional features so as to allow a consistent account of the known facts.

In particular, these basic constituents do not have a fixed size. We have shown, however, that if we postulate that the spreading velocity of free wave packets be less than the velocity of light, it follows that the spatial extension of any wave packet must be larger than half its Compton length $\lambda_C = \hbar/(mc)$. Hence any localization within a region smaller than half a Compton length is devoid of physical meaning.

The mathematical formalism of non-relativistic quantum mechanics is given a new interpretation. In traditional quantum mechanics the quantity $|\psi|^2 \, d^3x$ gives the probability for observing the particle in d^3x. In our interpretation, the similar quantity $|\psi|^2 \, d^3x \, dt$ is proportional to the probability that the wave packet interacts with another one within d^3x and within dt. The connection with the conventional interpretation is established by defining localization by means of an interaction of the wave packet under consideration with a wave packet of some detecting device. This latter wave packet must initiate a cascade process which leads to a macroscopic effect, say a black spot in a photographic emulsion. This spot then defines, as accurately as it can, the location of the original interaction, and from the location of

this interaction event the presence of the initiating wave packet can be concluded.

Since $|\psi|^2$ is no longer considered to be the probability for the detection of one particle, the time-independent quantity $A: = \int |\psi|^2 d^3x$ can no longer be interpreted as conserved probability and hence put equal to one. Instead, it is proposed to interpret it as conserved action. Of course, we assume that action is quantized in units of h. The quantity A is then proportional to the number of "three-dimensional" action quanta h^3 carried by the wave packet.

Generally, the quantity A is conserved if the Lagrangian is gauge invariant. The postulate of global gauge invariance for free as well as interaction Lagrangians is interpreted to mean conservation of action in any type of interaction. In the postulate of local gauge invariance, the feature that the gauge parameters are allowed to vary from one space point to the other means that they are allowed to vary from one wave packet to the other. This is in line with our idea that the extension of wave packets in some sense represent fundamental lengths, and with the absence of any phase relations between different wave packets.

The (anti)symmetrization postulate for states of identical particles (wave packets) is interpreted as expressing the existence of coalesced states in which the original wave packets completely loose their individuality. The coalesced packet is a function of variables that really pertain to one single system only. In the case of identical bose packets the coalesced packet is essentially the same as the original non-coalesced packets. In the case of identical fermi packets the coalesced packet is essentially different from any of the original non-coalesced

packets. This is Pauli's exclusion principle in our interpretation. The quantization of action in units of h then allows the derivation of Planck's formula by the standard method, where we only have to reinterpret the distribution of photons over phase-space cells as meaning the distribution of (three-dimensional) action quanta of size h^3 over the photon packets.

The problem of the measurement process in conventional quantum mechanics arises when, as usual, the measurement is conceived to be an interaction between two systems, object and measuring apparatus, which are described by one wave function each (or by mixtures), and when the interaction is described by the Schrödinger equation. The state of the combined system then is such that the object cannot be regarded to be in a definite eigenstate of the operator substantiated by the measuring apparatus. According to v. Neumann's reduction postulate, the system is thrown into an eigenstate only in an observation. In this connection Wigner has conjectured that the equations of motion must cease to be linear, as is the Schrödinger equation, when conscious beings enter the picture. Our conception is based on this conjecture. We only modify it in that we assume that the interactions that are described by the nonlinear laws are the core of every physical interaction, and that the Schrödinger equation with potential, actually, is only a phenomenological account of a series of these basic interactions. The wave packets emerging from a basic interaction are again completely independent of each other. The basic interactions have this property in common with the interactions conceived in the S-matrix approach. A mathematical specification cannot, however, be presented.

In our concept a measurement is completed when a macroscopic record of the result has been achieved. Some macroscopic body must be brought into a particular state from several possible different macroscopic states. A macroscopic body is conceived to be an aggregate. By an aggregate we mean a set of independent wave packets which incessantly undergo basic interactions with one another. Actually, not only a macroscopic body, but every system that can be described as a stable bound state, is an aggregate. Hence aggregates include also nuclei, atoms and molecules but exclude the elementary particles and resonances listed in the Rosenfeld tables. The concept of an aggregate means a violation of the superposition principle and hence goes beyond the framework of conventional quantum mechanics. This concept has, however, been shown to be essential for solving the problems of measurement, of spreading of macroscopic bodies and of screening in many-electron atoms.

In our conception there are different types of measurements. We have discussed three representative examples: a measurement of a position, of a momentum and of a spin component. (i) In the position measurement, the position of an event in a photographic emulsion is measured. The concepts of a macroscopic record, of an aggregate, of basic interactions and of macroscopic states are invoked here. (ii) The measurement of any physical quantity can be reduced to position measurements with the help of physical laws relating the quantity in question with the path the system will take. In the momentum measurement the average of the absolute value of a wave packet's momentum is determined by measuring the curvature of its track in a constant magnetic field. (iii) In the spin component measurement we consider a spin-1/2 atom passing through a Stern-Gerlach

magnet. In our conception the atom is forced into a definite path, either the upper or the lower one. This, however, is not effected by an observation but by the apparatus itself, by basic interactions of the incoming wave packet with the wave packets that in some way constitute the magnetic field. That the whole process is no longer time reversal invariant confirms this view. The Stern-Gerlach apparatus in this way acts as a polarizer, not as a proper measuring instrument. This apparatus can be employed for a spin component measurement by varying its orientation in space when one is given an ensemble of incoming wave packets, and it can be employed for a preparation if one of its two possible paths is blocked off. That the apparatus forces the incoming wave packet to become a definite eigenfunction of the spin component operator is regarded as a general feature of operators with discrete eigenvalue spectrum provided the operator can be substantiated by a physical apparatus.

The Einstein-Podolsky-Rosen problem in the view of orthodox quantum mechanics means that two particles, which after an interaction are completely separated beyond any possibility of dynamical influence, are nevertheless correlated: a measurement on the one particle will influence the outcome of a measurement on the other particle even if the two measurements are separated by a space-like interval. This peculiar quantum non-separability has been verified experimentally with photons and with protons, respectively. In our interpretation the experimentally observed quantum non-separability has, however, nothing to do with the EPR problem. The EPR problem arises from a description of the interaction

based on the Schrödinger equation. As already mentioned, in our view the Schrödinger equation with potential merely accounts for a series of basic interactions which are conceived to lead to complete separation of the outgoing wave packets. Hence these basic interactions cannot lead to the observed effect. Instead, the experimentally observed non-separability is ascribed to the additional process of coalescence of the incoming identical wave packets. The concept of coalescence was already introduced in connection with (anti)symmetrization. Thus, we predict that quantum non-separability will not be observed after the interaction of non-identical packets.

The existence of non-separability between identical wave packets is then taken as indicative of a general property of a wave packet, coalesced or not, namely that when it undergoes an interaction it is modified simultaneously over its whole extension. This property also means that a wave packet can induce correlated events at space-like separations. These features are in accord with the idea that the extensions of the wave packets in some sense represent fundamental lengths.

We have described in some detail why the possibility that a wave packet induces correlations over space-like distances does not necessarily mean that relativistic causality is violated, i.e. that signals can be transmitted with over-light velocity. Basic for the argumentation is the fact that one cannot predict with certainty the outcome of a quantum measurement.

All phenomena of quantum physics have been formulated in the language of philosophical realism, as repeatedly postulated by Einstein. No

observer is logically necessary. Our wave packets are real things with real properties, not merely observables. However, exact values of position and momentum are not counted among the properties of micro-objects, only ranges of these quantities are. Moreover, both a range of positions and a range of momenta are simultaneously ascribed to one micro-object. The difficulties that conventional quantum mechanics has with position and momentum arise from a misconception of the nature of the microscopic objects. Heisenberg's relation and any non-commutativity of operators have nothing to do with any measurement. They express the existence of correlations between properties of micro-objects. We have proposed to call this complementarity. It has been shown that such a complementarity is not a characteristic feature of quantum physics, but that those properties which are complementary in macro-physics are different from those properties which are complementary in quantum physics. It is a matter of experimental verification as to which properties can be attributed to the micro-objects and which of them are complementary to which.

With regard to determinism, the other basic postulate of Einstein, we have pointed out that, due to the finite velocity of signal transmission and the consumption of a large number of wave packets in the description of a few others, the future may be completely determined but still is not completely predictable, even if one knew all physical laws.

Finally we have pointed out that a Hilbert space exists, as proposed by Broyles, in which time and space are treated on equal footing and in which the wave function ψ used in the expression $|\psi|^2 \, d^3x \, dt$ is the result of a scalar product. We have reported on some of the basic

properties of Broyles' Hilbert space, and have shown that it is likely to provide a suitable covariant framework for a mathematical elaboration of the wave-packet picture.

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REFERENCES

1. Ludwig Wittgenstein: Philosophische Untersuchungen, Macmillan, New York (1958), No. 79, "Sage, was du willst, solange dich das nicht verhindert, zu sehen, wie es sich verhält. (Und wenn du das siehst, wirst du Manches nicht sagen)".

2. R. P. Feynman, R. B. Leighton, M. Sands: The Feynman Lectures on Physics, Addison-Wesley, Reading, Massachusetts (1965), Vol. III, Chap. 1.

3. W. Heisenberg, Z. Phys. $\underline{43}$, 172 (1927) p. 185: "Die 'Bahn' entsteht erst dadurch, daß wir sie beobachten" ("The 'trajectory' arises only by our observing it").

4. J. C. Maxwell writes: "It is mainly with the hope of making these [Faraday's] ideas the basis of a mathematical method that I have undertaken this treatise." (A treatise on electricity and magnetism, Oxford, At the Clarendon Press, 1891, 1892; reprinted by Dover Publications, Inc., New York (1954) vol. 2, p. 176).

5. Quantum field theory may be taken to have started with Dirac's paper "The Quantum Theory of the Emission and Absorption of Radiation", Proc. Roy. Soc. (London) $\underline{A114}$, 243-65 (1927), received February 2, 1927, see R. Jost in Ref. 6, p. 61-77. In: Fields and Quanta $\underline{3}$, 138 (1972), Dirac says, "I feel pretty sure that the changes which will be needed to get over the present difficulties facing quantum theory and appearing as a resistance between the quantum theory and relativity will be very drastic... and therefore one should not become too much attached to the present quantum mechanic."

6. A. Salam, E. P. Wigner (Eds.): Aspects of Quantum Theory, Cambridge (1972).

7. E. Schrödinger, Sci. Am. <u>189</u>, No. 3, 52 (1953).

8. A. Einstein: Elementare Überlegungen zur Interpretation der Grund-
    lagen der Quanten-Mechanik,in: Scientific Papers presented to Max
    Born on his retirement from the Tait chair of Natural Philosophy in
    the University of Edinburgh, Oliver & Boyd, Edinburgh, London (1953)
    p. 33-40.

9. M. Bunge: Philosophy of Physics, Reidel, Dodrecht-Holland (1973)
    especially p. 78-80, 84-86.

10. A. Einstein, "Quantenmechanik und Realität," Dialectica (Neuchâtel,
    Switzerland) <u>2</u>, 320-324 (1948); reprinted in Ref. 11, letter no. 88
    to Born (1948).

11. M. Born: Albert Einstein, Hedwig und Max Born, Briefwechsel 1916-1955,
    Nymphenburger Verlagshandlung, München (1969); english translation:
    The Born-Einstein Letters, Macmillan, London (1971). .

12. A. Einstein, "Elementare Überlegungen zur Interpretation der Grund-
    lagen der Quanten-Mechanik", in: A. George (ed.), Louis de Broglie,
    Physicien et Penseur (Les savants et le monde) Albin Michel, Paris
    (1953) p. 4-14, especially p. 6.

13. A. Einstein, in Ref. 14, p. 667-674.

14. P. A. Schilpp (ed.), Albert Einstein: Philosopher-Scientist, The
    Library of Living Philosophers, Inc., Cambridge University Press,
    London (1970).

15. M. Bunge, Dialectica (Neuchâtel, Switzerland) <u>19</u>, 195-222 (1965).

16. M. Bunge, Foundations of Physics, Springer, New York (1967) Chap. 5.

17. M. Bunge, Rev. Mod. Phys. <u>39</u>, 463-474 (1967).

18. M. Bunge, in Ref. 19, p. 1-6, 105-117.

19. M. Bunge (ed.): Quantum Theory and Reality (Studies in the Foundations, Methodology and Philosophy of Science, Vol. 2) Springer, Berlin (1967).

20. M. Bunge, A. J. Kálnay, in: R. Haller, J. Götschl (eds.), Philosophie und Physik, Vieweg, Braunschweig (1975) p. 25-37.

21. J. S. Bell, Subject and Object, in Ref. 22, p. 687.

22. J. Mehra (ed.): The Physicist's Conception of Nature (Symposium on the Development of the Physicist's Conception of Nature in the Twentieth Century, held at the International Centre for Theoretical Physics, Miramare, Trieste, Italy, 18-25 Sept. 1972) Reidel, Boston, USA (1973).

23. P. Jordan, Philosophy of Science 16, 269-278 (1949).

24. K. M. Popper, in Ref. 19, p. 7-44.

25. J. M. Jauch, in Ref. 26, p. 20-55.

26. D'Espagnat (ed.): Foundations of Quantum Mechanics, Academic Press, New York (1971), Proceedings of the International School of Physics "Enrico Fermi". Course 49, Varenna, 1970).

27. G. Ludwig, in Ref. 26, p. 287-315; in Ref. 22, p. 702-706.

28. R. Haag, Subject, Object and Measurement, in Ref. 22 , p. 691; and private communication.

29. J.-M. Lévy-Leblond, Dialectica (Neuchâtel, Switzerland) 30, 161 (1976); also printed in J. L. Lopes, M. Paty (eds.): Quantum Mechanics, A Half Century Later, Reidel, Dodrecht-Holland (1977) p. 171-206.

30. H. P. Stapp, Foundations of Physics (N.Y.) 7, 313 (1977).

31. K. V. Roberts, Proc. Roy. Soc. (London), A360, 135 (1978).

32. Cf. e.g.: M. L. Goldberger, K. M. Watson: Collision Theory, Wiley, New York (1967) p. 61-65, 624; D. I. Blokhintsev: Space and Time in the Microworld, ISBN 90 277 0240 3 (1973) Section 18.

33. W. Pauli: Ref. 34, P. 17-21; Pauli Lectures on Physics, Vol. 5: Wave Mechanics, MIT Press, London (1973) p. 7-10; D. Rapp: Quantum Mechanics, Holt, Rinehart and Winston, New York (1971) p. 22-25.

34. W. Pauli: Die allgemeinen Prinzipien der Wellenmechanik in: S. Flügge (ed.): Encyclopedia of Physics, Vol. V/1: Principles of Quantum Theory I, Springer, Berlin (1958) p. 1-168.

35. H. M. Bradford, Am. J. Phys. $\underline{44}$, 1058-1063 (1976); L. C. Baird, Am. J. Phys. $\underline{40}$, 327-329 (1972).

36. D. Ruelle, Helv. Phys. Acta $\underline{35}$, 147 (1962); R. Jost: The General Theory of Quantized Fields (Lectures in Applied Mathematics, Vol. IV), American Mathematical Society, Providence, Rhode Island (1965) p. 3, 4, 123-125.

37. See the analogous observation made for photons in quantum electrodynamics: A. I. Achieser, W. B. Berestetskii: Quantum Electrodynamics, Interscience, New York (1965) p. 10, 11.

38. B. Gerlach, D. Gromes, J. Petzold, Z. Phys. $\underline{202}$, 401 (1967).

39. See the review by A. J. Kálnay: The Localization Problem, in: Problems in the Foundations of Physics, M. Bunge (ed.), Springer, Berlin (1971) p. 93-110.

40. E. M. Henley, W. Thirring: Elementary Quantum Field Theory, McGraw Hill, New York (1962).

41. D. Lurié: Particles and Fields, Interscience, New York (1968).

42. For literature and historical review see E. W. R. Papp: Quantum Theory of the Natural Space-Time Units, in: The Uncertainty Principle and Foundations of Quantum Mechanics, W. C. Price, S. S. Chissick (eds.), Wiley, New York (1977) p. 29-50.

43. V. M. Gusev, O. N. Kompanets, A. R. Kukudzhanov, V. S. Letokhov, E. L. Mikhailov, Sov. J. Quant. Electron. $\underline{4}$, 1370 (1975).

44. For maximum likelihood methods see, e.g., S. Brandt: Statistical and Computational Methods in Data Analysis, North-Holland, Amsterdam (1976), Chap. 7.

45. A. Einstein in Ref. 14, p. 78, 88; "Physik und Realität", J. of the Franklin Inst. (Philadelphia) $\underline{221}$, 313-347 (1936); english translation in the same journal, p. 349-382; the German text is reprinted in A. Einstein: Aus meinen späteren Jahren, Deutsche Verlags-Anstalt, Stuttgart (1952) p. 63-104; the english text is reprinted in A. Einstein: Out of My Later Years, Greenwood Press, Westport, Connecticut (1970) (reprinted edition of Philosophical Library Inc., New York (1950) p. 59-97) and in A. Einstein: Ideas and Opinions, Dell (1973); P. Speziali (ed.): Albert Einstein, Michele Besso, Correspondance 1903-1955, Hermann, Paris (1972) letter 175 (1950).

46. See the review by T.-M. Yan, Ann. Rev. Nucl. Sci. $\underline{26}$, 199-238 (1976).

47. See the reviews by A. C. Scott, F. Y. F. Chu, D. Mc Laughlin, Proceedings of the IEEE $\underline{61}$, 1443 (1973); J. R. Klauder, Act. Phys. Austriaca, Suppl. XVIII (Wien) 1-62 (1977)(Proceedings of the XVI. Internationale Universitätswochen für Kernphysik 1977 der Karl-Franzens-Universität Graz at Schladming (Steiermark, Austria) 24th February-5th March 1977); V. D. Makhanov, Phys. Rep. $\underline{C35}$, 1-128 (1978); L. D. Faddeev, V. E. Korepin, Phys. Rep. $\underline{C42}$, 1-87 (1978).

48. See the review by P. Hasenfratz, J. Kuti, Phys. Rep. C40, 75-179

    (1978); F. Csikor, G. Preparata, Nuovo Cimento 44A, 265 (1978).

49. E. T. Jaynes, in: Coherence and Quantum Optics, L. Mandel, E. Wolf

    (eds.) Plenum Press, New York (1973) p. 35-81.

50. P.A.M. Dirac: The Principles of Quantum Mechanics, Clarendon Press,

    Oxford (1970) 4th edition, p. 9.

51. L. Mandel, E. Wolf, Rev. Mod. Phys. 37, 231 (1965); R. L. Pfleegor,

    L. Mandel, Phys. Rev. 159, 1084 (1962); D. B. Scarl, Phys. Rev. 175,

    1661 (1968).

52. E. P. Wigner: Relativistic Equations in Quantum Mechanics, in Ref. 22,

    p. 320-330.

53. Cf. Ref. 54 §34b.

54. R. Becker, Theorie der Wärme, Springer, Berlin (1961); english

    translation: Theory of Heat, Springer, New York (1967).

55. See for example N. N. Bogoliubov, D. V. Shirkov: Introduction to

    the Theory of Quantized Fields, Wiley, New York (1959) p. 85 f.

56. See for example E. S. Abers, B. W. Lee, Phys. Rep. 9C, 1-141 (1973);

    J. Iliopoulos: An Introduction to Gauge Theories, report CERN 76-11

    (15 June 1976) or CERN 77-18 (20 October 1977) p. 36-78.

57. P.A.M. Dirac: Fields and Quanta 3, 138-164 (1972) especially

    p. 154-158.

58. R. Mirman, Nuovo Cimento 18B, 110 (1973).

59. W. Heitler, F. London, Z. Physik 44, 455 (1927). The method is

    reproduced in the textbook of L. I. Schiff: Quantum Mechanics,

    McGraw-Hill, New York (1968) p. 376 f., 449 f., or in D. I. Blochinzew:

    Grundlagen der Quantenmechanik, Harri Deutsch, Frankfurt (1962)

    p. 479 f., 499f.

60. Compare Ref. 2, Chapt. 3-4, Table 3-1 with J. D. Bjorken, S. D. Drell: Relativistic Quantum Mechanics, McGraw-Hill, New York (1964) p. 136, formula (7.82).

61. S. N. Bose, Z. Physik, 26, 178 (1924); A. Einstein, Sitzungsberichte der Preussischen Akademie der Wissenschaften, Physikalisch-Mathematische Klasse, Berlin (1924) p. 261-267; Ref. 54, § 49 of the German edition; Ref. 62, Sections 4.3 and 9.5.

62. K. Huang: Statistical Mechanics, Wiley, New York (1963).

63. To compare with the usual presentations multiply (5.4) by 2 for the two polarization possibilities of photons, by $h\nu$ for obtaining the energy density $u(\nu, T)$, and by $c/4\pi$ for obtaining the energy emitted per sec into 1 steradian from an opening in the cavity of unit area.

64. A. Einstein, Sitzungsberichte der Preussischen Akademie der Wissenschaften, Physikalisch-Mathematische Klasse, Berlin (1925) p. 3-14, especially p. 6; E. Schrödinger, same journal (1925) p. 434-441, especially p. 440.

65. Ref. 54, p. 119, 156, 171; english translation p. 135, 136, 191, 207; Ref. 62, p. 197, 216.

66. Perhaps even the coherent states (Ref. 67) introduced by Glauber into quantum optics may be better described as states with a large average number of action quanta rather than of photons.

67. For a short introduction and literature see the book of J. M. Jauch, F. Rohrlich: The Theory of Photons and Electrons, Springer, New York (1976), 2nd edition, supplement S3.

68. E. P. Wigner, in Ref. 26, p. 1-20.

69. J. v. Neumann: Mathematische Grundlagen der Quantenmechanik, Springer, Berlin (1932, 1968); english translation: Mathematical Foundations of Quantum Mechanics, Princeton University Press, Princeton (1955), Chap. V, VI.

70. L. Landau, Z. Physik $\underline{45}$, 430 (1927); L. Landau, E. Lifschitz: Quantum Mechanics, Pergamon Press, London (1958) p. 35-38.

71. H. Margenau, Philosophy of Science $\underline{30}$, 1, 138 (1963).

72. H. D. Zeh, Foundations of Physics (New York) $\underline{1}$, 69 (1970); H. D. Zeh, O. Kübler, Ann. Phys. (New York) $\underline{76}$, 405 (1973).

73. See also E. P. Wigner, Am. J. Phys. $\underline{31}$, 6 (1963); J. v. Neumann in Ref. 69, p. 233 (Section VI, 3); A. Fine, Phys. Rev. $\underline{D12}$, 2783 (1970).

74. D. Bohm: Quantum Theory, Prentice-Hall, Englewood Cliffs, N.J., (1951) Chap. 22, Section 11.

75. K. Hepp, Helv. Phys. Acta $\underline{45}$, 237 (1972); For a comment on Hepp's paper see J. S. Bell, Helv. Phys. Acta $\underline{48}$, 93 (1975).

76. E. P. Wigner, Am. J. Phys. $\underline{31}$, 6 (1963).

77. E. Schrödinger, Die Naturwissenschaften $\underline{23}$, 807, 823, 844 (1935); For the cat see especially p. 812.

78. E. P. Wigner: Remarks on the Mind-Body Question in: I.J. Good (ed.) The Scientist Speculates, Heinemann, London (1961), and Basic Books, New York (1962) p. 284; reprinted in E. P. Wigner: Symmetries and Reflections, Indiana Univ. Press, London (1967) p. 171.

79. L. Wittgenstein, Tractatus logico-philosophicus, Suhrkamp, Frankfurt (1960) propositions 5.6 - 5.641; english translation: Routledge & Kegan Paul, London (1977).

80. J. S. Bell, CERN report Ref. TH 1424 - CERN, 27 October 1971.

81. J. S. Bell: Subject and Object,in Ref. 22, p. 687.

82. The opinion that quantum mechanics in its present form is unable to
    give a proper account of macroscopic systems has also been put
    forward by Ludwig, Ref. 83-85.

83. G. Ludwig, Physikalische Blätter, 11, 489-494 (1955).

84. G. Ludwig: Gelöste und ungelöste Probleme des Meßprozesses in der
    Quantenmechanik,in: F. Bopp (ed.): Werner Heisenberg und die Physik
    unserer Zeit, Vieweg, Braunschweig (1961) p. 150-181.

85. G. Ludwig, in Ref. 22, p. 702-708.

86. Ref. 14, p. 682, 683; Born, Einstein, Pauli, in Ref. 11, Letters
    No. 105-116.

87. N. Barash-Schmidt, A. Barbaro-Galtieri, C. Bricman, R. L. Crawford,
    C. Dionisi, R. J. Hemingway, C. P. Horne, R. L. Kelly, M. J. Losty,
    M. Mazzucato, L. Montanet, A. Rittenberg, M. Roose, T. G. Trippe,
    G. P. Yost, Phys. Lett. 75B, 1 (1978) "Review of Particle Properties."

88. A. Fridman, (ed.): Proceedings of the 4th European Antiproton Sym-
    posium, Barr (Strasbourg, France) (June 26-30, 1978); I. S. Shapiro,
    Phys. Rep. C35, 129-185 (1978).

89. Ref. 54, § 44; Ref. 62, Chapt. 9.1.

90. A. Daneri, A. Loinger, G. M. Prosperi, Nucl. Phys. 33, 297 (1962);
    Nuovo Cimento 44B, 119 (1966). In these papers a more detailed
    description of the unstable nature of some measuring apparatuses
    can be found. The further considerations of the authors are, how-
    ever, based on a concept of the measurement process that is different
    from ours. For a comment on the above papers see J. Bub, Nuovo
    Cimento 57B, 503 (1968).

91.   I. Prigogine, in Ref. 22, p. 561, 697, especially p. 698.

92.   D. B. Lichtenberg: Meson and Baryon Spectroscopy,in: Ergebnisse
      der exakten Naturwissenschaften, Vol. 36 (Springer Tracts in Physics)
      G. Höhler (ed.), Springer,Berlin (1964) p. 83-218.

93.   N. F. Mott, H. S. W. Massey: The theory of Atomic Collisions,
      Clarendon Press, Oxford (1965), 3rd edition, Chap. IX.

94.   M. Lamehi-Rachti, W. Mittig, Phys. Rev. $\underline{D14}$, 2543 (1976).

95.   See e.g. Ref. 93, p. 248, formulae (52), (53).

96.   A. Einstein, P. Ehrenfest, Z. Physik $\underline{11}$, 31 (1922).

97.   Ref. 93, p. 212, Ref. 34, p. 65-67.

98.   R. A. Beth, Phys. Rev., $\underline{50}$, 115 (1936); A. H. S. Holbourn, Nature
      $\underline{137}$, 31 (1936).

99.   W. Pauli: Über das H-Theorem vom Anwachsen der Entropie vom Stand-
      punkt der neuen Quantenmechanik, in: Probleme der modernen Physik,
      Arnold Sommerfeld zum 60. Geburtstage, gewidmet von seinen Schülern,
      S. Hirzel Verlag, Leipzig (1928) p. 30-45; reprinted in: Collected
      Scientific Papers by Wolfgang Pauli, R. Kronig, V. Weisskopf (eds.),
      Wiley, New York (1964) Vol. 1, p. 549-564.

100. See, e.g., the book of Bohm, Ref. 74, Chap. 14.16 and 22.6.

101. A. Messiah: Quantum Mechanics, North-Holland, Amsterdam (1972)
      Vol. I, p. 197.

102. A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. $\underline{47}$, 777 (1935).

103. N. Bohr, Phys. Rev. $\underline{48}$, 696 (1935).

104. W. H. Furry, Phys. kev. $\underline{49}$, 393 (1936).

105. N. Bohr: Discussion with Einstein on Epistemological Problems in
      Atomic Physics, in Ref. 14, p. 199-241, especially p. 224.

106. D. Bohm, B. Hiley, Foundations of Physics (New York) 5, 93-109 (1975);

reprinted in: J. L. Lopes, M. Paty (eds.): Quantum Mechanics, A

Half Century Later, Reidel, Dodrecht-Holland (1977) p. 207-225.

107. Ref. 108, part 3, Chapt. 8-13.

108. B. d'Espagnat: Conceptual Foundations of Quantum Mechanics,

Benjamin, Reading, Massachusetts (1976) 2nd edition.

109. D. Bohm, Y. Aharonov, Phys. Rev. 108, 1070 (1957); Nuovo Cimento

27, 964 (1960).

110. Ref. 108, p. 115.

111. F. J. Belinfante: A Survey of Hidden-Variables Theories, Pergamon

Press, Oxford (1973) p. 313.

112. M. Bruno, M. D'Agostino, C. Maroni, Nuovo Cimento 40B, 143 (1977).

113. Ref. 93, especially p. 223, 224.

114. R. F. Streater, A. S. Wightman: PCT, Spin and Statistics, and All

That, Benjamin, New York (1964) p. 100, 134.

115. E. Schrödinger, The British Journal for the Philosophy of Science

(Cambridge) 3, 109 (1952/53) especially p. 122.

116. E. Schrödinger: The Meaning of Wave Mechanics, in: A. George (ed.):
Louis de Broglie, Physicien et Penseur (Les savants et le monde)
Albin Michel, Paris (1953) p. 16-32.

117. E. Schrödinger, Endeavour (London) 9, 109-118 (1950); reprinted in:

E. Schrödinger: Was ist ein Naturgesetz, Oldenbourg, München (1962)

p. 121-143.

118. E. Schrödinger, The British Journal for the Philosophy of Science

(Cambridge) 3, 109, 233 (1952/53).

119. E. Schrödinger, Die Naturwissenschaften, 14, 664 (1926).

120. W. Heisenberg, Z. Physik 43, 172 (1927), in particular p. 184-185. Heisenberg also remarks that an orbiting narrow wave packet would imply that the frequencies emitted by the atom would be integer multiples of some basic frequency, which is evidently not the case. This argument does not hold in the case which Schrödinger considered, for in the range of large quantum numbers but small differences between them, the frequencies can in fact be written as integer multiples of some reference frequency.

121. M. Bunge, Am. J. Phys. 24, 272 (1956).

122. A. Einstein, Physikalische Zeitschrift (Leipzig) 18, 121 (1917).

123. H. P. Robertson, Phys. Rev. 35, 667A (1930); E. Schrödinger, Sitzungsberichte der Preussischen Akademie der Wissenschaften, Physikalisch-Mathematische Klasse, Berlin (1930) p. 296-304.

124. Cf. also M. Bunge Ref. 20, p. 28, 35, and British Journal for the Philosophy of Science (Cambridge) 18, 265-286 (1967); M. Planck, The Universe in the Light of Modern Physics, Allen and Unwin, London (1934); reprinted in M. Planck: The New Science (Greenwich editions published by Meridian Books Inc., Silver Spring, Maryland, USA, (1959)) p. 155-232; see especially p. 188.

125. E. Panarella, Foundations of Physics (New York) 4, 227 (1974); 7, 405 (1971); A. D. Allen, Phys. Today (Sept. 1977) p. 100.

126. A. Einstein in Ref. 14, p. 86, 667-668; in Ref. 11, Letters no. 48, 52 (1926); P. Speziali (ed.): Albert Einstein, Michele Besso, Correspondence 1903-1955, Hermann, Paris (1972) Letter no. 172 (1950) and many earlier letters; It seems not needless to point out that realism and determinism are independent of one another; see e.g. Bunge, Ref. 121, p. 278, 279, Ref. 15, p. 210.

127. See Ref. 26, 108, 111, and literature quoted there, also the book of de Broglie, Ref. 128.

128. L. de Broglie: Non-linear wave mechanics, a causal interpretation, Elsevier Publ. Comp., Amsterdam (1960).

129. E. T. Jaynes, American Scientist (Burlington, Vt.) $\underline{53}$, 136A (1965).

130. See for example S. Weinberg: Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity, Wiley, New York (1972) p. 489-491.

131. A. A. Broyles, Phys. Rev. $\underline{D1}$, 979 (1970).

132. See also W. Pauli, in Ref. 34, p. 139.

133. This suggests the question of whether in the relativistic case the Klein-Gordon density (2.23) with which $\int \rho \, d^3x$ is a Lorentz-scalar should perhaps be replaced by the quantity $\rho' = (\psi^*\psi + \lambda_C^2 \dot{\psi}^*\dot{\psi}/c^2 + \lambda_C^2 \nabla\psi\nabla\psi^*)/2$. This quantity obeys the continuity equation $\partial\rho'/\partial t + \text{div} \, \vec{j} = 0$ with $\vec{j} = -\lambda_C^2(\dot{\psi}\nabla\psi^* + \dot{\psi}^*\nabla\psi)/2$. In the non-relativistic limit ρ' and $\vec{j}$ go over into the Schrödinger expression, and $\int \rho' d^3x$ transforms as the zeroth component of a four-vector. See Pauli, Ref. 34, p. 139, footnote 3.

134. A manifestation of this basic feature has also been noticed in axiomatic quantum field theory. The generalized free fields of Greenberg, Ref. 135, which essentially are weighted continuous superpositions of free Klein-Gordon fields belonging to different mass values have been shown by Haag and Schroer, Ref. 136, to violate "primitive causality" by which these authors just mean that observations in an arbitrarily small time interval are sufficient to determine the state at all times.

135. O. W. Greenberg, Ann. Phys. (N.Y.) 16, 158 (1961).
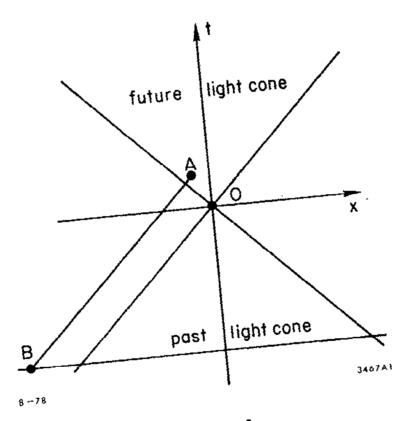
136. R. Haag, B. Schroer, J. Math. Phys. 3, 248 (1962).

137. N. N. Bogoliubov, D. V. Shirkov: Introduction to the Theory of
Quantized Fields, Wiley, New York (1959), p. 147-150; K. Nishijima:
Fields and Particles, Benjamin, New York (1974), p. 36.

138. M. Abramowitz, I. A. Stegun: Handbook of Mathematical Functions,
Dover, New York (1965), p. 378.

139. J. J. Sakurai, Ann. Phys. (New York) 11, 1 (1960); Currents and
Mesons, University of Chicago Press, Chicago (1969).

FIGURE CAPTION

1. The event at A can be influenced by an event at B lying outside the observer's O past light cone (cone of maximal knowledge).

Fig. 1