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QUARKS, QUATERNIONS, AND WEAKLY COUPLED GAUGE FIELDS*

B.F.L. Ward

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

and

Department of Physics
Purdue University, West Lafayette, Indiana 47907

ABSTRACT

Using the ring of complex quaternions, the more active view of weakly coupled gauge fields is formulated in such a way that, in the SU(2) × U(1) model of Salam, Ward, and Weinberg, for example, $\Delta S \neq 0$, $\Delta Q \doteq 0$ effects are properly suppressed without the appearance of a new quark field in the Lagrangian--only p, n, and λ quark fields appear. Here, S is strangeness and Q is electric charge. The success of the more active view of the SU(2) × U(1) model in describing $\Gamma(K_L \to \bar{\mu}\mu)/\left(\left(m_{K_L}-m_{K_S}\right)/m_K\right)$, charmed particles, ψ/J, ψ', Υ, Υ', etc., in terms of p, n, and λ quarks is not disturbed by this new formulation. Here, $m_K = \left(m_{K_L} + m_{K_S}\right)/2$ and m_a is the mass of a.

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I. INTRODUCTION

In a previous work, we pointed out that, in the more active view[1] of the gauge theoretic formulation of weak and electromagnetic interactions, it was possible, in the SU(2) × U(1) model[2] for example, to explain, entirely in terms of p, n, and λ quarks, the relationship between the rate for $K_L \to \bar{\mu}\mu$ and the K_L-K_S mass difference in the form $\left(m_{K_L} - m_{K_S}\right)/m_K$, where $m_K = \left(m_{K_L} + m_{K_S}\right)/2$ and m_a is the mass of a, $a = K_L, K_S$. The explanation resulted from taking the hadronic Lagrangian density to be as illustrated in the following expression for the Lagrangian density $\mathscr{L}$ of the SU(2) × U(1) model of Ref. 2:

$$\mathscr{L} = \sum_{q=\{p_n,p_\lambda,n,\lambda\}} \bar{L}_q i\not{D} L_q + \sum_{a=\{n,\lambda\}} \left(\bar{L}_{p_a} i\not{D} L_a + \text{h.c.}\right)$$

$$+ \sum_{a=\{n,\lambda\}} \left[\bar{p}_{a,R}(i\not{\partial} + (2g'/3)\not{B})p_{a,R} + \bar{a}_R(i\not{\partial} - (g'/3)\not{B})a_R\right]$$

$$+ \xi\left(\bar{L}_{p_n} i\not{D} L_\lambda + \bar{L}_{p_\lambda} i\not{D} L_n + \text{h.c.}\right) + \ldots \quad , \tag{1}$$

where h.c. denotes the hermitian conjugate,

$$p = p_n \cos\theta_C + p_\lambda \sin\theta_C \quad , \tag{2a}$$

$$L_a = \begin{cases} L\begin{pmatrix} a \\ 0 \end{pmatrix} \quad , & a = p_n, p_\lambda \\ L\begin{pmatrix} 0 \\ a \end{pmatrix} \quad , & a = n, \lambda \end{cases} \tag{2b}$$

with

$$L = (1-\gamma_5)/2 \quad , \tag{2c}$$

$$a_R = (1+\gamma_5)a/2 \quad , \quad a = \{n, \lambda, p_n, p_\lambda\} \quad , \tag{2d}$$

and

$$iD_\mu = i\partial_\mu - g\vec{\tau}\cdot\vec{A}_\mu + (g'/6)B_\mu \tag{2e}$$

where $\vec{A}$ are the usual SU(2) gauge fields, B is the U(1) gauge field, and $\vec{T}=\vec{\sigma}/2$ where $\vec{\sigma}$ are the Pauli matrices. As usual, g and g' are the respective SU(2) and U(1) coupling constants. The ... in (1) represents the Yukawa couplings of the quarks to the usual Higgs doublet in the model in the convention of Ref. 1 and the remaining part of $\mathscr{L}$. This remaining part of the Lagrangian (the lepton-boson part) is also taken in the standard form, as illustrated in Ref. 1 for example. The single parameter ξ was then shown in Ref. 1, in the free quark approximation, to be consistent with both the observed rate for $K_L \to \bar{\mu}\mu$ as well as $\left(m_{K_L} - m_{K_S}\right)/m_K$, to ten percent. For this reason, we do not take (1) lightly.

However, the motivation for constructing (1) was to explain the suppression of unwanted $\Delta S \neq 0$, $\Delta Q = 0$ effects, where S is the strangeness and Q is electric charge, without using new fields. Indeed, recently, it has been shown[3] that, in the theory of differential dispersion relations,[4] the ratio $m_{\psi/J}/m_D$ is computable to 1% in models of the hadrons involving only physical p, n, and λ quarks. Here, $m_{\psi/J}$ is the mass of the ψ/J particle[5] and m_D is the mass of the "so-called" charmed particles[6] D^0, D^+ in the SU(3) symmetric limit. Thus, it is an interesting question as to whether or not the charmed physical hadrons, as defined experimentally by their decay modes and their masses, require the use of a charmed quark in the hypothetical underlying quark field theory. The Lagrangian (1) was introduced to address this question. But, even though physical hadrons, according to (1), only consist of p, n, and λ quarks, the appearance of p_n and p_λ in (1) may lead some to believe that a new physical quark has been introduced after all. Our purpose here, therefore, is to reformulate (1) in a manner in which it only refers to (p, n, λ), explicitly.

Before turning to this reformulation, let us emphasize that the question of the ultimate use of a physical new quark in the problem of the description of hadrons by quarks will of course not be addressed here. But, we do wish to mention that the ability to compute[7] m_Υ from $m_{\psi/J}$ within 2% in the theory of differential dispersion relations,[4] without using a new quark, tends to indicate that new quarks are not necessary for the description of present day hadrons. Here, m_Υ is the mass of the upsilon particle.[8] Evidently, this indication deviates considerably from the lore.[9] Quite independent of the outcome of the discussions about the meaning of the new particles, we feel that our reformulation of (1) may be of interest in its own right.

Our work is presented as follows. In the next section, we give the desired reformulation of (1). Then, in Section III, we give a simple realization of the ideas in Section II.

II. QUATERNIONIC FORMULATION OF THE SU(2) × U(1) MODEL

Here, we proceed as follows. First, recall that, in arriving at (1), we looked at the one-loop Feynman diagrams in the free quark approximation for $K_L \to \bar{\mu}\mu$ and concluded that the rate for this process was so small that it must be very difficult for an $\bar{n}p$ vertex to be reached, by direct propagation of p, from a $\bar{p}\lambda$ vertex or for a $\bar{\lambda}p$ vertex to be reached, by direct propagation of p, from a $\bar{p}n$ vertex. This suggested that the n and λ aspects of p were orthogonal, as illustrated by (2a).

However, there is another way to express the empirical orthogonality of the n and λ aspects of the p—we can use quaternions. Specifically, we write $\mathscr{L}$ as (the ε_{abc} are the SU(2) structure constants)

$$i\mathscr{L} = \sum_{q=\{p,n,\lambda\}} i\left(\bar{L}_q i\not{D} L_q + \bar{q}_R(i\not{D}')q_R\right) + \cos\theta_C s_n\left(\bar{L}_p i\not{D} L_n + \text{h.c.}\right)$$
$$+ \sin\theta_C s_\lambda\left(\bar{L}_p i\not{D} L_\lambda + \text{h.c.}\right) - \frac{i}{4}\left[\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu - g\varepsilon_{\rightarrow bc} A^b_\mu A^c_\nu\right]^2$$
$$- \frac{i}{4}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right)^2 + \ldots \quad , \tag{3}$$

where s_n, s_λ are members of a quaternion ring with

$$s_n s_\lambda + s_\lambda s_n = 0 \ , \quad s_n^2 = is_n \ , \quad s_\lambda^2 = is_\lambda \ , \tag{4}$$

and the covariant derivatives are, again,

$$iD_\mu = i\partial_\mu - g\vec{\tau}\cdot\vec{A}_\mu + (g'/6)B_\mu \tag{5}$$

$$iD'_\mu = i\partial_\mu + Q_f g' B_\mu \ , \quad f=p,n,\lambda, \tag{6}$$

with g and g' again respectively equal to the SU(2) and U(1) coupling constants so that $Q_f|e_R|$ is the electric charge of fermion f; e_R is the renormalized electron charge. Here, ... again represents the remaining part of the Lagrangian involving leptons and the usual scalar fields (the Higgs doublet).[2] This latter part of $i\mathscr{L}$ we will continue to take after the convention of Refs. 1 and 2.

Thus, the space of numbers is contained in the ring of complex quaternions over the field of the usual complex numbers a+bi, a, b real, with the basis of the quaternions given by $(1,j,k,\ell)$:

$$j^2=k^2=\ell^2=-1 \ , \quad jk=\ell \ , \quad jk=-kj \ , \quad j\ell=-\ell j \ , \quad k\ell=-\ell k \ ; \tag{7}$$

the complex numbers commute with j,k,ℓ.

The Green's functions are most simply obtained from the Feynman path-integral: The generating functional iZ for connected Green's functions is (suppressing lepton and boson sources)

$$\exp[iZ] = \int (\mathscr{D}p\,\mathscr{D}\bar{p}\,\mathscr{D}n\,\mathscr{D}\bar{n}\,\mathscr{D}\lambda\,\mathscr{D}\bar{\lambda}\,\mathscr{D}\{\vec{A}\}\,\mathscr{D}\{B\}\ldots) \exp\left\{ i\int d^4x \left[\mathscr{L} + \sum_{a=\{p,n,\lambda\}} (\bar{\eta}_a a + \bar{a}\eta_a) \right] \right\} \tag{8}$$

where ... represents the measure over the remaining field variables and over the gauge constraint. As usual, η_a, $\bar{\eta}_a$ are the sources of $\bar{a}$ and a, respectively.

Since the coefficients of the quaternions in the arguments of the exponential on the RHS of (8) are "real" in our path-space field theory formulation (and would be hermitian in the operator field theory formulation), the functional iZ necessarily corresponds to a unitary, relativistically invariant set of transition amplitudes. It should be compared with the work of Edmonds and others on more general quaternion formalisms,[10] wherein one attempts to represent the Minkowski space and its Lorentz group on a complex quaternion algebraic structure or wherein one considers a quaternionic generalization of ordinary quantum mechanics. We do not find these more comprehensive algebraic structures necessary or desirable for our purposes here. Only linear combinations of s_n and s_λ can occur in (8).

Indeed, the suppression of unwanted $\Delta S \neq 0$, $\Delta Q = 0$ effects at the one-loop level is now an immediate consequence of the fact that

$$s_n s_\lambda + s_\lambda s_n = 0 \quad . \tag{9}$$

For, to second order in the flavor changing interactions in (3) one has

$$\frac{1}{2!}\int d^4x_1 \Big[\cos\,\theta_C s_n\Big(\bar{L}_p(x_1)i\not{\partial}L_n(x_1) + h.c.\Big) + \sin\,\theta_C s_\lambda\Big(\bar{L}_p(x_1)i\not{\partial}L_\lambda(x_1) + h.c.\Big)\Big]$$

$$\int d^4x_2 \Big[\cos\,\theta_C s_n\Big(\bar{L}_p(x_2)i\not{\partial}L_n(x_2) + h.c.\Big) + \sin\,\theta_C s_\lambda\Big(\bar{L}_p(x_2)i\not{\partial}L_\lambda(x_2) + h.c.\Big)\Big]$$

$$= \frac{1}{2!}\int d^4x_1 d^4x_2 \Big[s_n^2 \cos^2\,\theta_C\Big(\bar{L}_p(x_1)i\not{\partial}L_n(x_1) + h.c.\Big)\Big(\bar{L}_p(x_2)i\not{\partial}L_n(x_2) + h.c.\Big)$$

$$+ s_\lambda^2 \sin^2\,\theta_C\Big(\bar{L}_p(x_1)i\not{\partial}L_\lambda(x_1) + h.c.\Big)\Big(\bar{L}_p(x_2)i\not{\partial}L_\lambda(x_2) + h.c.\Big)$$

$$+ \sin\,\theta_C \cos\,\theta_C\Big\{s_n s_\lambda\Big(\bar{L}_p(x_1)i\not{\partial}L_n(x_1) + h.c.\Big)$$

$$\Big(\bar{L}_p(x_2)i\not{\partial}L_\lambda(x_2) + h.c.\Big) + s_\lambda s_n\Big(\bar{L}_p(x_1)i\not{\partial}L_\lambda(x_1) + h.c.\Big)$$

$$\Big(\bar{L}_p(x_2)i\not{\partial}L_n(x_2) + h.c.\Big)\Big\}\Big] \quad . \tag{10}$$

Interchanging the labels 1 and 2 on the x_i in one of the two terms proportional to $\sin\,\theta_C \cos\,\theta_C$ in (10) shows that the possible second order $\Delta S \neq 0$, $\Delta Q = 0$ interaction is contained in

$$\frac{1}{2!}\sin\,\theta_C \cos\,\theta_C\,(s_n s_\lambda + s_\lambda s_n)\int d^4x_1\int d^4x_2\Big[\Big(\bar{L}_p(x_1)i\not{\partial}L_n(x_1) + h.c.\Big)$$

$$\Big(\bar{L}_p(x_2)i\not{\partial}L_\lambda(x_2) + h.c.\Big)\Big]$$

$$= 0 \quad . \tag{11}$$

Thus, there is no unwanted second order $\Delta S \neq 0$, $\Delta Q = 0$ transition.

Looking at fourth order, we have, using (10) and (11),

$$\frac{1}{4!}\int d^4x_1 d^4x_2 d^4x_3 d^4x_4 \Big[s_n^2 \cos^2\theta_C \left(\bar{L}_p(x_1) i\not{\partial} L_n(x_1) + \text{h.c.}\right)\left(\bar{L}_p(x_2) i\not{\partial} L_n(x_2) + \text{h.c.}\right)$$

$$+ s_\lambda^2 \sin^2\theta_C \left(\bar{L}_p(x_1) i\not{\partial} L_\lambda(x_1) + \text{h.c.}\right)\left(\bar{L}_p(x_2) i\not{\partial} L_\lambda(x_2) + \text{h.c.}\right)\Big]$$

$$\Big[s_n^2 \cos^2\theta_C \left(\bar{L}_p(x_3) i\not{\partial} L_n(x_3) + \text{h.c.}\right)\left(\bar{L}_p(x_4) i\not{\partial} L_n(x_4) + \text{h.c.}\right)$$

$$+ s_\lambda^2 \sin^2\theta_C \left(\bar{L}_p(x_3) i\not{\partial} L_\lambda(x_3) + \text{h.c.}\right)\left(\bar{L}_p(x_4) i\not{\partial} L_\lambda(x_4) + \text{h.c.}\right)\Big]$$

$$= \frac{1}{4!}\int d^4x_1 d^4x_2 d^4x_3 d^4x_4 \Big[s_n^4 \cos^4\theta_C \prod_{j=1}^{4} \left(\bar{L}_p(x_j) i\not{\partial} L_n(x_j) + \text{h.c.}\right)$$

$$+ s_\lambda^4 \sin^4\theta_C \prod_{j=1}^{4} \left(\bar{L}_p(x_j) i\not{\partial} L_\lambda(x_j) + \text{h.c.}\right)$$

$$+ s_n^2 s_\lambda^2 \cos^2\theta_C \sin^2\theta_C \left(\bar{L}_p(x_1) i\not{\partial} L_n(x_1) + \text{h.c.}\right)\left(\bar{L}_p(x_2) i\not{\partial} L_n(x_2) + \text{h.c.}\right)$$

$$\left(\bar{L}_p(x_3) i\not{\partial} L_\lambda(x_3) + \text{h.c.}\right)\left(\bar{L}_p(x_4) i\not{\partial} L_\lambda(x_4) + \text{h.c.}\right)$$

$$+ s_\lambda^2 s_n^2 \sin^2\theta_C \cos^2\theta_C \left(\bar{L}_p(x_1) i\not{\partial} L_\lambda(x_1) + \text{h.c.}\right)\left(\bar{L}_p(x_2) i\not{\partial} L_\lambda(x_2) + \text{h.c.}\right)$$

$$\left(\bar{L}_p(x_3) i\not{\partial} L_n(x_3) + \text{h.c.}\right)\left(\bar{L}_p(x_4) i\not{\partial} L_n(x_4) + \text{h.c.}\right)\Big] \quad . \tag{12}$$

However, interchanging the labels on the x_j in one of the two terms in (12) proportional to $\sin^2\theta_C \cos^2\theta_C$ shows that the possible $\Delta S \neq 0$, $\Delta Q = 0$ interaction at fourth order is proportional to

$$s_n^2 s_\lambda^2 + s_\lambda^2 s_n^2 = is_n is_\lambda + is_\lambda is_n = (-)(s_n s_\lambda + s_\lambda s_n) = 0 \quad . \tag{13}$$

Thus, there is no fourth order $\Delta S \neq 0$, $\Delta Q = 0$ interaction. It follows that there are no unwanted one-loop $\Delta S \neq 0$, $\Delta Q = 0$ interactions.

The absence of $\Delta S \neq 0$, $\Delta Q = 0$ transitions to all orders is easily established by induction from (4) and (11). This, as we pointed out earlier, is too restrictive, since, for example, the mass difference $m_{K_L} - m_{K_S}$ is not identically zero--rather, it is suppressed. However, just as we showed in Ref. 1, the additional interaction density $\mathscr{L}_r$ given by

$$\mathscr{L}_r = \xi\left[\bar{L}_p i\not{\partial} L_n + \bar{L}_p i\not{\partial} L_\lambda + h.c.\right] \tag{14}$$

with $\xi^2 \doteq 4 \times 10^{-6}$ allows one to relate $\Gamma(K_L \to \bar{\mu}\mu)$ to $\left(m_{K_L} - m_{K_S}\right)/m_K$ to ten percent, where $\Gamma(K_L \to \bar{\mu}\mu)$ is the rate for $K_L \to \bar{\mu}\mu$.

At this point, the reader may wonder if there exist two quaternions s_n and s_λ which have the properties (4). We construct them as follows. Writing

$$\begin{aligned} s_n &= \left(a_0 i + a_1 j + a_2 k + a_3 \ell\right)/2 \\ s_\lambda &= \left(b_0 i + b_1 j + b_2 k + b_3 \ell\right)/2 \end{aligned} \tag{15}$$

where a_α, b_α are real numbers, we see that

$$s_n s_\lambda + s_\lambda s_n = 0$$

$$\Rightarrow \quad \sum_{\alpha=0}^{3} a_\alpha b_\alpha = 0 \ , \quad b_0 a_1 + a_0 b_1 = 0 \ , \quad b_0 a_2 + b_2 a_0 = 0 \ , \quad b_0 a_3 + a_0 b_3 = 0 \ . \tag{16}$$

Further,

$$s_n^2 = i s_n \ , \quad s_\lambda^2 = i s_\lambda$$

$$\Rightarrow \quad a^2/2a_0 = a_0 \ , \quad a_0 = 1 \ , \quad b^2/2b_0 = b_0 \ , \quad b_0 = 1 \ , \tag{17}$$

where

$$a^2 = \sum_{\alpha=0}^{3} a_\alpha a_\alpha \ , \quad b^2 = \sum_{\alpha=0}^{3} b_\alpha b_\alpha \ .$$

Solving (16) and (17) we find

$$a = (1,\hat{n}) \ , \quad b = (1,-\hat{n}) \ , \tag{18}$$

where $\hat{n} = (a_1,a_2,a_3)$ is a unit 3-vector:

$$\hat{n}^2 = a_1^2 + a_2^2 + a_3^2 = 1 \ . \tag{19}$$

The reason for the choice (15) for s_n and s_λ is that we wish to maintain, for example, the time reversal property that the pure imaginary number i goes to -i under a time reversal transformation so that the more general pure imaginary "numbers" s_n and s_λ will be taken to have the same property: $s_a \to -s_a$, $a=n,\lambda$, under time reversal. The norms of s_n, s_λ may be taken to be

$$\| s_n \| = \| s_\lambda \| = 1 \tag{20}$$

with the agreement that

$$\| z_0 + z_1 j + z_2 k + z_3 \ell \|^2 = \sum_{\alpha=0}^{3} |z_\alpha|^2 , \tag{21}$$

where z_α are ordinary complex numbers and $|z_\alpha|$ is the usual norm on complex numbers. Obviously, the norm (21) is not a morphism of multiplication. But, this appears to be of little consequence, physically.

In closing this section, we note that, like i, s_n and s_λ satisfy the equation

$$z^2 = iz \quad . \tag{22}$$

This appears to be sufficient to make the quantum field theory sensible. This last remark is made more manifest by the explicit construction in the next section.

III. EXPLICIT REALIZATION

We wish now to be more explicit about effecting computations with s_n and s_λ. Specifically, it is well known that $(1,j,k,\ell)$ may be realized by $(1, i\sigma_1, i\sigma_2, -i\sigma_3)$, where σ_i are the Pauli matrices. Taking, for example,

$$\hat{n} = (0,0,-1) \tag{23}$$

in (18) we see that, in the representation

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{24}$$

we have

$$s_n = \frac{1}{2}(i+i\sigma_3) = \frac{i}{2}(1+\sigma_3) = i\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{25}$$

$$s_\lambda = \frac{1}{2}(i-i\sigma_3) = \frac{i}{2}(1-\sigma_3) = i\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{26}$$

To make the connection with transition amplitudes we replace the usual plane waves

$$U_a = \frac{e^{-ik\cdot x}}{(2\pi)^{3/2}} \frac{u(k)}{\sqrt{k_0/m_a}} \tag{27}$$

with

$$U'_a = U_a\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \text{for } a=n,\lambda \tag{28}$$

and

$$U'_a = U_a\begin{pmatrix} \cos\theta_C \\ \sin\theta_C \end{pmatrix}, \quad \text{for } a=p, \tag{29}$$

where, in choosing (29), we have made the replacements $\cos\theta_C s_n \to s_n$, $\sin\theta_C s_\lambda \to s_\lambda$ in (3) for convenience. The analogous definitions hold for the antiparticles. Here,

$$k_0 = \left|\left(\vec{k}^2 + m_a^2\right)^{1/2}\right| \tag{30}$$

and u(k) is the usual positive energy spinor solution of the free Dirac equation for a particle of mass m_a and four momentum k in the convention of Bjorken and Drell,[11] for example, $a=n,\lambda,p$. The obvious scalar product is then understood on the space of s_n, s_λ: For n, for example, defining

$\bar{U}'_n = \bar{U}_n(0\ 1)$ in the standard notation of Ref. 11 for $\bar{U}_n$, we have

$$\bar{U}'_n U'_n = \bar{U}_n U_n (0\ 1) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \bar{U}_n U_n \text{ , etc.} \tag{31}$$

The tree and one-loop phenomenology associated with (14) and (25)-(31) is not unequivocally inconsistent with observation, as one can see from Refs. 1 and 12. We would like to emphasize that this rather simple realization of s_n, s_λ may not suffice for the ultimate description of nature, although it appears to be sufficient for present day observations.

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