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AMPLITUDE ANALYSIS OF Y*(1385) PRODUCTION IN THE LINE-REVERSED REACTIONS: $\pi^+p \to K^+Y^*(1385)$ AND $K^-p \to \pi^-Y^*(1385)$ AT 7 AND 11.5 GeV/c†

J. Ballam, J. Bouchez,[a] J. T. Carroll, C. V. Cautis,
G. B. Chadwick, V. Chaloupka, R. C. Field, D. R. Freytag,
R. A. Lewis,[b] M. N. Minard, K. C. Moffeit, and R. A. Stevens[a]

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305 U.S.A.

P. A. Baker, J. S. Chima, P. J. Dornan, D. J. Gibbs,
G. Hall, D. B. Miller, T. S. Virdee, and A. P. White

Imperial College
London, England

ABSTRACT

We have measured for the first time in one detector the complete decay angular distribution on $Y^* \to \Lambda\pi$, $\Lambda \to p\pi^-$ in the two line-reversed reactions: $\pi^+p \to K^+Y^*(1385)$ and $K^-p \to \pi^-Y^*(1385)$. Our experiment was conducted in the SLAC 1 m rapid cycling bubble chamber (15 Hertz) triggered by electronic detectors and an online algorithm. The Extended Maximum Likelihood method was used to obtain the transversity amplitudes of the Y*(1385). Our results are in good agreement with both the quark model and Stodolsky-Sakurai model predictions. Finite helicity non-flip contributions at the Y* vertex observed in our data can be associated with double quark scattering in the forward direction.

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(a) Present address: DPhPe, CEN-Saclay, B.P. No. 2, F-91190, Gif-Sur-Yvette, France.
(b) Present address: Physics Department, Michigan State University, East Lansing, Michigan 48824 U.S.A.

1. INTRODUCTION

We present in this paper results from a model independent analysis of Y*(1385) production in the line-reversed reactions:

$$\pi^+ p \rightarrow K^+ Y^*(1385) \qquad (1)$$

$$K^- p \rightarrow \pi^- Y^*(1385) \qquad (2)$$

at two energies, 7 and 11.5 GeV/c.

The present experiment is the first one to measure in a single detector the complete decay angular distribution of the Y*(1385) for both reactions (1) and (2). Some of the results relevant to Exchange Degeneracy (EXD) predictions have been published elsewhere.[1,2] In the following we present details of the amplitude structure of the Y*(1385).

In the next section we give a short description of the experimental technique and in Section 3 we describe the method used in extracting production amplitudes. The results are presented and discussed in terms of the additive quark model and Regge phenomenology in Section 4. Our conclusions are summarized in Section 5.

2. EXPERIMENTAL TECHNIQUE

The experiment was conducted at the SLAC Hybrid Facility,[2] which consists of the SLAC 1 m rapid cycling bubble chamber (15 Hertz) triggered by data from a downstream spectrometer. The electronic data was processed online by a DGC-840 computer. The electronic fast trigger was given by an incoming π^+ (K^-) and a fast forward K^+ (π^-) as defined by pulse height analysis of the upstream and downstream Cerenkov counters. The triggering tracks were reconstructed online using thirteen planes of Proportional Wire Chambers (PWC). The online algorithm triggered the bubble chamber camera lights after eliminating low momentum tracks, interactions outside the fiducial volume and noninteracting beam tracks. For the K^- run, a μ-hodoscope behind 1 m of iron was used to reduce the triggering rate from K^- decays. The experimental setup and the trigger are discussed in more detail elsewhere.[2,3]

The film was scanned for all events with a visible strange particle decay. These events were measured in three views on precision measuring tables and reconstructed by our geometry program. Tracks passing through the downstream system were constrained to fit the PWC data, giving a momentum resolution of $\sim$1.5%. Events belonging to reactions (1) and (2) were identified by seven constraint kinematic fits at the primary and strange particle decay vertices. The mass resolution of constrained events is ±8 MeV in the Y*(1385) region.

The detector has $\sim$100% acceptance in the interval $.02 < t_{min}-t < .4$ GeV^2 at 7 GeV/c and in the interval $.01 < t_{min}-t < 1.0$ GeV^2 at 11.5 GeV/c. The small angle loss is due to the triggering algorithm which eliminates noninteracting beam tracks. The large angle limitation is due to the geometry of the detector. In addition, we find small losses in the Λ sample, which bias some of the angular distributions: asymmetric vees in which one of the tracks (mostly π^-) is too short to be measured properly and vees with a small opening angle which are misidentified as γ conversions. These losses amount to $\lesssim$3% and have been taken into account when fitting the transversity amplitudes.

The sample of events used in the present analysis is described in Table I. The distributions of $\Lambda\pi^+$ invariant masses are shown in Fig. 1. The data shows a strong Y*(1385) peak over a background level less than 10% of the signal.

3. METHOD OF ANALYSIS

We have made a model independent analysis of Y*(1385) production in reactions (1) and (2). The six independent variables which we choose to describe the four-particle final state are: t, the square of the four-momentum transfer from the beam to the fast forward particle (K^+ or π^-), $m_{\Lambda\pi^+}$, the invariant mass of the $\Lambda\pi^+$ system, and Ω, a set of four polar angles describing the cascade decay: $Y^* \to \Lambda\pi^+$, $\Lambda \to p\pi^-$, as defined below.

For a fixed incoming energy and fixed region of momentum transfer, we write the probability density function as an incoherent sum of two terms:

$$\omega = C_1 \cdot BW_{1385}(m_{\Lambda\pi^+}) \cdot W(\Omega) + C_2 \tag{3}$$

where C_1 and C_2 are constants, $BW(m_{\Lambda\pi^+})$ is a Breit-Wigner propagator and $W(\Omega)$ is the complete decay angular distribution of the Y*(1385). For the Breit-Wigner function we use[4]:

$$BW(m) = \frac{m}{p} \frac{\Gamma}{\left(m^2 - m_0^2\right)^2 + m_0^2 \Gamma^2}$$

$$\Gamma = \Gamma_0 \left(\frac{p}{p_0}\right)^{2\ell+1} \cdot \frac{a^2 + p_0^2}{a^2 + p^2} \tag{4}$$

where m_0 and Γ_0 are the mass and width of the Y*(1385), p is the momentum of the Λ in the Y* rest frame, ℓ is the orbital angular momentum of the $\Lambda\pi^+$ system (ℓ=1 for the Y*(1385)) and a is constant (we have used a=.1 GeV). The decay angular distribution of the Y* is measured in the transversity frame defined with the z axis along the normal to the production plane: $\hat{z} = \hat{B} \times \hat{M}$ where $\hat{B}$ is the direction of the beam and $\hat{M}$ is the direction of the system recoiling against the $\Lambda\pi^+$. The y axis is taken along the direction of the $\Lambda\pi^+$ system in the overall center-of-mass frame. The Λ decay is measured in the frame with $\hat{z}'=\hat{\Lambda}$ in the Y* rest frame and $\hat{y}'=\hat{z}\times\hat{z}'$. With this choice of axes, the decay angular distribution of the Y*(1385) can be written as[5,6]:

$$W(\Omega) = \sum_{\substack{\lambda,\mu \\ \lambda'\mu'}} D^{*3/2}_{\lambda\mu}(\phi,\theta)\cdot D^{3/2}_{\lambda'\mu'}(\phi,\theta)\cdot M_{\mu\mu'}(\theta'\phi')\cdot\rho_{\lambda\lambda'}$$

$$M_{\pm\frac{1}{2}\pm\frac{1}{2}} = \frac{1}{4\pi}(1 \pm \alpha\cos\theta') \qquad (5)$$

$$M_{\pm\frac{1}{2}\mp\frac{1}{2}} = \frac{\alpha}{4\pi}\sin\theta'\; e^{\pm i\phi'}$$

where λ is the transversity of the Y*(1385) and μ is the helicity of the Λ and α is the Λ decay asymmetry parameter (α=.647). The spin density matrix ρ_{ij} can be written in terms of transversity amplitudes T_{ij} as:

$$\rho_{ij} = \sum_k T_{ik}\cdot T^*_{jk} \qquad (6)$$

where k is the transversity of the incoming proton. This shows that the maximum rank of ρ is two.

Parity conservation in the production process requires for the Y*(1385):

$$T_{ij} = (-1)^{i-j}\, T_{ij} \qquad (7)$$

This leaves four non-zero transversity amplitudes:

$$T_{3/2\ -1/2},\quad T_{-1/2\ -1/2},\quad T_{1/2\ 1/2},\quad T_{-3/2\ 1/2} \qquad (8)$$

The real and imaginary parts of the transversity amplitudes are parameters in the fit. This parametrization insures the positivity and proper rank of ρ. By relaxing the rank condition, we have verified that the data does not require a rank greater than two even when integrated over large intervals of momentum transfer.

From the decay angular distributions, we can determine the absolute values of the four amplitudes and two of the relative phases. The overall phase and the phase between proton spin "up" and "down" cannot be measured in this experiment. The unknown phases were fixed by imposing $\mathrm{Im}\, T_{1/2\ 1/2} = \mathrm{Im}\, T_{-1/2\ -1/2} = 0$.

We use the extended maximum likelihood method[7] to estimate from the data the total amount of Y*(1385) production and the contribution of the four transversity amplitudes as functions of momentum transfer. We write the log-likelihood function as:

$$\log L = \sum_{i=1}^{N} \log \omega_i - \int \omega \, d\tau \qquad (9)$$

where N is the total number of events in the sample and the integral is performed over the same region of phase-space as the one used to select the experimental sample. This method insures the normalization of each amplitude over the phase-space region selected for the fit. All cuts imposed on the experimental sample in order to eliminate the biased regions are taken into account in calculating the integral.

The maximization of the log-likelihood function was done using the program OPTIME.[8] After each fit we have plotted the result of the fit on top of different experimental distributions and found good agreement with the data (see Fig. 2).

After each fit we also calculated the spin polarization of the Y*(1385) in the transversity frame and spin density matrix elements in the s-channel helicity frame. We define the polarization of a state with total angular momentum J as:

$$P_J = \frac{1}{J} \sum m \cdot \rho_{mm} \qquad (10)$$

where m is the projection of J along the quantization axis. The helicity density matrix elements were calculated from the transversity density matrix according to the rotation:

$$\rho^H_{mn} = \sum_{i,j} D^{3/2}_{mi}(-R) \cdot D^{3/2}_{jn}(R) \cdot \rho^T_{ij}$$

where R = $(\pi/2, \pi/2, \pi/2)$. All errors were calculated by propagation of the complete covariance matrix.

4. RESULTS OF THE AMPLITUDE ANALYSIS

The fits described in the previous section are compared to the $\Lambda\pi$ invariant mass distributions in Fig. 1. To obtain a good description of the mass spectrum up to 2 GeV, we tried several parametrizations for the background. The one shown in Fig. 1 includes two simple Breit-Wigner functions in the $m_{\Lambda\pi^+} \sim 1.7$ GeV region. The results for the Y*(1385) do not depend on the parametrization used for the background.

For the amplitude analysis, we have selected the mass range $m_{\Lambda\pi^+} < 1.55$ GeV. In this region an isotropic phase-space as given in Eq. (3) is an adequate background.

The absolute values of the four transversity amplitudes are shown for reactions (1) and (2) at 7 GeV/c in Fig. 3(a) and at 11.5 GeV/c in Fig. 3(b).

The Stodolsky-Sakurai[9] and additive quark models[10] predict for each of reactions (1) and (2) independently that double-flip transversity amplitudes $T_{3/2\ -1/2} = T_{-3/2\ 1/2} = 0$ and that the remaining two amplitudes should be equal ($T_{1/2\ 1/2} = T_{-1/2\ -1/2}$).

These predictions are indicated as dashed lines in Fig. 3. The data for both reactions and at both energies are in agreement with quark model predictions. The approximate symmetry of the Y*(1385) spin density matrix about the second diagonal, is also in agreement with strong EXD predictions.

The only significant non-zero double-flip values are at small t in the 11.5 GeV/c data similar to what has been observed at 4.2 GeV/c in K^-p interactions.[6] This effect may be associated with a finite helicity non-flip contribution to the Y*(1385) vertex. At $t=t_{min}$ all helicity flip amplitudes go rigorously to zero. Any remaining non-flip contribution forces the transversity amplitudes to the values:

$$|T_{-3/2\ 1/2}| = |T_{3/2\ -1/2}| = \sqrt{\frac{3}{8}}\ ; \quad |T_{1/2\ 1/2}| = |T_{-1/2\ -1/2}| = \sqrt{\frac{1}{8}}$$

in the forward direction. The trend in our data is in qualitative agreement with these values. In quark model language the non-zero values of $T_{3/2\ -1/2}$ and $T_{-3/2\ 1/2}$ imply double quark scattering which is expected to contribute at small angles.[11] The 7 GeV/c data, however, show no departure from simple quark model predictions in the forward direction but do not rule out the behavior seen at 11.5 GeV/c and 4.2 GeV/c.

Additional confirmation of helicity non-flip contribution to the Y*(1385) vertex comes from the study of the differential cross section and the spin density matrix elements in the helicity frame. Fits to the Y*(1385) differential cross sections indicate a 16-20% helicity non-flip contribution.[1,12] The s-channel helicity frame matrix elements shown in Fig. 4 are also consistent with an increased non-flip contribution in the forward direction as seen in the larger value of ρ_{11} at low t values. We have verified that the effects observed in the forward direction in the transversity amplitudes and in the helicity density matrix elements agree in magnitude and t-dependence with the helicity non-flip contribution estimated from the differential cross sections.[1]

The double-flip amplitudes which show the strongest deviation from zero are the $T_{3/2\ -1/2}$ in reaction (1) and $T_{-3/2\ 1/2}$ in reaction (2). This reflection symmetry for the line-reversed reactions is in agreement with weak EXD predictions.

5. CONCLUSION

We have performed a model independent amplitude analysis of Y*(1385) production in two line-reversed reactions (1) and (2) at 7 and 11.6 GeV/c. Our results indicate that the quark model and exchange degeneracy predictions are in agreement with the main features of the data. However, small violations are observed at low momentum transfer. While the Y*(1385) vertex is helicity flip dominated, the nonvanishing of $T_{3/2\ -1/2}$ and $T_{-3/2\ 1/2}$ suggests some finite helicity non-flip contribution in the forward direction.

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TABLE I

The number of events assigned to reaction (1) and (2) at 7 and 11.5 GeV/c.

Reaction	7 GeV/c	11.5 GeV/c
$\pi^+ p \to K^+ Y^*(1385)$	509	936
$K^- p \to \pi^- Y^*(1385)$	656	911

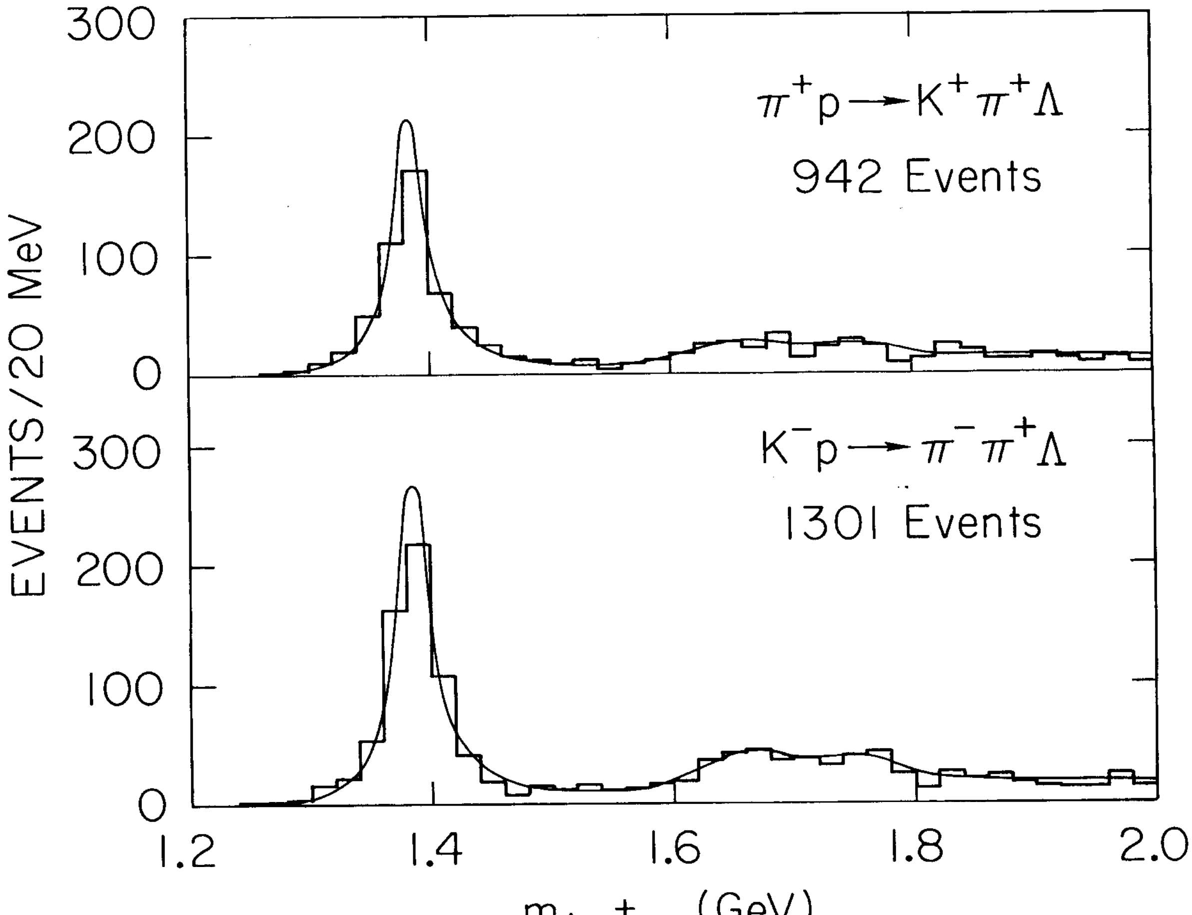
FIGURE CAPTIONS

1. Invariant mass distribution of the $\Lambda\pi^+$ system: (a) at p_{beam}=7 GeV/c and (b) at p_{beam}=11.5 GeV/c. The solid lines are the results of maximum likelihood fits described in the text.

2. Angular distributions of the cascade decay: $Y^* \to \Lambda\pi^+$, $\Lambda \to p\pi^-$, in the region $m_{\Lambda\pi} < 1.55$ GeV, $-t < 1$ GeV^2. The data come from reaction (1) at 11.5 GeV/c. The solid lines are the results of fits. Similar distributions are obtained from the other reactions.

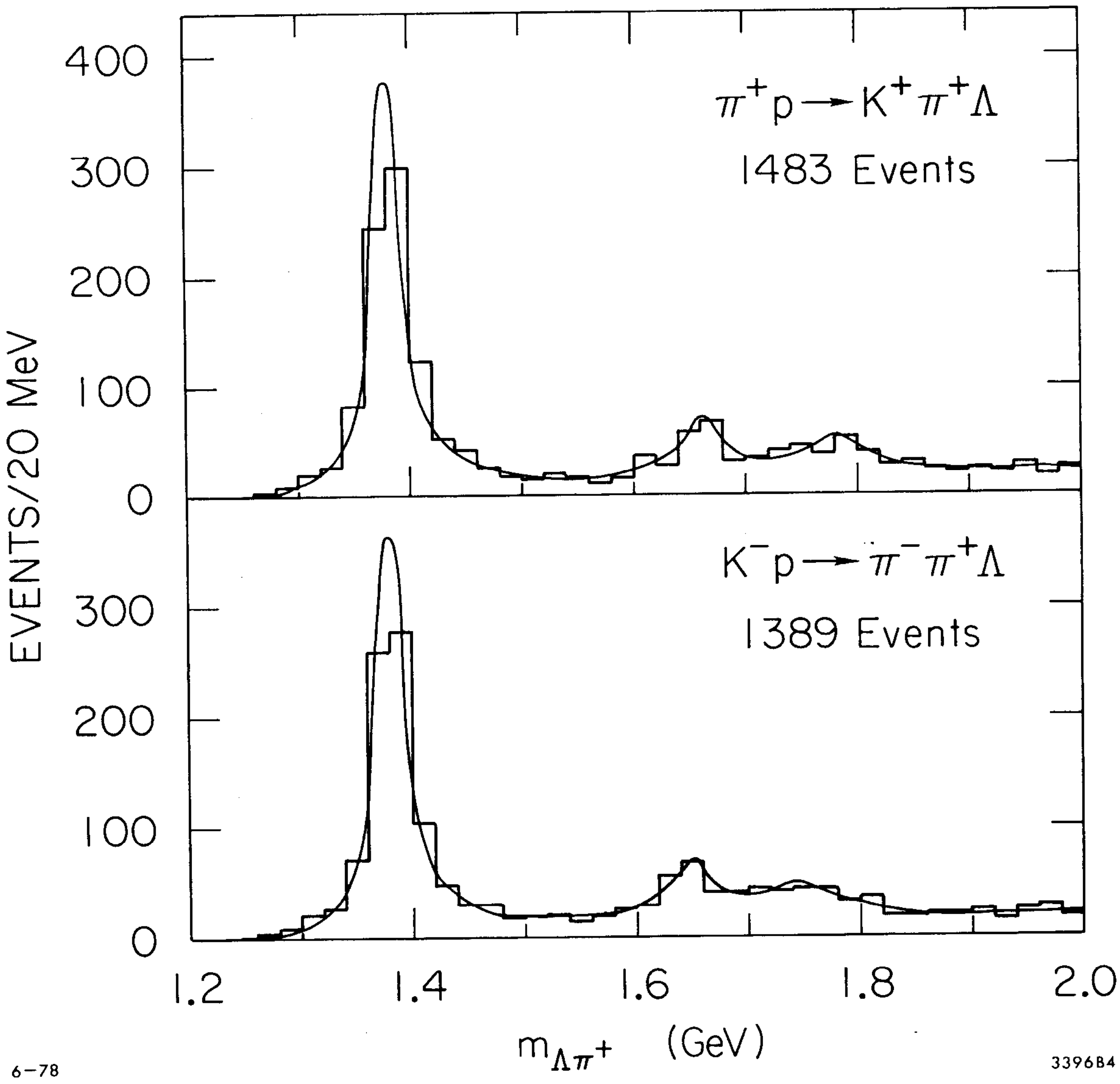
3. Absolute values of the Y*(1385) transversity amplitudes as function of momentum transfer: (a) at p_{beam}=7 GeV/c and (b) at p_{beam}=11.5 GeV/c. The dashed lines are quark model prediction.

4. Density matrix elements of the Y*(1385) in the s-channel helicity frame as functions of momentum transfer: (a) at p_{beam}=7 GeV/c and (b) at p_{beam}=11.5 GeV/c. The dashed lines are quark model predictions.



300
200
100
0
300
200
100
0
EVENTS/20 MeV
π⁺p → K⁺π⁺Λ
942 Events
K⁻p → π⁻π⁺Λ
1301 Events
1.2
1.4
1.6
1.8
2.0
m_Λπ⁺ (GeV)
8-78
3443A6

Fig. 1a



400
300
200
100
0
300
200
100
0
EVENTS/20 MeV
π⁺p → K⁺π⁺Λ
1483 Events
K⁻p → π⁻π⁺Λ
1389 Events
1.2
1.4
1.6
1.8
2.0
m_Λπ⁺ (GeV)
6-78
3396B4

Fig. 1b

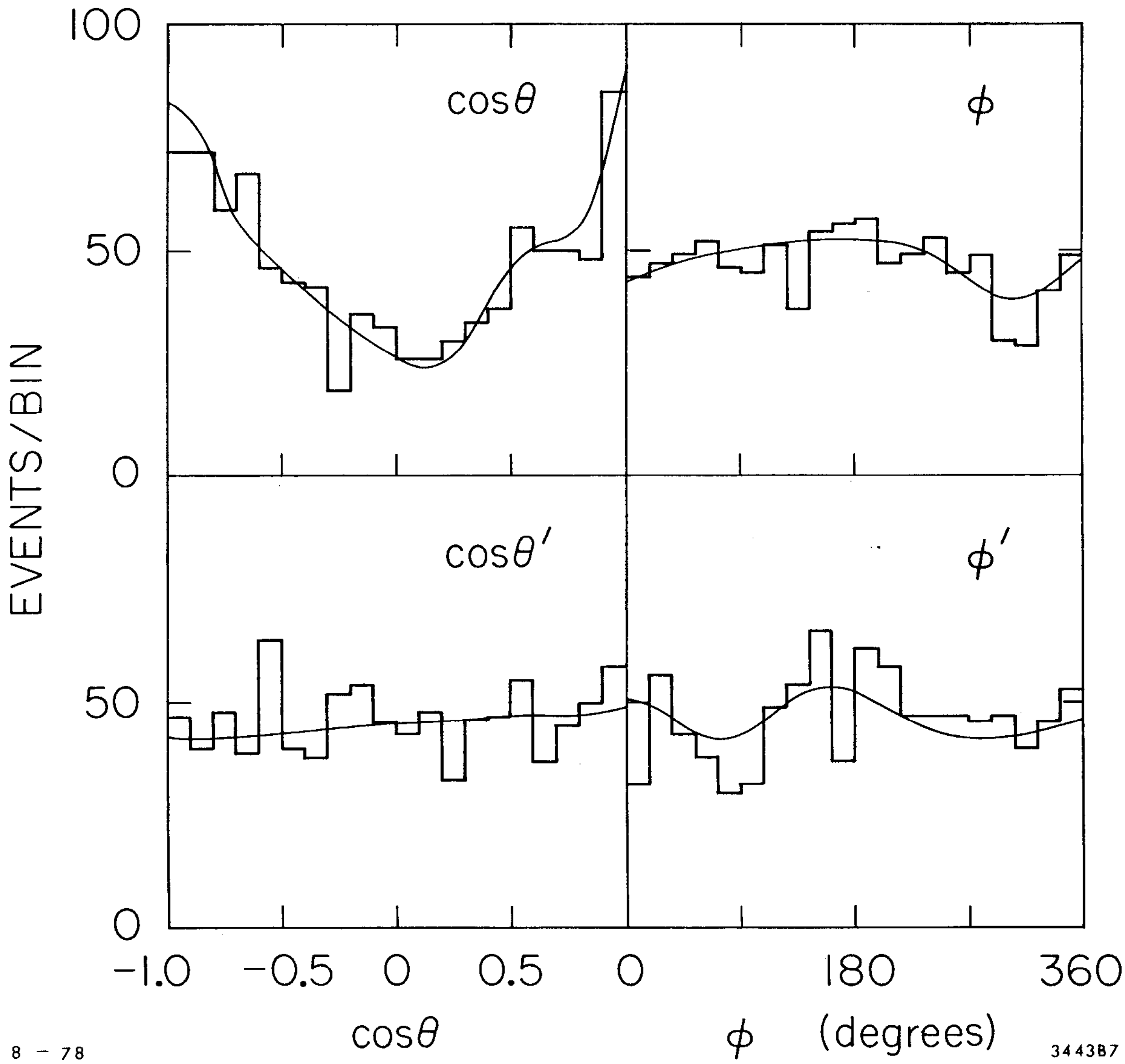
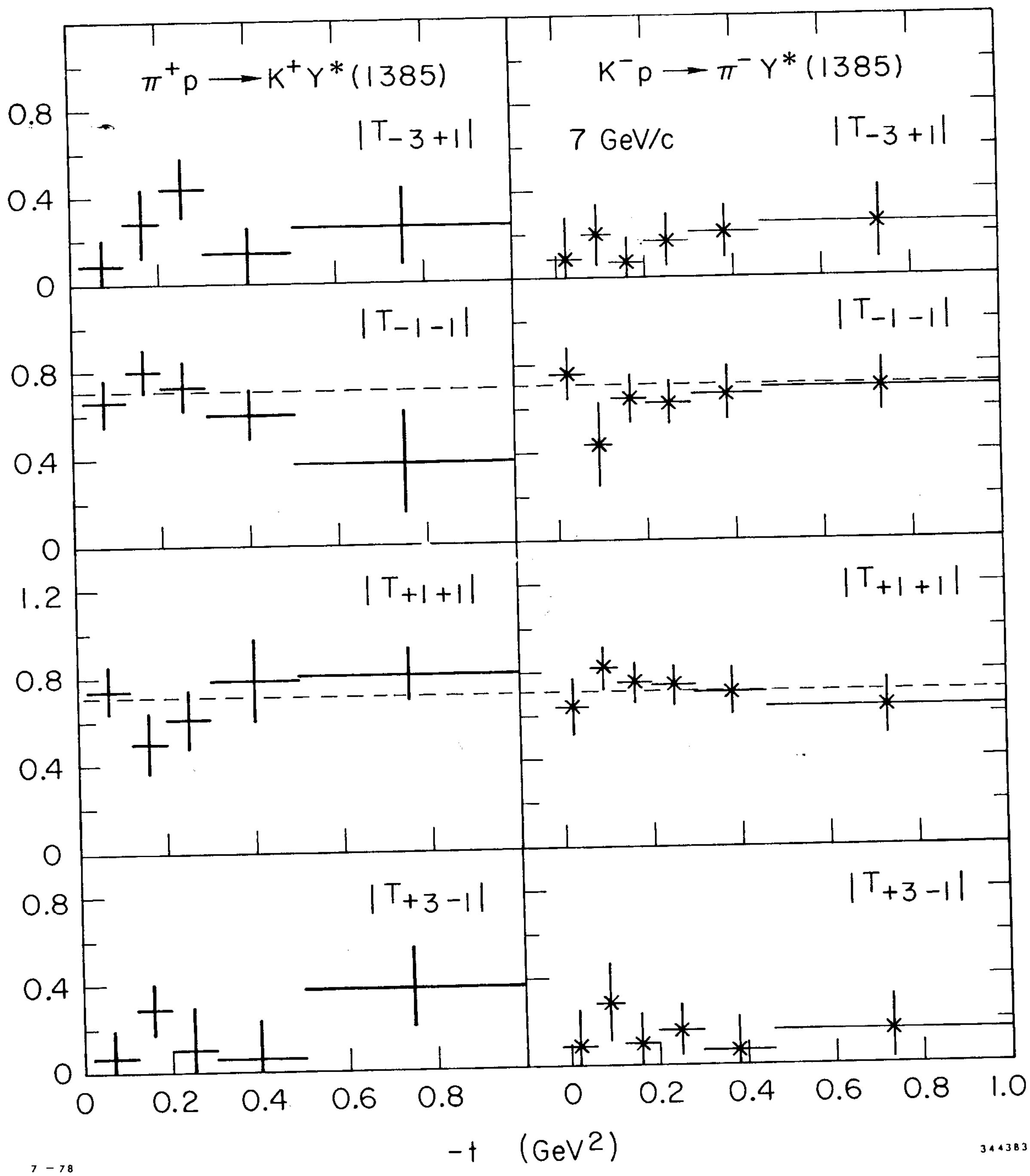
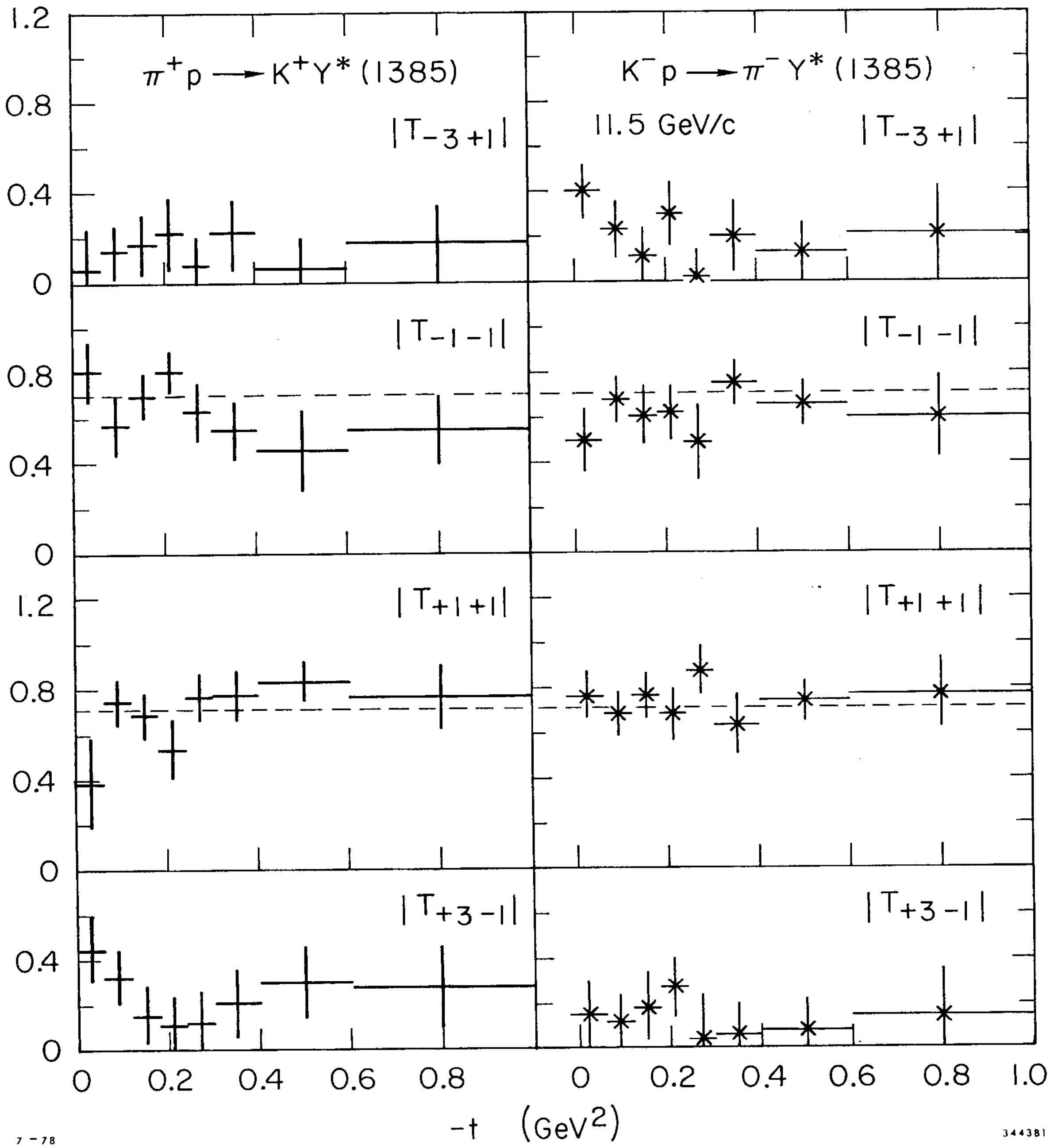

100
50
0
50
0
EVENTS/BIN
cosθ
φ
cosθ′
φ′
−1.0
−0.5
0
0.5
0
180
360
cosθ
φ (degrees)
8 − 78
3443B7

Fig. 2



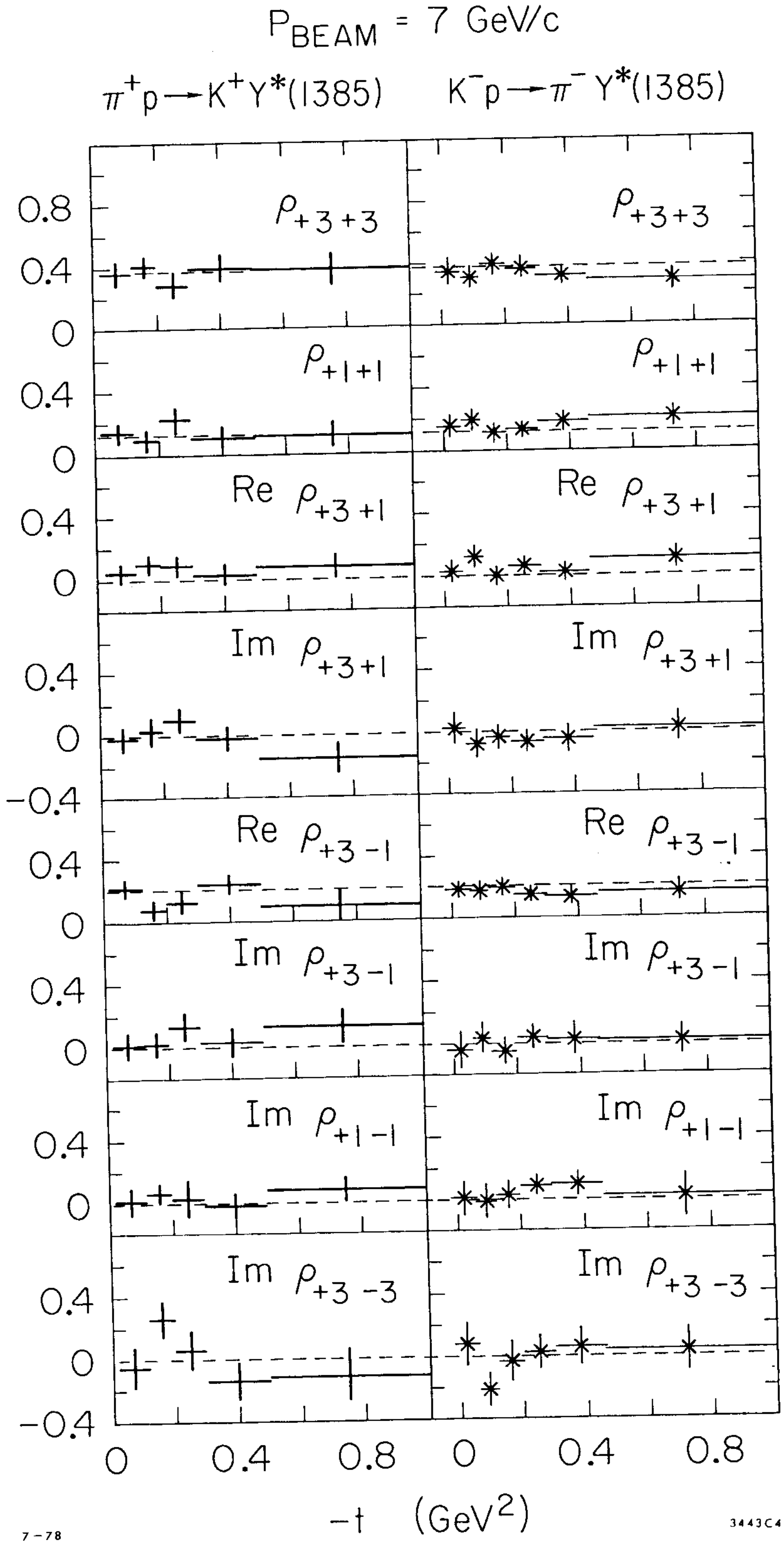
π⁺p → K⁺Y*(1385)
K⁻p → π⁻ Y*(1385)
7 GeV/c
|T₋₃ ₊₁|
|T₋₁ ₋₁|
|T₊₁ ₊₁|
|T₊₃ ₋₁|
0.8
0.4
0
1.2
0.2
0.6
1.0
−t (GeV²)
7 − 78
3443B3

Fig. 3 a



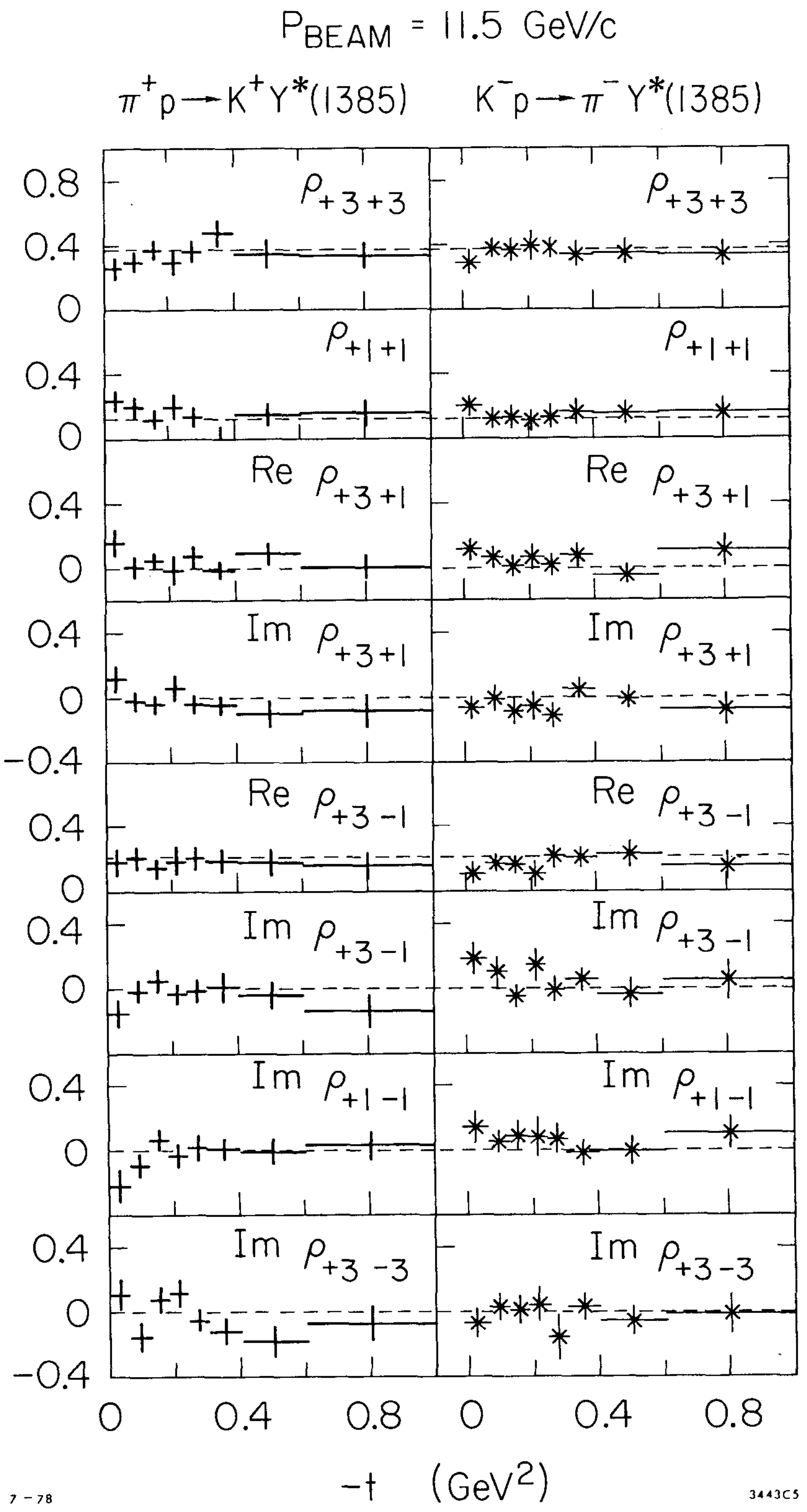
$\pi^+ p \longrightarrow K^+ Y^*(1385)$
$K^- p \longrightarrow \pi^- Y^*(1385)$
11.5 GeV/c
$|T_{-3+1}|$
$|T_{-1-1}|$
$|T_{+1+1}|$
$|T_{+3-1}|$
1.2
0.8
0.4
0
0.2
0.6
1.0
$-t$ (GeV^2)
7 - 78
3443B1

Fig. 3 b



P_BEAM = 7 GeV/c
π⁺p → K⁺Y*(1385)
K⁻p → π⁻ Y*(1385)
ρ+3+3
ρ+1+1
Re ρ+3+1
Im ρ+3+1
Re ρ+3-1
Im ρ+3-1
Im ρ+1-1
Im ρ+3-3
0.8
0.4
0
-0.4
-t (GeV²)
7-78
3443C4

Fig. 4a



P_{BEAM} = 11.5 GeV/c
$\pi^+ p \to K^+ Y^*(1385)$
$K^- p \to \pi^- Y^*(1385)$
ρ_{+3+3}
ρ_{+1+1}
Re ρ_{+3+1}
Im ρ_{+3+1}
Re ρ_{+3-1}
Im ρ_{+3-1}
Im ρ_{+1-1}
Im ρ_{+3-3}
0.8
0.4
0
-0.4
$-t$ (GeV^2)
7-78
3443C5

Fig. 4 b