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THE RADIATION OF THE POSITRONS CHANNELED BETWEEN THE PLANES OF A CRYSTAL*

S. Kheifets

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

The case of the channeling radiation for electrons was considered in the previous work[1]. The use of the electrons for producing and detecting this type of radiation has the advantage that available electron beams have much smaller transverse phase space than those of the positrons. In addition, the intensity of radiation of electrons is somewhat larger in comparison to the positrons simply due to the fact that the electron is captured by an attractive ion potential of the crystal and, therefore, it moves in a strong electric field. The positron, on the other hand, is captured by a repulsive potential formed by the two next ion planes. It moves in a rather weak field oscillating around the middle plane. This remoteness from ions increases the ratio of the channeling radiation intensity to the bremsstrahlung one making the experiment with the positrons easier to interpret. So it is worthwhile to fulfill the calculations also for the positrons.

Our aim is to find the full average intensity of channeling radiation. This can be done easily for any given potential. This approach does not give the spectra of emitted radiation (but it allows one to find the position of the spectra maximum). The last one can be found only for equivalent harmonic oscillator. The shape of the emitted line can be found in work [2], where such a shape was calculated for Si in parabolic approximation for potential corrected for unharmonicity. Since all

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characteristics of the radiation are of interest, the two approaches supplement each other and shed more light on the phenomena.

1. The Continuum Potential for Positrons

To find the potential for positrons we use the Lindhard's potential [3] for electrons and the fact that it is decreasing quite rapidly with the increased distance from an ion plane. Therefore, for the vicinity of the middle plane the average potential for positrons is simply the sum of the potentials produced by the two nearest ion planes:

$$U(y) = A\left\{\sqrt{\left(\frac{d}{2}_p+y\right)^2+C^2a^2} - \sqrt{\left(\frac{d}{2}_p+y\right)^2} + \sqrt{\left(\frac{d}{2}_p-y\right)^2+C^2a^2} - \sqrt{\left(\frac{d}{2}_p-y\right)^2}\right\}. \quad (1)$$

Here y is the distance from the middle plane: $-d_p/2 < y < d_p/2$ All other parameters are the same as in [1] except that A is now positive to provide the repulsive force on the positron.

Let us now introduce a new variable

$$x = 2y/d_p\ , \qquad |x| < 1 \quad (2)$$

and a parameter $b = 2Ca/d_p < 1$.

Notice that the variable x differs from the one introduced in [1]. The continuum potential and, consequently, all other functions for the electrons depends only on one parameter, a, while the continuum potential for the positrons depends on two unrelated parameters: a and d_p. The potential (1) can be rewritten now $U(y) = \frac{Ad_p}{2} U_b(x)$, where

$$U_b(x) = \sqrt{(1+x)^2+b^2} + \sqrt{(1-x)^2+b^2} - 2\ . \quad (3)$$

Fig.1 represents as an example $U_b(x)$ for $b^2 = 0.0552$ (solid curve). For small x ($x << 1$) $U_b(x)$ can be expanded into the series:

$$U_b(x) \simeq 2\sqrt{1+b^2} - 2 + \frac{b^2\, x^2}{(1+b^2)^{3/2}} \equiv U_0(x) \quad (4)$$

The function $U_o(x)$ for the same value of b^2 is plotted also in Fig. 1 for comparison (dashed curve). The value $\left(1 - \frac{U_o(1)}{U_b(1)}\right)$ might serve as a measure of unharmonicity. For the $b^2 = 0.055$ this value equals 37%.

2. The Intensity of Radiation

To calculate the radiation intensity of the positrons we use the same approach as in [1]. For the average intensity, we now get:

$$I = I_o F_b(x_m) , \tag{5}$$

where $I_o = 2\, e^2\gamma^2A^2/3\, m^2c^3$, and

$$F_b(x_m) = \frac{\int_o^{x_m}\left[\frac{1+x}{\sqrt{(1+x)^2+b^2}} - \frac{1-x}{\sqrt{(1-x)^2+b^2}}\right]^2 P_b(x,x_m)\,dx}{\int_o^{x_m} P_b\,(x,x_m)\,dx} \tag{6}$$

Here x_m is maximum excursion of the trapped positron from the middle plane (in units of the half distance between planes d_p, (2)). The function $P_b(x,x_m)$ is defined by:

$$P_b(x,x_m) = \left[\sqrt{(1+x_m)^2+b^2} + \sqrt{(1-x_m)^2+b^2} - \sqrt{(1+x)^2+b^2} - \sqrt{(1-x)^2+b^2}\right]^{-\frac{1}{2}} \tag{7}$$

For small x_m we get:

$$P_b(x,x_m) = \frac{(1+b^2)^{3/4}}{b\sqrt{x_m^{\,2}-x^2}} \tag{8}$$

The denominator in formulae (6) is proportional to the period T of the positron oscillations:

$$T(x_m) = 2\sqrt{\frac{m\gamma d_p}{A}}\, T_b(x_m) \tag{9}$$

where $T_b(x_m) = \int_0^{x_m} P_b\,(x,x_m)\,dx$. For small x_m, T is independent from x_m:

$$T = \pi\sqrt{\frac{m\gamma d_p}{A}}\;\frac{(1+b^2)^{3/4}}{b} \qquad (10)$$

The functions $F_b(x_m)$ and $T_b(x_m)$ are presented in Figs. 2 and 3, respectively, for different values of the parameter b^2. For small x_m, $F_b(x_m)$ equals:

$$F_b^o(x_m) = \frac{2\,b^4 {x_m}^2}{(1+b^2)^3}\,, \qquad x_m \ll 1\,. \qquad (11)$$

This value gives the estimate for the case of parabolic well approximation.

3. The Position of the Spectra Maximum

For the interesting case of small ratio of trajectory angle to the radiation characteristic angle, $1/\gamma$, the maximum of the spectra occurs at the frequencies [1] :

$$\hat{\omega} \simeq 2\pi\gamma^2/T(x_m) \qquad (12)$$

As can be seen from Fig. 3 the function $T_b(x_m)$ has rather slow dependance on x_m at least for not very small values of parameter b^2. So the value for T(o) from (10) which is the value for the corresponding harmonic oscillator is a good approximation:

$$\hat{\omega}_o = \sqrt{\frac{A}{m\,d_p}}\;\frac{2b\,\gamma^{3/2}}{(1+b^2)^{3/4}}\,. \qquad (13)$$

For very small b^2 and for large x_m, the values of $T_b(x_m)$ should be taken from Fig. 3.

4. Discussion and Numerical Example

The dependance of F_b on x_m suggests an interesting conclusion: the intensity of the channeling radiation for the positrons grows very rapidly with x_m , which means that the main part of the radiation is produced by positrons with large angles in the plane of the channel. One must remember, of course, that this angle should not exceed the limit at which the energy of oscillations will be greater than the barrier energy of the potential [1] . Or, in other words, the maximum excursion x_m of the particle from the middle plane should be less than 1. This conclusion is exactly opposite the one for the electrons where the main part of the radiation comes from electrons with small amplitudes.

Such features of radiation once again stress that the calculations of the intensity of radiation in the parabolic approximations are too rough and can underestimate the effect.

Let us again, as in [1] , take an example of the crystal with parameters: $A = 2.13 \times 10^4$ MeV/cm, $d = 3.75 \times 10^{-8}$ cm, $Ca = 0.42 \times 10^{-8}$ cm. Then $b = 0.235$, $I_o = 1.5 \times 10^{13} \gamma^2$ eV/sec.

For any given γ and the trajectory angle α with the crystal axis in the plane of oscillation one can find first the value of x_m from the equation:

$$\alpha = \sqrt{\frac{Ad_p}{mc^2\gamma} \left(U_b(x_m) - U_b(o)\right)} \,. \tag{14}$$

Then it is easy to find the corresponding values of $F_b(x_m)$ and $T_b(x_m)$. Table 1 gives the function $x_m = f_1(\gamma,\alpha)$ for different values of γ and α and for $b^2 = 0.0552$. Table 2 contains values of F_b for the same values of γ and α. Dashes for big values of γ and α mean that for these parameters there is no channeling.

Table 3 presents the comparison of radiation intensities and the spectra maximum for different γ and α calculated by means of formulae (6) and (9) ("exact") and (12) and (13) ("harmonic oscillator"), respectively.

One can clearly see that the bigger x_m is, the bigger is the difference between results which one gets by means of exact solution and in the approximation of the parabolic well. This is especially true for the whole intensity of radiation where the discrepancy can be two orders of magnitude. On the other hand, the spectra maximum frequency, if corrected for unharmonicity, is determined by approximate solution quite well.

Acknowlegements

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References

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TABLE 1

x_m for Different γ and α (b^2 = 0.0552)

γ \ α	10^{-3}	0.3×10^{-3}	10^{-4}	0.25×10^{-4}
10^2	.79	.33	.10	.0
10^3	-	.77	.33	.10
10^4	-	-	.79	.28
2×10^4	-	-	.93	.38
4×10^4	-	-	-	.51

TABLE 2

F_b for Different γ and α (b^2 = 0.0552)

γ \ α	10^{-3}	0.3×10^{-3}	10^{-4}	0.25×10^{-4}
10^2	0.028	0.7×10^{-3}	0.5×10^{-4}	0.0
10^3	-	0.027	0.7×10^{-3}	0.5×10^{-4}
10^4	-	-	0.028	0.5×10^{-3}
2×10^4	-	-	0.11	0.10
4×10^4	-	-	-	0.25

TABLE 3. The Comparison between "Exact" and "Approximate" Solutions for Radiation Intensity (F_b) and the Spectra Maximum Frequencies for Different γ and α.

γ	γ^{-1}	$\alpha = 10^{-3}$				$\alpha = 0.3 \times 10^{-3}$				$\alpha = 10^{-4}$				$\alpha = 0.25 \times 10^{-4}$			
		F_b	F_b^o	$\hat{\omega}$	$\hat{\omega}_o$	F_b	F_b^o	$\hat{\omega}$	$\hat{\omega}_o$	F_b	F_b^o	$\hat{\omega}$	$\hat{\omega}_o$	F_b	F_b^o	ω	$\hat{\omega}_o$
10^2	10^{-2}	28×10^{-3}	3.2×10^{-3}	2.42×10^{19}	1.40×10^{19}	0.7×10^{-3}	0.6×10^{-3}	1.52×10^{19}	1.40×10^{19}	0.5×10^{-4}	0.5×10^{-4}	1.40×10^{19}	1.40×10^{19}	0.	0.	1.4×10^{19}	1.40×10^{19}
10^3	10^{-3}	---	---	---	---	27×10^{-3}	3.1×10^{-3}	7.3×10^{20}	4.43×10^{20}	0.7×10^{-3}	0.6×10^{-3}	4.82×10^{20}	4.43×10^{20}	0.5×10^{-4}	0.5×10^{-4}	4.47×10^{20}	4.43×10^{20}
10^4	10^{-4}	---	---	---	---	---	---	---	---	28×10^{-3}	3.1×10^{-3}	2.32×10^{22}	1.40×10^{22}	0.5×10^{-3}	0.4×10^{-3}	1.48×10^{22}	1.40×10^{22}
2×10^4	0.5×10^{-4}	---	---	---	---	---	---	---	---	---	---	---	---	1×10^{-1}	0.75×10^{-3}	4.34×10^{22}	3.96×10^{22}
4×10^4	0.25×10^{-4}	---	---	---	---	---	---	---	---	---	---	---	---	2.5×10^{-1}	1.4×10^{-3}	1.33×10^{23}	1.12×10^{23}
		Form.(5), (6)	Form.(11)	Form.(12), sec^{-1}	Form.(13), sec^{-1}	Form.(5), (6)	Form.(11)	Form.(12), sec^{-1}	Form.(13), sec^{-1}	Form.(5), (6)	Form.(11)	Form.(12), sec^{-1}	Form.(13), sec^{-1}	Form.(5), (6)	Form.(11)	Form.(12), sec^{-1}	Form.(13), sec^{-1}

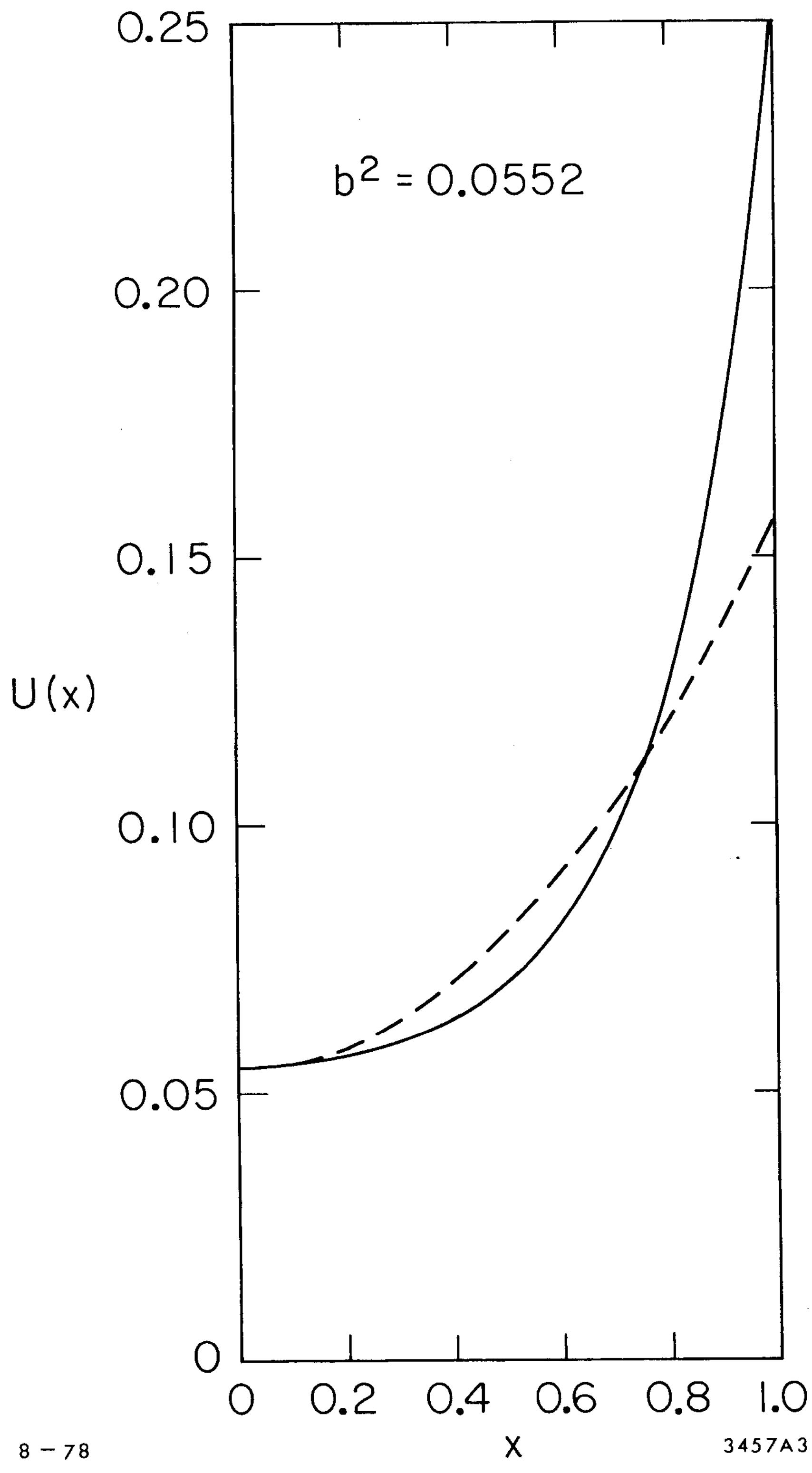

Fig. 1 - The Potential U(x) for the Channeled Positron (Solid Curve). The dashed curve represents corresponding parabolic well.

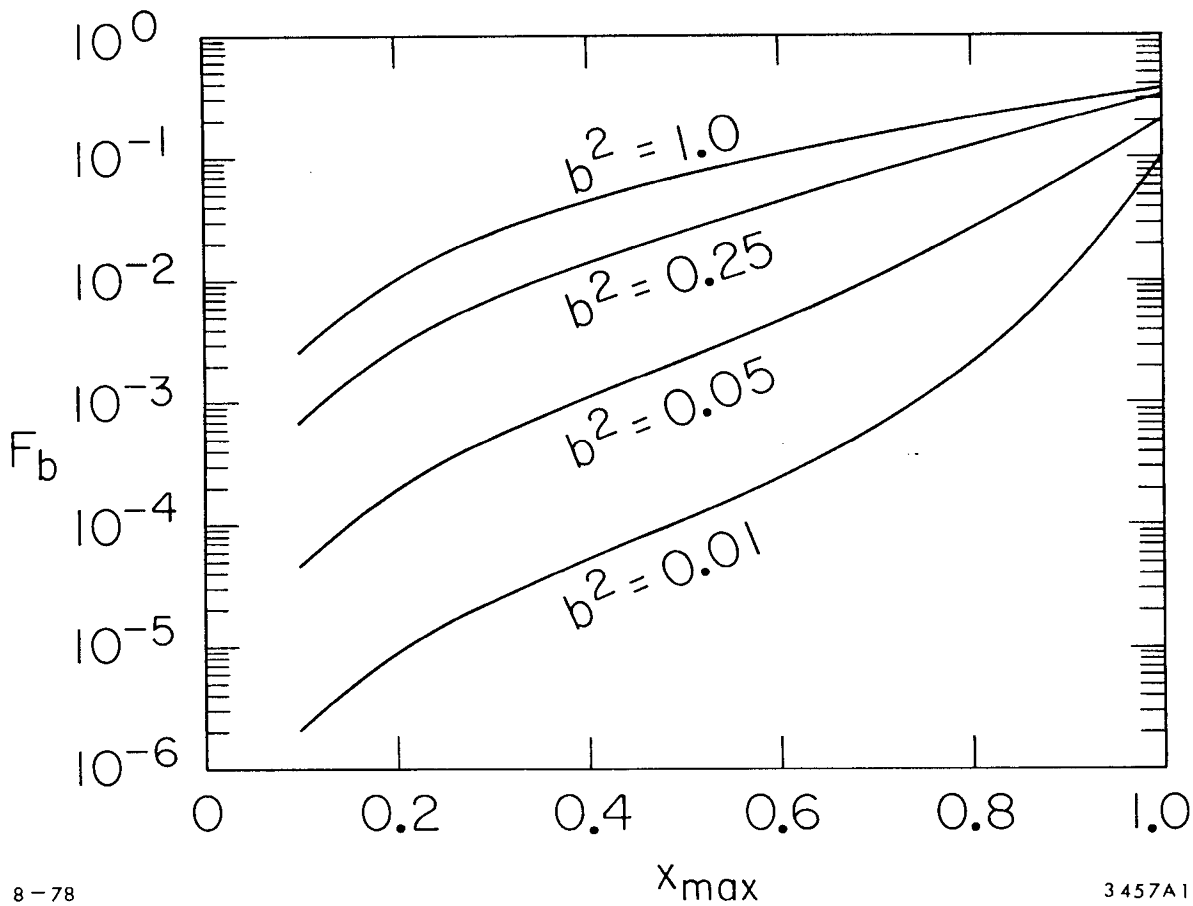

Fig. 2 - The intensity of the channel radiation of the positron in the units $2\ e^2\gamma^2A^2/3m^2c^3$ for different values of the parameter $b = 2\ Ca/d_p$.

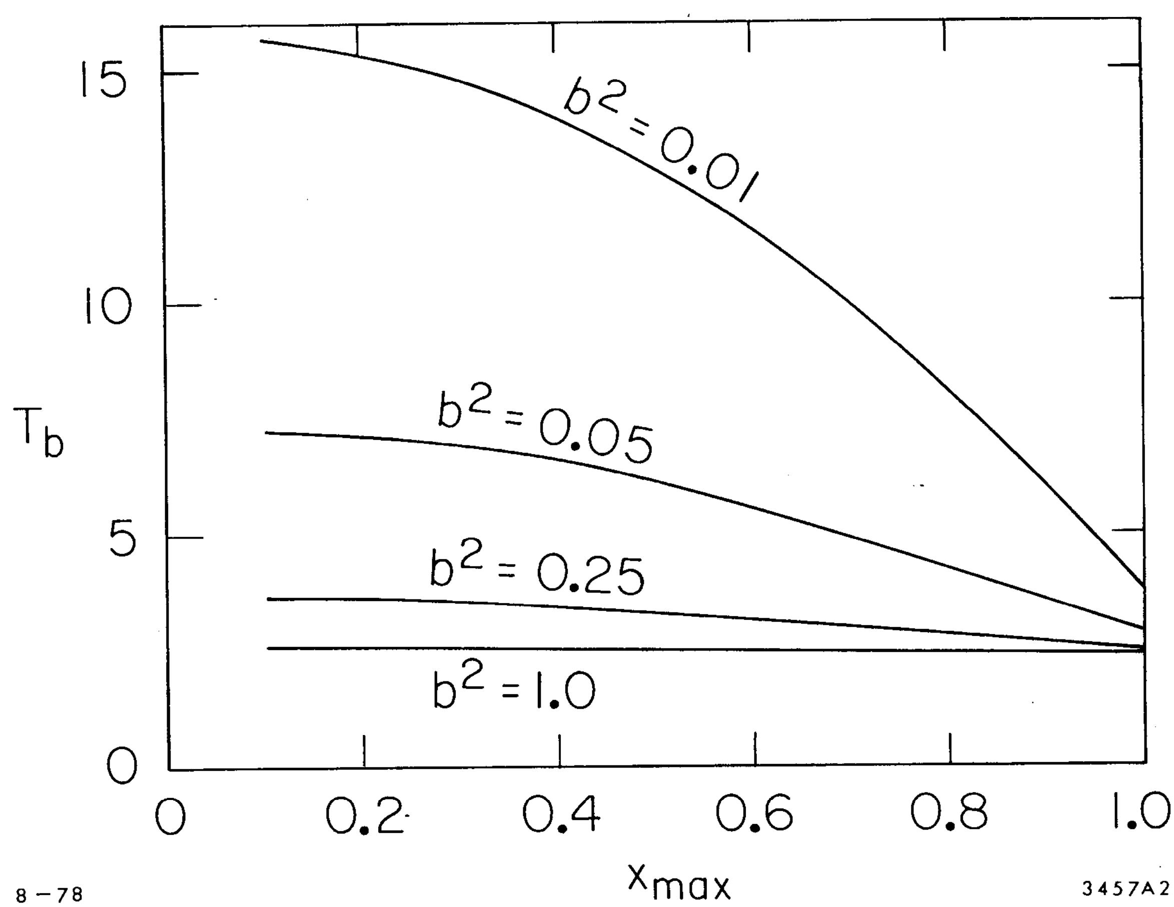

Fig. 3 - The period of oscillations of the channeled positron in the units $2\sqrt{m\gamma d_p/A}$ for different values of the parameter $b = 2Ca/d_p$.