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DEEP INELASTIC STRUCTURE FUNCTIONS FOR BOUND STATES IN THE LADDER APPROXIMATION*

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ABSTRACT

A simple relativistic expression for the structure function of a 2-body S-wave bound state in $\lambda\phi^3$ theory is obtained at all values of the scaling variable in terms of the DGSI spectral function of the Bethe-Salpeter wave function in the ladder approximation. Similar result is given for a fermion-antifermion bound state with approximate wave function derived from conformal invariance. At small x the structure function has the expected peak broadened by binding effect and for a fermion-antifermion S-wave bound state it behaves like $(1-x)^2$ as $x \to 1$.

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I. INTRODUCTION

There have been many attempts in the past to understand the behavior of the structure function of nucleons measured as a function of the scaling variable x in deep inelastic electron and neutrino scattering experiments. The structure function $F_2(x)$ is peaked around $x = \frac{1}{3}$ and decreases like $(1-x)^3$ as $x \to 1$.[1] The behavior near $x = \frac{1}{3}$ seems to suggest that the nucleon is composite and made of 3 quarks weakly bound by some sort of a confinement potential which may be derived from asymptotic freedom gauge theory. This effect can be easily accounted for in a phenomenological way by using a nonrelativistic wave function for the nucleon[2] and the result does not depend very much on the choice of the nonrelativistic wave function which at present is not directly known as the dynamics of the quark confined in the nucleon is not well understood. Totally unrelated to this is the behavior of the structure function as $x \to 1$. As can be seen, this is related to the behavior of the wave function at large relative momenta of the bound constituents such that the relativistic Bethe-Salpeter (B.S.) wave function[3] must be used to get the correct behavior. This $(1-x)^3$ behavior is reproduced in certain model,[4] for example, for a nucleon taken to be a bound state of a spin ½ and a spin 0 constituents. However the lack of knowledge of the relativistic B.S. wave function at all relative momenta prevents us from making a full dynamical calculation of the bound state structure function. At first sight, it seems hopeless to derive the structure function from the B.S. wave function, the solution of which is never completely given. In fact, as will be seen below, the main features of the structure function at all values of x can be derived exactly for the simplest model describing a spinless bound state system of two spinless constituents interacting via the exchange of a massless scalar meson in the ladder approximation.

In this paper we shall first present a complete calculation of the structure function in this model using the Deser-Gilbert-Sudarshan-Ida (DGSI) representation[5] of the B.S. wave function (Section II). This representation embodies in a simple way both the analytic properties and the asymptotic behavior of the wave function in the relative momentum and the smooth transition to the nonrelativistic limit. It is found that the structure function for all values of x can be written as

$$F_2(x) = \frac{x^2(1-x)^3 Z(x)}{(x-\frac{1}{2})^2 + \gamma^2} \quad , \gamma^2 = \frac{\rho^2}{m^2}, \rho^2 = m^2 - \frac{1}{4}\mu^2$$

(μ and m being the constituent and bound state masses respectively). $Z(x)$ is a slowly varying function of x and behaves like a constant as $x \to 1$ provided that the spectral function $\tilde{g}$ in the DGSI representation behaves like $\beta(1-\beta)$ as $\beta \to 0,1$. Assuming a similar DGSI representation for the fermion-anti-fermion (pion) bound state structure function as suggested by the conformal leading solution in non-gauge theory, the pion structure function is shown to have terms of the form

$$\frac{x(1-x)^2}{(x-\frac{1}{2})^2 + \gamma^2}$$

and similar terms which go like $(1-x)^{5/2}$ and $(1-x)^3$ as $x \to 1$ in agreement with previous calculations[6]. We now proceed to the calculation of the structure function.

II. STRUCTURE FUNCTION FOR BOUND STATES IN THE LADDER APPROXIMATION

A. Two-body S-wave Bound State in $\lambda\phi^3$ Theory

In $\lambda\phi^3$ theory, Bjorken scaling is valid as $q^2 \to \infty$ and it is convenient to work with the bilocal operator $\theta_{\mu\nu}(o,x)$ in the scaling limit. In this limit, the hadron structure function describing deep inelastic electron (neutrino) hadron scattering is related to $\theta_{\mu\nu}(o,x)$ by the following relation:[7]

$$\langle p|\theta_{\mu\nu}(o,x)|p\rangle = p_\mu p_\nu f_2(p\cdot x,\ x^2{=}0) + \ldots \tag{1}$$

with

$$f_2(p\cdot x,\ x^2{=}0) = \int_0^1 d\alpha\ \cos(\alpha p\cdot x) F_2(\alpha) \tag{2}$$

$F_2(\alpha)$ is the usual deep inelastic structure function and terms not relevant to $F_2(\alpha)$ have been dropped. Comparing Eq. (2) with the Wilson's short distance operator product expansion for $f_2(p\cdot x,\ x^2{=}0)$, we have

$$p_\mu p_\nu f_2(px,\ x^2{=}0) = \sum_{n\ \text{even}} \frac{i^n}{n!} x^{\mu_1}\ldots x^{\mu_n} \langle p|\theta_{\mu\nu},_{\mu_1\ldots\mu_n}(o)|p\rangle \tag{3}$$

where

$$\langle p|\theta_{\mu\nu},_{\mu_1\ldots\mu_n}(o)|p\rangle = A_{n+2}\Pi_{\mu\nu},_{\mu_1\ldots\mu_n},\ n\ \text{even}$$

and

$$\Pi_{\mu\nu},_{\mu_1\ldots\mu_n} = p_\mu p_\nu p_{\mu_1}\ldots p_{\mu_n} - \text{Track terms} \tag{4}$$

The trace terms contain all possible combinations of $g_{\mu_i\mu_j}$ and do not contribute to $f_2(p\cdot x,\ x^2 = 0)$ in the scaling limit.

The A_{n+2} are the n-moments of the structure function defined as:

$$A_{n+2} = \int_0^1 d\alpha\ \alpha^n F_2(\alpha) \tag{5}$$

As will be seen below, it is more convenient to define a function $f(p\cdot x,\ x^2)$ as:

$$f(p\cdot x,\ x^2=0) = \int_0^1 d\alpha\ \exp(i\alpha p\cdot x)\, F_2(\alpha) \tag{6}$$

we have then

$$p_\mu p_\nu f(p\cdot x,\ x^2) = \sum \frac{i^n}{n!} x^{\mu_1} \cdots x^{\mu_n} \langle p|\theta_{\mu\nu,\mu_1\cdots\mu_n}(o)|p\rangle \tag{7}$$

and

$$f_2(p\cdot x,\ x^2) = \mathrm{Re} f(p\cdot x,\ x^2) \tag{8}$$

Thus $F_2(\alpha)$ can be obtained immediately once $f(p\cdot x,\ x^2=0)$ is given in the form of Eq. (6).

The problem of computing the structure function is thus reduced to the calculation of a set of matrix elements of twist-2 local operators $\theta_{\mu\nu,\mu_1\cdots\mu_n}(o)$ which can be calculated in the same manner as those for other static quantities of hadron such as charge densities or magnetic moment etc ... .

Consider now a neutral boson bound state of two spinless charged constituents which interact with each other via the exchange of a massless boson according to the interaction Lagrangian $\mathscr{L}_I = g\phi\Phi^\dagger\Phi$. The twist-2 spin n local operators $\theta_{\mu\nu,\mu_1\cdots\mu_n}(x)$ are given by

$$\theta_{\mu\nu,\mu_1\cdots\mu_n}(x) = \Phi^\dagger(x)\overleftrightarrow{\partial}_\mu\overleftrightarrow{\partial}_\nu\overleftrightarrow{\partial}_{\mu_1}\cdots\partial_{\mu_n}\Phi(x) \tag{9}$$

Since the impulse approximation is valid for Compton scattering of virtual photons in the scaling limit,[4] one needs only to calculate the matrix elements of $\theta_{\mu\nu,\mu_1\cdots\mu_n}(o)$ in this approximation. In this approximation these matrix elements are given by the familiar triangle diagram according to Mandelstam's prescription[8] (Fig. 1).

Let $\Gamma_p(q_1,q_2)$ be the B.S. "vertex" function for the bound state with total momentum p, we have from Eq. (7)

$$p_\mu p_\nu f(p\cdot x,\ x^2) = -\partial_\mu \partial_\nu I(p\cdot x,\ x^2) \tag{10}$$

$$I(p\cdot x,\ x^2) = \frac{i}{(2\pi)^4}\int \frac{d^4k\ \Gamma_p(k,p-k)^2 e^{ik\cdot x} \exp(ik\cdot x)\bar{\Gamma}_p(p-k,k)}{\left[(k^2-\mu^2-i\varepsilon)^2\ (p-k)^2-\mu^2-i\varepsilon\right]}$$

where $\bar{\Gamma}_p(p-k,k) = -\ \Gamma(\bar{k},\bar{p}-\bar{k})^*$, $\left(\bar{k} = (k_0^*,\ \vec{k})\right)$ is the final state vertex function.[5,8] Writing $I(p\cdot x,\ x^2=0)$ as

$$I(p\cdot x,\ x^2=0) = \int_0^1 d\alpha\ e^{i\alpha p\cdot x} g(\alpha) \tag{11}$$

we have

$$f(p\cdot x,\ x^2=0) = \int_0^1 d\alpha\, e^{i\alpha p\cdot x} \alpha^2 g(\alpha) \tag{12}$$

(the trace terms do not contribute to $I(p\cdot x,\ x^2=0)$). The Fourier transform of $f(p\cdot x,\ x^2=0)$ which is given by $\alpha^2 g(\alpha)$ and is nonvanishing for $0 \leq \alpha \leq 1$ can now be identified with the structure function.

The "vertex" function is defined as the B.S. wave function with the free propagators removed. For a neutral bound state of a particle-antiparticle system in a definite charge conjugation state, $\Gamma_p(q_1,q_2)$ is either symmetric or antisymmetric under the exchange of the momenta of the two particles, for a S-wave C = +1 bound state we have:

$$\Gamma_p(q_1,q_2) = \Gamma_p(q_2,q_1)$$

and $\Gamma_p(q_1,q_2)$ is given by:

$$\Gamma_p(q_1,q_2) = \left(q_1^2-\mu^2\right)\left(q_2^2-\mu^2\right)\psi_p(q_1,q_2) \tag{13}$$

with

$$\psi_p(q_1,q_2) = \frac{1}{\sqrt{2}}\int dx\ e^{iq_1\cdot x}\left[\langle o|T\{\Phi^+\ (x)\Phi^-(o)\}|p\rangle + (+ \leftrightarrow -)\right]$$

satisfying the B.S. wave equation:

$$\left(q_1^2-\mu^2\right)\left(q_2^2-\mu^2\right)\psi_p(q_1,q_2) = \frac{i}{(2\pi)^4}\int d^4k'K(k',k)\psi(p, \frac{1}{2}\,p-k') \tag{14}$$

$\psi(p, \frac{1}{2}\,p-k')$ being $\psi_p(q_1,q_2)$ written in the relative momentum of the two constituents ($q_1=k$, $q_2=p-k$). The kernel $K(k', k)$ in a generalized ladder approximation is given by:[9]

$$K(k',k) = g^2\int \frac{\sigma(\lambda^2)d\lambda^2}{(k'-k)^2-\lambda^2-i\varepsilon}$$

with the property that the asymptotic behavior for large $(k'-k)^2$ is determined by the behavior of $\sigma(\lambda^2)$ for λ^2 large. In $\lambda\phi^3$ theory, $\sigma(\lambda^2)\sim(\lambda^2)^{-1}$ so $K(k',k) \sim \frac{1}{(k'-k)^2}$. To simplify the discussion we shall take the kernel in the lowest order ladder approximation as:

$$K(k',k) = \frac{g^2}{(k'-k)^2-i\varepsilon} \tag{15}$$

resulting from the exchange of a massless scalar meson. For this interaction, the solution of the B.S. equation can be written in a convenient form as derived by Wick[10] and generalized by Ida and Maki by means of the Deser, Gilbert, Sudarshan (DGSI) representation. For the study of the structure function we shall mention some of the relevant properties of the B.S. wave function, taken from Wick's paper.

Let q now be the relative momentum of the two particles in the center of mass system, $\psi(p,q)$ is given by

$$\psi(p,q) = i\int_{-1}^{1} dz\int_{0}^{\infty} dt\, \frac{g(z,t)}{(q^2+p\cdot qz-\rho^2-t)^3}\ , \quad \rho^2=\mu^2 - \frac{1}{4}\,p^2 \tag{16}$$

with $g(z,t)$ can be chosen to be real. If the mass of the exchanged particle vanishes, $g(z,t)=g(z)\delta(t)$ as derived by Wick and $g(z)$ satisfies an integral equation of the Fredholms's type:

$$g(\xi) = \frac{1}{2}\, g^2 \int_{-1}^{1} R(\xi, z) \frac{g(z)}{\left(\frac{1}{4}\, z^2 p^2 + \rho^2\right)} dz \tag{17}$$

Equation (17) can be converted to the following differential equation:

$$g''(z) + g^2 \frac{g(z)}{(1-z^2)\left(\frac{1}{4}\, z^2 p^2 + \rho^2\right)} = 0 \tag{18}$$

with $g(+1)=0$, $g(-1)=0$.

As will be seen below, the behavior of the structure function as $x \to 1$ is related to the behavior of $g(z)$ as $z \to \pm 1$. In this model, we have as $z \to \pm 1$,

$$g(z) \sim (1-z^2) \quad .$$

In particular for the zero energy solution (tight binding limit, $p^2 \to 0$), the Wick's solution is:

$$g(z) = (1-z^2) \tag{19a}$$

and in the weak binding limit ($\rho^2/\mu^2 << 1$), it is given by

$$g(z) = \frac{1-z^2}{1+|z|} \tag{19b}$$

The B.S. vertex function $\Gamma(p,q)$ is given by:

$$\Gamma(p,q) = i\, \frac{1}{2} \int_{-1}^{1} dz \frac{g(z)}{\left(\frac{1}{2} z^2 p^2 + \rho^2\right)} \frac{1}{(q^2 + p\cdot qz - p^2 - i\varepsilon)} \tag{20}$$

which can be easily obtained after a Feynman integration over d^4k' of the r.h.s. of (14) using the representation (16) for $\psi(p,q)$ ($g(z)$ should not be confused with the quantities $g(\alpha)$ related to the structure function defined by (11)).

In terms of $\beta = \frac{1}{2}(1+z)$, we have

$$\Gamma_p(k,p-k) = \Gamma(p,q) = i\int_0^1 d\beta\tilde{g}(\beta)\,\frac{1}{\left[k^2-2\beta p\cdot k + \beta p^2-\mu^2-i\varepsilon\right]} \tag{20'}$$

where

$$\tilde{g}(\beta) = \frac{g(\beta)}{\left(\beta - \frac{1}{2}\right)^2 p^2+\rho^2}$$

The final state vertex function is given by

$$\bar{\Gamma}_p(p-k,k) = \Gamma_p(k,p-k)$$

In general $\tilde{g}(\beta)$ is not necessarily symmetric around $\beta = \frac{1}{2}$ $\left(\text{i.e. } \tilde{g}(\beta) \neq \tilde{g}(1-\beta)\right)$. For an S-wave bound state of two identical particles or for an S-wave C = +1 state of a charged particle-antiparticle system, because of Bose symmetry, $\Gamma_p(k,p-k) = \Gamma_p(p-k,k)$ hence $\tilde{g}(\beta)$ is symmetric around $\beta = \frac{1}{2}$. In the weak binding limit $\rho^2 << \mu^2$, $\tilde{g}(\beta)$ is peaked at $\beta = \frac{1}{2}$ almost like $\delta\left(\beta - \frac{1}{2}\right)$ and most of the contribution to Γ_p comes from this region.

For the C = +1 bound state, each particle contributes the same amount to $I(p\cdot x,\, x^2)$. For one particle, we have

$$I(p\cdot x,\, x^2) = \frac{i}{(2\pi)^4}\int \frac{d^4k}{(k^2-\mu^2-i\varepsilon)^2}\,\frac{e^{ik\cdot x}}{\left[(p-k)^2-\mu^2-i\varepsilon\right]}\,\Gamma_p(k,p-k)\bar{\Gamma}_p(p-k,k) \tag{21}$$

By means of the following identities:

$$\frac{1}{A-i\varepsilon} = i\int_0^\infty dx\, e^{-i(A-i\varepsilon)x}$$
$$\left(\frac{1}{A-i\varepsilon}\right)^2 = -\int_0^\infty x dx e^{-i(A-i\varepsilon)x} \tag{22}$$

(for $\varepsilon>o$ and infinitesimal) $I(p\cdot x,\ x^2)$ can be written in the form:

$$I(p\cdot x,\ x^2) = \int_0^1 d\beta \int_0^1 d\beta'\ \tilde{g}(\beta)\tilde{g}(\beta')V(p\cdot x,\ x^2) \tag{23}$$

where

$$V(p\cdot x,\ x^2) = \frac{i}{(2\pi)^4}\ \pi^2 \int_0^\infty \lambda d\lambda \int_0^\infty \frac{d\rho d\sigma d\tau}{(\lambda+\rho+\sigma+\tau)^2} \exp\Bigg[ip\cdot x\ \frac{(\rho+\beta\sigma+\beta'\tau)}{(\lambda+\rho+\sigma+\tau)}$$

$$+ i\ \frac{x^2}{4}\ \frac{1}{(\lambda+\rho+\sigma+\tau)} + i\ p^2\ \frac{(\rho+\beta\sigma+\beta'\tau)^2}{(\lambda+\rho+\sigma+\tau)} - ip^2(\rho+\beta\sigma+\beta'\tau)$$

$$+ i(\lambda+\rho+\sigma+\tau)\mu^2 - i\varepsilon\Bigg] \tag{24}$$

By means of a change of variables

$$\lambda \to \lambda' = \frac{\rho+\beta\sigma+\beta'\tau}{\lambda+\rho+\sigma+\tau}$$

$$\rho \to \rho' = \rho+\beta\sigma+\beta'\tau$$

$$\sigma \to \sigma' = \frac{\sigma}{\rho'}$$

$$\tau \to \tau' = \frac{\tau}{\rho'}$$

and inserting $\int\delta(\alpha-\lambda')d\alpha$ into (24) we have finally:

$$V(p\cdot x,\ x^2) = -\frac{i}{(2\pi)^4}\ \pi^2 \int_0^1 d\alpha\ \exp(i\,\alpha\, p\cdot x) \int_0^\infty \rho^2 d\rho\ \alpha^3\ \frac{1-\alpha}{\alpha} \int d\sigma d\tau$$

$$\left[1 - \frac{\alpha}{1-\alpha}\ \Big((1-\beta)\sigma+(1-\beta')\tau\Big)\right] \times \exp\left[i\rho\left(\mu^2-\alpha(1-\alpha)p^2\right) + i\ \frac{x^2}{4\rho}\right] \tag{25}$$

The integration over $d\sigma d\tau$ is restricted to the following region:

$$\sigma(1-\beta) + \tau(1-\beta') \le \frac{1-\alpha}{\alpha}$$

$$\beta\sigma + \beta'\tau \le 1 \tag{26}$$

Carrying out this integration we finally obtain the expression for $I(p\cdot x,\ x^2)$:

$$I(p\cdot x,\ x^2) = \frac{-i}{(2\pi)^4}\pi^2\int_0^1 d\alpha\ \exp(i\alpha p\cdot x)\int_0^1 \rho^2\ d\rho\ \exp\left[i\rho\left(\mu^2-\alpha(1-\alpha)p^2\right)+i\frac{x^2}{4\rho}\right]$$

$$\times (1-\alpha)^3 Z(\alpha) \tag{27}$$

with

$$(1-\alpha)^3 Z(\alpha) = \alpha^3\left(\frac{1-\alpha}{\alpha}\right)\int_0^1 d\beta\tilde{g}(\beta)\int_0^1 d\beta'\tilde{g}(\beta')\int d\sigma d\tau\left[1-\frac{\alpha}{1-\alpha}\left((1-\beta)\sigma+(1-\beta')\tau\right)\right] \tag{28}$$

and

$$Z(\alpha) = \frac{1}{6}\left[\left(\int_0^\alpha \frac{d\beta}{1-\beta}\tilde{g}(\beta)\right)^2 + \left(\frac{\alpha}{1-\alpha}\right)^2\int_\alpha^1 \frac{d\beta}{\beta}\tilde{g}(\beta)\int_\alpha^1\frac{d\beta'}{\beta'}\tilde{g}(\beta')\times\right.$$

$$\left(1+\frac{\beta-\alpha}{\beta(1-\alpha)}+\frac{\beta'-\alpha}{\beta'(1-\alpha)}\right) + 2\int_0^\alpha \frac{d\beta\tilde{g}(\beta)}{(1-\beta)}\times\int_\alpha^1 \frac{d\beta'\tilde{g}(\beta')(\alpha-\beta)}{(1-\alpha)(\beta'-\beta)}$$

$$\left. + 2\frac{\alpha}{(1-\alpha)^2}\int_0^\alpha d\beta\tilde{g}(\beta)\times\int_\alpha^1 d\beta'\tilde{g}(\beta')\left(\frac{\beta'-\alpha}{\beta'(\beta'-\alpha)}\right)\left(1+\frac{\beta'-\alpha}{(1-\alpha)\beta'}\right)\right] \tag{29}$$

The integration over $d\rho$ can be written as:[11]

$$\int_0^\infty \rho^2 d\rho\ \exp\left[i\rho\left(\mu^2-\alpha(1-\alpha)p^2\right)+i\frac{x^2}{4\rho}\right] = \frac{\partial^2}{\partial(K^2)^2}G(x^2,K^2) \tag{30}$$

with

$$G(x^2,K^2) = \int_0^\infty d\rho\ \exp\left[-i\rho(-K^2-i\varepsilon)+i\frac{x^2}{4\rho}\right] \tag{30'}$$

and

$$K^2 = \mu^2-\alpha(1-\alpha)p^2 \quad .$$

The structure function can now be obtained readily by setting $x^2=0$ in (30). The result is

$$F_2(x) = N^2\left(\frac{1}{4\pi}\right)^2 x^2(1-x)^3 Z(x)\left(\frac{1}{p^2\left(x-\frac{1}{2}\right)^2+\rho^2}\right)^3 \tag{31}$$

The normalization constant N is determined by the condition

$$-i\partial_\mu I(p\cdot x,\ x^2) = p_\mu \ ,\ x=0$$

which gives

$$\int_0^1 \frac{F_2(x)}{x}\,dx=1 \qquad (32)$$

This is essentially the Adler sum rule[4] in the two-body bound state approximation. The presence of multi-particle configuration in the wave function will reduce the two-body contribution to the sum rule and the normalization constant N can be most simply determined by comparing with experiments the expression (31) for $F_2(x)$ at $x \geq \frac{1}{2}$ where gluons and multi-parton contributions are negligible. Equation (31) is the complete relativistic expression for the structure function at all values of x.

Note that in the impulse approximation, the parton distribution function $p(x) = \frac{F_2(x)}{x}$ is not symmetric around $x = \frac{1}{2}$, (i.e. $p(x) \neq p(1-x)$), although $\tilde{g}(\beta)$ is symmetric. Because of the lack of this symmetry property, in the 2-body ladder approximation (i.e. only valence quarks in the wave function) each parton in general does not carry half the total momentum. This can be easily seen by looking at Eqs. (28) and (31) for p(x). Writing

$$p(x) = p_S(x) + p_A(x)$$

with

$$p_S(x) = \frac{1}{2}\left[p(x) + p(1-x)\right]$$

$$p_A(x) = \frac{1}{2}\left[p(x)-p(1-x)\right]$$

We obtain after substitution $x \to 1-x$ in (28):

$$p(1-x) = p(x) - 2p_A(x) \qquad (33)$$

where

$$p_A(x) = \left(\frac{1}{2}\right)N^2\left(\frac{1}{4\pi}\right)^2 \frac{x^3}{\left[p^2\left(x-\frac{1}{2}\right)^2+\rho^2\right]^3}\int_0^1 d\beta\tilde{g}(\beta)\int_0^1 d\beta'\tilde{g}(\beta')\int d\sigma d\tau \left[(\beta-x)\sigma+(\beta'-x)\tau\right] \tag{33'}$$

which is nonvanishing ($\int d\sigma d\tau$ is extended over the region given by (26)). Hence $p(x)$ is not symmetric.

Since $p_A(x)$ is antisymmetric under $x \to 1-x$, it does not contribute to the Adler's sum rule and to the normalization of the charge in the ladder approximation. However the fraction of the momentum carried by each parton now depends on $p_A(x)$ and given by:

$$\langle x\rangle = \frac{\int_0^1 F_2(x)dx}{\int_0^1 p(x)dx} = \frac{1}{2} + \frac{\int_0^1 x\, p_A(x)dx}{\int_0^1 p_S(x)dx}$$

which does not amount to 50% of the total momentum. Obviously in the 2-body ladder approximation with no neutral boson component in the wave function, the matrix elements of $\theta_{\mu\nu,\mu_1\cdots\mu_n}(0)$ and $\theta_{\mu\nu}(0)$ (the energy momentum tensor) cannot be evaluated by the lowest order impulse approximation. Radiative corrections and off-shell effect of these matrix elements between virtual parton states must be included. Also the interaction energy which provides the binding energy for the constituents must also be included. Because of the radiative corrections due to the emission and absorption of gluon by the same constituent, the constitutents can no longer be treated as elementary and the factor $e^{ik\cdot x}$ in Eq. (21) for $I(p\cdot x,\, x^2)$ must now be replaced by $\int_0^1 dy\; g(y,k^2)\exp(iyk\cdot x)$:

$$I(p\cdot x,\ x^2) = \frac{i}{(2\pi)^4}\int_0^1 dy \int \frac{d^4k\ g(y,k^2)e^{iyk\cdot x}}{(k^2-\mu^2-i\varepsilon)^2\left[(p-k)^2-\mu^2-i\varepsilon\right]}\ \Gamma_p(k,p-k)\bar{\Gamma}_p(p-k,k) \quad (21')$$

$g(y,k^2)$ is defined as the structure function for the constituent off the mass shell. In particular $g(y,k^2)$ can be calculated to any order in perturbation theory. If these radiative corrections arise from the emission and absorption of the same gluon which is exchanged between the constituents in the ladder approximation then part of the gluon component in the B.S. wave function can be included in $g(y,k^2)$. The free field approximation for $g(y,k^2)$ is thus only consistent with the valence parton approximation for the B.S. wave function within the ladder approximation and one will not obtain a consistent result without going beyond the ladder approximation. We note in passing that these corrections give nonleading terms to the parton structure function as $x \to 1$ since the gluons are radiated from the valence parton and due to phase space limitation, the correction terms behave at least as $(1-x)$ relative to the leading term as $x \to 1$.

Consider now the weak binding limit $(\rho^2/p^2 << 1)$, $\tilde{g}(\beta)\sim\delta\left(\beta - \frac{1}{2}\right)$, $p(x)\sim\delta\left(x - \frac{1}{2}\right)$ and the second term is zero. $p(x)$ is then symmetric and since there is no interaction each parton carries half the total momentum of the bound state. The deviation from the zero binding limit (δ-function approximation) is proportional to the binding which is of the order $O(g^2)$, for g^2 small. In fact a straightforward calculation gives $\langle x\rangle = \frac{1}{2} - \frac{8}{3}\ \gamma^2$. $(\gamma^2 = \rho^2/p^2 << 1)$. It is necessary to calculate the radiative corrections (off-shell effect, interaction energy etc...) to $O(g^2)$ for $\langle p|\theta_{\mu\nu}(0)|p\rangle$ to check the momentum sum rule.

In the tight binding limit, using the solution (19a) for $g(z)$ we obtain the following expression for the structure function:

$$F_2(x) = N^2 x^2 (1-x)^3 Z(x) \tag{34}$$

$$Z(x) = \left(\frac{1}{6}\right)\left[\frac{1}{4} x^4 + \frac{1}{4} x^2(1-x)^2 + \left(\frac{1}{2} x^2(1-x)^2 + \frac{x^3}{1-x} \left(\ell n x + 1-x + \frac{1}{2}(1-x)^2\right) + x^3\left(\frac{1}{2}(1-x) - \frac{1}{3} - \frac{x^2}{(1-x)^3} \left(\ell n x + 1-x + \frac{1}{2}(1-x)^2\right)\right)\right) + \frac{2}{3} x^4(I_1+I_2) + 2I_3\right] \tag{35}$$

where, $\left(\text{putting } \lambda = \frac{1-x}{x}\right)$,

$$I_1 = \frac{4}{5}\lambda - \frac{1}{10}\left(1 - \frac{\ell n(1+\lambda)}{\lambda}\right) - \frac{1}{4}\lambda\left(1 - \frac{\ell n(1+\lambda)}{\lambda}\right) + \frac{1}{2}\lambda^2\left(1 - \frac{3}{20} + \frac{3}{10}\lambda\right) - \frac{1}{2}\lambda^2\left(1+\lambda+\frac{3}{10}\lambda^2\right)\ell n\left(\frac{1+\lambda}{\lambda}\right)$$

$$I_2 = \frac{4}{5}\lambda - \frac{1}{4}\lambda\left(1-\lambda\,\ell n\left(\frac{1+\lambda}{\lambda}\right)\right) - \frac{\lambda^2}{10}\left(1-\lambda\,\ell n\left(\frac{1+\lambda}{\lambda}\right)\right) + \frac{1}{2}\left(1 - \frac{1}{\lambda}\,\ell n(1+\lambda)\right) - \frac{1}{2}\,\ell n(1+\lambda) + \frac{3}{20}\left(\frac{1}{\lambda}\left(1 - \frac{1}{\lambda}\,\ell n(1+\lambda)\right) - \frac{1}{2}\right)$$

$$\begin{aligned} I_3 = {} & \frac{4}{5} x^3\left[\frac{1}{3}(1-x) + \frac{x}{(1-x)^3}\left(\ell n x + 1-x + \frac{1}{2}(1-x)^2 + \frac{1}{3}(1-x)^3\right)\right] \\ & - \frac{1}{10} x^3\left[\frac{1}{2}(1-x) + \frac{x}{(1-x)^2}\left(\ell n x + 1-x + \frac{1}{2}(1-x)^2\right)\right] \\ & - \frac{1}{30} x^3\left[(1-x) + \frac{x}{1-x}(\ell n x + 1-x)\right] \\ & - \frac{1}{30} x^2(1-x)\left(1 + \frac{1-x}{x}\,\ell n(1-x)\right) \end{aligned} \tag{35'}$$

It is clear that the structure function behaves like $(1-x)^3$ as $x \to 1$. The full expression for $F_2(x)$ however is rather complicated. There are also terms like $x \ln x$ and $(1-x)\ln(1-x)$ which go to zero as $x \to 0,1$.

The result of a numerical calculation is given in Fig. 3 with $F_2(x)$, $p(x)$ and $Z(x)$ calculated as functions of x. As can be seen, $p(x)$ is not symmetric around $x = \frac{1}{2}$ and it turns out that each parton carries only 47.5% of the total momentum. The missing momentum therefore must be due partly to the interaction energy between the constituents.

In general, the solution of Eq. (18) will give $\tilde{g}(\beta)$ and the structure function is then completely determined. Since $\tilde{g}(\beta) \sim \beta(1-\beta)$ as $\beta \to 0,1$, the integral $Z(x)$ is well defined. As $x \to 1$, only the first term on the r.h.s. of (29) survives, hence $Z(x) \to C$ (a constant) as $x \to 1$. It is interesting to note that the expressions for $\tilde{g}(\beta)$ and $F_2(x)$ look similar if one identifies x with β. Both $\tilde{g}(x)$ and $F_2(x)$ are peaked at $x = \frac{1}{2}$ due to the presence of the factor

$$\left(\frac{1}{p^2\left(x-\frac{1}{2}\right)^2+\rho^2}\right) .$$

The physical meaning of $\tilde{g}(\beta)$ can be seen by looking at (20') for $\Gamma_p(k,p-k)$. In the Bjorken's limit $\left(k^2 \to \infty,\ p\cdot k \to \infty, \xi = \frac{k^2}{2p\cdot k} \text{ fixed}\right)$, we have:

$$\Gamma_p(k,p-k) \longrightarrow \frac{1}{2p\cdot k}\,\Gamma_p(\xi) \tag{36}$$

where

$$\Gamma_p(\xi) = i\int_0^1 d\beta\,\tilde{g}(\beta)\,\frac{1}{[\xi-\beta-i\varepsilon]} \tag{37}$$

is defined as the "structure function" for Γ_p in the scaling limit. From this integral representation we see that the behavior of $\Gamma_p(\xi)$ depends on that of $\tilde{g}(\xi)$ as $\xi \to 1$. It follows then that the behavior of $\Gamma_p(\xi)$ determines the behavior of the structure function as $x \to 1$. This connection between $\tilde{g}(\beta)$ and the short distance behavior of the B.S. wave function can be established in a more general manner by means of the operator product expansion and has been studied by Callan and Gross,[12] by Ciafaloni and Menotti,[13] Menotti[14] and more recently by Goldberger, Soper and Guth.[15] These analyses show that the behavior of $\tilde{g}(\beta)$ as $\beta \to 0,1$ is determined by the Bjorken limit of the B.S. wave function which is given by the leading light cone ($x^2=0$) singularities and by the region $\xi \to 1$.

The behavior of the electromagnetic form factor $F(q^2)$ at large momentum transfer q^2 is also determined[16] by the behavior of $\tilde{g}(\beta)$ as $\beta \to 0,1$. Hence the relation between the form factor at large q^2 and the structure function as $x \to 1$ is clearly visible through the behavior of $\tilde{g}(\beta)$ as $\beta \to 0,1$.

So far we have presented a detailed relativistic calculation of the structure function of a S-wave bound state system of two spinless bosons interacting via the exchange of a massless boson in $\lambda\phi^3$ theory. If the exchanged particle has a non-vanishing mass, the expression for $F_2(x)$ is now rather complicated and no simple form can be found. We have:

$$F_2(x) = N^2\left(\frac{1}{4\pi}\right)^2 x^2(1-x)^3\int_0^\infty dt\,dt'\ Z(x;t,t') \tag{38}$$

where

$$Z(\alpha;t,t') = \int_0^1 d\beta\tilde{g}(\beta,t)\int_0^1 d\beta'\tilde{g}(\beta',t')\int d\sigma d\tau\left[1 - \frac{\alpha}{1-\alpha}\left((1-\beta)\sigma+(1-\beta')\tau\right)\right]$$

$$\times \frac{1}{\left[p^2\left(\alpha - \frac{1}{2}\right)^2+\rho^2+\alpha(\sigma t+\tau t')\right]^3} \qquad (38')$$

(integration over $d\sigma d\tau$ is carried out in the region defined by Eq. (26)) and

$$\tilde{g}(\beta,t) = \frac{g(\beta,t)}{\left[p^2\left(\beta - \frac{1}{2}\right)^2+\rho^2+t\right]}$$

and $g(\beta,t)$ satisfies[16]

$$\int_0^\infty |g(\beta,t)|dt \sim \beta(1-\beta)$$

(for a regular solution in $\lambda\phi^3$ theory).

The resulting integration over dt dt' is then finite and $F_2(x) \sim (1-x)^3$ as $x \to 1$.

Assuming the DGSI representation is valid for a spin $\frac{1}{2}$ bound state, similar results can also be obtained for a composite nucleon considered to be a bound state of a spin $\frac{1}{2}$ and a spinless constituent. However for all realistic physical problems, where the nucleons and mesons are considered to be bound states of quark and antiquark due to the exchange of massless vector gluons in Quantum Chromodynamics (QCD), such features for the structure function cannot be easily derived as the interaction is singular and the non-zero energy solutions for the B.S. equation are not known at present. The zero energy solution is useful for the study of some of the low energy aspects of the pion wave function (e.g. the pion decay constant f_π etc...) but not suitable for use in the study of the structure function. It is hoped that a similar DGSI representation could be derived for all the independent amplitudes in the decomposition of the fermion-antifermion B.S. wave function by Feldman, Fulton and Townsend.[17] In non-gauge theory, since the light cone limit ($x^2 \to 0$) determines the behavior of the structure function as $x \to 1$, expression for the pion B.S. wave function obtained by Callan and Gross[12] and by Menotti[14] can now be used to calculate the pion structure function. In the following we shall limit ourselves to non-gauge theory for a fermion-antifermion bound state and give a calculation of the pion structure function using this wave function.

B. Structure Function for a Fermion-Antifermion Bound State

Let q_1, q_2 be the momenta of the two particles, the fermion-antifermion B.S. wave function in momentum space is defined as:

$$\psi_p(q_1,q_2) = \int d^4x\, e^{iq_1\cdot x} \langle 0|T\{\psi(x)\bar{\psi}(0)\}|p\rangle \gamma_5 \quad . \tag{39}$$

The pseudoscalar character of the bound state has been explicitly displayed by the factor γ_5. $\psi_p(q_1,q_2)$ satisfies the B.S. equation:

$$\Gamma_p(q_1,q_2) = (\not{q}_1-m)\psi_p(\not{q}_2-m) = \frac{\lambda}{\pi^2}\int \frac{d^4k'}{(k'-k)^2-i\varepsilon}\,\psi_p(k') \tag{40}$$

λ is the coupling constant (squared) and $\Gamma_p(q_1,q_2)$ is the B.S. vertex function. ($\psi_p(q_1,q_2)$ has to be symmetrised for $^1S_0, C = +1$ state.). Also the final state wave function $\bar{\psi}_p(q_2,q_1)$ is defined as:

$$\bar{\psi}_p(q_2,q_1) = \int dx\, e^{-iq_1\cdot x} \langle p|T\{\psi(0)\bar{\psi}(x)\}|0\rangle \gamma_5 \tag{41}$$

which can be shown to be related to $\psi_p(q_1,q_2)$ using PT invariance. We have for a negative parity state:

$$\bar{\psi}(q_2,q_1) = -(C\gamma_5)^T \psi^T(q_1,q_2)(C\gamma_5) \tag{42}$$

where C is the charge conjugation matrix.

The DGSI representation for ψ_p is not known at present, however, it has been shown by Callan and Gross[12] and by Menotti[14] using Conformal Invariance, that this representation is valid for $q_1^2 \to \infty$, $q_2^2 \to \infty$ and in a theory with massless particles. Following Menotti we have

$$\Gamma_p(q_1,q_2) = \not{q}_1\not{q}_2\int_0^1 du\left[q_1^2 u + q_2^2(1-u)\right]^{1-d'-\bar{d}/2}\left[u(1-u)\right]^{\bar{d}/2-1/2}$$
$$+ \frac{d'-\bar{d}/2-2}{d'+\bar{d}/2-2}\int_0^1 du\left[q_1^2 u + q_2^2(1-u)\right]^{2-d'-\bar{d}/2}\left[u(1-u)\right]^{\bar{d}/2-1/2} \tag{43}$$

for the γ_5-even term (even number of γ_μ); d' and $\bar{d}$ being respectively the dimensions of the fermion and the composite fields. In the following we shall assume the fermion and the pion fields to have canonical dimension (in the limit of small coupling constant λ) and put $d' = \frac{3}{2}$, $\bar{d} = 3$. In terms of q_1=k and q_2=p-k, one can then write $\Gamma_p(k,p-k)$ as follows (including the nonleading term $\not{q}_1$ and $\not{q}_2$):

$$\Gamma_p(k,p-k) = \int_0^1 du\,\frac{u(1-u)}{\left[k^2-2up\cdot k+up^2-i\varepsilon\right]} + \not{q}_1\not{q}_2\int_0^1 du\,\frac{u(1-u)}{\left[k^2-2up\cdot k+up^2-i\varepsilon\right]^2}$$
$$+ \not{q}_1\int_0^1 du\,\frac{\frac{1}{3}(2-u)\left[u(1-u)\right]^{\frac{1}{2}} + \frac{2}{3}u(1-u)\left[u^{\frac{1}{2}}(1-u)^{\frac{1}{2}}\right]'}{\left[k^2-2up\cdot k+up^2-i\varepsilon\right]^{3/2}}$$
$$+ \not{q}_2\int_0^1 du\,\frac{\frac{1}{3}(1+u)\left[u(1-u)\right]^{\frac{1}{2}} - \frac{2}{3}u(1-u)\left[u^{\frac{1}{2}}(1-u)^{\frac{1}{2}}\right]'}{\left[k^2-2up\cdot k+up^2-i\varepsilon\right]^{3/2}} \tag{44}$$

where the notation [ ]' means the derivative with respect to u. This is precisely the DGSI representation for the B.S. wave function. Notice that expression (44) is similar to that for a bound state of two spinless particles in Eq. (20') with u(1-u) replaced by $\tilde{g}(u)$. This suggests a representation for $\Gamma_p(k,p-k)$ can be approximately given by:

$$\begin{aligned}\Gamma_p(q_1,q_2) &= \int_0^1 d\beta \frac{\tilde{g}_1(\beta)}{[k^2-2\beta p\cdot k+\beta p^2-m^2-i\varepsilon]} \\ &+ \not{q}_1\not{q}_2 \int_0^1 d\beta \frac{\tilde{g}_2(\beta)}{[k^2-2\beta p\cdot k+\beta p^2-m^2-i\varepsilon]^2} \\ &+ \not{q}_1 \int_0^1 d\beta \frac{\tilde{g}_3(\beta)}{[k^2-2\beta p\cdot k+\beta p^2-m^2-i\varepsilon]^{3/2}} \\ &+ (\not{q}_2-\not{q}_1)\int_0^1 d\beta \frac{(1-\beta)\tilde{g}_3(\beta)}{[k^2-2\beta p\cdot k+\beta p^2-m^2-i\varepsilon]^{3/2}} \\ &+ \frac{2}{3}(\not{q}_2-\not{q}_1)\int_0^1 d\beta \frac{(1-2\beta)\tilde{g}_3(\beta)+(1-\beta)\tilde{g}_3'(\beta)}{[k^2-2\beta p\cdot k+\beta p^2-m^2-i\varepsilon]^{3/2}}\end{aligned}$$

with

$$\tilde{g}_1(\beta) = \frac{g_1(\beta)}{\left(\beta-\frac{1}{2}\right)^2 p^2+\rho^2}$$

$$\tilde{g}_2(\beta) = \frac{g_2(\beta)}{\left(\beta-\frac{1}{2}\right)^2 p^2+\rho^2}$$

$$\tilde{g}_3(\beta) = \frac{g_3(\beta)}{\left(\beta-\frac{1}{2}\right)^2 p^2+\rho^2}$$

and $g_1(\beta)$, $g_2(\beta)\sim\beta(1-\beta)$, $g_3(\beta)\sim[\beta(1-\beta)]^{\frac{1}{2}}$ as given by conformal invariance. This is also the solution for $g(\beta)$ obtained by Wick for the spinless case in the tight binding limit.

This should be a good approximation to the B.S. vertex function for a pseudoscalar fermion-fermion bound state as it satisfies conformal invariance in the limit of large q_1^2/m^2 and q_2^2/m^2 and it exhibits all the binding effect and the nonrelativistic limit for a weakly bound system.

The structure function can now be calculated from the quantity $f(p\cdot x,x^2)$ defined in Eq. (10) in terms of $I(p\cdot x,x^2)$.

We have:

$$p_\mu p_\nu f(p\cdot x,x^2) = -\frac{i}{(2\pi)^4}\int \frac{d^4k\ e^{ik\cdot x}}{[k^2-m^2-i\varepsilon]^2}\ \frac{\mathrm{Tr}\left\{\bar{\Gamma}_p\,(\not{k}+m)\gamma_\mu k_\nu(\not{k}+m)\Gamma_p\ (\not{k}-\not{p}+m)\right\}}{[(p-k)^2-m^2-i\varepsilon]} \tag{46}$$

For the scalar term (no γ matrix), the trace is given by

$$\mathrm{Tr}\{\ldots\} = \left[p_\mu k_\nu(k^2-m^2) + k_\mu k_\nu(k^2-m^2-2p\cdot k+4m^2)\right] S^2 \tag{47}$$

where

$$S = \int_0^1 d\beta\ \frac{\tilde{g}_1(\beta)}{[k^2-2\beta p\cdot k + \beta p^2-m^2-i\varepsilon]} \tag{48}$$

The first term in Eq. (47) gives rise to a structure function proportional to x, it is obtained from the quantity $I_1(p\cdot x,x^2)$ defined as

$$I_1(p\cdot x,x^2) = \frac{-i}{(2\pi)^4}\int \frac{d^4k\ e^{ik\cdot x}\ S^2}{(k^2-m^2-i\varepsilon)\ [(p-k)^2-m^2-i\varepsilon]} \tag{49}$$

Using (48) for S, we obtain:

$$I_1(p\cdot x,x^2) = \frac{-i}{(2\pi)^4}\ \pi^2\int_0^1 d\beta\int_0^1 d\beta'\tilde{g}(\beta)\tilde{g}(\beta')\int_0^1 d\alpha\ \exp(i\alpha p\cdot x)$$

$$\times\int_0^\infty \rho\ d\rho\int d\sigma d\tau\ \alpha^2\ \exp\left[i\rho(m^2-\alpha(1-\alpha)p^2) + i\,\frac{x^2}{4\rho}\right]$$

Carrying out the integrations over $d\rho d\sigma d\tau$ in the same manner as before, we obtain a contribution to $F_2(x)$ a term:

$$A_1(x) = N^2\left(\frac{1}{4\pi}\right)^2 x(1-x)^2 Z_1(x) \frac{1}{\left[\left(x - \frac{1}{2}\right)^2 p^2+\rho^2\right]^2} \tag{50}$$

where

$$Z_1(\alpha) = \frac{1}{2}\left[\left(\int_0^\alpha \frac{d\beta}{1-\beta}\tilde{g}_1(\beta)\right)^2 + \left(\frac{\alpha}{1-\alpha}\right)\left(\int_\alpha^1 \frac{d\beta}{\beta}\tilde{g}_1(\beta)\right)^2 \right.$$
$$\left. + \frac{2}{(1-\alpha)^2}\int_0^\alpha d\beta \int_\alpha^1 d\beta' \frac{\tilde{g}_1(\beta)\tilde{g}_1(\beta')}{(\beta'-\beta)} \left\{\frac{\alpha(\beta'-\alpha)}{\beta'} + \frac{(1-\alpha)(\alpha-\beta)}{1-\beta}\right\}\right] \tag{51}$$

Since $\tilde{g}_1(\beta)\sim\beta(1-\beta)$ as $\beta \to 0,1$, as $\alpha \to 1$, $Z_1(\alpha) \to C$ (a constant) hence $A_1(x)\sim(1-x)^2$ as $x \to 1$. The second term on the r.h.s. of (47), being proportional to $k_\mu k_\nu$ gives a contribution to the structure function a term proportional to x^2 as in the case of a spinless bound state. This contribution is obtained from the quantity $I_2(p\cdot x,x^2)$:

$$I_2(p\cdot x,x^2) = \frac{-i}{(2\pi)^4}\int d^4k \frac{e^{ik\cdot x}\; S^2}{(k^2-m^2-i\varepsilon)^2}\left[1 + \frac{4\left(m^2 - \frac{1}{4}p^2\right)}{(p-k)^2-m^2-i\varepsilon}\right] \tag{52}$$

The second term in the integral of (52) contains the same denominator as that of $I(p\cdot x,x^2)$ in Eq. (10') and gives $F_2(x)$ a term

$$B(x) = N^2\left(\frac{1}{4\pi}\right)^2 4\rho^2x^2(1-x)^3 Z(x) \left(\frac{1}{\left(x - \frac{1}{2}\right)^2 p^2+\rho^2}\right)^3 \tag{53}$$

which is much smaller than $A_1(x)$ for $x \to 1$ in the weak binding limit $p^2 << m^2$ and gives the nonleading $(1-x)^3$ behavior for $F_2(x)$ as $x \to 1$. It may be however quite comparable to $A_1(x)$ as $x \to \frac{1}{2}$. It is clear then that the mass terms cannot be neglected in the nonrelativistic limit. Similarly, the first integral in (52) gives $F_2(x)$ a term:

$$A_2(x) = N^2\left(\frac{1}{4\pi}\right)^2 x^2(1-x)^2 Z_2(x) \left[\frac{1}{\left(x-\frac{1}{2}\right)^2 p^2+\rho^2}\right]^2 \tag{54}$$

where

$$Z_2(\alpha) = \frac{1}{2}\int_0^\alpha d\beta\tilde{g}_1(\beta)\int_0^\beta d\beta'\tilde{g}_1(\beta')d\beta' + \int_\alpha^1 d\beta\tilde{g}_1(\beta)\,\frac{\alpha}{(1-\alpha)^2}\int_\beta^1 d\beta'\tilde{g}_1(\beta')\left(\frac{\beta-\alpha}{\beta}\right)^2$$
$$+\left(\frac{\alpha}{1-\alpha}\right)\int_\alpha^1 d\beta\tilde{g}_1(\beta)\int_\alpha^\beta d\beta'\tilde{g}_1(\beta')\left(\frac{1-\beta}{\beta}\right)^2\left[1+\frac{\beta-\alpha}{\beta(1-\alpha)}\right] \tag{55}$$

The contribution from the terms with $\not{q}_1\,\not{q}_2$, $\not{q}_1$ and $\not{q}_2$ can be obtained in a similar manner. The calculation is quite complicated and will be given elsewhere. In addition to terms with $(1-x)^2$ and $(1-x)^3$ behavior, there are also terms which behave like $(1-x)^{5/2}$ which come from the $\not{q}_1$ and $\not{q}_2$ term.

Thus in a rather simple manner we have been able to calculate the deep inelastic structure function for a pseudoscalar fermion-antifermion bound state in non-gauge theory with the help of the DGSI representation and conformal invariance. The result is a structure function $F_2(x)$ for the pion which displays all the features of relativistic and nonrelativistic limit. The leading $(1-x)^2$ behavior as $x \to 1$ agrees with that obtained previously[6] by Ezawa, Farrar and Jackson and by Soper. We have not discussed the case of anomalous dimension in the ladder approximation. This case has been studied by Gross and Callan and by Menotti and Ciafaloni. Using their results and carrying out a similar calculation as above, we found that $F_2(x)$ behaves like

$$(1-x)^{2+\frac{1}{2}\gamma_\theta}$$

with γ_θ the anomalous dimension of the mass insertion term (in the lowest order approximation $\gamma_\theta \approx \frac{-g^2}{4\pi^2}$, for g small)

(The $(1-x)^D$ behavior, with $D=2+\frac{1}{2}\gamma_\theta$ comes from the integrals of the form $\int d\sigma d\tau\, \sigma^{D-2}\tau^{D-2}$ in $I(p\cdot x,x^2)$). It is not known how the factor $\left(\frac{1}{\left(\beta-\frac{1}{2}\right)^2+\rho^2}\right)$

should be replaced by some similar factor with anomalous dimension. For this reason we have suggested that in the limit of small coupling constant

factor $\frac{1}{\left(\beta-\frac{1}{2}\right)^2 p^2+\rho^2}$

is not modified by the anomalous dimension, hence the structure function can still be proportional to $\frac{1}{\left(\beta-\frac{1}{2}\right)^2 p^2+\rho^2}$.

III. CONCLUSION

In the ladder approximation the structure function for a bound state system can be expressed in a simple manner in terms of the spectral function in the DGSI representation of the solutions of the B.S. equation. We have found the remarkable result that the structure function bears some resemblance to the spectral function and the behavior of the structure function as $x \to 1$ is related to the large q^2 behavior of the electromagnetic form factor by the spectral function. The relativistic expression obtained is useful as a guide to the nonrelativistic limit and should be used in any experimental fit with data. Though conformal invariance is useful as a means of getting an approximate solution, more work on the DGSI representation for different amplitudes in the fermion-antifermion bound state is needed before any rigorous calculation of the structure function in both non-gauge and gauge theory.

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Note Added

After completion of the paper we became aware of a similar calculation of the structure function using the DGSI representation by P. Fishbane and I. Muzinich (P.M. Fishbane and I. J. Muzinich, Phys. Rev. D8, 4015 (1973)) and similar results obtained by R. Blankenbecler, S. J. Brodsky and J. Gunion. (R. Blankenbecler, S. J. Brodsky and J. Gunion, Phys. Rev. D12,3469 (1975)).

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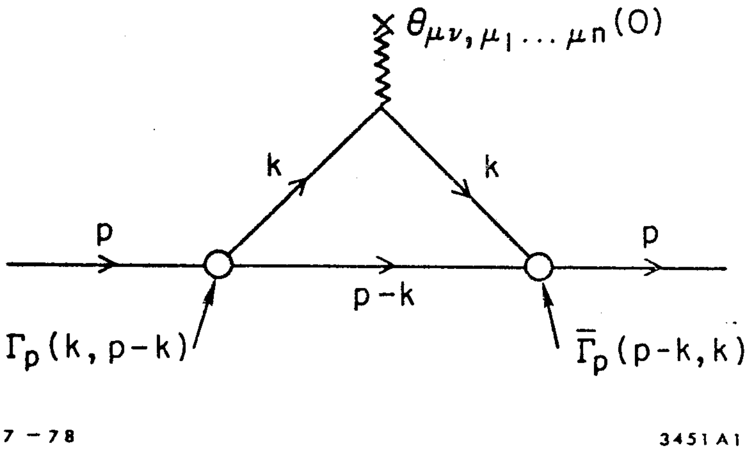
FIGURE CAPTIONS

1. The matrix element of $\theta_{\mu\nu,\mu_1 \cdots \mu_n}(0)$ in the impulse approximation.
2. The same with radiative corrections.
3. The structure function $F_2(x)$ and the parton distribution function $p(x)$ in the tight binding limit $(p^2 \to 0)$.



$\theta_{\mu\nu,\mu_1\ldots\mu_n}(0)$
k
k
p
p
p−k
$\Gamma_p(k, p-k)$
$\bar{\Gamma}_p(p-k, k)$
7 − 78
3451A1

Fig. 1

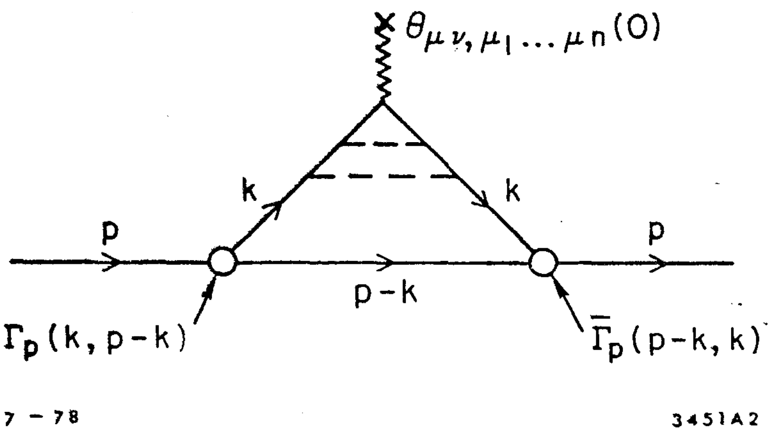

$\theta_{\mu\nu,\mu_1\ldots\mu n}(0)$
k
k
p
p
p-k
$\Gamma_p(k, p-k)$
$\bar{\Gamma}_p(p-k, k)$
7 - 78
3451A2

Fig. 2

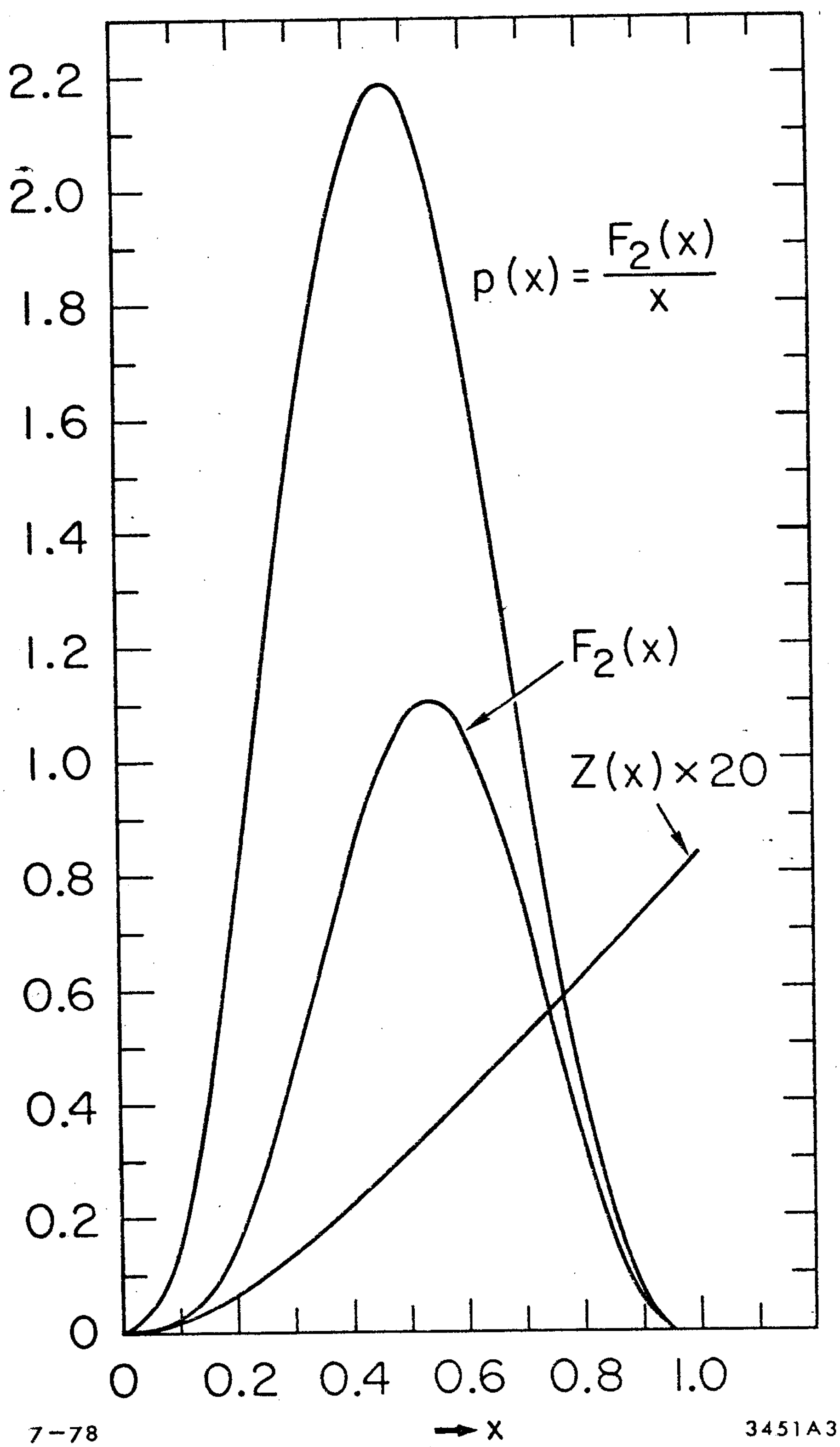

2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
$p(x) = \frac{F_2(x)}{x}$
$F_2(x)$
$Z(x) \times 20$
0
0.2
0.4
0.6
0.8
1.0
7-78
→ x
3451A3

Fig. 3