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GAUGE THEORIES OF THE WEAK INTERACTIONS*

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Lecture I

Nowadays it seems almost unnecessary to motivate a discussion of gauge theories of the weak interactions--they are fast becoming the accepted dogma. Furthermore one particular version, the Weinberg-Salam version,[1] or (more specifically in the context of charm) the Weinberg-Salam-Glashow-Iliopoulos-Maiani[2] version seems to be able to explain most of the relevant data, though some areas are still unclear, especially the question of parity violation in atomic physics. My lectures will focus on this model. I will try to leave you with some feeling for how it is put together, which along the way will allow comment on some possible variations, many of which exist in the literature.

In spite of my first disclaimer let me begin with a short discussion of the improvement afforded by gauge theories over their predecessor, the four fermi theory of weak interactions. That theory was successful in describing the phenomenology of low energy weak interactions (such as angular and energy distributions of product particles in β-decay) but was not completely satisfactory for two (closely related) reasons: At sufficiently high energy ($\sim$ 300 GeV) the predictions violate unitarity, and any attempt to perform higher order calculations is plagued by infinities which cannot be removed by renormalization. We needed a theory which could remove these two problems without changing

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the low energy predictions--gauge theories provide such a theory. In addition, each gauge model one writes down makes a host of new and testable predictions. Weinberg in 1967 wrote down "a theory of leptons" as a first simple example of such a theory. This model, extended to incorporate hadronic weak interactions by inserting the quarks by analogy to the leptons, gives a remarkably successful phenomenology.

When Weinberg wrote his model he hoped it would solve the above mentioned problems of renormalizability and unitarity--that it did indeed do so was shown somewhat later.[3] In a four-fermi or current-current theory we start with a weak charge-changing current, empirically determined to be of the V-A type, for example for leptons

$$j_\alpha = \bar{\mu}\gamma_\alpha \frac{(1-\gamma_5)}{2}\nu_\mu + \bar{e}\gamma_\alpha \frac{(1-\gamma_5)}{2}\nu_e \tag{I.1}$$

(I shall use Bjorken and Drell conventions throughout, and notice that V-A = $1-\gamma_5$ with my definitions. Also I will often write particle names to stand for the Dirac spinor for that particle.) The weak interaction amplitude is then

$$\frac{4G}{\sqrt{2}}\, j_\alpha j^{\alpha\dagger} \tag{I.2}$$

(The factor of 4 may look strange, it compensates for the fact that I have written j_μ with $(1-\gamma_5)/2$ rather than the old-fashioned $(1-\gamma_5)$. This definition will be convenient to maintain when we get to gauge theories since $(1-\gamma_5)/2$ is the correct projection operator for left-handed fermions, in fact one usually sees the shorthand μ_L for $((1-\gamma_5)/2)\mu$ in gauge theory papers.)

The interaction (I.2) can be represented diagramatically, for example, the process shown in Fig. 1 is part of the cross term between the electron and muon pieces of the current. The idea of introducing an intermediate vector boson to try to damp the high energy growth of this amplitude predates its gauge theory realization by some time. Naively one might hope the diagram of Fig. 2 for which the amplitude is given by

$$A = g^2 \, j_\alpha \, D^{\alpha\beta} j_\beta^\dagger \tag{I.3}$$

would give a suppression of $m_W^2/(s-m_W^2)$ for large s when g^2/m_W^2 is adjusted to give the correct low energy strength. Clearly this requires m_W^2 to be large enough that at present energies the propagator is effectively a constant, in order to maintain the good results of the current-current theory. That is easily enough achieved, however in this simple form the idea does not work for all possible processes. In this process $e^-\bar{\nu}_e \to \mu^-\bar{\nu}_\mu$ it provides the necessary suppression, but when looking at other processes, for example, $e^+e^- \to W^+W^-$ and even $e^+e^- \to W^+W^-\gamma$ one finds again problems with unitarity. The problem is that the propagator for a massive vector particle has the form

$$D_{\alpha\beta}(q) = -i\left(g_{\alpha\beta} - \frac{q_\alpha q_\beta}{m^2}\right)/(q^2-m^2) \tag{I.4}$$

The term proportional to $g_{\alpha\beta}$ has indeed the desired behavior in all cases but the $q_\alpha q_\beta/m^2$ term in some processes can give terms of order q^2/m^2 which cancel out any large q^2 suppression from the denominator.

After gauge theories had been found to be a workable way to circumvent this problem several people asked the question "Are they the only way?" in the following form: Suppose I start with the vectors and quarks coupled as in the process (I.3) and allow in addition neutral vector and scalar particles in the theory with arbitrary masses and coupling constants. Now I require tree graph unitarity,[4] this is that the partial wave amplitudes generated by the sum of tree graph diagrams for a given process should not grow more rapidly than s^{2-m} for $2 \to m$ particle processes. Imposing this condition on a sufficiently large set of amplitudes gives relationships among the masses and coupling constants (Yukawa couplings and vector-scalar couplings as well as vector-vector couplings). In every case the set of couplings so determined are a set which one could derive by building a gauge theory with the same particle content!

Having come so far, let me now explain how to build a gauge theory. The recipe is simple[5]

I. Choose a gauge group.

II. Choose fermion representation content.

III. Choose Higgs scalar representation content.

IV. Arrange for spontaneous symmetry breaking to give a non-vanishing vacuum expectation value for some scalar or set of scalars.

Of course all this needs some further explanation to be meaningful--and some cleverness in following the steps to arrive at a possible theory of the weak and electromagnetic interactions--there are many theories I could write following steps I to IV which would not be viable for this purpose--for example it is trivial to arrange that only one massless vector survives after the spontaneous symmetry breaking but it is somewhat more complicated to arrange that that vector has the correct couplings to be a photon.

Let us start with step I. What do I do when I choose a gauge group. In a gauge theory the vector mesons are always in the adjoint representation of the group, so choosing a group tells me how many vector mesons I have, and defines the way they couple to one another. In less group theoretic language "in the adjoint representation" means there is one vector meson for each independent structure matrix λ^{α}. In SU(2) the structure matrices are the set of traceless unitary 2×2 matrices, the familiar Pauli σ-matrices, of which there are three $((2\times2)-1)$ so SU(2) means three vectors. A product of groups such as SU(2) $\times$ U(1) has as many vectors as needed for each factor group separately so SU(2) $\times$ U(1) has four vectors, SU(3) has eight $((3\times3)-1)$, etc.

In deriving vector couplings it is convenient to define the matrix

$$A_{\mu} = A_{\mu}^{\alpha}\, \lambda^{\alpha} \quad . \tag{I.5}$$

Since every term in the Hamiltonian (or Lagrangian) must be a scalar (singlet) under the gauge group we can readily construct possible terms from the objects (I.5) by taking traces, for example

$$\mathrm{TR}\ (A_{\mu} A_{\nu} A_{e}) = if^{\alpha\beta\gamma}\ A_{\mu}^{\alpha} A_{\nu}^{\beta} A_{e}^{\gamma} \tag{I.6}$$

is a group singlet three-vector term. The structure function $f^{\alpha\beta\gamma}$ is defined by

$$\left[\lambda^{\alpha}, \alpha^{\beta}\right] = if^{\alpha\beta\gamma}\ \lambda^{\gamma} \quad . \tag{I.7}$$

Of course the Lorentz indices in (I.6) must also be contracted in some way to give it the correct Lorentz invariance properties.

Now we come to step II, choosing the representation content of the fermions. Let us discuss this and subsequent steps in the context of SU(2) $\times$ U(1) in order to give concrete examples. Choosing representation content simply means choosing which multiplets of fermions we are to introduce. The Weinberg SU(2) is often called weak isospin, a priori we may choose fermions as weak isospin singlets, doublets, triplets, etc. In doing this one treats the left- and right-handed components of the fermions completely separately. The choice we make is guided by experiment. Let us start by examing Weinberg's choices for the leptons. He chose left-handed doublets

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \tag{I.8}$$

and right-handed singlets e_R, μ_R and the standard Weinberg-Salam model extends this choice to quarks

$$\begin{pmatrix} u \\ d_c \end{pmatrix}_L \quad \begin{pmatrix} c \\ s_c \end{pmatrix}_L \quad \cdots \quad u_R,\ d_R,\ s_R,\ c_R \ \cdots \tag{I.9}$$

where

$$d_c = d \cos \theta_c + s \sin \theta_c \quad \text{and} \quad s_c = -d \sin \theta_c + s \cos \theta_c$$

Why these choices? For the leptons they are clearly the simplest possible choice, which allows us to couple to the SU(2) vectors, to left-handed fermions. Using the group singlet quantity

$$\left(\nu_e e\right)_L^\dagger \gamma_0 \gamma^\mu \left(A_\mu^\alpha \cdot \sigma^\alpha\right) \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \tag{I.10}$$

while the U(1) vector can couple to both left- and right-handed fermions

$$\left(\nu_e e\right)_L^\dagger \gamma_0 \gamma^\mu B_\mu \cdot I \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \text{and} \quad e_R^\dagger \gamma_0 \gamma^\mu B_\mu e_R \tag{I.11}$$

All I am doing here is constructing group singlet objects of the form

$$\sum_{a,b} \bar{\psi}_a \gamma^\mu \left(A_\mu^\alpha, M^\alpha\right)_{ab} \psi_b$$

Clearly if my fermions are in triplets the matrices M_{ab} must be the 3×3 representations of SU(2) and so on.

It is immediately clear from (I.10) why Weinberg did not stop at SU(2). If we write out this expression we have

$$\frac{g}{\sqrt{2}} \left(A_\mu^+ \bar{\nu}_L \gamma^\mu e_L + A_\mu^- \bar{e}_L \gamma^\mu \nu_L\right)$$

$$+ \frac{g}{2} A_\mu^o \left(\bar{\nu}_L \gamma_\mu \nu_L - \bar{e}_L \gamma_\mu e_L\right) \qquad \text{(I.12)}$$

The charge-carrying vectors A^+ and A^- have been constructed to couple to the correct weak currents of (I.4), but the neutral particle is not a good photon candidate, it couples to the electron with a V-A coupling, and it also couples to the neutrino. Weinberg added the U(1) factor, thus introducing an additional neutral vector B. Now by astutely choosing the relative strengths of the left- and right-handed couplings of the B it can be arranged that there is a linear combination of A^o and B which has pure vector coupling to the electron and which does not couple to the neutrino--thus this linear combination is a candidate photon. However there is then inevitably another (orthogonal) linear combination of A^o and B, call it the Z, which couples with some well defined set of couplings, a mixture of vector and axial, to both neutrinos and electrons. It is only a matter of algebra to find it out. I recommend that you should carry through this exercise, starting from (I.10) and (I.11). Defining g as the coupling of the SU(2) vectors to the fermion doublet and g'/2 for the B-coupling to the left-handed doublet coupling one finds the relationship

$$e = gg'/(g^2 + g'^2)^{1/2} \tag{I.13}$$

One free parameter is left, it is usually written as

$$\sin\theta_W = g'/(g^2 + g'^2)^{1/2} \tag{I.14}$$

Now to the quarks, or even the muon; what determines that I should make the same assignments for them, especially for the right-handed parts, since clearly I have enough right-handed quarks to put some or all of them in nontrivial multiplets too. The answer is phenomenology; the following points are important:

(i) Cabibbo universality

The relationship between μ-decay and β-decay is most readily achieved by the choice (I.9). For example if I put the u and d quarks as neighboring members of a triplet then their coupling to the W^+ would have a factor of $\sqrt{2}$ relative to the muon and electron couplings (simply a Clebsch Gordon coefficient which is different for different isotopic spin assignments.)

(ii) The u and d couplings are left-handed, at least at present energies. Thus if u_R or d_R are members of nontrivial multiplets of the SU(2) they must be in different multiplets, paired with heavier quarks. As we will see later presently existing data from ν-scattering does not allow a doublet right-handed assignments for u and d with quarks of mass less than about 5 GeV.

(iii) The repetition of the $(u,d_c)_L$ by the $(c,s_c)_L$ is a manifestation of the Glashow-Ilioupolous-Maiani mechanism to avoid strangeness changing neutral currents. That must be such an old story around here these days that it scarcely needs to be mentioned. What may not be so well known is that naive generalizations to further flavors such as $(t,b)_L$, t_R, b_R avoids all flavor changing neutral currents--the rule is that I must assign all left-handed quarks of the same charge to the same multiplet (position and type) and similarly for the right-handed quarks to avoid the generation of flavor changing neutral currents.[6] So far we have little experimental evidence on the subject, but the theoretical literature is heavily biased in this direction.

I am trying to make clear the ad hoc nature of the construction. Within the basic recipe many variations are possible, even once I complete step I there are many choices at step II, etc. The beauty of the game is that each choice gives many predictions. The history of the field is a tribute to the experimentalists, who seem to be able to eliminate models almost as fast as the theorists can cook them up (following the recipe). Of course, the more that is known the harder the game of cooking becomes--there are more and more constraints that a model must satisfy before it is even worth discussing. More remarkable yet, the one model which seems to be doing best is the original $SU(2) \times U(1)$. There are some murky points, about which we will no doubt hear much more in the next week or so. In particular, in atomic physics parity violations and $\bar{\nu}_\mu e$ scattering experiments differ, but there is possible conflict with the models. However the model is doing well enough that I will continue to treat it here as the prime candidate theory.

Let us then proceed to steps III and IV of the recipe which introduce the Higgs sector. Why put in Higgs at all? The question can be asked at various levels of sophistication. Let me begin by proceeding naively, which in this context really means perturbatively. From the unitarity arguments given earlier, in particular one finds the scalars are needed if the W and Z are assumed to be massive. From a theorists viewpoint it is a question of writing a Lagrangian with a given non-Abelian gauge invariance, which a priori means massless vectors, and in addition a chiral invariance which means also massless fermions. Now we want a way to introduce vector and fermion masses without destroying the renormalizability of that theory. The only way to do this which gives perturbatively calculable predictions is to introduce elementary scalars which couple gauge-invariantly to the vectors and via Yukawa couplings to the fermions. The "Higgs" trick involves arranging the mass (ϕ^2) and self-interaction (ϕ^4) parameters of these scalars so that a nonvanishing vacuum expectation appears for some scalar--this is called spontaneous symmetry breaking, despite the fact that it is about as spontaneous as the appearance of a horse in a corral. (I first build the corral and herd the horses if I wish to have the effect occur.)

What does a nonvanishing vacuum expectation value for a field mean? It means that quanta of the theory, to which I can give a particle interpretation, are simply quantum fluctuations about zero of the variable

$$\rho = \phi - v \qquad \text{(I.15)}$$

where $v = \langle\phi\rangle$ is the vacuum expectation value, as opposed to fluctuations of ϕ itself about zero. Hence it is convenient to change variables and rewrite the Lagrangian in terms of ρ. I can represent this process diagrammatically by writing

v = ---x

for any term v which appears. For example a Yukawa coupling term is shown in Fig. 3. Clearly

$$Y\bar{\psi}\phi\psi = Y\bar{\psi}\rho\psi + (Yv)\bar{\psi}\psi \tag{I.16}$$

and we see that a quark mass term (Yv) has appeared. Similarly the terms

$$g^2\phi^2 A^\alpha_\mu A^{\alpha\ \mu}$$

gives a gluon mass term as shown in Fig. 4 with

$$m^2 \propto g^2 v^2 \tag{I.17}$$

How do I achieve a nonvanishing vacuum expectation value? Everyone by now must have seen the picture many times. I want a potential $V(\phi)$ which has the form shown in Fig. 5. Since we are talking about breaking a continuous symmetry, the phase symmetry of $\phi \to e^{i\theta}\phi$, the picture is three-dimensional--the Mexican hat potential. In a scalar field theory

$$V(\phi) = \mu^2\phi^2 + \lambda\phi^4 \tag{I.18}$$

where μ and λ are the parameters appearing in the Lagrangian. Obviously, negative values of μ^2 give the desired shape. Notice that although μ^2 looks like a mass parameter when we change variables there are additional scalar mass terms proportional to λv^2, so that there is no problem of negative $(\text{mass})^2$ for physical scalar particles.

Before I get too far from this picture let me comment on another obvious feature of it. The choice of the direction of vacuum expectation value in the (ϕ_{Re}, ϕ_{Im}) space is arbitrary, no phase is preferred. This means that for any value I choose there is one mode of oscillation about that value which has zero frequency, it is along the minimum of the potential. This is the Goldstone phenomenon which happens whenever a continuous symmetry is spontaneously broken. There is a zero mass particle associated with such a zero frequency mode. The trick of the Higgs scheme is that this zero mass scalar (one degree of freedom) can be eaten up by the zero mass vector (two degrees of freedom) to give a massive vector (three = 2 + 1 degrees of freedom). Since there do not appear to be any real zero mass scalars in the world we must arrange our Higgs sector in such a way that every such Goldstone boson corresponds to a symmetry which is gauged, and hence that there is a vector available to eat it up. (The pseudo-Goldstone[7] boson is a possible evasion of this rule, it may happen that there is a symmetry of Higgs Lagrangian which is not a symmetry of the full Lagrangian. If such a symmetry is spontaneously broken it will appear in a lowest order calculation of the type just discussed that there is a massless scalar, but keeping higher order effects from the vector mesons will give this particle a mass.)[8]

After all these preliminaries we are ready to perform steps III and IV. In SU(2) × U(1) with the fermion assignments which we have just made we need at least one Higgs doublet. Yukawa couplings are of the form

$$Y\bar{\psi}_R\phi^*\psi_L + \text{hermitian conjugate} \tag{I.19}$$

The right-handed electron is in an SU(2) singlet and the left-handed electron is in a doublet. The only scalar representation choice which allows such a coupling is a doublet. Here is yet another reason for making quark multiplet assignments mirror fermion assignments: it allows one to be economical in the Higgs sector. Suppose I were to choose to put the right-handed up quark in a high isospin multiplet. First I would have to introduce peculiar new quarks (charges other than -1/3 or

2/3) to fill up the multiplet, and then I would need additional Higgs content to contrive to give the up quark and the rest of its new cousins their masses. Such games usually rapidly proliferate in particles and in ugliness.

With standard SU(2) × U(1) assignments I can get by with only Higgs doublets of the form

$$\phi = \begin{pmatrix} \phi^{o} \\ \phi^{-} \end{pmatrix} \tag{I.20}$$

The charge conjugate doublet

$$\tilde{\phi} = \begin{pmatrix} \phi^{+} \\ -\phi^{o*} \end{pmatrix} \tag{I.21}$$

is then also present. Up-type quarks get mass from Yukawa terms of the type $\bar{u}_R \phi^* \begin{pmatrix} u \\ d \end{pmatrix}_L$, and down quarks (like electrons) need $\bar{d}_R \phi^* \begin{pmatrix} u \\ d \end{pmatrix}_L$ couplings.

I have defined the ϕ charges in relationship to my previously defined photon. That photon can only stay massless if only the neutral part of ϕ has a nonvanishing vacuum expectation value. (Remember the photon was defined simply as that linear combination of the A^o and B particles which coupled to the electron with a vector coupling and decoupled from the neutrino.) The U(1) factor is a hypercharge, in general this photon couples to electric charge, defined as

$$Q = T_3 + Y/2 \tag{I.22}$$

and we can arrange the hypercharge to get the standard quark charges, and the charges defined above for the scalars.

In my next lecture I will write out the Lagrangian to show how all this works. A few more comments can be made without doing so. I have said we need at least one complex Higgs doublet, for most of the rest of my lectures I will talk as if there is only one doublet. The existence of additional scalar doublets does not change the phenomenology of the lepton quarks and vector mesons very much, though it becomes important when finer points such as CP invariance and of course scalar particle phenomenology are discussed--John Ellis will talk about these things later in the school. However the matter of whether there are in addition to the doublet other scalar representations such as triplets does indeed affect the phenomenology. We will shortly see that assuming only Higgs doublets leads to the mass relationship

$$m_W/m_Z = \cos\theta_W \tag{I.23}$$

Adding a Higgs triplet with a nonvanishing vacuum expectation value for its neutral member would change this relationship, allowing the Z-mass to be increased arbitrarily, thus weakening the effective strength of the low energy ($s \ll m_Z^2$) neutral current effects. Using only doublets the SU(2) × U(1) theory predicts the curve shown in Fig. 6 for the ratio of neutral current to charge current total cross sections for neutrinos and for antineutrinos. Each point on the curve corresponds to a value for $\sin^2\theta_W$. As the figure shows the experimental values[9] are consistent with this prediction for a value

$$\sin^2\theta_W \sim .2 - .3 \tag{I.24}$$

so apparently we do not need to add any triplet Higgs. To do so would relax the prediction of the model, instead of the line we could adjust parameters to yield any point in the cone enclosed by the two dotted lines and the Weinberg-Salam prediction.

All this is just a brief introduction to the rules of the game of model building. The main points I want to stress in this lecture are that the idea of a gauge theory of the weak interactions is very general and allows many specific realizations, of which the standard SU(2) × U(1) model is only one. The structure is very rich and flexible, but flexibility is usually obtained at the price of introducing more and more particles. The beauty of Weinberg-Salam-GIM is that so far it has fit a lot of data while being quite economical in particle content. If it survives the parity violation test[10] (which means if either the Novosibirsk experiment and the theoretical atomic physics calculations, or the Oxford and both the Washington experiments, are wrong) we will have a remarkable candidate weak interaction theory. If not then the theorists must go back to work to produce a model which can fit the SLAC results for parity violation in polarized electron scattering and the atomic physics--no doubt several people are already working on such models.

As John Ellis will discuss next week there is at least one area where the predictions of these theories remain virtually untested--the Higgs sector. So far no one has seen any direct effect of these particles. They have been introduced in a somewhat arbitrary fashion to allow us to write a renormalizable theory with vector and fermion masses; one with which we can perform perturbative calculations. There is a school of thought among theorists which says that elementary scalars are ugly, perhaps the same effects can occur dynamically from formation of boundstates in the scalar channels. The problem is that we cannot do much more than suggest the possibility, the idea takes us beyond the realm of perturbation theory and hence, for the most part, beyond the range of our ability to calculate.

One could go even further and add that we have no direct evidence for the vector sector. (Again John Ellis will discuss the phenomenology of this sector later this week.) Bj[11] for one, has tried to introduce a note of caution into the general bandwagon acceptance of gauge theories as dogma by discussing how much of the phenomenology can be obtained by making weaker assumptions--such as symmetry properties without necessarily assuming gauge realizations of them--and he concludes that nothing in the present data compels us to accept the gauge theory picture. However neither does anything preclude us from doing so, so for the next week we will continue to ignore all alternatives and discuss, as the title of this lecture series states, only the gauge alternatives.

Lecture II

Yesterday I managed to be very general and avoided writing any detailed algebra. Today's lecture will be much more detailed, as we investigate all those generalities in the context of the Weinberg-Salam SU(2) × U(1) and see how one arrives at specific experimental predictions, a few of which I have already mentioned.

There are two types of exercise which we must pursue. The first is, once I have told all there is to tell about gauge group and particle content, to read off from that whatever we can about the physical couplings and mass relationships. The second is, given the couplings, to compute cross sections.

I will write down the full Weinberg-Salam theory and then we will investigate it piece by piece to see the phenomena discussed yesterday at work.

Let me define

$$F^{\alpha}_{\mu\nu} = \partial_{\mu}A^{\alpha}_{\nu} + \partial_{\nu}A^{\alpha}_{\mu} + gf^{\alpha\beta\gamma}\, A^{\beta}_{\mu}A^{\gamma}_{\nu} \text{ for the SU(2) vectors} \tag{II.1}$$

and

$$G_{\mu\nu} = \partial_{\mu}B_{\nu} + \partial_{\nu}B_{\mu} \quad \text{for the U(1) vector .} \tag{II.2}$$

Further let

$$\psi^{i}_{L} = \begin{pmatrix} a^{i}_{L} \\ b^{i}_{L} \end{pmatrix} \tag{II.3}$$

where i runs over both leptons and quarks. Then

$$\begin{aligned}
\mathscr{L} = &- \frac{1}{4} F^{\alpha}_{\mu\nu}F^{\alpha,\mu\nu} - \frac{1}{4} G_{\mu\nu}G^{\mu\nu} \\
&+ \sum_{k} \left|\left(\partial_{\mu} - igA^{\alpha}_{\mu}\sigma^{\alpha} + i\,\frac{g'}{2}\, B_{\mu}\right)\phi_{k}\right|^{2} \\
&+ \sum_{i} \Bigg\{\bar{\psi}^{i}_{L}\gamma^{\mu}\left(\partial_{\mu} - igA^{\alpha}_{\mu}\sigma^{\alpha} - i\,\frac{g'}{2}\,\alpha_{i}B_{\mu}\right)\psi^{i}_{L} \\
&+ \bar{a}^{i}_{R}\gamma_{\mu}\left(\partial_{\mu} - i\,\frac{g'}{2}\,\beta_{i}B_{\mu}\right)a^{i}_{R} + \bar{b}^{i}_{R}\gamma_{\mu}\left(\partial_{\mu} - i\,\frac{g'}{2}\,\delta_{i}B_{\mu}\right)b^{i}_{R}\Bigg\} \\
&+ \sum_{ijk} \left[Y_{ijk}\; \bar{a}^{i}_{R}\phi^{*}_{k}\;\psi^{j}_{L} + \tilde{Y}_{ijk}\; b^{i}_{R}\tilde{\phi}^{*}_{k}\;\psi^{j}_{L} + \text{h.c.}\right] + V(\phi_{k})
\end{aligned} \tag{II.4}$$

Now we shall proceed through a set of trivial exercises in algebra with this Lagrangian, assuming the Higgs potential is such that the vacuum expectation value

$$\left\langle \sum_{k} \phi_{k} \right\rangle_{vac} = \begin{pmatrix} v \\ 0 \end{pmatrix} \tag{II.5}$$

For simplicity we will carry out these exercises as if there is only one term in this sum, that is as if there is only one doublet. If there are many doublets we can simply define that (normalized) linear combination which gets a nonvanishing vacuum expectation value to be ϕ_1 and then the following discussion is valid for k=1.

Exercise I. What are the vector mass terms? We have in the Lagrangian

$$\left|\left(g\vec{A}\cdot\vec{\sigma} + \frac{g'}{2} BI\right) \begin{pmatrix}\phi_0 \\ \phi^-\end{pmatrix}_{vac}\right|^2$$

$$= \left|\left(\frac{g}{2}\begin{bmatrix} A_0 & \sqrt{2}A^+ \\ \sqrt{2}A^- & -A_0\end{bmatrix} + \frac{g'}{2}\begin{bmatrix} B & 0 \\ 0 & B\end{bmatrix}\right)\begin{pmatrix} v \\ 0\end{pmatrix}\right|^2 \tag{II.6}$$

$$= \left|\begin{pmatrix}\frac{1}{2}\left(gA_0+g'B\right) v \\ \frac{g}{\sqrt{2}} A^- v\end{pmatrix}\right|^2 = \frac{1}{2} g^2 v^2 A^+ A^- + \frac{1}{4} (g^2+g'^2)v^2 \left[\frac{gA_0 + g'B}{(g^2+g'^2)^{\frac{1}{2}}}\right]^2$$

Thus, identifying the massive neutral state as Z, we have

$$Z = \frac{gA_0 + g'B}{(g^2+g'^2)^{\frac{1}{2}}} = \cos\theta_W A_0 + \sin\theta_W B \tag{II.7}$$

we can read off the masses from (II.6)

$$m_Z = \frac{1}{\sqrt{2}} (g^2+g'^2)^{\frac{1}{2}} v \qquad m_W = \frac{1}{\sqrt{2}} gv$$

This gives the advertized ratio $m_W/m_Z = \cos\theta_W$ and the orthogonal combination to the Z, the photon,

$$A = -\sin\theta_W A_0 + \cos\theta_W B \tag{II.9}$$

clearly has zero mass, by construction.

Here we have defined a photon as the linear combination of A_0 and B which gets no mass. Now for Exercise II we can go back and check how to choose α_i, β_i and δ_i so that this particle has pure vector couplings with the right coefficients, that is

$$q = \begin{cases} -1 \text{ for electron, muon, tau, etc.} \\ 2/3 \text{ for u quark, c quark, etc.} \\ -1/3 \text{ for d quark, s quark, etc.} \end{cases} \tag{II.10}$$

To do this we start by simply rewriting the relevant terms in terms of $\mathscr{L}$ and using

$$A_0 = Z \cos\theta_W - A\sin\theta_W, \quad B = Z\sin\theta + \gamma\cos\theta$$

We find the fermion couplings to neutral vectors are

$$-\bar{a}_i\gamma^\mu\left\{\left(\frac{g}{2}A_{0\mu} + \frac{g'}{2}\alpha_i B_\mu\right)\left(\frac{1-\gamma_5}{2}\right) + \frac{g'\beta_i}{2}B_\mu\left(\frac{1+\gamma_5}{2}\right)\right\}a_i$$

$$-\bar{b}_i\gamma^\mu\left\{\left(\frac{-g}{2}A_{0\mu} + \frac{g'}{2}\alpha_i B_\mu\right)\left(\frac{1-\gamma_5}{2}\right) + \frac{g'\delta_i}{2}B_\mu\left(\frac{1+\gamma_5}{2}\right)\right\}b_i$$

$$= -\frac{(g^2+g'^2)^{\frac{1}{2}}}{2}\left\{\bar{a}_i\gamma^\mu\left[\left(\cos^2\theta_W + \sin^2\theta_W\alpha_i\right)\left(\frac{1-\gamma_5}{2}\right) + \sin^2\theta_W\beta_i\left(\frac{1+\gamma_5}{2}\right)\right]Z_\mu a_i\right.$$

$$+ \bar{a}_i\gamma^\mu(\sin\theta_W\cos\theta_W)\left[(-1+\alpha_i)\left(\frac{1-\gamma_5}{2}\right) + \beta_i\left(\frac{1+\gamma_5}{2}\right)\right]A_\mu a_i$$

$$+ \bar{b}_i\gamma^\mu\left[\left(-\cos^2\theta_W + \sin^2\theta_W\alpha_i\right)\left(\frac{1-\gamma_5}{2}\right) + \sin^2\theta_W\delta_i\left(\frac{1+\gamma_5}{2}\right)\right]Z_\mu b_i$$

$$\left.+ \bar{b}_i\gamma^\mu(\sin\theta_W\cos\theta_W)\left[(1+\alpha_i)\left(\frac{1-\gamma_5}{2}\right) + \delta_i\left(\frac{1+\gamma_5}{2}\right)\right]A_\mu b_i\right\} \tag{II.1}$$

The requirement of absence of γ_5 couplings for the photon immediately gives

$$\beta_i = -1 + \alpha_i \quad \text{and} \quad \delta_i = 1 + \alpha_i \tag{II.12}$$

The charges of a_i and b_i are then given by

$$\left.\begin{aligned} eq_a &= \frac{gg'}{(g^2+g'^2)^{\frac{1}{2}}}\left(\frac{1-\alpha_i}{2}\right) \\ eq_b &= \frac{gg'}{(g^2+g'^2)^{\frac{1}{2}}}\,\frac{-1-\alpha_i}{2} \end{aligned}\right\} = \frac{gg'}{(g^2+g'^2)^{\frac{1}{2}}}\left(T_3 - \frac{\alpha_i}{2}\right) \tag{II.13}$$

where T_3 is the weak isospin assignment of the left-handed fermion. Hence we can identify the coupling of the photon

$$e = \frac{gg'}{(g^2+g'^2)^{\frac{1}{2}}} \tag{II.14}$$

My parameter α_i is the negative of the hypercharge. We arrive at the right charge assignment for leptons with $\alpha_i=1$ giving $q_a=0$ and $q_b=-1$. (Notice that this gives $\beta_i=0$ as it must since there is no right-handed neutrino to form a β type coupling with the B.) For quarks we set $\alpha_i = \frac{-1}{3}$ giving $q_a = \frac{2}{3}$ and $q_b = \frac{-1}{3}$. Furthermore we have now specified the Z-couplings which with a little further algebra we can rewrite as

$$-(g^2+g'^2)^{\frac{1}{2}} \left\{T_3 \left(\frac{1-\gamma_5}{2}\right) - Q \sin^2 \theta_W\right\} \tag{II.15}$$

Clearly the couplings of the Z are in general a mixture of V and A although a peculiar accident may happen to remove the V part, for example the negative leptons e, μ, etc. would have pure axial coupling to the Z if $\sin^2 \theta_W = 0.25$. (Experimentally we will find the preferred value of $\sin^2 \theta_W$ is not very far from this value.) I could at this point proceed to the next set of terms--the Yukawa coupling terms, and carry out exercise III, which is to find the quark and lepton mass matrices. I will not do more than make a few comments on this exercise--carry it out as a homework problem if you wish. I remark that the Y_{ijk} and $\tilde{Y}_{ijk}$ do not require that i=j--this has the consequence that the mass eigenstates, the quarks, u,c... and d,s... may be linear combinations of the a^i and b^i respectively. This phenomenon has already been mentioned, we find Cabibbo combinations

$$\begin{aligned} b_1 &= \cos \theta_c d + \sin \theta_c s \\ b_2 &= -\sin \theta_c d + \cos \theta_c s \end{aligned} \tag{II.16}$$

are the weak eigenstates. If we introduce further quarks with the same charges then they could, in principle, also mix with the d and s. Experimentally the success of Cabibbo universality tells us that the amount b in the doublet with u must be small, as Stan Wojcicki discussed in Monday's lecture. The Yukawa couplings must then be arranged so that this is so.

Just a few more comments on the rules for putting together theories of the Weinberg-Salam type and then on to real physics--that is to cross section calculations. One of the advertized virtues of gauge theories compared to the old four fermi theory is renormalizability. In fact the Weinberg-Salam theory as I have written it is not necessarily renormalizable--because of anomalies, which means processes involving the triangle of Fig. 7. One can take the attitude that this does not much matter. We have to go to such high order before there is any problem that we might be being unreasonably optimistic to hope that our present theory is valid to that accuracy. However the dogma says we must get rid of these anomalies; that is to say we must have a renormalizable theory. We can arrange to do so by having a number of such triangle diagrams with their sum vanishing identically. In general this is achieved by requiring the sum of the fermion charges to vanish. In Weinberg-Salam, with SU(3) color, this happens if one has as many flavors of quark doublet as there are lepton doublets, e.g.,

$$\begin{pmatrix} \nu_c \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

$$\begin{pmatrix} u \\ d_c \end{pmatrix} \quad \begin{pmatrix} c \\ s_c \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

For each pair of doublets $\Sigma q = (0 + -1)$ for leptons + $n_c \times \left(\frac{2}{3} + -\frac{1}{3}\right)$ where n_c is the number of colors of each quark flavor. For color SU(3), $n_c=3$ and $\Sigma q=0$ with this arrangement.

As I stressed yesterday there is no a priori reason for the continuing replication of similar multiplets. Assuming such replication leads to a prediction that there are no flavor changing neutral currents. In the context of this theory the masses of the various fermions are achieved quite arbitrarily by adjusting Yukawa couplings.

We have now written a model which tells us everything there is to know about the weak interactions of leptons and of quarks. For leptons the rest is completely straightforward, we can simply calculate any process we choose. For hadron physics we need something more to relate this model to experiment--we need to know how the quarks are put together to make hadrons. That we do not really know, so we are left somewhat up in the air by our beautiful theory of the weak interactions. However there is a great deal we can do, in the framework of the quark-parton model. We define a set of functions called the structure functions which describe at least part of what we need to know--they are a description, at least in the high energy limit, of hadron composition in terms of quarks. We can then calculate cross sections for a number of processes in terms of these same functions, and hence test the theory by the consistency between the various rates--testing whether all experiments can be fit with the same set of structure functions.

Let us therefore discuss the familiar example of deep inelastic scattering. For sufficiently high energy and momentum transfer we can neglect lepton and quark masses, though clearly if we come to a new quark threshold that rule will be in abeyance for a while. This means we only have to do very few calculations, since the interactions γ_μ and $\gamma_\mu\gamma_5$ each preserve helicity up to corrections of order m/E. The calculations are simple enough. I will not go through them here; I will simply state the results for deep inelastic scattering. I define the usual set of variables for the process shown in Fig. 8.

$$\nu = q\cdot p \qquad x = \frac{-q^2}{2\nu} \qquad y = p\cdot q/p\cdot k \tag{II.17}$$

In terms of the quark-parton model the cross sections for various deep-inelastic processes can be obtained by assuming incoherent scattering off the individual quark constituents of the target and defining structure functions $f_q(x)$ which, in the high energy limit, represent the probability of finding a quark of type q carrying a fraction x of the proton's momentum in a frame in which the proton is moving with very large momentum. This parton picture interpretation of the structure function is of course frame dependent, but the cross sections which we write down are functions of the invariants and hence are not. In a more general picture one finds that the structure functions could in fact be functions of q^2 as well as x, the fact that to a first approximation they should be q^2 independent was first suggested by Bj and hence is known as Bjorken scaling.[12] In the context of a specific model of the strong interactions, namely QCD, one can obtain more detailed predictions about these functions and their q^2 dependence[13]--these predictions will be discussed tomorrow by John Ellis. For the moment however let us take the naive parton model point of view and treat these as functions of x alone. Neglecting lepton and quark masses one obtains a very simple set of predictions, namely scattering left-handed fermion on left-handed fermion or right on right gives

$$\frac{d\sigma}{dxdy} \propto xf(x) \tag{II.18}$$

scattering left-handed on right-handed gives

$$\frac{d\sigma}{dxdy} \propto xf(x)\ (1-y)^2 \tag{II.19}$$

Let us look at this for $\nu(\bar{\nu})$ nucleon $\rightarrow \mu^-\ (\mu^+)$ anything. The charged weak current sees only left-handed quarks and thus only right-handed antiquarks, so the above rule gives the familiar predictions

$$\begin{aligned} \frac{d\sigma^{\nu}}{dxdy} &= \frac{G^2ME}{\pi}\, 2x \sum_q \left[f_q(x) + f_{\bar{q}}(x)(1-y)^2\right] \\ \frac{d\sigma^{\bar{\nu}}}{dxdy} &= \frac{G^2ME}{\pi}\, 2x \sum_q \left[f_q(x)(1-y)^2 + f_{\bar{q}}(x)\right] \end{aligned} \tag{II.20}$$

with

$$G^2 = \frac{1}{8}\left(\frac{g^2}{2m_W^2}\right)^2 = \frac{1}{8v^2} \tag{II.21}$$

in Weinberg-Salam.

The same calculations can be made for the deep inelastic neutral current neutrino scattering. To do so it is convenient to write the Z couplings as

$$-(g^2+g'^2)^{\frac{1}{2}} \left\{\frac{\varepsilon_L^i(1-\gamma_5)}{2} + \frac{\varepsilon_R^i(1+\gamma_5)}{2}\right\} \tag{II.22}$$

For SU(2) $\times$ U(1) theories we find

$$\begin{aligned} \varepsilon_L^i &= T_{3L}^i - q^i \sin^2\theta_W \\ \varepsilon_R^i &= T_{3R}^i - q^i \sin^2\theta_W \end{aligned} \tag{II.23}$$

In the standard version we had $T^i_{3R}=0$ for all fermion types. We notice that these formulae apply either for quarks or antiquarks, and imply the relationships

$$\varepsilon^q_L = -\varepsilon^{\bar{q}}_R \;; \qquad \varepsilon^q_R = -\varepsilon^{\bar{q}}_L \tag{II.24}$$

The strength of the neutral current processes can be compared to those of charged currents. For charged currents the amplitude is proportional to $g^2/2m^2_W = 1/v^2$ whereas for neutral currents the comparable factor is $(g^2+g'^2)\ \varepsilon^a\varepsilon^b/m^2_q = 2\varepsilon^a\varepsilon^b/v^2$. Thus for example we obtain for neutrino deep inelastic scattering, using (II.24),

$$\frac{d\sigma^{\nu\to\nu}}{dxdy} = \frac{G^2ME}{\pi}\, 8x \sum_q \left\{\left(\varepsilon^q_L\right)^2 \left[f_q(x) + f_{\bar{q}}(x)(1-y)^2\right] + \left(\varepsilon^q_R\right)^2 \left[f_q(x)(1-y)^2 + f_{\bar{q}}(x)\right]\right\} \tag{II.25}$$

and similarly

$$\frac{d\sigma^{\bar{\nu}\to\bar{\nu}}}{dxdy} = \frac{G^2ME}{\pi}\, 8x \sum_q \left\{\left(\varepsilon^q_L\right)^2 \left[f_q(x)(1-y)^2 + f_{\bar{q}}(x)\right] + \left(\varepsilon^q_R\right)^2 \left[f_q(x) + f_{\bar{q}}(x)(1-y)^2\right]\right\} \tag{II.26}$$

(An obvious note, if the target contains neutrons and protons then

$$f^{\text{target}}_q(x) = N_P f^P_q(x) + N_N f^N_q(x) \tag{II.27}$$

where N_P (N_N) is the number of protons (neutrons) in the target. Isospin invariance tells us that

$$f^P_u(x) = f^N_d(x)$$

$$f^P_d(x) = f^N_u(x)$$

$$f_s^P(x) = f_s^N(x) \tag{II.28}$$

and similarly for antiquarks.)

I can treat photon exchange in this same formalism, the photon couplings can be written in analogy to the Z-couplings as

$$e\beta_L^i \frac{(1-\gamma_5)}{2} + e\beta_R^i \frac{(1+\gamma_5)}{2} \tag{II.29}$$

where obviously

$$\beta_L^i = \beta_R^i = q^i \tag{II.30}$$

The strength factor $g^2/2m_W^2$ is replaced by e^2/q^2.

For deep inelastic electron scattering I can treat the left- and right-handed parts of the electron incoherently, but I must remember that photon and Z exchanges add coherently. Thus I have

$$\frac{d\sigma}{dxdy} = \frac{d\sigma^L}{dxdy} + \frac{d\sigma^R}{dxdy} \tag{II.31}$$

and

$$\frac{d\sigma^L}{dxdy} = \frac{e^4 ME}{4\pi} \times \sum_i \left\{ \left[\frac{-Q^i}{q^2} + \frac{\varepsilon_L^e \varepsilon_L^{q_i}}{\sin^2\theta_W \cos^2\theta_W (q^2-m_Z^2)} \right]^2 \left[f_{q_i}(x) + f_{\bar{q}_i}(x)(1-y)^2 \right] \right.$$
$$\left. + \left[\frac{-Q^i}{q^2} + \frac{\varepsilon_L^e \varepsilon_R^{q_i}}{\sin^2\theta_W \cos^2\theta_W (q^2-m_Z^2)} \right]^2 \left[f_{q_i}(x)(1-y)^2 + f_{\bar{q}_i}(x) \right] \right\} \tag{II.32}$$

For $d\sigma^R/dxdy$ one simply makes the replacements

$$\varepsilon_L^e \to \varepsilon_R^e \ ; \qquad \varepsilon_L^{q_i} \leftrightarrow \varepsilon_R^{q_i} \tag{II.33}$$

in (II.32).

Lecture III

Note for the reader--this lecture followed after Lecture I by John Ellis.

I want to start this lecture with some comments on what we have discussed so far. In my first lecture I told you how to build a gauge theory model. I remind you that it is an extremely ad hoc process, good and bad models are distinguished by experimental tests, not by theoretical reasoning. Even when a model can be constructed to fit all present data it makes no definite prediction about how many heavier quarks there might be, and there is similarly much arbitrariness the predictions about the scalar sector. These things will be discussed further by John Ellis in subsequent lectures, and by Mary Kay Gaillard in the topical conference.

In the second lecture I told you how to calculate: Given a model, one can read off W and Z masses and couplings and from them proceed directly to predictions for deep inelastic scattering processes. These calculations are valid in the naive form only when it is reasonable to neglect both the lepton and the quark masses. Near a threshold, for example, where charm production begins to enter in the allowed final states, the model is not capable of giving clear predictions. There exist a number of slightly different suggestions for including quark mass corrections in the near threshold region. They all interpolate smoothly between the scaling prediction below threshold and the new scaling prediction sufficiently far above. They differ somewhat in how rapidly the new value is achieved--in other words in how far above threshold is sufficiently far. I will not go into this discussion here. The quantity y plotted as a function of energy for $\bar{\nu}$ scattering has been used in the literature as a particularly sensitive test for the appearance of a threshold corresponding to a right-handed coupling of a u or d quark to a heavier quark. The reason for the choice is obvious enough. With only left-handed couplings the valence quark contribution to antineutrino scattering is proportional to $(1-y)^2$, so a right-handed coupling, giving a term proportional to 1 would give a marked increase in $\langle y \rangle$. However the scaling corrections discussed yesterday by John Ellis also tend to increase $\langle y \rangle$ with increasing energy. The reason for this is that the contribution of antiquarks in the target increases, due to the glue $\rightarrow$ quark-antiquark terms which John discussed, giving also an increasing contribution of y independent cross section. I think it is now generally agreed that these corrections are sufficient to account for the observed variation of $\langle y \rangle$ with energy, thus excluding right-handed coupling of the u or d quarks to any quark with mass less than about 5 GeV.

For the theorists in the audience I want to add one warning (it is obvious to the experimenters)--every experiment makes certain cuts in the data for purely experimental reasons. In comparing experiment with theory one must know about these cuts and take them into account. We theorists have a bad habit of trying to extract numbers from the experiments to compare directly with the simplest theoretical calculations. What should be done is the other way around, one extracts numbers from the theory (if necessary via Monte Carlo calculations) to compare directly with what has actually been measured.

Let me now go on to discuss further predictions which can be obtained from a gauge theory model, as before continuing to use Weinberg-Salam $SU(2) \times U(1)$ as the sample model. Obviously purely leptonic processes such as $\nu_\mu e$ scattering can be calculated by the same rules as deep inelastic, simply replacing structure functions by a delta-function at x=1. For $\bar{\nu}_e e$ or $\bar{\nu}_\mu \mu$ scattering one must remember that there is a direct channel W-exchange diagram to include as well as the t-channel Z-exchange. The predictions are usually given in terms of g_V and g_A, in terms of the previously defined Z-couplings

$$\left.\begin{aligned} g_V &= \varepsilon_L^e + \varepsilon_R^e \to -\frac{1}{2} + 2\sin^2\theta_W \\ g_A &= \varepsilon_L^e - \varepsilon_R^e \to -\frac{1}{2} \end{aligned}\right\} \text{for standard Weinberg-Salam.} \qquad \text{(III.1)}$$

The experimental situation is shown in Fig. 9. There is one further result from Gargamelle which is in conflict with the other experiments, and with the Weinberg-Salam prediction, however, it appears that the analysis of the second half of the data will significantly change the result, so I do not include it here.

The next area where the theory can be tested is in elastic νp scattering experiments. One new unknown function enters--the axial form factor of the proton. However, one can make a reasonable model for this, in parallel to the behavior of the vector form factor. In the context of such a model the Weinberg-Salam prediction is in good agreement with the measurements,[15] for $\sin^2\theta_W$ in the range .2 to .3.

Recently Mike Barnett and Larry Abbott[16] have made a very nice systematic study of predictions of neutral current process, including semi-inclusive processes. They find this gives them a good tool for distinguishing between gauge theory models. Mike will be talking about this in the topical conference, so I will not discuss it further here.

Now we come to the topic of the first morning of the topical conference, parity violations. Let me start with the easy cases first. I refer you to a paper by Bob Cahn and Fred Gilman[17] for the details of the calculations. Using the deep inelastic scattering formulae given in Lecture II one arrives at the following predictions

$$A(x,y) = \left(\frac{d\sigma^L}{dxdy} - \frac{d\sigma^R}{dxdy}\right) \Big/ \left(\frac{d\sigma^L}{dxdy} + \frac{d\sigma^R}{dxdy}\right) \tag{III.2}$$

For deuterium, keeping only valence quark contributions

$$A_{ed} = \frac{-Gq^2}{2\sqrt{2}\ \pi\alpha} \cdot \frac{9}{10} \cdot \Big[\left(1 + 2T^e_{3R}\right)\left(1 - \frac{20}{9}\sin^2\theta_W + \frac{4}{3}T^u_{3R} - \frac{2}{3}T^d_{3R}\right)$$
$$+ \left(1-4\sin^2\theta_W - 2T^e_{3R}\right)\left(1 - \frac{4}{3}T^u_{3R} + \frac{2}{3}T^d_{3R}\right)\left(1-(1-y)^2\right) \Big/ \left(1+(1-y)^2\right)\Big] \tag{III.3}$$

Notice A is x independent. For any target

$$\begin{aligned} f_u(x) &= N_P f^P_u(x) + N_N f^P_d(x) \\ f_d(x) &= N_P f^P_d(x) + N_N f^P_u(x) \end{aligned} \tag{III.4}$$

Thus we see that if $N_P = N_N$ then $f_u = f_d$ and hence f(x) cancels out in the ratio A. I remark also that with right-handed singlet assignment for all quarks and leptons the prediction becomes y-independent for $\sin^2\theta_W = .25$, or slowly varying with y for $\sin^2\theta_W$ near that value; and the present best values are quite close to .25. This is in marked contrast to some other models, for example, models with nontrivial T_{3R}. Models such as $SU(2)_L \times SU(2)_R \times U(1)$ (Ref. 18) have also been constructed to reproduce the standard Weinberg-Salam predictions for deep inelastic ν-scattering, but can give quite different predictions for parity violating effects, in particular for the atomic physics experiments they predict no effect. The result of the SLAC-Yale experiment[19] along with the predictions of Weinberg-Salam and of a theory with the right-handed electron in a doublet is shown in Fig. 10. This result is also in conflict with the version of $SU(2)_L \times SU(2)_R \times U(1)$ which gives no atomic physics parity violations. Further information, in particular on the relative u and d couplings is gained from data on hydrogen.[19] The SLAC-Yale collaboration intends to make further measurements for smaller y. The results of such measurements, if they can be made with errors comparable to those of the existing measurement, will provide very interesting further information.

Cahn and Gilman have also calculated predictions for asymmetries for elastic ep and ep $\to \ell\Delta(1236)$. These predictions, like those for elastic νp total cross sections, depend on some assumptions about form factors, but one could obtain some further tests of the model by measuring these quantities.

Now we come to the "Mares Nest" for Weinberg-Salam, the question of parity violations in atomic physics. These are of course tests of some of the same parameters in the model as occur in ep and ed scattering at y=0. (One needs both ep and ed to be able to test up and down quark couplings separately.) In the atomic physics experiments what is measured is the optical rotation of light in a laser induced atomic transition. This effect is proportional to the matrix element for the mixing of a "wrong parity" state due to the axial coupling of the Z to an electron. In Weinberg-Salam $g_A^e = -1/2$. At the nucleus we need a g_V

coupling, which is given by

$$N_u\left(\frac{1}{2} - \frac{4}{3}\sin^2\theta_W\right) + N_d\left(-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W\right)$$

$$= (2N_P+N_N)\left(\frac{1}{2} - \frac{4}{3}\sin^2\theta_W\right) + (2N_N+N_P)\left(-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W\right)$$

$$= -Z\left(\sin^2\theta_W + \frac{A-2Z}{2Z}\right) \qquad \text{(III.5)}$$

However this is the easy part of the calculation, the hard part is the constant of proportionality, which is to say the calculation of the atomic physics matrix elements

$$\phi = \begin{pmatrix}\text{known}\\ \text{coefficient}\end{pmatrix} \sum_n \left\{ \frac{\langle f|D|n\rangle\langle n|\bar{e}\gamma_\mu\gamma_5 e|i\rangle}{E_i - E_n} + \frac{\langle f|\bar{e}\gamma_\mu\gamma_5 e|n\rangle\langle n|D|i\rangle}{E_f - E_n} \right\} \qquad \text{(III.6)}$$

where D is an electric dipole operator. To calculate this one needs to know the energy levels and the relevant wave functions for the atom in question, which is Bismuth in all experiments carried out to this date.

The energy levels are well measured, but the wave functions are not as easily obtained. One makes models for them, and the models are tested by their ability to reproduce certain measured results, such as energy levels. I display in Table I as an example a table from a paper by Henley, Kaplisch and Wilets.[20] CI in this table means "configuration interaction." The point of the paper is that the original calculations by Henley and Wilets of the expected parity violating effect used a Hartree-Fock independent-particle model, including the configuration interaction corrections changes the predicted effect by as much as 0.65. You may judge for yourselves from the table the extent to which the energy levels confirm these corrections.

There are independent calculations by Novikov, Sushkov, and Khriplovich[21] which take what they call a semi-empirical approach. This means

TABLE I. Some energy levels in Bi I of $J = \frac{3}{2}$ (in inverse centimeters).

Level	Without CI	CI including 7*s*	CI including 6*s*	Expt.[a]
$6p_{3/2}{}^2(2)6p_{1/2}$	11 658	11 598	11 770	11 419
$6p_{3/2}{}^3$	34 903	34 694	33 364	33 164
$6p_{1/2}6p_{3/2}(1)7s$	42 674	42 710		44 865
$6p_{1/2}6p_{3/2}(2)7s$	49 696	49 595		49 456
$6s6p_{1/2}{}^2 6p_{3/2}{}^2$	81 299		84 828	

[a]C. E. Moore, *Atomic Energy Levels*, National Bureau of Standards Circular No. 467 (U.S. GPO, Washington, D. C., 1958), Vol. III.

that adjustments are made in the model to correct certain predictions to match measured values. Unfortunately some corrections have to be made based on measurements in Thallium rather than Bismuth, since the relevant measurement is not available for Bi. The relevant quantity is

$$\rho_6 = \int dr\ r^3\ R_{6s} R_{6p_{3/2}} \tag{III.6}$$

where R is the radial part of relevant wave function. For Thallium the model predicts

$$\rho_6 = -2.9\ a_0$$

and photoionization measurements give $|\rho_0| = 1.8\ a_0$. Hence the effect of a $6s \to 6p$ electron transition in Bismuth is corrected by a factor (1.8/2.9) from the theoretical prediction. There are other relevant contributions coming from $6p \to 7s$ and $6p \to$ (higher states including continuum) for which the estimates are made similarly, but with reference to tests in Bi. In calculating the total predicted effect the relative signs of these various contributions are very important. (The above discussion was given, with some further detail, in a talk by Peter Rosen at the Workshop on Weak Interactions at Ames, Iowa last month.)[22]

Where does all this leave us--after all corrections have been applied the best value for the predicted effect, for either the 876 or 648 nm line is of order -10×10^{-8} using $\sin^2 \theta_W = .2 - .25$. The experimental situation will be discussed in detail at the Topical Conference next week. There are now four experiments, two from Seattle, one from Oxford and one from Novosibirsk. Of these, three including the second generation Seattle experiment, give an upper limit about an order of magnitude below the prediction while the fourth, from Novosibirsk finds an effect in agreement with the predicted value. Obviously not everyone is right--there are several options, among them

1. Novosibirsk, Atomic Theory and Weinberg-Salam are right and Oxford and Seattle are wrong.
2. Novosibirsk and Atomic Theory are wrong and Weinberg-Salam, Oxford, and Seattle are right.
3. Novosibirsk and Weinberg-Salam are wrong and Atomic Physics, Oxford, and Seattle are right.
4. Everyone is wrong.

I do not intend my previous discussion to be a judgment on the atomic physics theory. I have not studied it carefully enough to make such a judgment. Clearly there are some uncertainties, but the question is whether they are at the factor of 2 level or as much as an order of magnitude. One must also look very carefully at the experiments to try to understand what might possibly be going wrong in any one of them, since they disagree. These are difficult measurements but I do not know of any telling point which has been raised against any one of them, all I can say is the discussion next Wednesday promises to be interesting. The situation may also be resolved by further experiments. An experiment in Thallium is being worked on at Berkeley, which has the virtue that certain cross-checks of the model can be made at the same time. From the theorists point of view the ideal experiment is of course in hydrogen. This will come; groups at Michigan, Seattle, and Yale are working on it. Results are not expected for some time. (Predictions vary from a few months to more than a year.)

For the most part the composite quark picture of hadrons, together with a gauge theory of the weak interactions, gives us a good description of the observed weak interactions. Let me list some salient points:

We do not see second class currents.[23] (Their existence would be a serious problem, if not a disaster for these theories.)

Deep inelastic neutrino scattering data is for the most part well fit by the model; we do not need to invoke scalar component, which would give a term proportional to (1-y) in $d\sigma/dxdy$, though such a contribution is also not excluded by the present data. One outstanding problem here is the ratio σ_L/σ_T which is found in electron scattering which, even including higher order gluon effects, is predicted to be somewhat smaller than the measured value.[24] This quantity must be dominated by terms involving mass corrections, terms dropped in all the standard asymptotic (scaling) treatments. Various attempts[25] have been made to estimate such effects, it is a pretty grubby business. From a pragmatic point of view it is fair to keep the magnitude of σ_L/σ_T in mind as a measure of the order of magnitude of possible corrections to the quark model plus QCD treatment which we have discussed.

There are some areas where the theory ceases to be useful. It is a theory of the weak interactions of quarks and not of physical hadrons. In deep inelastic scattering we could absorb our ignorance of the hadron wave functions into a few structure functions and then compare experiments. For explaining hyperon decays however we need to know more. Certain absolute rules like $\Delta Q = \Delta S$ arise as a natural consequence of the structure of the quark currents. However the $\Delta I = 1/2$ enhancement, which Stan Wojciki discussed on Monday, is a detailed property of the hadronic matrix elements of two quark currents. There are both $\Delta I = 1/2$ and $\Delta I = 3/2$ operators formed from these currents. Empirically we find the $\Delta I = 1/2$ parts dominated by a factor of 50-100. Keeping higher order gluon corrections, anomalous dimensions as discussed in the context of scaling violations by John Ellis, gives some $\Delta I = 1/2$ enhancement,[26] but it is my judgment that with reasonable values of the parameters involved it is not enough to fit the data, it is more like a factor of 5 than the factor experimentally observed. That does not mean the theory is wrong, simply that the effect is dominated by the part which we cannot calculate, the long distance part, rather than by the short distance part for which this calculation can be made. If we really understood hadrons as quark bound states we should be able to explain the effect, but that of course is a strong interaction problem, gauge theories of the weak interactions can at present only make useful predictions where such problems can be avoided.[27]

There is another area of weak phenomenology which I have barely mentioned--the area of CP violation. As Stan Wojcicki told you on Monday a six quark version of Weinberg-Salam in general has some CP violating phase in the quark-mixing matrix which defines weak eigenstates in terms of mass-eigenstates (or vice versa). Adding more than one Higgs doublet can also introduce CP violating effects. John Ellis will tell you more about how these things work. I just want to comment that these theories naturally incorporate CP violating effects without having to add anything radically new. The simplest Weinberg-Salam theory with just four quark flavors and one complex Higgs doublet does not have CP-violations, but experimental results are already pushing us beyond that model anyway.

A complicated Higgs sector can lead to CP violations of the milliweak type, with a predicted value for the neutron dipole moment not much below the present experimental upper bound.[28] The CP violations coming from phases in the quark sector are typically superweak in character. The CP violating phase in this case, like everything else coming from the Yukawa coupling terms, is a free parameter in the model.

I have tried in these lectures to give you some feeling for the generality of the gauge theory idea, as well as of the status of the "standard model". There clearly are some questions yet to be settled, but in the last year much progress has been made. A year ago there were many candidate models to discuss--now there is just one, and that is a very economical one. A viable model must at least reproduce the neutrino phenomenology of the Weinberg-Salam model. There is a large class of models of the type SU(2) × U(1) × G which do so;[29] the parity violation situation may force us to extend the model in this way. There are many areas yet to be explored. I have focused on what we know now, leaving John Ellis with the problem of spending his next three lectures talking about things we know practically nothing about, at least experimentally speaking.

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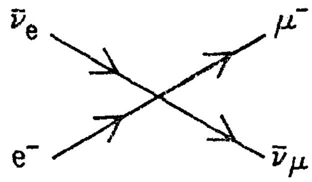

Fig. 1. Typical process in four-fermi theory.

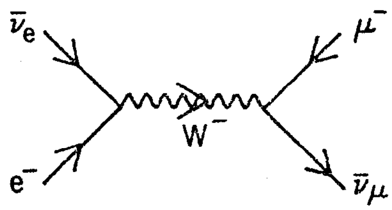

Fig. 2. Introduction of an intermediate vector boson to modify the amplitude shown in Fig. 1.

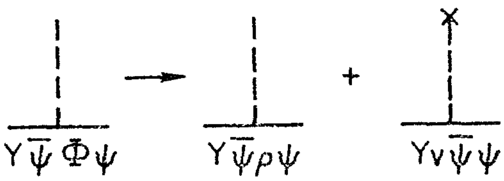

Fig. 3. Diagramatic representation of the change of variables $\phi = \rho + v$.

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Fig. 4. Effective gluon mass term generated by vacuum expectation value v.

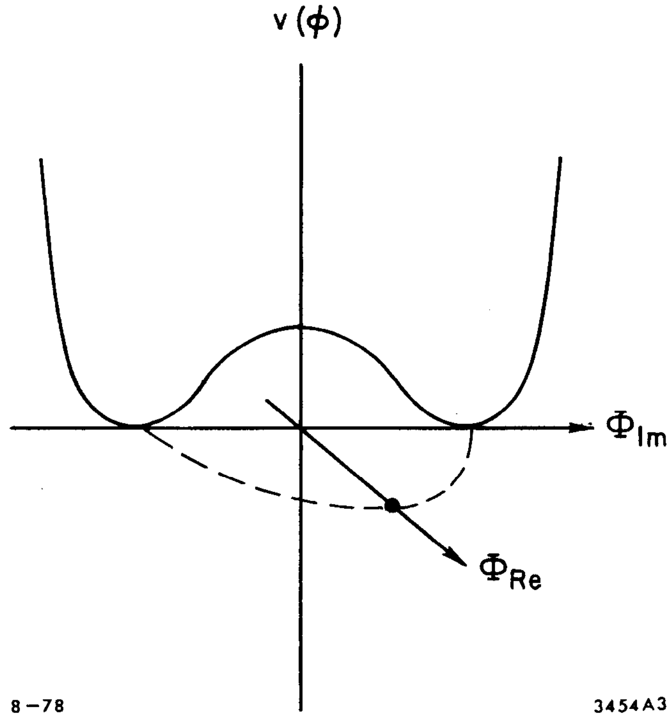

Fig. 5. Typical scalar potential for theory with spontaneous symmetry breaking.

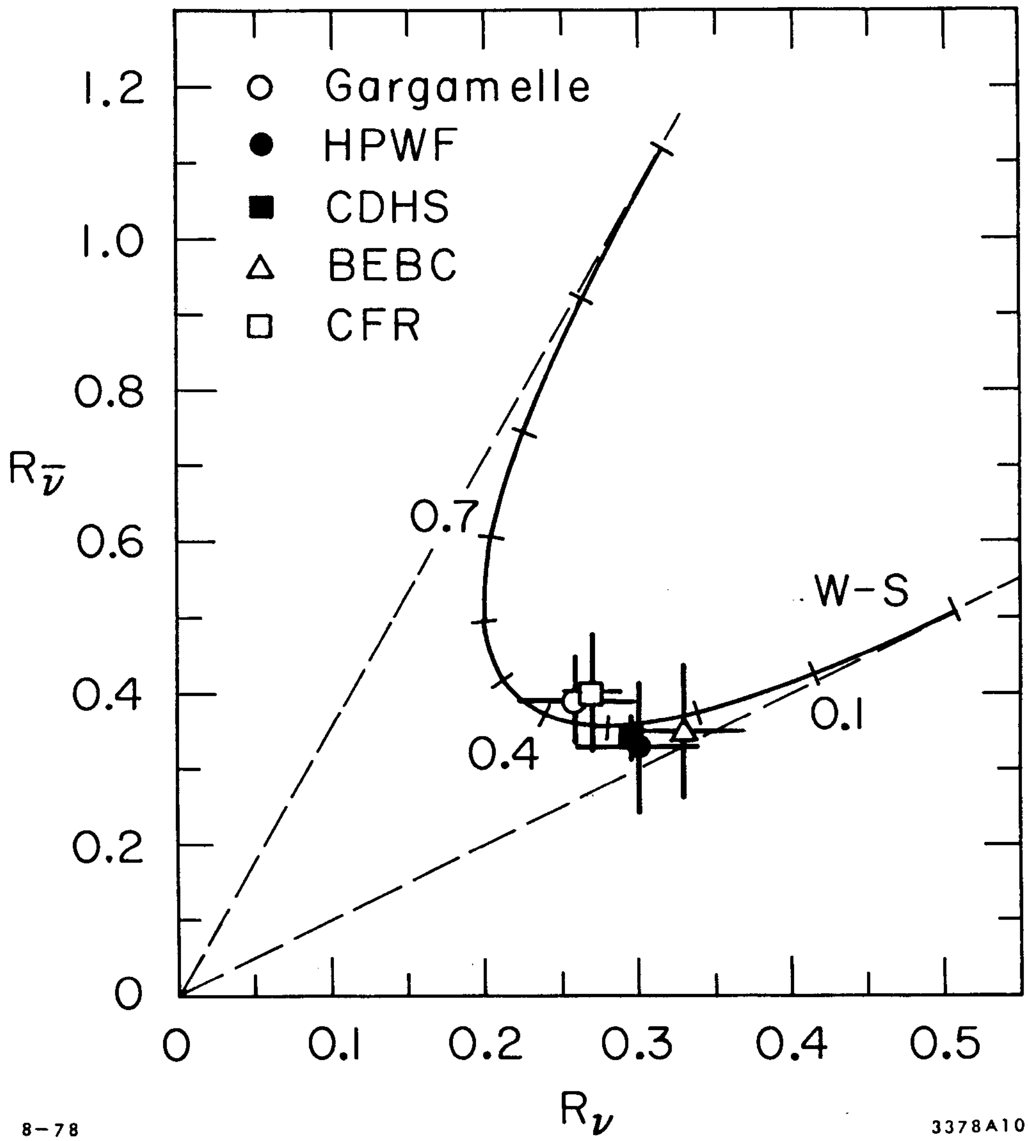

Fig. 6. The ratio of neutral-current to charged-current total cross sections for neutrinos (R_{ν}) and antineutrinos ($R_{\bar{\nu}}$) scattering of equal numbers of neutrons and protons. The solid line is the standard model prediction for various $\sin^2 \theta$ values. The dashed lines enclose the area allowed by adding a triplet of Higgs bosons.

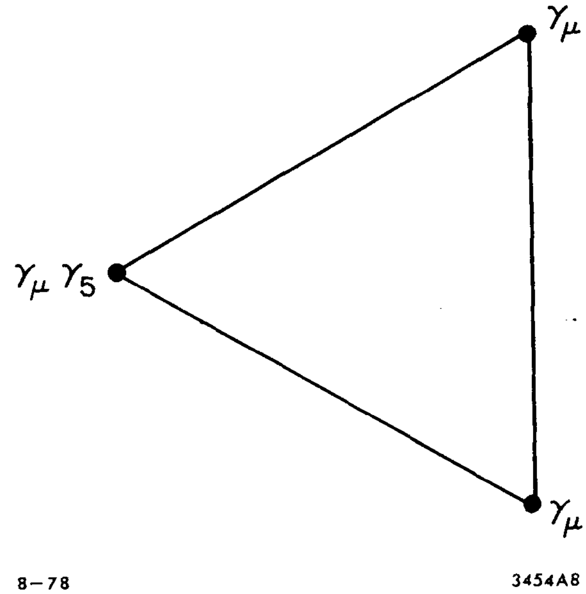

Fig. 7. The anomalous triangle graph.

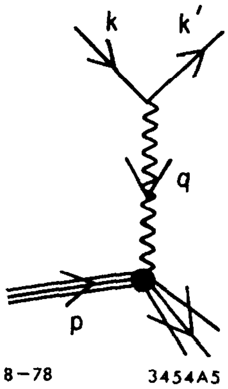

Fig. 8. Labeling of momenta in deep inelastic scattering processes.

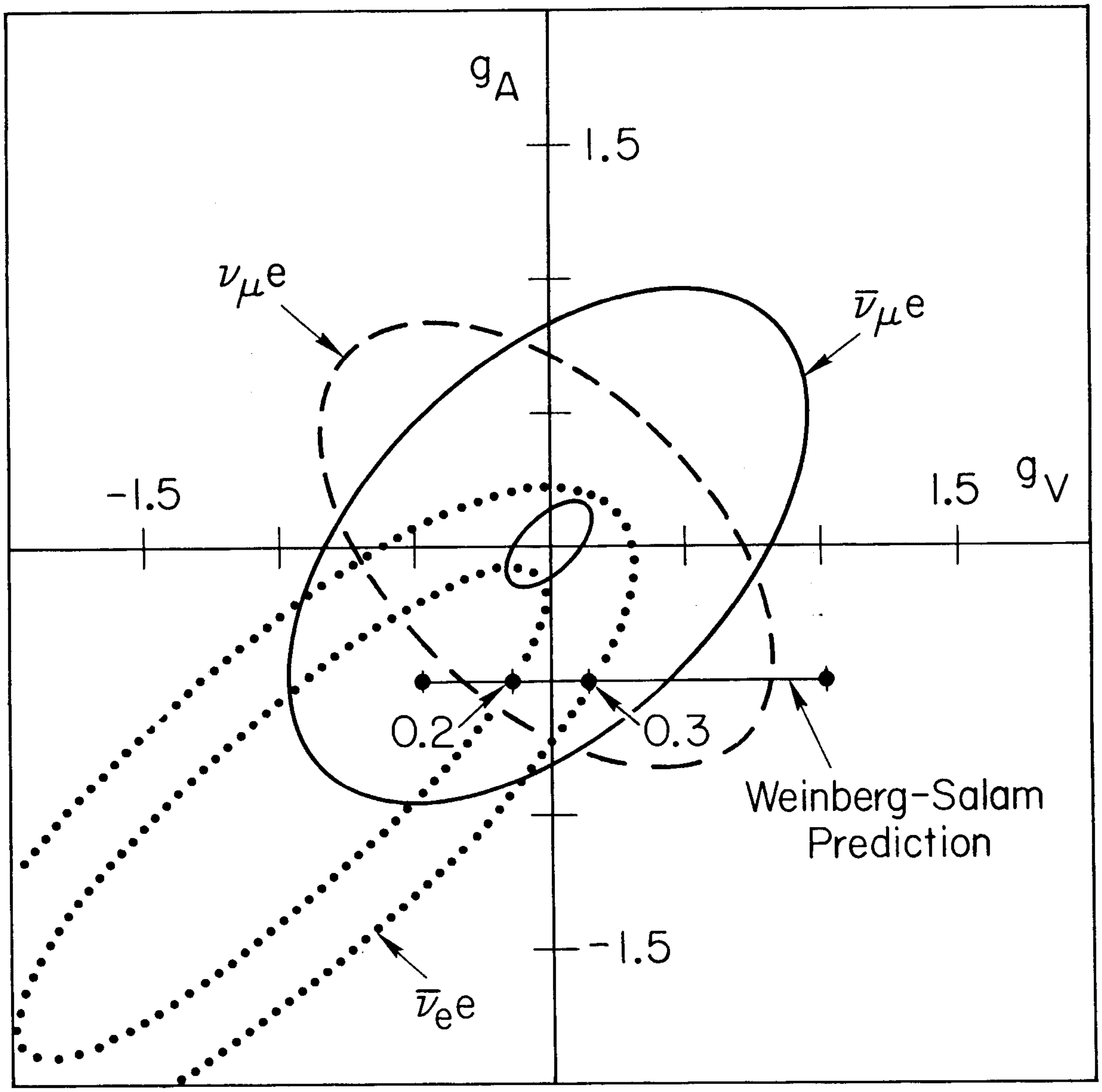

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Fig. 9. Experimental constraints on g_V and g_A from lepton scattering data.[19]

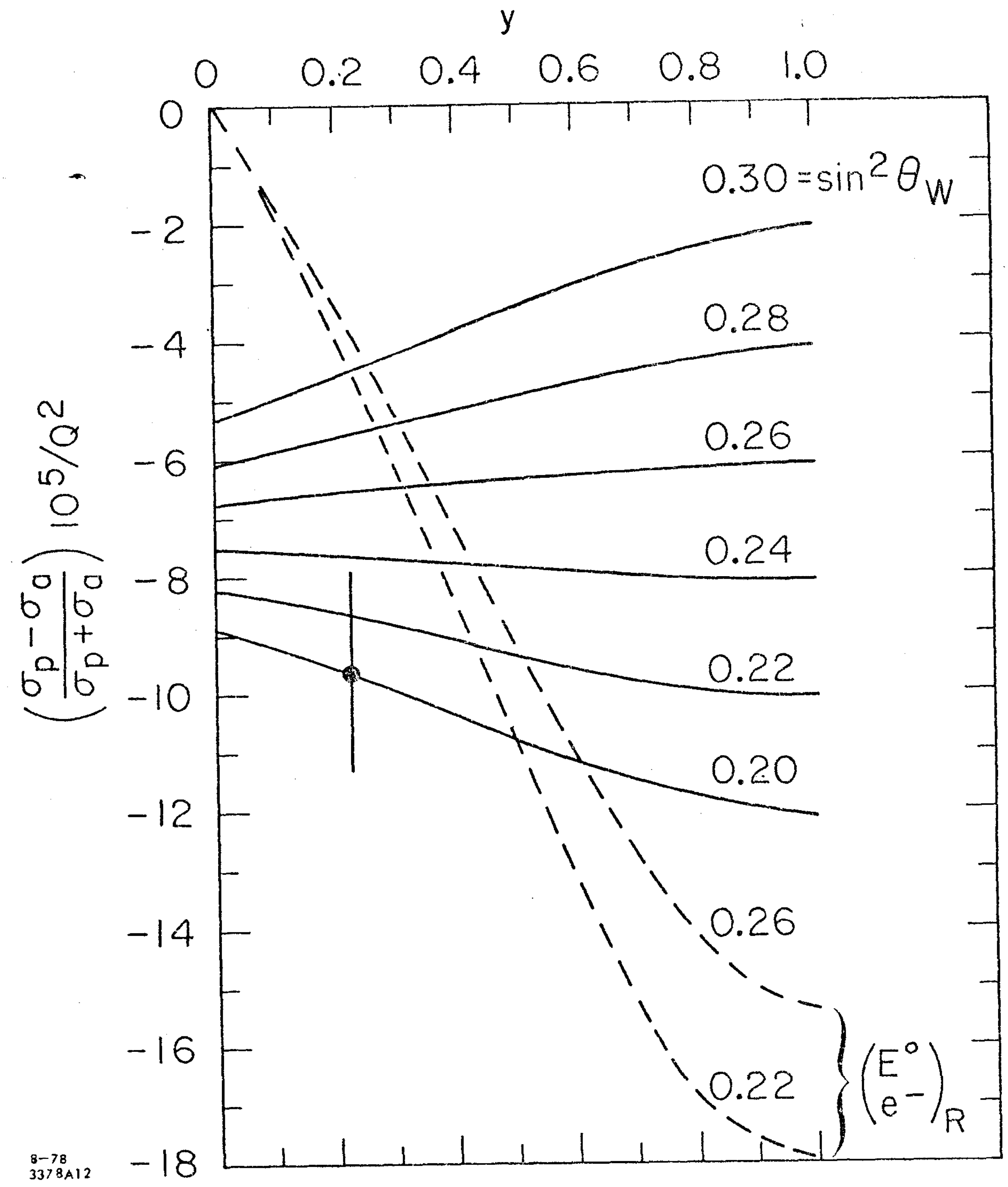

Fig. 10. Comparison of SLAC-Yale ed asymmetry measurement with predictions of Weinberg-Salam and a model with the right-handed electron in a doublet.