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THE GLUON DISTRIBUTION IN HADRONS*

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ABSTRACT

We consider the effects of hadron size and quark composition on the distribution of gluons in mesons and baryons. Coherence effects in the color singlet bound state eliminate the usual quark-mass infrared singularity. The color cancellations are important for all x_{gluon} unless the gluon transverse momentum is large compared to the inverse hadron size. Using a simple model for the meson bound state, we relate the scale-size of color coherence to the scale of the electromagnetic form factor. Applications to the flavor and quark number dependence of total cross sections and gluon-induced reactions are discussed.

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I. INTRODUCTION

A remarkable feature of quantum chromodynamics, first noted by Gross and Wilczek,[1] is that the gluon momentum fraction in hadrons

$$\int_0^1 dx\cdot x\ G_{g/H}(x,Q^2) = f_{g/H}(Q^2) \qquad (1)$$

approaches a universal value $f_g(\infty)$ as $\log Q^2/\Lambda^2 \to \infty$, where $-Q^2$ is the momentum transfer squared of the probe. The value $f_g(\infty) = n_g/\left(n_g + \frac{1}{2}n_q\right)$ depends on the number of gluons (8 in SU(3)) and the number of quarks (flavor and color), but is independent of the nature of the target H, holding for mesons, nucleons, nuclei and even glue-balls. The rate of approach to asymptopia in $\log Q^2/\Lambda^2$ is also in principle computable from QCD. However, it should be noted that as $f_g(Q^2)$ reaches its asymptotic value, the structure functions $G_{g/H}(x,Q^2)$ and $G_{q/H}(x,Q^2)$ will each vanish for all x except for x near zero.

In this paper we shall consider the effects of hadronic size and structure on the value of $G_{g/H}(x,Q^2)$ at moderate Q^2. One possible approach to the hadronic gluon distribution has already been considered in detail in Ref. 2. In these papers an estimate of the gluon distribution at a reference point Q_0^2 is computed (self-consistently) from a convolution over the quark and anti-quark distribution functions $G_{q/H}(x,Q_0^2)$, as dictated by the gluon bremsstrahlung ($q \to qg$) and pair production ($g \to q\bar{q}$, $g \to gg$) processes.[3] In these approaches, knowledge of the quark distribution function is sufficient to fix the gluon distribution. However, as the gluon wavelength becomes large compared to the hadronic size, the ability to resolve the internal hadronic structure becomes lost and the gluon will tend to decouple from the color singlet source. Thus (destructive) interference effects due to the emission of low momentum gluons from different quarks within a hadron must occur. The simple convolution approach, which treats each quark incoherently, clearly will fail in the low momentum region. It is also clear that the size of the hadron (as determined, for example, by the electromagnetic form factor) will be an important parameter for determining the shape and magnitude of the preasymptotic gluon distribution, and the gluon momentum fraction $f_{g/H}(Q^2)$ is in general target dependent. The coherent cancellations in the infrared for gluons is of course analogous to the suppression of long wavelength radiation from neutral bound states such as positronium in QED. More generally, the absence of infrared mass singularities in hadronic (color singlet) amplitudes for hadrons of fixed size and mass follows from the Kinoshita,[4] Lee, Nauenberg[5] theorem.

Phenomenologically, information on the gluon distribution for mesons, etc., should be obtainable from high p_T reactions where gluon-initiated subprocesses such as $gq \to qg$, $gg \to gg$, and $gq \to \gamma q$ can become important.[6] Furthermore, the production of heavy particles, ψ/J, η_c, Ψ, glue-balls, etc., in hadronic collisions may be attributed to gluon-induced reactions.[7] It has also been proposed that the Pomeron is directly related to the exchange of gluons[8] or sea-quarks;[9] both mechanisms are sensitive to the gluon distribution in the infrared region.

In addition to the infrared coherent cancellations for gluon emission, there is the additional complication of final state interactions in QCD: the gluon, being colored, can continue to interact with the quarks after the bremsstrahlung production (see Fig. 1a). Since the hadronic system has strong binding forces such "final state interactions" will evidently tend to equalize the rapidities of all the colored constituents. We note that there is a (semi-) infinite amount of time available for such interactions before the current probe acts. Only for large $\log Q^2/\Lambda^2$ where asymptotic freedom sets in, or at $x_g \to 1$, where soft exchange is a relatively negligible effect can one argue that final state interactions are unimportant. We also note that the rapidity-equalizing effects of final state interactions on the sea quarks (see Fig. 1b), would tend to eliminate the "hole" parton flavor correlations.[10] For example, if a sea quark q is probed at a given rapidity y_q, its flavor-balancing anti-quark partner will tend to have the rapidity of the hadron rather than y_q.[11]

The complete calculation of the gluon distribution within hadrons in the framework of QCD is clearly very complicated. A representative set of perturbation theory graphs is shown in Fig. 2. The final state interaction graphs (c), (d), (e)... are unavoidable because of gauge-invariance. Furthermore it is not clear that such distributions can be calculated without considering non-perturbative effects, especially in the infrared region. Our goal in this paper is more modest; we wish to investigate the effects of coherent cancellations due to the overall neutrality of the hadron. Note that the amplitudes (a) and (b) (as well as (c) and (d)) of Fig. 2 tend to cancel for low momentum gluons because of the opposite sign of the gluon coupling to quarks and anti-quarks. The gluons emitted from internal lines (such as (c), (d), (e)) are suppressed in the infrared (for gluons of wavelength large compared to hadronic size) by the classical Yennie, Frautschi and Suura[12] arguments. Thus, to simplify the discussion, we will consider a simple gauge theory model with scalar quarks, where gauge invariance is satisfied by the amplitudes of Fig. 2a and 2b above.

II. THE EFFECTS OF HADRONIC SIZE

The simplest model for a hadronic wavefunction which can illustrate the effects of hadronic size consists of an SU(3) color singlet scalar meson which couples to spin zero color triplet q and $\bar{q}$, with a constant vertex function, Γ. The quark struction function is[13]

$$G_{q/M}(x) = \int \frac{d^2k_\perp}{2(2\pi)^3} \frac{|\psi(\vec{k}_\perp,x)|^2}{x(1-x)} , \qquad 0<x<1 \tag{2.1}$$

The meson wavefunction is

$$\psi(\vec{k}_\perp,x) = \frac{\Gamma}{M^2 - \frac{\vec{k}_\perp^2 + m_q^2}{x(1-x)}} \tag{2.2}$$

and is normalized to satisfy the momentum and "charge" sum rules

$$\sum_{q,\bar{q}} \int_0^1 dx\ x\ G_{q/M}(x) = \int_0^1 dx\ G_{q/M}(x) = 1 \tag{2.3}$$

As usual $x = (k_0+k_3)/(p_0+p_3)$ is the light-cone/infinite momentum fraction. The contribution to the M form factor from quark q can be written in the Drell-Yan form,[14]

$$F(q^2) = \int \frac{d^2k_\perp}{2(2\pi)^3} \int_0^1 \frac{dx}{x(1-x)}\ \psi(\vec{k}_\perp,x)\ \psi(\vec{k}_\perp + (1-x)\vec{q}_\perp,x) \tag{2.4}$$

(with $q^2 = -Q^2 = -q_\perp^2$) which $\to 1$ as $q^2 \to 0$ from (2.3). All the above formulas apply equally to the $\bar{q}$; this follows from the $x \leftrightarrow (1-x)$ symmetry of $\psi(\vec{k}_\perp,x)$.

Despite the simplicity of the model, it gives the standard behavior expected for mesons:

$$G_{q/M}(x) \sim (1-x) \text{ as } x \to 1 \text{ and } F(Q^2) \sim (Q^2)^{-1} \log Q^2$$

as $Q^2 \to \infty$.

The lowest order coupling of vector gluons to the meson is computed from the two diagrams of Fig. 3. The amplitude is gauge-invariant without final state interactions. We shall compute the distribution $G_{g/M}(z,\ell_\perp) = dN/d^2\ell_\perp dz$ for transverse gluons ($\vec{k}_g \cdot \hat{\epsilon}_g = 0$). Using standard light-cone/infinite momentum frame techniques we obtain

$$G_{g/M}(z,\vec{\ell}_\perp) = \frac{4}{3}\frac{\alpha_s}{\pi^2}\int \frac{d^2k_\perp}{2(2\pi)^3}\int_z^1 dx\,\frac{1}{z^3(1-x)(x-z)}$$

$$\left[M_H^2 - \frac{\vec{k}_\perp^2+xm_q^2}{x(1-x)} - \frac{(x\vec{\ell}_\perp - z\vec{k}_\perp)^2 + xzm_q^2}{xz(x-z)}\right]^{-2}$$

$$\left[\psi^2(\vec{k}_\perp,x)\,\frac{(x\vec{\ell}_\perp - z\vec{k}_\perp)^2}{x^2} - \frac{\psi(\vec{k}_\perp,x)\;\psi(\vec{k}_\perp-\vec{\ell}_\perp,x-z)\,(x\vec{\ell}_\perp-z\vec{k}_\perp)\cdot((1-x)\vec{\ell}_\perp+z\vec{k}_\perp)}{x(1-x+z)}\right]$$

$$+\ (x \to 1-x,\ \vec{k}_\perp \to -\vec{k}_\perp) \qquad (2.5)$$

The two terms in the last line of (2.5) correspond to the diagonal and off-diagonal terms of Fig. 4. The factor 4/3 is from SU(3) color, with the standard definition of α_s.

In the limit of small z, Eq. (2.5) becomes

$$G_{g/M}(z,\vec{\ell}_\perp) \to (2)\,\frac{4}{3}\frac{\alpha_s}{\pi^2}\,\frac{1}{z}\,\frac{1}{\vec{\ell}_\perp^{\,2}}\int \frac{d^2\vec{k}_\perp}{2(2\pi)^3}\int_0^1 \frac{dx}{x(1-x)}$$

$$\cdot\left[\psi^2(\vec{k}_\perp,x) - \psi(\vec{k}_\perp,x)\;\psi(\vec{k}_\perp-\vec{\ell}_\perp,x)\right] \qquad (2.6)$$

The explicit factor of (2) in (2.6) corresponds to the emission from both quark and anti-quark. Thus the off-diagonal term regulates the infrared behavior at $\vec{\ell}_\perp \to 0$. We can make this quantitative by writing the meson form factor given in Eq. (2.4) in the form ($q^2 = -Q^2$)

$$F(Q^2) \equiv \int_0^1 dx\ f(x,(1-x)^2\ Q^2) \quad . \tag{2.7}$$

Then for $z \to 0$,

$$G_{g/M}(z,\vec{\ell}_\perp) = (2)\ \frac{4\alpha_s}{3\pi^2}\,\frac{1}{z}\,\frac{1}{\vec{\ell}_\perp^{\,2}} \int_0^1 dx \left[f(x,0) - f(x,\vec{\ell}_\perp^{\,2})\right] \tag{2.8}$$

which is valid for $\vec{\ell}_\perp^{\,2}/z >> M_H^2 - \left(k_\perp^2 + xm_q^2\right)/x(1-x)$. Using the mean value theorem, we can write

$$G_{g/M}(z,\vec{\ell}_\perp) = \frac{8}{3}\,\frac{\alpha_s}{\pi^2}\,\frac{1}{z}\,\frac{1}{\vec{\ell}_\perp^{\,2}} \left[1 - F\left(\vec{\ell}_\perp^{\,2}/(1-\bar{x})^2\right)\right]$$

$$\cong \frac{8}{3}\,\frac{\alpha_s}{\pi^2}\ \frac{1}{z}\,\frac{1}{\vec{\ell}_\perp^{\,2} + M_V^2/4} \tag{2.9}$$

where we used $F(Q^2) \sim 1/(1+Q^2/M_V^2)$ and took $1-\bar{x} \sim \frac{1}{2}$. Thus the scale of the meson form factor M_V^2 sets the scale of color coherence. This form explicitly shows the absence of infrared singularities in $zG_{g/M}(z)$. For $M_V^2 \cong 0.5\ \mathrm{GeV}^2$, the coherence size is $\lambda_M \equiv 2/M_V \cong 1/350$ MeV. If we integrate Eq. (2.9), then for small z

$$zG_{g/M}(z) \cong \frac{8}{3}\ \frac{\alpha_s}{\pi}\ \log\left[1 + \lambda_M^2(\vec{\ell}_\perp^{\,2})_{max}\right] \tag{2.10}$$

which sets the magnitude of wee gluon emission. The value of $(\vec{\ell}_\perp^{\,2})_{max}$ in this model is in principle only set by kinematics: $(\vec{\ell}_\perp^{\,2})_{max} \propto s$ in hadronic collisions and $(\vec{\ell}_\perp^{\,2})_{max} \propto (q+p)^2 = W^2$ in current induced reactions.

The modifications at large and small W^2 from asymptotic freedom effects in QCD are discussed in the Appendix. To the order computed here, one can argue that α_s in Eq. (2.10) should be evaluated at the renormalization point $\mu^2 = \lambda_M^{-2}$. We note that if $zG_{g/M} \propto (1-z)^3$ then (2.10) implies that ~40% of the meson momentum is carried by gluons at $\left(\vec{\ell}_\perp^{\,2}\right)_{max} \doteq 50\ \text{GeV}^2$, if $\alpha_s\left(\lambda_M^{-2}\right) \cong 0.33$. The crucial point here is the hadronic size (λ_H) dependence; for the same available energy, the distributions, for gluons and sea quarks, depend logarithmically on the hadronic size. Thus the gluon and sea-quark induced reactions discussed in Section I will depend logarithmically on λ_H^2. For the case of gluon exchange reactions (which involve the integration of <u>two</u> $G_{g/H}(z,\vec{\ell}_\perp^{\,2})$ over $\vec{\ell}_\perp^{\,2}$, the total meson-meson cross section depends linearly on λ_H^2. Thus even though the basic gluon interactions are flavor-independent, the flavor-content of hadrons indirectly affects the magnitude of cross sections, with heavier (i.e., smaller) hadrons interacting least. (The precise relation between hadron mass and size can be model dependent. In the simplest models based on vector dominance $\lambda_H^2 \sim 1/M_V^2$ where M_V is the lowest mass vector meson for quarks of the type composing H. In linear potential models $\lambda_H^2 \sim 1/M_V$. For the MIT bag model,[15] the variation is even slower.) We discuss some numerical results below.

Let us return again to the perturbation theory result (2.5) for $G(z,\vec{\ell}_\perp)$ and consider the region $z \to 1$, where the gluon carries off a large fraction of the hadron momentum. Since $x \geq z$, let $(1-x) = (1-z)(1-\tau)$.

Then for $(1-z) \to 0$, we find

$$G_{g/M}(z,\vec{\ell}_\perp) \to \frac{4}{3}\frac{\alpha_s}{\pi^2}(1-z)\int\frac{d^2k_\perp}{2(2\pi)^3}\int_0^1\frac{d\tau}{\tau(1-\tau)}$$

$$\cdot\left[\frac{\vec{k}_\perp^2+m_q^2}{1-\tau}+\frac{(\vec{\ell}_\perp-\vec{k}_\perp)^2+m_q^2}{\tau}\right]^{-2}$$

$$\cdot\left[\psi^2(\vec{k}_\perp,x)(\vec{\ell}_\perp-\vec{k}_\perp)^2-\psi(\vec{k}_\perp,x)\ \psi(\vec{k}_\perp-\vec{\ell}_\perp,x-z)(\vec{\ell}_\perp-\vec{k}_\perp)\cdot\vec{k}_\perp\right]$$

$$+\ \text{(symmetric terms)}\quad, \tag{2.11}$$

In our model ψ is symmetrical about $x \to 1-x$. For any $\vec{\ell}_\perp$ (2.11) predicts that $G_{g/M}(\ell_\perp,z) \overset{z\to 1}{\sim} (1-z)^3$. (More generally, if the power law dependence of $G_{q/H}(x)$ is $(1-x)^a$ at $x \to 1$, then the power fall-off of $G_{g/H}(z)$ at $z \to 1$ in perturbation theory is $(1-z)^{2+a}$ in the case of spin 0 quarks and $(1-z)^{1+a}$ in the case of spin 1/2 quarks.) For $\vec{\ell}_\perp \to 0$, both terms in the last line of (2.11) add coherently. Thus even for hard gluons with $z \to 1$, there is constructive interference when $\vec{\ell}_\perp^2$ is small compared to the hadronic scale λ_H^{-2}. The coherence is absent only for large $\vec{\ell}_\perp$ where for any z, we have

$$G_{g/M}(z,\vec{\ell}_\perp) \xrightarrow[\vec{\ell}_\perp^2 >> \lambda_M^{-2}]{} \frac{8}{3}\frac{\alpha_s}{\pi^2}\int\frac{d^2k_\perp}{2(2\pi)^3}\int_z^1 dx\,\frac{1}{zx(1-x)}\,\frac{(1-z/x)}{\left(\vec{\ell}_\perp-\frac{z}{x}\vec{k}_\perp\right)^2}\,\psi^2(\vec{k}_\perp,x) \tag{2.12}$$

This is in fact the expected convolution rule

$$G_{g/M}(z,\vec{\ell}_\perp) \to \sum_{q,\bar{q}}\int_z^1\frac{dx}{x}\int d^2\vec{k}_\perp\ G_{g/q}\left(\frac{z}{x},\vec{\ell}_\perp-\frac{z}{x}\vec{k}_\perp\right)G_{q/M}(x,\vec{k}_\perp) \tag{2.13}$$

where the gluon distribution from a single scalar quark at large $\vec{\ell}_\perp$ is

$$G_{g/q}(z,\vec{\ell}_\perp) = \frac{4}{3}\frac{\alpha_s}{\pi^2}\ \frac{1-z}{z}\ \frac{1}{\vec{\ell}_\perp^2}\quad. \tag{2.14}$$

(For spin 1/2 quarks, 1-z in (2.14) becomes $\frac{1}{2}(1+(1-z)^2)$.) Thus the region of gluon momentum where coherent effects can be ignored (and the simple impulse approximation becomes valid) always entails large $\ell_\perp$ even for gluons with large light-cone fraction z. In particular it is not in general correct to calculate the gluon momentum fraction $\int_0^1 dz\ z\ \bar{G}_{g/H}(z) = f_{g/H}$ using formulas based on convolutions over the quark distributions. Coherent corrections which are sensitive to the bi-quark distributions are necessary. The coherent effects replace the usual dependence on quark mass in $zG_{g/q} \sim \left(\ell_\perp^2 + z^2 m_q^2\right)^{-1}$ by the size effects indicated by Eq. (2.9).

It should, however, be emphasized that the QCD renormalization group analysis which gives the logarithmic dependence of the gluon distribution moments on Q^2 and their approach to the asymptotic values only requires the large $\ell_\perp^2$ region of integration and the convolution formulae[3] are valid for this application. The bi-quark correction terms correspond to higher-twist operators with extra power-law fall-off and can be neglected in the ultraviolet, high $\ell_\perp^2$ region. The coherent terms are necessary for discussing the starting values of $G_{g/M}(z,\ell_\perp)$ and $G_{g/M}(z)$ at initial values of Q_0^2.

Finally, we note that in our perturbation theory model, to first approximation $G_{g/H}(z)$, at small z, depends linearly on the number of valence quarks in the hadron as in Eq. (2.6). In the case of baryons, there are three diagonal and three off-diagonal terms, and the latter are controlled by the communication between two quarks in the hadron wave function. The corresponding diquark "form factor" is expected by dimensional counting to have the same monopole behavior as the meson form factor and should have a similar scale for quarks of the same mass. Thus we expect the size parameter which controls the coherence effects to be the same for the pion and proton and hence at small z,

$$G_{g/p} \sim \frac{3}{2} G_{g/\pi} \tag{2.15a}$$

and

$$G_{q_{sea}/p} \sim \frac{3}{2} G_{q_{sea}/\pi} \quad . \tag{2.15b}$$

Again, this indicates that the gluon momentum fraction is in general dependent on the hadronic parameters.

A less direct experimental manifestation of this dependence on the hadronic parameters appears through measurements of total hadronic cross sections, although the precise results depend on the detailed quark/gluon scattering mechanism. Four basic models can be distinguished: (a) gluon exchange, (b) gluon annihilation, (c) wee quark exchange, and (d) wee quark anti-quark annihilation (Fig. 5). In the latter three cases the number of gluons (or wee quarks) available for collision in each hadron is proportional to $G_{g/A}$ and $G_{g/B}$. Hence the ratio of total cross sections (which obviously are constant or rising with log (s) due to the 1/z behavior of $G_{g/H}$) is determined by the number of quarks in A and B and the logarithmic size dependence factors for A and B. For example we find for $(\vec{\ell}_\perp^{\,2})_{max} \sim s \sim 50\ \mathrm{GeV}^2$ that

$$\sigma_T(\rho p):\sigma_T(\phi p):\sigma(\psi p):\sigma_T(\Upsilon p)$$

are in the ratio 6:5:3:1 in a vector dominance model where we take $\lambda_V^2 = 4/M_V^2$, $\lambda_p^2 = 4/m_\rho^2$ and ignore the variation of $\alpha_s\left(\lambda_V^{-2}\right)$ (which minimizes the ratios).

The gluon exchange mechanism is the leading contribution to the cross section to lowest order in the running coupling constant α_s. Unlike the total Coulomb cross section, the gluon exchange cross section for color singlets is finite because the coherent distribution at $z \sim 0$

$$zG_{g/M}(z,\vec{\ell}_\perp) \cong \frac{4n_H}{3\pi^2} \frac{\alpha_s(\vec{\ell}_\perp^2)}{\vec{\ell}_\perp^2 + \lambda_H^{-2}} \tag{2.16}$$

is finite at $\vec{\ell}_\perp=0$ (assuming that $\alpha_s(\vec{\ell}_\perp^2)$ is regular at $\vec{\ell}_\perp^2 \to 0$). Here n_H is the number of quarks and anti-quarks in H. The gluon exchange contribution to the total cross section is essentially obtained by convoluting two $G_{g/H}(0,\ell_\perp)$ with one another. To lowest order in α_s^2,

$$\sigma_{AB}^{(g)}(s) = \frac{4\pi\left(\frac{4}{3}\right)^2 n_A n_B}{\left(\lambda_A^{-2} - \lambda_B^{-2}\right)} \left[\alpha_s^2\left(\lambda_B^{-2}\right) \log\left(1+\lambda_B^2 s\right) - \alpha_s^2\left(\lambda_A^{-2}\right) \log\left(1+\lambda_A^2 s\right)\right] \tag{2.17}$$

and

$$\sigma_{BB}^{(g)}(s) = 4\pi\left(\frac{4}{3}\right)^2 n_B^2 \lambda_B^2 \alpha_s^2\left(\lambda_B^{-2}\right) \tag{2.18}$$

Although this contribution will be modified by the higher order corrections, it may give a rough guide to the Pomeron contribution in QCD. If we assume that $zG_{g/p}(z) \propto (1-z)^4$ and take $\left(\vec{\ell}_\perp^2\right)_{max} = 50 \text{ GeV}^2$ and $\alpha_s\left(\lambda_\rho^{-2}\right) \cong 0.33$ (as we did for the meson) one obtains 50% for the gluon momentum fraction in protons using Eq. (2.10). This gives $\sigma_{pp} \cong 58$ mb if $\lambda_\rho \equiv 2/m_\rho$.

In conclusion, we reemphasize that the complete picture of quark and gluon distributions will require attention to coherent effects. It will be necessary to extend the QCD calculations to include realistic bound states, wavefunctions, higher order interactions, and final state interaction effects. However, the minimal effects of coherence and hadron size are already evident in the results from the simple model calculations considered here: the pre-asymptotic momentum fraction carried by gluons is dependent on the number of valence quarks and on the size of the hadron.

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APPENDIX. THE VALUE OF α_s

The general prescription, consistent with renormalization group analyses, is to use the running coupling constant $\alpha_s(\ell^2)$ in Eqs. (2.5)-(2.9) where ℓ^μ is the off-shell gluon four momentum ($\ell^2 = -\vec{\ell}_\perp^{\,2}$ for $z \to 0$). Equation (2.10) for $z \to 0$ then becomes in QCD

$$zG_{g/M}(z) = \frac{8}{3\pi} \int_0^{\left(\vec{\ell}_\perp^{\,2}\right)_{max}} \frac{d\vec{\ell}_\perp^{\,2}}{\vec{\ell}_\perp^{\,2} + \lambda_M^{-2}} \alpha_s\left(\vec{\ell}_\perp^{\,2}\right) \qquad \text{(A1)}$$

Here $\left(\vec{\ell}_\perp^{\,2}\right)_{max} = W^2 = Q^2(1-z)/z$. We shall need to assume that $\alpha_s(\ell^2)$ is regular at $\ell^2=0$. If $\alpha_s(\ell^2)$ is slowly varying over the range of the integral and is characterized by its value at $\ell^2 \sim \lambda_M^{-2}$, then we can write

$$zG_{g/M}(z) \cong \frac{8}{3\pi} \alpha_s\left(\lambda_M^{-2}\right) \log\left(1+W^2 \lambda_M^2\right) \qquad \text{(A2)}$$

which should be valid for moderate values of W^2. For large W^2, the logarithmic variation of $\alpha_s(\ell^2)$ in (A1) gives $zG_{g/M}(z) \sim \log\log W^2$. An approximate expression which incorporates these two limiting behaviors is

$$zG_{g/M}(z) \cong \frac{32}{3\left(11 - \frac{2}{3} n_F\right)} \log \frac{\alpha_s\left(\lambda_M^{-2}\right)}{\alpha_s\left(W^2+\lambda_M^{-2}\right)} \qquad \text{(A3)}$$

where n_F is the number of quark flavors, and we have used the one-loop QCD equation[1]

$$\frac{1}{\alpha_s\left(W^2+\lambda_M^{-2}\right)} = \frac{1}{\alpha_s\left(\lambda_M^{-2}\right)} + \frac{\left(11 - \frac{2}{3} n_F\right)}{4\pi} \log\left(1+W^2 \lambda_M^{-2}\right) \qquad \text{(A4)}$$

In fact Eq. (A3) follows from (A1) if the argument of α_s is taken as $\vec{\ell}_\perp^{\,2} + \lambda_M^{-2}$. For moderate values of W^2 the second term in (A4) is small, and Eq. (A2) follows from (A3). This is in agreement with Eq. (2.10) if we identify $\alpha_s = \alpha_s\left(\lambda_M^{-2}\right)$.

FIGURE CAPTIONS

1. Final-state-interaction corrections to the gluon and $q\bar{q}$ sea distributions in protons.

2. A representative set of QCD perturbation theory diagrams for gluon emission from a meson $q\bar{q}$ bound state.

3. Lowest order diagrams for the simplified QCD model discussed in the text for gluon emission from a meson.

4. Diagonal and off-diagonal contributions to the gluon distribution for a hadronic system.

5. The four simplest mechanisms for the hadron-hadron total cross section in QCD: (a) gluon exchange, (b) gluon annihilation into a low mass hadronic state, (c) wee quark exchange, and (d) wee quark, wee anti-quark annihilation into a low mass hadronic state.

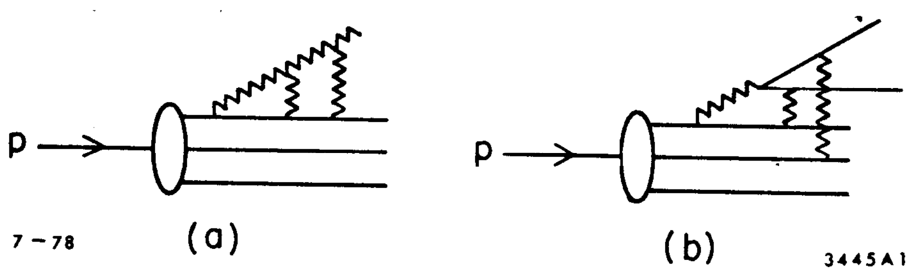


Fig. 1

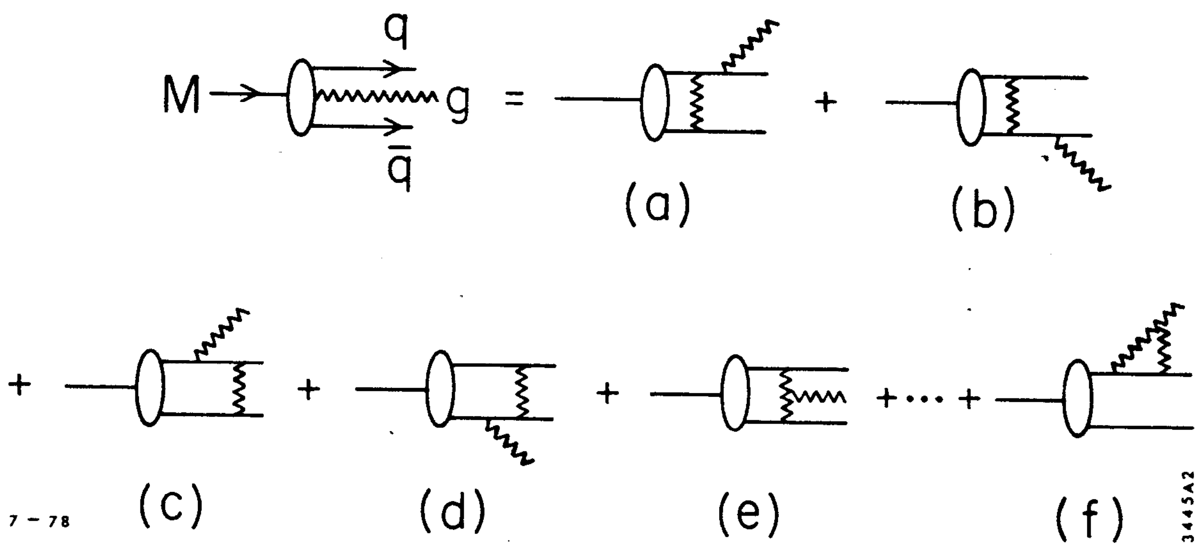


Fig. 2

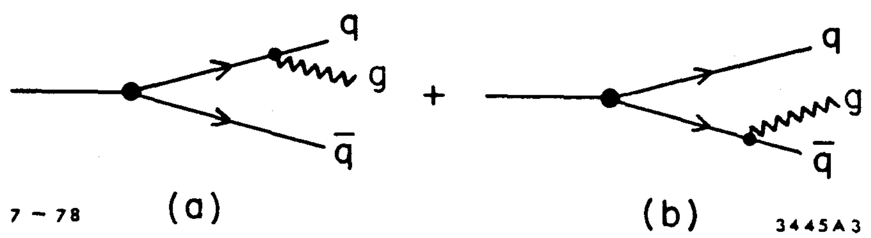
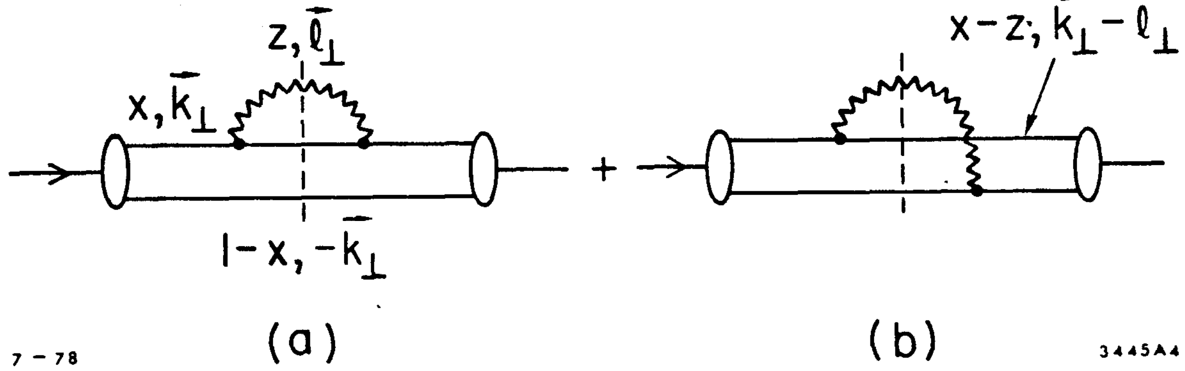


Fig. 3



$z, \vec{\ell}_\perp$
$x, \vec{k}_\perp$
$1-x, -\vec{k}_\perp$
+
$x-z, \vec{k}_\perp - \vec{\ell}_\perp$
(a)
(b)
7 - 78
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Fig. 4

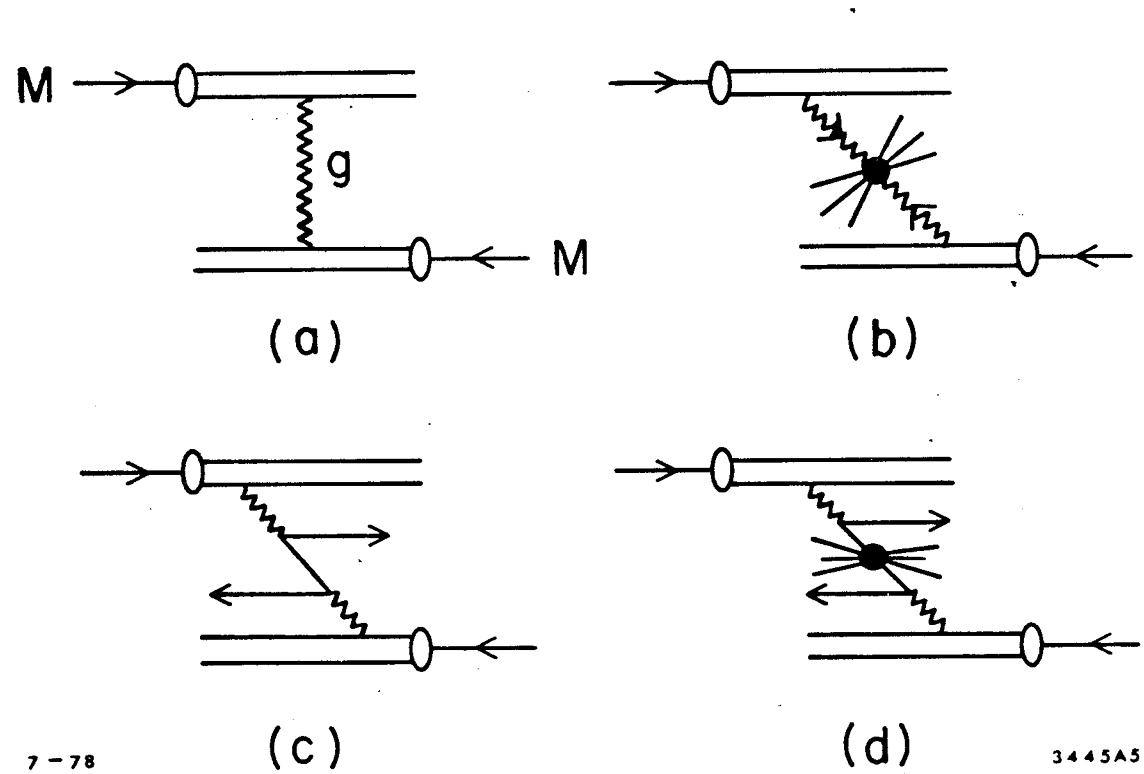

Fig. 5