SLAC-PUB-2160
July 1978
(T/E)

ARE GLUON JETS OBLATE?*

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ABSTRACT

We consider a model for gluon jet fragmentation based on QCD in which the fast hadrons in the jet are produced by the sequential reaction gluon$\to q\bar{q}\to$hadrons. The resulting jet shows an oblate transverse momentum structure, with a major axis preferentially oriented normally to the direction of linear polarization of the gluon. We discuss jet-jet oblateness angular correlations in decays of heavy $Q\bar{Q}$ pseudoscalar and vector systems.

(Submitted to Physics Letters)

*Work supported by the Department of Energy.

A critical prediction of QCD is the existence of gluon jets - jets of hadrons which are produced in the wake of an $SU(3)_c$-octet spin 1 quantum. Such jets are expected to be initiated in leptonic processes such as $e^+e^- \to q\bar{q}g$,[1] $eq \to eqg$,[2] hadron scattering subprocesses such as $q\bar{q} \to gg$, $gq \to gq$, $gg \to gg$,[3] and the decay of heavy quark bound states, $\Upsilon \to gg\gamma$, ggg.[4] An important problem will be to enumerate the properties of gluon jets which distinguish them from quark $SU(3)_c$-triplet jets.

Among such discriminants are flavor retention[5] in the fragmentation region: the flavor of quark jets should be evident in the total quantum number of its fragments up to a universal constant; a gluon jet should show an absence of leading flavor correlations. It is also anticipated that gluon jets are associated with high multiplicity events. For example, the multiplicity of soft gluon bremsstrahlung from a color octet in lowest order perturbation theory is $2/(1 - \frac{1}{n_c^2}) = 9/4$(with $n_c=3$) compared to that of a triplet.[6]

An important question is whether or not the gluon spin can be determined directly from its jet properties. A definitive answer requires a detailed understanding of the dynamics which control color confinement and the evolution of the hadronic final state. In this letter we consider a specific model for gluon jet evolution which in fact does imply that the gluon jet will be oblate with central axes correlated with the direction of the linear polarization of the gluon.

The dynamics of hadron formation in $e^+e^- \to q\bar{q}$ have been discussed from the point of an inside-outside cascade by Kogut, Sinclair, and

Susskind,[7] Bjorken,[8] and Brodsky and Weiss.[5] The separating q and $\bar{q}$ state radiates soft gluons, each of which ultimately pair up into hadrons. The hadronic cloud in the center of mass is produced along a hyperboloid $t^2-x^2 = \tau^2$ which links up with the original quark lines $x=\pm\beta t$ at a time $t = \sqrt{s}\,\tau/2m_q$ where τ is a typical soft $g \to q\bar{q}$ decay time.

We compare the evolution of gluon and quark jets in this model pictorially in Fig. 1. In the case of gluon jets, the original fast gluon radiates soft gluons, again creating $q\bar{q}$ pairs and hadrons along a hyperboloid. The elementary emission spectrum for $g \to g+g$ is of the form $G_{g/g}(x)\sim(1-x)/x + x/(1-x) + x(1-x)$ which favors the emission of a soft gluon at each stage: $g_{fast} \to (g_{slow} + g_{fast}) + (g_{fast} + g_{slow})$. The gluon polarization is preserved by the fast gluon with $\cos^2\theta$ likelihood. (The two terms in the emission again indicate the higher multiplicity of gluon versus quark jets.) At any stage the fast gluon can split into a $q\bar{q}$ pair each with a relatively flat spectrum

$$G_{q/g}(x) \propto x^2 + (1-x)^2 \tag{1}$$

The q and $\bar{q}$ in turn can continue to radiate. The gluonic jet system in this picture looks very much like the combination of coherent q and $\bar{q}$ jets. (Alternatively the fast gluon could produce hadrons by fragmentation into leading SU(3) flavor singlet states[9] such as the η' with a $D_{\eta'/g}(z)\sim(1-z)$ spectrum, or possibly produce a gluonic bound state. We suspect that these may be less important than $g \to q\bar{q}$ for pions at large x, since the η' is mostly a $\bar{q}q$ state and gluonic bound states are thought to have masses of between 1 and 2 GeV and thus will decay into high multiplicity states with a steeply falling momentum distribution.)

Because the $g \to q\bar{q}$ process in Fig. 1b has a hard dQ^2/Q^2 spectrum, the mean mass squared Q^2 of the $q\bar{q}$ system should scale with s. We thus expect that if the tensor $\sum_{a=had} \vec{p}^{\,i}_a \vec{p}^{\,j}_a$ for a gluon jet is diagonalized, the ratio of its large to small transverse eigenvalues will be of order $\langle Q^2 / k_\perp^2 \rangle$ where $\langle k_\perp^2 \rangle$ is the size of the transverse eigenvalues associated with the soft decay of quark jets. We thus expect that a typical gluon jet will be oblate with its largest transverse principle axis aligned along the production plane of its $q\bar{q}$ jet components.

An important question is the degree of correlation of the plane of oblateness of a gluon jet and its initial polarization. For the hadrons H which arise from the decay of the q or $\bar{q}$, we have

$$\frac{dN_H}{dz d\phi} \equiv D_{H/g}(z,\phi) = \int_z^1 D_{H/q}\left(\frac{z}{x}\right) D_{q/g}(x,\phi) \frac{dx}{x} + q \to \bar{q} \tag{2}$$

The fraction of the gluon's momentum carried off by hadrons

$$f \equiv \sum_H \int_0^{2\pi} d\phi \int_0^1 z dz\; D_{H/g}(z,\phi) \tag{3}$$

would be the 1 if all the hadrons came from this mechanism. Here $\cos\phi = \hat{\varepsilon}_q \cdot \hat{p}_q$ is the cosine of the angle of the q-$\bar{q}$ production with the decaying gluon's linear polarization. Note that Eq. (2) ignores the smearing of ϕ due to the quark decay, and also possible interference effects from the q and $\bar{q}$ jets (which should become small for very oblate jets). We now use $D_{q/g}(x,\phi) \propto (1-4\,x(1-x)\cos^2\phi)$ to obtain (for f=1) the sum rule

$$\frac{d\mathscr{E}}{d\phi} \equiv \sum_H \int_0^1 dz\ zD_{H/g}(z,\phi) = \frac{1+2\ \sin^2\phi}{4\pi} \quad . \qquad (4)$$

This gives the correlation of the production plane of the hadrons weighted by the hadronic (light-cone) momentum fraction

$$z = \left(p_H^o + p_H^3\right)\Big/\left(p_J^o + p_J^3\right) \quad . \qquad (5)$$

Thus according to (4), if a gluon can be produced with a specific linear polarization, its jet will be oblate with a principal axis three times as likely to be orthogonal rather than parallel to $\hat{\varepsilon}$. However this correlation is clearly an <u>upper limit</u>, since it is reduced by hadrons not obtained from q or $\bar{q}$ decay (i.e., f is less than 1), the smearing effects of hadronic decay, and the depolarization effect of soft gluon emission before the $g \to q\bar{q}$ decay.

The last effect from gluon "straggling" can be estimated as follows. Since the square of the matrix element (hard gluon, polarization ε_i → hard gluon, polarization ε_f plus soft gluon) has the form $\alpha_c(\hat{\varepsilon}_i \cdot \hat{\varepsilon}_f)^2$, the hard gluon suffers a depolarization, and to lowest order

$$\sin^2\phi \longrightarrow \frac{\sin^2\phi + 0\left(\frac{\alpha_c}{\pi}\right)\left[\frac{1}{4} + \frac{1}{2}\sin^2\phi\right]}{1 + 0\left(\frac{\alpha_c}{\pi}\right)} \qquad (6)$$

in Eq. (4). However, we expect that in this case, $\alpha_c = \alpha_c(Q^2)$ sets the scale of the running coupling constant, and gluon jets with high oblateness (Q^2 large) have the smallest depolarization. Thus at high s and Q^2 the predicted correlation between the oblate axis and gluon polarization should become more and more accurate.

The sum rule definition for $d\mathscr{E}/d\phi$ weights hadrons by their energy fraction in the jet, and thus biases against soft hadrons emitted before it decays into the $q\bar{q}$ which show no azimuthal correlations.[10] One can further enhance the likelihood of seeing the oblateness of the gluon jet by excluding events in which a final state hadron has $z=E_{had}/E_{jet} > \frac{1}{2}$. This favors jets in which the gluon fragments into a $q\bar{q}$ pair of roughly equal momentum, $z \sim 1-z$, biasing the event toward higher $Q^2=(p_\perp{}^q)^2/z(1-z)$ and biasing Eq. (2) toward a $\sin^2\phi$ dependence.

Let us now consider applications from QCD perturbation theory where the gluon polarization and correlations with the oblate jet principal axis can be predicted. The simplest application is to the decay of a heavy pseudoscalar η_C, η_B, η_T, etc. which can decay to two gluon jets. The calculation of the quark decay places is identical to that for double Dalitz decay $\pi^o \to \gamma\gamma \to e^+e^- + e^+e^-$ which has a matrix element proportional to $\hat{\varepsilon}_1 \times \hat{\varepsilon}_2$. We find in the η center of mass,

$$\frac{dN}{dx_1 dx_2 d\psi} \propto \left[x_1^2 + \left(1-x_1\right)^2\right] \left[x_2^2 + \left(1-x_2\right)^2\right] + (x_1)(1-x_1)(x_2)(1-x_2)(1-2\cos^2\psi) \tag{7}$$

where ψ is angle between the quark-pair decay planes:

Again we can use sum rules as in Eqs. (3-4) to obtain

$$\frac{d\mathscr{E}}{d\psi} = \sum_{H_a H_b} \int_0^1 dz_a z_a \int_0^1 dz_b z_b \frac{dN}{dz_a dz_b}(z_a, z_b, \psi) = \frac{15 + 2\sin^2\psi}{32\pi} \tag{8}$$

which gives the correlation in azimuthal angle summed over all opposite hadron pairs in $\eta \to g_a + g_b \to H_A + H_B + X$. Again this result should be considered as an upper bound for the correlation. Equation (20) predicts at most a 20 percent effect between perpendicular and parallel planes.

As a final example consider the decay of the Υ (or other heavy $q\bar{q}$ system) into $\gamma + g_1 + g_2$ or $g_1 + g_2 + g_3$. The correlation of the polarization of any one of the final vector particles relative to the normal to the decay plane can be obtained by a standard positronium $\to 3\gamma$ calculation. We find the distribution

$$\frac{dN}{dx_1 dx_2 d\phi} = \frac{\left(X_1^2 + X_2^2 + X_3^2\right) + (2\cos^2\phi - 1)X_1X_2X_3}{\int dx_1 dx_2 \left(X_1^2 + X_2^2 + X_3^2\right)} \qquad (9)$$

where $X_1 = (1-\cos\theta_{23}) = 2(1-x_1)/x_2x_3$, etc. and $x_1+x_2+x_3=2$. Here $\cos\phi = \hat{\varepsilon}\cdot\hat{n}$ is projection of any one of the polarization vectors with the normal $\hat{n} = \hat{p}_1 \times \hat{p}_2$.

We then can compute the corresponding angular distribution of hadrons which come from the decay $g \to q + \bar{q} \to H$ + anything. The sum rule is

$$\frac{d\mathcal{E}}{dx_1 dx_2 d\chi} = \sum_H \int_0^1 z dz \, \frac{dN_H}{dz dx_1 dx_2 d\chi}$$

$$= \int_0^{2\pi} d\phi \, \frac{\left[1 + 2\sin^2(\chi - \phi)\right]}{4\pi} \, \frac{dN_g}{dx_1 dx_2 d\phi} \qquad (10)$$

where again each hadron is weighted by its momentum fraction z. We thus obtain

$$\frac{d\mathcal{E}}{dx_1 dx_2 d\chi} \propto \left(X_1^2 + X_2^2 + X_3^2\right) + \frac{1}{4}\,(1 - 2\cos^2\chi)X_1X_2X_3 \qquad (11)$$

where $\cos\chi = \hat{p}_H \cdot \hat{n}$ computed for each hadron in the gluon jet. The maximal effect occurs for equal angles of the gluon jets $\left(\theta_c = 120^o,\ X_i = \frac{3}{2}\right)$. This gives a relative weight of 9/7 for hadrons in a given gluon jet aligned in the plane rather than normal to the plane. In fact, since the correlation in $\cos\phi$ is idential for each gluon polarization, the distribution in $\cos\chi$ holds summing over all hadrons in the decay $\Upsilon \to \gamma$ + hadron or $\Upsilon \to$ hadrons, as the identification of each hadron with individual gluon jets is not necessary. [Integrating over x_1 and x_2 results in a much smaller effect, $d\mathscr{E}/d\chi \cong 1 + .038\ (1 - 2\cos^2\chi)$, due to the contribution from the regions of phase space where any $x_i \approx 1$.] These results suggest that while the ggg or ggγ decays of the Υ form a pancake, it is a very thick pancake, since the predicted correlation in χ is small.

Again we emphasize that the results presented here should be taken as an upper bound for possible correlations, since all hadrons in the gluon jet are assumed to come from q or $\bar{q}$ fragmentation, and straggling effects are neglected. Aside from these model-dependent considerations, it seems a viable possibility that the decay product of gluons will form oblate jets with angular correlations which reflect the linear polarization of the parent.

ACKNOWLEDGEMENTS

We would like to thank John Ellis for an interesting conversation. This work was supported by the Department of Energy.

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FIGURE CAPTIONS

1. (a) The evolution of (a) a quark jet or (b) a gluon jet into hadrons.

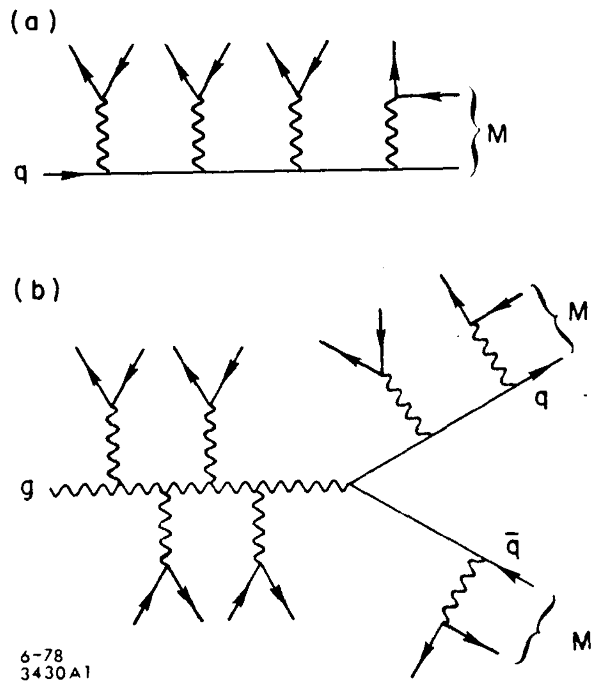

Fig. 1