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FOUR-QUARK MODEL FOR THE MESONS*†

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ABSTRACT

A four-quark mechanism is proposed which generates virtually the entire known spectrum of (non-charm) meson states to remarkable accuracy. The ground state 0^- and 1^- nonets are taken as input; exclusive of octet-singlet mixing there are no free parameters.

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Previous attempts to derive the meson spectrum on the basis of the quark model have focused almost entirely on the $q\bar{q}$ system. Such approaches invariably involve the introduction of a potential (and/or a bag), with associated free parameters. Unfortunately, our experience indicates that the potential must be very complex (and the number of parameters large) in order to generate a detailed description (e.g., at the level of this paper). Recently, several authors have studied 4-quark ($q\bar{q}q\bar{q}$) systems with regard to a few select conventional states (0^{++}), and 0^{--} exotics.[1] However, this approach is again within the potential/bag framework, and is not (at present) quantitative. In contrast, taking the ground state 0^- and 1^- nonets as input, the 4-quark mechanism proposed in this Letter involves no free parameters, and generates every well-determined meson state to remarkable accuracy. This implies that the mechanism responsible for confining the $q\bar{q}$ s-waves, whatever its nature, does not directly produce low mass states with $\ell \geq 1$.

Specifically, I propose that the excitations commonly associated with $\ell \geq 1$ arise as the consequence of a simultaneity condition involving the pairwise masses of all four $q\bar{q}$ combinations. In order to understand this condition, we first consider Fig. 1a, which depicts three mesons m_1, m_2, m_3 resonating in pairs to produce particles A (m_1m_2) and B (m_1m_3) simultaneously. This can occur only if the invariant three-body mass takes on a particular value M_0 determined by the masses of A, B and the three mesons. Almost a generation back, Peierls noted the sharp energy-dependence of this effect, and proposed that it could be responsible for generating the N*(1512).[2] Others extended Peierls' treatment to produce excellent predictions for the masses of the A_1, Q and E mesons. Physically, there is nothing strange about this effective "force"; for example, in the singly ionized hydrogen molecule, in which a single electron can be bound to either of two protons, the molecular binding is produced by exactly such an (exchange) mechanism. At the technical level, however, subsequent analysis by several authors demonstrated that the associated S-matrix singularity is on the wrong Riemann sheet, and hence cannot be related to a resonance.[3]

On the other hand, I recently discussed an alternative mechanism in which particles m_1, m_3 again form resonance B, but particles m_2, m_3 are taken to be relatively at rest. The related singularity is now on the correct sheet, and closely similar values are generated for M_0 in many cases (e.g., A_1, Q, D, E).[4] This is easy to see, since particles m_2 and m_3 must be identical in the case of a single type of resonance (A=B), and hence the pairwise masses M_{12} and M_{13} are equal; $M_{12}=M_{13}=m_A$. In the A_1, for example, A, B are the ρ, and m_1, m_2, m_3 are all pions. I have shown that for broad states A(characteristic of physical mesons) this effect will be important only under rather special circumstances, but in the limit that A has zero width it is guaranteed to be strong (i.e., at the level of the quark model). For our purposes it is important to note that nothing has changed physically with respect to the Peierls idea; this reformulation is aimed at the mathematical subtleties.

We now turn to Fig. 1b, representing a 4-quark system which we regard as arising from the excitation of an original 2-quark state $(q_1\bar{q}_2)$. We thus suppose that q_1, $\bar{q}_2$ are moving originally as free particles in the center of a bag or very deep potential well, and that a fluctuation occurs in which an additional pair $\bar{q}_3$, q_4 is created (also in the bag/well). The energy for this process will be minimal if the pair $\bar{q}_3$, q_4 is relatively at rest; to conserve charge they must both clearly be of the same flavor ($u\bar{u}$, $d\bar{d}$, $s\bar{s}$), and form an I=0 singlet. In analogy with the above, we shall assume that the extremely strong forces responsible for confinement will attempt to force as many $q\bar{q}$ pairs as possible into the unique energy levels observed in the 0^-, 1^- nonets. We would then expect the favored energy levels to correspond to the pairs $(q_1\bar{q}_3)$, $(\bar{q}_2q_4)$, and $(q_1\bar{q}_2)$ forming meson states a, b, and c, respectively. Applying the conditions $(p_1+p_3)^2=m_a^2$, etc. to the 4-body system, it is simple to derive the expression

$$M^2 = m_c^2+2(m_a^2-m_{q_1}^2) + 2(m_b^2-m_{q_2}^2) \tag{1}$$

for the mass M of the excited state, where m_{q_i} is the mass of quark q_i. Below I demonstrate that this simple formula correctly generates the observed spectrum of meson states.

Of course, it is not sufficient just to calculate a sequence of masses; one must show that each excitation has the right quantum numbers to be identified with the corresponding physical state. Fortunately, given our picture of the excitation mechanism, we have a number of facts to work with. In the first place, the isospin of the system must be identical with that of meson c ($I=I_c$). Secondly, for neutral states the effect of the charge conjugation operator C is to rotate the system about the horizontal axis bisecting the diagram of Fig. 1b. Thus, if mesons a and b have relative angular momentum L, we associate the state with $C=(-)^L$. Together, these facts determine the g-parity; $g=(-)^{L+I}$. If we then make the reasonable assumption that the particular combination of a and b is only important if their quantum numbers allow them to be emitted as actual decay products, we deduce the additional constraint $g=g_a g_b$; this is useful if $I_a \neq \frac{1}{2}$, $I_b \neq \frac{1}{2}$. From our identification of a, b, c as either vectors or pseudoscalars, we can also determine the most likely value of the total spin S of the excited state. Thus, if a, b, c are all 0^-, the individual quark spins are alternately up or down, and we infer S=0 (other choices suggest S=0,1,2). Given L and S, we severely limit the possible values of total angular momentum J. The parity P is simply $(-)^L P_a P_b$. Finally, it is natural to assume that the purely pseudoscalar combinations a, b, c with L=0 generate the lowest-lying excitations ($J^{PC}=0^{++}$), and that successively replacing each with a vector leads (in general) to a state of higher J. Taking all these rules together, it is possible to achieve an almost totally unambiguous classification of states (with an exception discussed below).

As a specific example, let us first consider the possible excitations involving just π's and ρ's. In doing so, it is useful to introduce the notation (ab)c for the particles involved, and to represent the common mass of the u and d quarks by m. According to Eq. (1), $(\pi\pi)\pi$ leads to the relation $M^2=5m_\pi^2-4m^2$. At this state we clearly need a value for m, which we fix by requiring that this lowest level excitation leads simply back to the pion; i.e., $m=m_\pi$. While this is certainly not the only choice one might make _a priori_, it turns out to be the only acceptable one in our scheme, and has some very intesting consequences. Thus $(\pi\pi)\pi$ with L=0 has I=1, C=+, g=−; it hence cannot decay into two pions a and b (whatever the value of m). On the other hand, $(\pi\pi)\rho$ yields an acceptable vector (1^{--}) with L=1, and predicts $M=m_\rho$ _if_ $m=m_\pi$. Therefore, the pion is stable against the lowest excitation, but the ρ has an acceptable mechanism for decay; $\rho \to (\pi\pi)\rho \to \pi\pi$. To go beyond the π and ρ, the next level is $(\rho\pi)\pi$. For this the possibility of $\rho\pi$ decay requires g=-; with I=1 we need L even. Since S=1 the lowest choice L=0 yields 1^{++}, with a predicted mass $M^2=2m_\rho^2-m_\pi^2$. Numerically, $M(1^{++})=1080$ MeV, which may be compared with $A_1(1100)$.[5] The next state is $(\rho\pi)\rho$; again S=1 and L=even. Since this is a higher level we take L=2, yielding possibilities 1^{++}, 2^{++}, 3^{++}. Having a (lower) 1^{++} already, the most likely choice is 2^{++}; we then predict $M(2^{++})=1319$ MeV, to be compared with $A_2(1310)$. Finally, $(\rho\rho)\rho$ has S=2, I=1, g=+ (implying L=odd); for L=1 one may have 1^{--}, 2^{--}, 3^{--}. Having a 1^{--} (and a better candidate for 2^{--} below), we predict $M(3^{--})=1699$ MeV, to be compared with g(1680). Thus we achieve the sequence π, ρ, A_1, A_2, g_2 with the correct quantum numbers, decay modes, and masses (to the level of 1% error).

In order to generate the I = ½ states, we next consider the diagram in which q_1 is taken to be the s quark. In precisely the same fashion as the above, we obtain the K and K^* from $(K\pi)K$ and $(K\pi)K^*$, providing that the s mass is equal to m_K. The next excitation (L=0) has two modes $(K^*\pi)K$ and $(K\rho)K$, leading to the respective values 1161 and 1179 for $M(1^{++})$. This splitting arises from the fact that $m_\rho^2-m_\pi^2$ is not precisely equal to $m_{K^*}^2-m_K^2$ empirically; at the level of our argument it seems reasonable to neglect this and predict a single state at the average value 1170. This agrees nicely with the value $Q_a(1180)$ extracted from a recent analysis of (1^+) $K\pi\pi$ data.[6] A continuation of this sequence leads to $M(2^{++})=1388$, $M(3^{--})=1745$; the experimental states are $K^*(1420)$ and $K^*(1780)$.

Having established the rules of our game, it is straightforward to generate the I=0 partners of these states, and other J^{PC} combinations. For this reason I shall defer noncritical details to a subsequent (more

lengthy) article, and concentrate on some points of particular interest.
(1) Results are displayed in Tables I and II for the lowest lying J^{PC} states. For reference purposes, an index N has been introduced to characterize the particular 4-quark configuration involved. Using the symbol m to represent either d or u, the diagrams (Fig. 1b) with $\bar{q}_3 q_4 = \bar{m}m$ and $q_1\bar{q}_2 = m\bar{m}$, $s\bar{m}$, $s\bar{s}$ are labeled by N=1,2,3, respectively. The diagrams with $\bar{q}_3 q_4 = s\bar{s}$ and $q_1 q_2 = \bar{m}m$, $\bar{m}s$, $\bar{s}s$ are labeled by N=4,5,6.
(2) The I=0 $m\bar{m}$ and $s\bar{s}$ configurations are regarded as projections onto the pure octet and singlet states. At the level of unmixed SU(3), the excitations are thus computed using the appropriate octet and singlet masses. In order to compare with experiment, however, one must allow for some mixing of states such as $(a\eta_8)c$ and $(a\eta_1)c$. The simplest assumption is that mixing occurs precisely as it appears empirically in the 0^-, 1^- ground states. Due to the nature of our mass formula, this means that one may simply employ the <u>physical</u> masses of ω, φ, η, η' in Eq. (1). With a few exceptions, this postulate leads to generally excellent results. The overall quality can be considerably improved, however, by allowing for slightly different mixing of η, η'. The values quoted have thus been computed using the effective masses m_η=617 MeV, $m_{\eta'}$=920 Mev.
(3) The 0^{++} ground states are crucial as input for generating certain higher J^{PC} excitations. For example, the first I=1 entry in Table II shows a state I denoted as "b" at 861 MeV; this arises from $(\eta\pi)\pi$. It has not been seen experimentally, but may be difficult to disentangle from the δ. It is presumably of comparably narrow width, and I have therefore assumed that it can be employed as one of the a,b,c states (communicating via the $q\bar{q}$ p-wave). Correspondingly, an I=½ partner (b')

is generated at 983 MeV by $(K\eta)K$. This can decay to $K\pi$ and $K\pi\pi\pi$, but is also unobserved. For our purposes, the predicted b, b' states are vital in generating well-determined states such as B, Q_1, h, U (in addition to A_3, L, etc.); their experimental verification would provide a crucial test of this theory. The use of relatively broad excitations as input is perhaps questionable; as an example I have listed 4^{++} states arising from A_1 and Q_a. The results are clearly encouraging, yielding h, U and X(1900).

(4) In Table II I have listed four nonets of 0^{++} states; there are additional I=0 scalars of higher mass. Among the former are good candidates for relatively narrow states corresponding to $\delta(980)$, $S^*(980)$, $\kappa(1400)$;[5] and to $\delta'(1260)$ and $S^{*\prime}(1310)$.[7] Others may be associated with broad enhancements $\kappa(1250)$, $\varepsilon(1300)$ in the $K\pi, \pi\pi$ s-waves. In addition, states degenerate in mass with η and ω which couple strongly to $\pi\pi$ are predicted (labeled η^*, ω^*); possible evidence for such states has been noted in the decays $\psi \to \phi\pi^+\pi^-$ and $\psi \to \omega\pi^+\pi^-$.[8]

(5) In addition to those states normally attributed to $q\bar{q}$, it is possible to construct exotics such as the 0^{--} nonet shown. These (lightest) excitations are all close to the busy 1400 MeV region, and will not be easy to sort out.

(6) The 1^+ $I=\frac{1}{2}$ system is particularly striking. A recent analysis by this author suggested two Q states; $Q_a(1180) \to K^*\pi$, ρK and $Q_1(1290) \to \rho K$,[6] whereas previous analyses found the same Q_1, but a higher mass state $Q_2(1400) \to K^*\pi$, εK.[9] The present theory suggests that all of these exist, with Q_a occurring in C = +, and Q_1, Q_2 associated with C = − combinations of $K^+\pi^+\pi^-$ and $K^-\pi^+\pi^-$. Note that the L=1 (Kb) and $(K\delta)$ configurations may well prefer to decay into the s-wave $K\rho$ or $K^*\pi$ configurations (i.e.,

horizontally, in the sense of Fig. 1b). This Q triplet is quite unexpected from the standpoint of $q\bar{q}$ dynamics, and is an important signature of this mechanism.

In summary, I feel that this model must be regarded as a serious alternative to the simple picture of $q\bar{q}$ excitations. For those states which are best determined experimentally and uncomplicated by mixing ambiguities (e.g., δ, B, A_2, E, Q_1, g, U, h, $K^*(1420)$, L, $K^*(1780)$) the predictions are essentially exact. With slight differences in mixing, and considering experimental errors, this could conceivably be true for the entire spectrum. In addition, new effects such as the Q splitting, the b, b' particles, and the low mass $\pi\pi$ states are predicted, and could provide a definitive test of the theory. A generalization to the baryon isobars, and (comparable) results for charm particles, will be reported in subsequent articles.

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TABLE I

J^{PC}	LS	I=1 N(ab)c	I=1 T	I=1 E	I=½ N(ab)c	I=½ T	I=½ E	I=0 (8) N(ab)c	I=0 (8) T	I=0 (8) E	I=0 (1) N(ab)c	I=0 (1) T	I=0 (1) E
1^{++}	01	$1(\rho\pi)\pi$	1080	$A_1(1100)$	$2(K^*\pi)K$	1161	$Q_a(1180)$	$3(K^*\bar{K})\eta$	1220	D(1285)	$3(K^*\bar{K})\eta'$	1396	E(1420)
					$2(K\rho)K$	1179							
1^{+-}	11	$1(b\pi)\pi$	1210	B(1235)	$2(Kb)K$	1299	$Q_1(1280)$	$3(b'\bar{K})\eta$	1351	--	$3(b'\bar{K})\eta'$	1514	--
					$2(b'\pi)K$	1299							
					$2(K\delta)K$	1439	$Q_2(1400)$						
2^{++}	21	$1(\rho\pi)\rho$	1319	$A_2(1310)$	$2(K^*\pi)K^*$	1378	$K^*(1420)$	$3(K^*\bar{K})\phi$	1464	f'(1515)	$3(K^*\bar{K})\omega$	1310	f(1270)
					$2(K\rho)K^*$	1394							
2^{-+}	21	$1(b\eta)\rho$	1660	$A_3(1640)$	$2(b'\eta)K^*$	1721	L(1770)	$3(b'\bar{K})\phi$	1576	--	$1(b\pi)\omega$	1434	--
3^{--}	12	$1(\rho\rho)\rho$	1699	g(1680)	$2(K^*\rho)K^*$	1745	$K^*(1780)$	$3(K^*\bar{K}^*)\phi$	1802	--	$3(K^*\bar{K}^*)\omega$	1679	$\omega(1670)$
4^{++}	42	$1(A_1\omega^*)\pi$	1894	X(1900)	$2(Q_a\omega^*)K$	1931	--	$1(A_1b)\eta$	2051	h(2040)	$1(A_1b)\eta'$	2161	--
								$3(Q_a\bar{b}')\eta$	2030		$3(Q_a\bar{b}')\eta'$	2142	
4^{++}	42	$4(Q_a\bar{b}')\pi$	2159	--	$2(b'A_1)K$	2017	--	$4(Q_a\bar{b}')\eta$	2241	--	$4(Q_a\bar{b}')\eta'$	2342	U(2350)

TABLE II

J^{PC}	LS	I=1			I=½			I=0 (8)			I=0 (1)		
		N(ab)c	T	E	N(ab)c	T	E	N(ab)c	T	E	N(ab)c	T	E
0^{++}	00	1(ηπ)π 4($K\bar{K}$)π	861 958	(b) δ(980)	5(Kη)K	983	(b')	1(ππ)η	617	(η*)	1(ππ)ω	783	(ω*)
0^{++}	00	1(ηπ)ρ	1146	--	5(Kη)K*	1232	κ(1250)	4($K\bar{K}$)η	1131	--	3($K\bar{K}$)η'	920	S*(980)
0^{++}	00	1(η'π)π	1294	δ'(1260)	5(Kη')K	1378	κ(1400)	6(ηη)φ	1260	ε(1300)	4($K\bar{K}$)η'	1321	S*'(1310)
0^{++}	00	1(η'π)ρ	1499	--	5(Kη')K*	1565	--	6(ηη')φ	1587	--	1(ηη)ω	1435	--
0^{-+}	01	1(S*π)π	1294	--	2(KS*)K	1378	K'(1400)	1(bπ)η	1351	--	1(bπ)η'	1512	--
0^{--}	11	1(ωπ)b	1388	--	2(Kω)b'	1468	--	4($K^*\bar{K}$)η*	1544	--	4($K^*\bar{K}$)ω*	1617	--
1^{--}	10	1(ρρ)π	1521	ρ'(1600)	2(K*ρ)K	1579	K*'(1650)	3($K^*\bar{K}^*$)η	1608	--	3($K^*\bar{K}^*$)η'	1747	ω'(1780)

FIGURE CAPTION

1. (a) Three-meson system with pairs forming resonances A and B.

(b) Four-quark system with pairs forming mesons a, b and c. The pair $\bar{q}_3 q_4$ are relatively at rest.

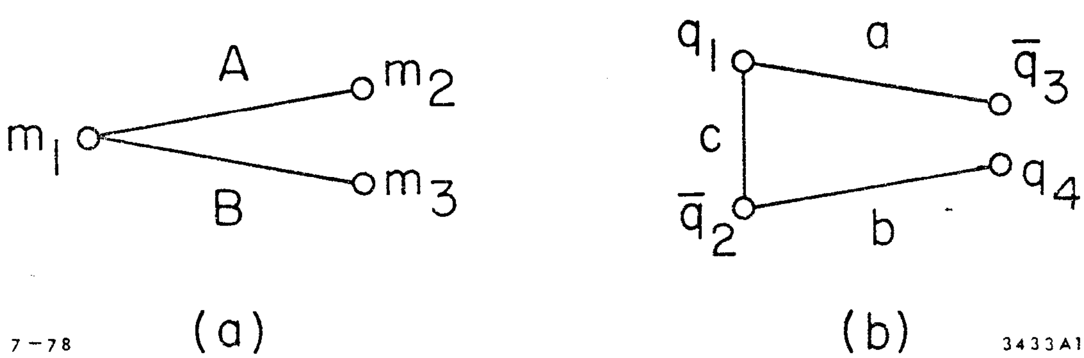

Fig. 1