LEPTON WIDTH SUPPRESSION IN VECTOR MESONS*

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ABSTRACT

QCD radiative corrections to the vector meson lepton width are discussed. It is shown how these modify the 0th order equation of Van Royen and Weisskopf, thereby leading to a significant suppression, by about a factor of 2, of the calculated value of Γ. Consequences for phenomenology are examined. In particular, a prediction is made for the width of T', as well as for widths of heavier mesons.

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I.   INTRODUCTION

Over the past few years there has been a proliferation of quark-potential models.[1] These have generally been quite successful in predicting properties of the charmonium system and some[2] can accommodate the masses of the recently found T particles.[3] Most of these models[4] are, however, parametrized not only by the masses of certain mesons, but also by their leptonic decay widths, based on the Van Royen-Weisskopf[5] formula,

$$\Gamma(V \to e^+e^-) = \frac{16\pi \, \alpha^2 |\phi(0)|^2}{m_V^2} \, e_Q^2 \tag{1}$$

where e_Q is the quark charge, m_V is the vector-meson mass, and ϕ is the $q\bar{q}$-wave function.

In what follows, we hope to cast doubt on the reliability of this equation by showing, through an estimate of QCD radiative corrections, that these cause a large suppression (of order 50%) of Γ.

Phenomenological implications of the modified formula will be presented in the final section of this paper.

II.   CALCULATION OF THE LEPTON WIDTH

The effect of lepton width suppression can be clearly seen in a QED   calculation   where orthomuonium, $\bar{0}$, decays into $e\bar{e}$. The radiative correction to the 0th order process is due to the exchange of a photon, renormalized by a counterterm (see Fig. 1). (Notice that part of the photon contribution iterates the Coulomb potential, thus repeating the 0th order decay rate.) The calculation can be done following the methods

outlined in Karplus and Klein[6] and the result,[6] suppressed as advertised, is

$$\Gamma(\overline{0} \to e^+ e^-) = \frac{16\pi\alpha}{3} \frac{|\phi(0)|^2}{m_0^2} \left(1 - \frac{4\alpha}{\pi}\right) \qquad (2)$$

For the purposes of comparison with QCD, we define three new quantities, Γ_0, Γ_T, Γ_L so that

$$\Gamma = \Gamma_0 \left\{1 + \alpha(\Gamma_T + \Gamma_L)\right\} \quad .$$

Γ_0 is the 0th order rate, Γ_T is the contribution of the transverse photon, and Γ_L is the relative-order-α contribution of the longitudinal photon (here, the standard Coulomb-gauge terminology is employed). The decomposition into Γ_T and Γ_L is not given in Karplus and Klein[6] but (tedious) calculation shows $\Gamma_T = -\frac{2}{3\pi}$ and $\Gamma_L = -\frac{10}{3\pi}$. We note that both Γ_T and Γ_L cause suppression, but that the main effect is due to Γ_L. Furthermore, we find when doing the calculation, that the radiative correction $(\Gamma_T + \Gamma_L)$ arises from the exchange of <u>hard</u> photons (where the momentum scale is of order m_μ). More specifically, in Fig. 1, the small k region of the loop integration contributes only to the iteration of the wave function and to corrections of $0(\alpha^2)$.[7] Terms which are $0(\alpha)$ are due to the region of integration where $|k| >> \alpha m$. As a final observation, we note that the wave function, ϕ, (the q-loop in Fig. 1) factors out when k is large, hence $\Gamma_T + \Gamma_L$ is not dependent on ϕ.

These above observations are important in trying to understand the analogous QCD calculation. As an ansatz, let us suppose that the first-order QCD correction arises in precisely the same way as above--namely, that the effect is due to single hard gluon exchange, and soft gluons (as

well as some other processes involving more gluons) will be assumed to contribute (through $O(\alpha_S)$) only to the bound state. (Notice that this ansatz is of the same nature as that made by de Rújula et al.[8] in computing the spin-dependent $O(\alpha_S)$ perturbation to a confining potential.) The resulting decay width is, then, by the remarks of the previous paragraph,

$$\Gamma(V \to e^+ e^-) = \frac{16\pi \, \alpha \, e_Q^2 |\phi(0)|^2}{m_V^2}$$

$$\times \left(1 - \left(\frac{4}{\pi}\right)\left(\frac{4}{3}\right) \alpha_S \, (m_Q)\right) \tag{3}$$

where color factors have been included, and the strong coupling constant is renormalized at m_Q, which, as we saw, is the typical momentum for this term. Equation (3) has been noted by Barbieri et al.[9] although the precise nature of the underlying ansatz was not made clear in their paper.[10] It is unfortunate that this observation of theirs has escaped attention for it does in fact imply considerable suppression of Γ. Actual values will be shown later, but first we shall discuss the validity of the assumptions leading to (3).

The main difficulty encountered in calculating the QCD corrections is, of course, the fact that we do not know what diagrams contribute to the confinement. For the case of muonium, the Coulomb potential is given by iteration of the longitudinal photons. In the same way, iterations of the longitudinal gluon would lead to a Coulomb potential which does not have the confining behavior expected in QCD. This means that the low momentum Coulomb gluon does not iterate the potential and the computation of Γ_L does not proceed as in QED. We can (and do) still assume that the

soft gluon serves as part of the iterating term (and does not contribute
to $O(\alpha)$ corrections)but such a separation between iteration and correction
is not at all clear. The transverse gluon is not binding. Thus it ap-
pears that Γ_T may be less questionable. Unfortunately, as was observed
previously, $\Gamma_T = \Gamma_L/5$ (in muonium) and so it appears that the main con-
tribution to the correction may come from the less reliable term (although,
in point of fact, we do not really know that the transverse gluon does
not enter as part of the iteration--our intuition from QED is not reliable
in this matter).

There is a further difference between the QCD and QED calculations--
namely, tri-gluon contributions could contribute in $O(\alpha_S)$, as is shown in
Fig. 2. This is, in principle, calculable, but the calculation is long,
and, in light of the previous problem (regarding the iterations of the
potential) may not be very revealing. In fact, the multigluon couplings
are probably the source of confinement and so it might be hoped that Fig. 2
should be ignored for our purposes.[11]

One last point must be raised in the analysis of Eq. (3). This re-
gards the question of the validity of a perturbative expansion in this
equation. $\alpha_S \gtrsim 0.3$ implies $\Gamma_T + \Gamma_L \gtrsim 0.5$, suggesting that higher orders
will be important and thus that the $O(\alpha_S)$ result is somewhat unreliable.
In particular, note that for $\alpha_S > 0.7$, Eq. (3) implies $\Gamma < 0$. In order
to assure positivity of Γ in the phenomenological analysis to be done,
(3) will be replaced by

$$\Gamma(V \to e^+e^-) = \frac{16\pi\alpha^2 \, e_Q^2 |\phi(0)|^2}{m_V^2} \left(1 - \frac{8}{3\pi} \alpha_S \, (m_Q)\right)^2 \qquad (4)$$

This is equivalent through $O(\alpha_S)$ to (3) and has the advantage of positivity.

III. PHENOMENOLOGICAL IMPLICATIONS

We here adopt the ansatz made in the last section and consider Eq. (4). To estimate the suppression we use[12]

$$\alpha_S(M) = \frac{12\pi}{33 - 2n} \frac{1}{\log (M^2/\Lambda^2)} \tag{5}$$

where n is the number of flavors with quark mass < 2M, and Λ is found from deep inelastic scattering[13] to be about 500 MeV. Taking[1,2] $M = m_c \sim 2$ GeV, we find $\alpha_S(m_c) \sim 0.50$ and $\left(1 - \frac{8}{3\pi} \alpha_S(m_c)\right)^2 \sim 0.33$. This is, as claimed, a very large suppression and so we see that Eq. (1) should not be used in constructing a potential.

One possible indication of the suppression of Γ has to do with the mystery of the large splitting between $\psi(3.1)$ and X(2.83).[14] If X is η_c (paracharmonium), then it is difficult[15] to reconcile such a large hyperfine splitting with the lepton width of $\psi(\Gamma = 4.8 \pm 0.6$ KeV).[16] The reason for this is that the Breit-Fermi types of formulas[17] imply that the hyperfine splittings derive their main contribution from $|\phi(0)|^2$ which, as has been mentioned, is usually fixed by Eq. (1).[18] But we have just seen that Eq. (1) must be modified and that a larger value of $|\phi(0)|^2$ (called for by the ψ-X difference) is compatible with the observed lepton width of ψ. Unfortunately, neither the status of the X (as paracharmonium)[19] nor the questioned reliability of the hyperfine calculation really lends much weight to this "evidence" for suppression.

Another possible hope for seeing the radiative correction is to measure $\Gamma(\eta_c \to 2\gamma)/\Gamma(\psi \to e^+e^-)$. This experiment may soon be done[20] and, if sufficiently accurate, could measure the higher order effects. The 0th order prediction for the ratio[21] is

$$\Gamma(\eta_C \to 2\gamma)/\Gamma(\psi \to e^+ e^-) = 3 e_Q^2 \frac{M_\psi^2}{M_{\eta_C}^2} = \frac{4}{3} \frac{M_\psi^2}{M_{\eta_C}^2} \tag{6}$$

and any deviations from this would be evidence for QCD-radiative corrections. The effect could be large if $\Gamma(\eta_C \to 2\gamma)$ were _enhanced_ by its $O(\alpha_S)$ perturbation but there is some indication from QED that this might not be the case. Namely, in parapositronium, P, Harris and Brown[22] have shown that the radiative corrections to $\Gamma(P \to 2\gamma)$ suppress, rather than enhance, the rate. If the suppression should also occur in QCD, then it might turn out that experiment will confirm Eq. (6) even though there are radiative effects (which just happen to cancel).

Finally, we leave the question of directly testing (3) and turn to the predictions that can be made for lepton widths in a potential model. Perhaps the most reliable of these are the ratios $\Gamma(V' \to e^+ e^-)/\Gamma(V \to e^+ e^-)$ where V' is a radial excitation of V. We expect this ratio to be approximately independent of radiative corrections. Because Eq. (4) and (1) give the same value for this ratio, all models should agree for this quantity.

We can also make predictions of the lepton width of heavy-quark states, without being committed to the precise value of the radiative correction. Namely, we will use the equation

$$\Gamma(V \to e^+ e^-) = \Gamma_0 \Big(1 - a\alpha_S(m_Q)\Big)^2 \tag{7}$$

where "a" is determined from $\Gamma(\psi \to e^+ e^-)$. Using (7), predictions for lepton widths were made for a potential derived from a Padé approximation to the β-function[23] (this potential yields a spectrum in agreement with charmonium and upsilonium). Results are tabulated in the Table and in

Figure 3. Observe that "a" is found to be 0.95 in reasonable agreement with the coefficient $\left(\frac{8}{3\pi}\right)$ of Eq. (4). Also the computed value of $\Gamma(T \to e^+e^-)$ is in good agreement with experiment.[24,25] (Here, the role of the radiative correction is important, since $\left(1 - a\,\alpha_S(m_T)\right)^2 / \left(1 - a\,\alpha_S(m_C)\right)^2 = 1.9$). Of special interest is the prediction of $\left(\Gamma(T' \to e^+e^-)/\Gamma(T \to e^+e^-)\right) = 0.41$, which has yet to be measured. This prediction is to be compared with that made by Thacker, Quigg and Rosner[26] who, using Eq. (1) and a very general class of potentials, find (based on $\Gamma(\gamma \to e^+e^-) = 1.3$ KeV) $0.23 \leq \left(\Gamma(T' \to e^+e^-)/\Gamma(T \to e^+e^-)\right) \leq 0.31$. (Note that at PETRA energies, we predict $\Gamma(V' \to e^+e^-)/\Gamma(V \to e^+e^-) \sim 0.3$.)

IV. SUMMARY

By studying the leptonic decay of orthomuonium, we have seen that QED radiative corrections cause an $O(\alpha)$ <u>suppression</u> to the 0th order rate, Γ_0. Furthermore, more detailed examination of that calculation leads us to believe that QCD radiative corrections will have a similar form in the computation of $\Gamma(V \to e^+e^-)$. If so, then Eq. (4) should be used. In charmonium, we find that $\Gamma \sim 0.3\,\Gamma_0$, i.e., a very large suppression is effective. When applied to a specific model previously proposed, based on a Padé approximation to the β-function,[23] the above computed Γ is in close agreement with experimental values of the decay rates of ψ, ψ' and T. Other predictions for heavy-quark states are made and in particular, the prediction for T' is $\Gamma(T' \to e^+e^-)/\Gamma(T \to e^+e^-) \sim 0.4$ which may be larger than the ratio expected in other potential models which correctly give ψ, ψ' and γ rates.[26]

The most important implication of such a significant suppression factor is that the lepton width should <u>not</u> be used as an experimental

parameter with which to determine potential models. The above results indicate that doing so can (and probably does) lead to an underestimation of $|\phi(0)|^2$ by as much as a factor of 3.

ACKNOWLEDGMENTS

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10. In fact, in Ref. 9, it is implied that $\Gamma_L = 0$ although that could admittedly be due to a difference of conventions.

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25. To be compared with the predicted value of $\Gamma(T \rightarrow e^+ e^-) \sim 0.8$ KeV from E. Eichten and K. Gottfried, Phys. Letters 66B, 286 (1977).

26. See the work of H. B. Thacker, C. Quigg and J. Rosner, Ref. 2.

27. This is the region soon to be explored by PETRA.

TABLE I

Predictions of lepton widths for heavy-quark vector mesons using the equation $\Gamma_0\left(1 - a\alpha_s(m_Q)\right)$ where $\Gamma_0 = 16\pi\alpha^2|\phi(0)|^2(e_Q/m_V)^2$, m_Q is the quark mass, m_V is the vector meson mass, and "a" is fit to the ψ-width. The potential used is that of Ref. 23. Masses are in GeV.

m_Q	m_V	$\Gamma(e_Q = 1/3)$ (KeV)	$\Gamma(e_Q = \frac{2}{3})$ (KeV)	Experiment (KeV)
2.0	3.1(ψ)	1.2	4.8	4.8 ± 0.6 [a]
	3.7(ψ')	0.5	2.0	1.9 ± 0.4 [a]
5.4	9.4 (T)	1.46	5.85	1.3 ± 0.4 [b]
	10.0 (T')	0.60	2.41	-
	10.4	0.41	1.64	-
	10.7	0.35	1.39	-
8.0	14.5	1.53	6.11	-
	15.2	0.57	2.30	-
	15.6	0.40	1.59	-
12.0	22.4	1.60	6.39	-
	23.1	0.56	2.24	-
	23.5	0.36	1.43	-
16.0	30.1	1.71	6.83	- [c]
	30.9	0.56	2.25	-
	31.3	0.34	1.40	-
25.0	47.8	1.94	7.76	-
	48.7	0.60	2.40	-

(a) Ref. 16; (b) Ref. 24; (c) Ref. 27

FIGURE CAPTIONS

1. Leptonic decay of orthomuonium including first-order radiative
    corrections.

2. A trigluon correction to lepton width of orthocharmonium.

3. Lepton widths for quarkonia, using the equation $\Gamma_0\left(1 - a\alpha_s(m_Q)\right)$
    where Γ_0, "a" and α_s are given in the text. The potential is that
    of Ref. 23. The three labelled curves refer to the levels of radial
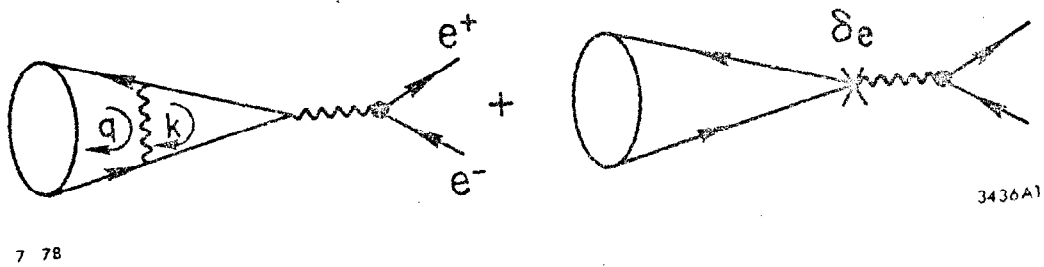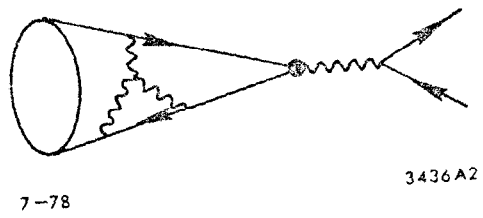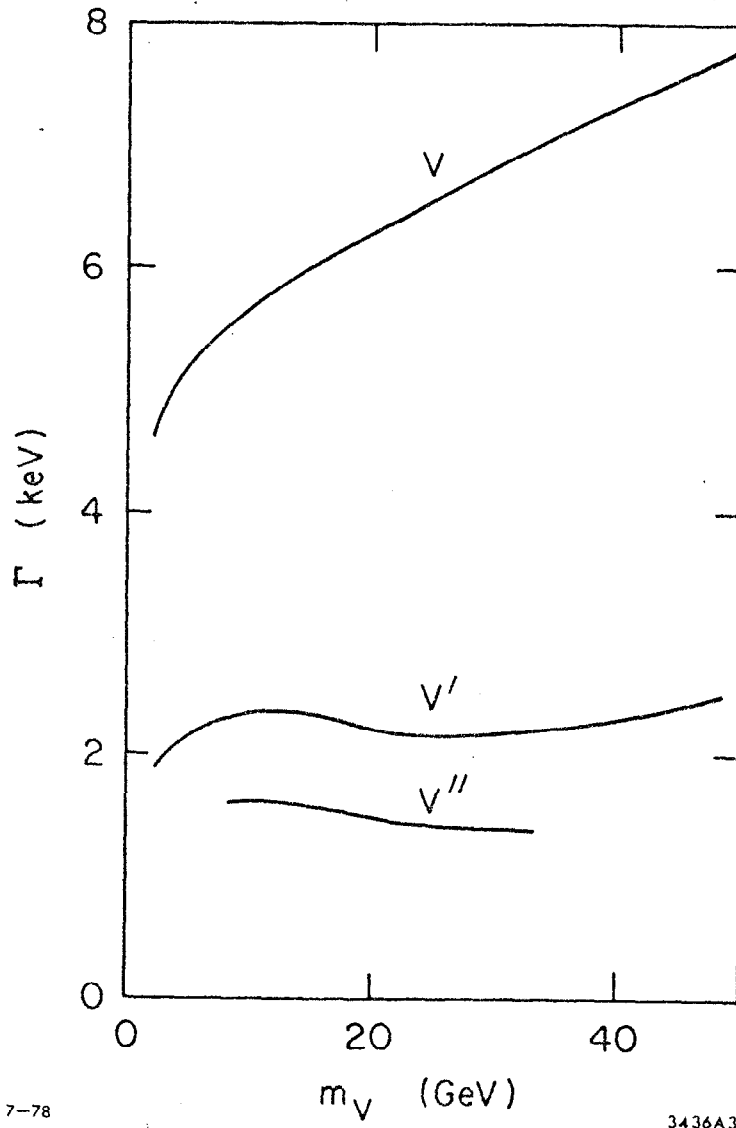   excitation of the state. The quark charge is $e_Q = 2/3$.

e^+

δe

e^-

$+$

q k

7 78

3436A1

Fig. 1

7−78

3436A2

Fig. 2

Fig. 3