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MODEL-INDEPENDENT REMARKS ON ELECTRON-QUARK PARITY-VIOLATING NEUTRAL-CURRENT COUPLINGS*

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ABSTRACT

Present experiments on atomic parity-violation in bismuth and on polarization asymmetry in electron-nucleon deep-inelastic scattering should allow determination of three out of four of the phenomenological parity violating Fermi-couplings of electrons to up and down quarks. The fourth may be obtained via measurement of atomic parity violation in deuterium (but not hydrogen). We also examine the question of corrections to the parton model in the deep-inelastic asymmetry measurements and argue, from a model independent starting point, that they should be small.

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Recent model-independent studies of neutrino-induced semileptonic neutral current processes[1] have largely succeeded in determining the phenomenological couplings and have found them in accord with the Weinberg-Salam $SU(2) \times U(1)$ gauge theory.[2] As data accumulate on weak interactions of the electron-quark system, it will likewise be of interest to analyze that situation from a model-independent point of view. We believe it of particular interest to avoid single Z^o exchange or factorization hypotheses in such studies, inasmuch as even within the gauge theory framework the most probable complication to the simplest model is addition of more neutral gauge bosons. In this note we take a modest step toward such generalized studies.[3] The main assumption underlying this work is that the effective Lagrangian for these processes has a four-fermion V and A structure similar to what is found for charged-current weak processes. Furthermore, we assume that for foreseeable experiments only the electron couplings to the vector and axial currents of up and down quarks are of importance. There are then four phenomenological parity-violating couplings to determine. We argue that the combination of the atomic parity violation measurements in bismuth[4] (or other heavy nuclei) and the deep-inelastic polarized electroproduction data[5] should soon provide determinations of three out of four of the basic couplings. Atomic parity violation measurements in deuterium[6] may be necessary to complete the determination. We also examine the applicability of the parton model to the electroproduction measurements, made at relatively low $Q \sim 1.5 \text{ GeV}^2$. We argue that because of some fortuitous cancellations corrections such as additional Q^2 dependence should be very small.

Section I summarizes what we do. Sections II and III examine kinematic considerations for polarized electroproduction and generalizations of the phenomenology free from assumptions such as the parton model or scaling behavior. Section IV discusses atomic parity-violation experiments performed with heavy atoms and the possible significance of future experiments using hydrogen and deuterium.

I. Summary of Results

Let the effective Lagrangian for the parity-violating electron-quark interaction be written[7]

$$-\mathscr{L} = \frac{G}{\sqrt{2}} \left[\begin{array}{l} \bar{e}\gamma^{\lambda} e \left\{ \epsilon_{VA}(e,u)\,\bar{u}\gamma_{\lambda}\gamma_5 u + \epsilon_{VA}(e,d)\,\bar{d}\gamma_{\lambda}\gamma_5 d \right\} \\ + \bar{e}\gamma^{\lambda}\gamma_5 e \left\{ \epsilon_{AV}(e,u)\,\bar{u}\gamma_{\lambda} u + \epsilon_{AV}(e,d)\,\bar{d}\gamma_{\lambda} d \right\} \end{array} \right] \tag{1.1}$$

We now discuss in turn model-independent determinations of axial and vector electron couplings:

A. Axial electron couplings:

Atomic parity-violation in bismuth measures $\left\{\epsilon_{AV}(e,u) + 1.15\,\epsilon_{AV}(e,d)\right\}$, while the deep inelastic electron-deuteron polarized scattering asymmetry at $y = 0$ measures the almost orthogonal combination $\left\{2\,\epsilon_{AV}(e,u) - \epsilon_{AV}(e,d)\right\}$. Thus, provided there is no unexpectedly strong y-dependence of the deep inelastic asymmetry, the existing combination of measurements already determine rather well both couplings. It also turns out that the deep inelastic asymmetry at $y = 0$ is very insensitive to the parton-model assumptions. Variations on the present measurements in deuterium such as resonance production, measurements at larger x, or deep-inelastic measurements in hydrogen (but still at $y = 0$) determine approximately the same linear combination of $\epsilon_{AV}(e,u)$ and $\epsilon_{AV}(e,d)$ and require rather high accuracy in order to provide new information. However, elastic scattering on deuterium or on hydrogen (provided the kinematics is such that G_E dominates over G_M) does probe a significantly different linear combination of couplings.

B. Vector electron couplings:

In general, the contribution of $\epsilon_{VA}(e,q)$ to the deep-inelastic asymmetry A vanishes at y = 0. The slope of the asymmetry at y = 0, $dA/dy\big|_{y=0}$, measures the vector electron couplings. Again only the combination $\left\{2\,\epsilon_{VA}(e,u) - \epsilon_{VA}(e,d)\right\}$ and approximations thereto are measurable via deep-inelastic asymmetries. In principle the model-dependence of the isovector part of this asymmetry can be controlled by comparison with the properties of the closely related parity-violating structure function W_3 in the charged-current νN and $\bar{\nu} N$ reactions.[8]

Measurements of the orthogonal combination of $\epsilon_{VA}(e,q)$ couplings is possible by observing parity-violating mixings of the $M_F = \pm\frac{1}{2}$, $2S_{\frac{1}{2}}$ and $2P_{\frac{1}{2}}$ levels in deuterium.[6] The hydrogen measurements do not provide information unattainable by the deep-inelastic asymmetry measurements.

In the next sections we elaborate on each of these points. Let us keep in mind that in the standard SU(2) × U(1) model (i.e. left-handed e and q in weak doublets; right-handed e and q weak singlets)

$$\begin{aligned}
\epsilon_{VA}(e,u) &= \tfrac{1}{2}\left(1-4\sin^2\theta_W\right)\\
\epsilon_{VA}(e,d) &= -\tfrac{1}{2}\left(1-4\sin^2\theta_W\right)\\
\epsilon_{AV}(e,u) &= \tfrac{1}{2}\left(1-\tfrac{8}{3}\sin^2\theta_W\right)\\
\epsilon_{AV}(e,d) &= -\tfrac{1}{2}\left(1-\tfrac{4}{3}\sin^2\theta_W\right)
\end{aligned} \qquad (1.2)$$

II. Axial Electron Couplings: Kinematic Considerations

In general the deep-inelastic asymmetry $A = A_{AV} + A_{VA} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$ depends upon the three kinematic variables Q^2, ν and $y = \nu/E$. Direct calculation shows that the contribution from axial electron coupling is

$$\frac{A^{eD}(Q^2,\nu,y)_{AV}}{Q^2} \sim \frac{\ell_{\mu\nu}\int\langle D|\{j^{\mu}(x)J^{\nu}(0)+J^{\mu}(x)j^{\nu}(0)\}|D\rangle e^{iq\cdot x}d^4x}{\ell_{\mu\nu}\int\langle D|j^{\mu}(x)j^{\nu}(0)|D\rangle e^{iq\cdot x}d^4x} \tag{2.1}$$

where $\ell_{\mu\nu} = \mathrm{Tr}\, \not{p}'\gamma_{\mu}\not{p}\gamma_{\nu}$ is the lepton trace, and j_{μ} and J_{ν} are electromagnetic and weak vector current respectively. One can decompose these currents into isovector and isoscalar currents V_{μ} and S_{μ}: $V_{\mu} = (\bar{u}\gamma_{\mu}u - \bar{d}\gamma_{\mu}d)$ and $S_{\mu} = (\bar{u}\gamma_{\mu}u + \bar{d}\gamma_{\mu}d)$. Letting

$$\langle VV\rangle \equiv \ell_{\mu\nu}\int\langle D|V^{\mu}(x)\,V^{\nu}(0)|D\rangle\, e^{iq\cdot x}d^4x\,, \tag{2.2}$$

etc., and noting that in deuterium $\langle VS\rangle = 0$, one gets

$$\frac{A^{eD}_{AV}}{Q^2} \sim \frac{\langle VV\rangle\left[\epsilon_{AV}(e,u)-\epsilon_{AV}(e,d)\right]+\frac{1}{3}\langle SS\rangle\left[\epsilon_{AV}(e,u)+\epsilon_{AV}(e,d)\right]}{\langle VV\rangle+\frac{1}{9}\langle SS\rangle}\,. \tag{2.3}$$

We expect for large Q^2 that $\langle VV\rangle \approx \langle SS\rangle$ because the difference term

$$\langle(V-S)(V+S)\rangle \sim \ell_{\mu\nu}\int\langle D|\bar{u}(x)\gamma^{\mu}u(x)\,\bar{d}(0)\gamma^{\nu}d(0)|D\rangle\, e^{iq\cdot x}d^4x \tag{2.4}$$

is a correlation function between different quark types which strictly vanishes in the parton-model scaling limit.[3] However we may in full generality write

$$\begin{aligned}\langle SS\rangle &= W(1+\delta)\\ \langle VV\rangle &= W(1-\delta)\end{aligned} \tag{2.5}$$

with

$$|\,\delta(Q^2,\nu,y)\,| \le 1\,. \tag{2.6}$$

Under most circumstances we expect $|\delta| \ll 1$.

As we shall see in Section IV, empirically $\epsilon_{AV}(e,u)$ and $\epsilon_{AV}(e,d)$ are very likely to be of opposite sign; the axial electron current couples predominantly to isovector hadron current. Equation (2.3) then implies very little Q^2, ν or y dependence of A^{eD}_{AV}/Q^2 in deuterium. As corollary, it follows that <u>for deuterium A^{eD}/Q^2 at y=0 is, to good approximation independent of Q^2 and ν.</u>

In general, upon expanding to first order in δ, one obtains[9]

$$\left.\frac{A^{eD}}{Q^2}\right|_{y=0} = -\frac{3G}{10\pi\alpha\sqrt{2}}\left\{2\epsilon_{AV}(e,u)\left[1+\frac{3}{10}\delta\right] - \epsilon_{AV}(e,d)\left[1-\frac{6}{5}\delta\right]\right\}. \tag{2.7}$$

If $|\delta| < 0.3$ and $\epsilon_{AV}(e,u) \approx -\epsilon_{AV}(e,d)$, the effect of dropping terms involving δ is less than 6%.

For asymmetries from hydrogen, the situation is complicated only by the presence of the $\langle VS \rangle$ contribution, which can be related directly to the ratio of en and ep scattering cross sections under the same kinematic circumstances. Defining

$$f(Q^2,\nu,y) = \frac{\dfrac{d\sigma_{ep}}{dQ^2 d\nu} - \dfrac{d\sigma_{en}}{dQ^2 d\nu}}{\dfrac{d\sigma_{ep}}{dQ^2 d\nu} + \dfrac{d\sigma_{en}}{dQ^2 d\nu}} \equiv \frac{P-N}{P+N} \tag{2.8}$$

one finds for hydrogen (keeping terms linear in δ)

$$\left.\frac{A^{ep}}{Q^2}\right|_{y=0} = -\frac{3G}{10\pi\alpha\sqrt{2}(1+f)}\left\{2\epsilon_{AV}(e,u)\left[1+\frac{3}{10}\delta+\frac{5}{3}f\right]-\epsilon_{AV}(e,d)\left[1-\frac{6}{5}\delta-\frac{5}{3}f\right]\right\} \tag{2.9}$$

For the present measurements[5] ($\langle x\rangle \sim 0.15$), the neutron-to-proton ratio

N/P is $\gtrsim 0.8$; hence f $\lesssim 0.1$. If $\epsilon_{AV}(e,u) \approx -\epsilon_{AV}(e,d)$, then the difference between A^{ep} and A^{eD} is $\lesssim 10\%$.

There are cases, of course, where the approximation $\langle VV \rangle \approx \langle SS \rangle$ breaks down. For elastic electron-deuteron scattering, evidently $\langle VV \rangle = 0$. Also for the electric (G_E) contribution to elastic electron-nucleon scattering $\langle SS \rangle \cong 9 \langle VV \rangle$. In these cases one measures a predominantly isoscalar combination of $\epsilon_{AV}(e,u)$ and $\epsilon_{AV}(e,d)$. However for the magnetic (G_M) contribution to elastic scattering one has $\langle VV \rangle >> SS$, while of course for $\Delta(1238)$ production $\langle SS \rangle = 0$. In these cases one sees there is no major change in A_{AV}/Q^2 to be expected.

III. Vector Electron Couplings: Kinematic Considerations

The general structure of the asymmetry associated with the vector electron couplings is, schematically

$$\frac{A^{eD}(Q^2,\nu,y)_{VA}}{Q^2} \sim \frac{\ell^5_{\mu\nu} \int \langle D \,|\, j^\mu(x) J^\nu_5(0) \,|\, D \rangle \, e^{iq\cdot x} d^4x}{\ell_{\mu\nu} \int \langle D \,|\, j^\mu(x) j^\nu(0) \,|\, D \rangle \, e^{iq\cdot x} d^4x} \tag{3.1}$$

where J^ν_5 is the weak axial neutral current of the hadrons, and where $\ell^5_{\mu\nu} = \mathrm{Tr}\, \not{p}' \gamma_\mu \not{p} \gamma_\nu \gamma_5$ is the appropriate lepton trace. The hadronic matrix element in the numerator has a unique tensor structure: it is analogous to the V-A interference term W_3 present in neutrino phenomenology. It is thus possible to factor out all exterior kinematics not depending on Q^2 or ν. One finds, therefore (this is the same structure as obtained in the parton-model limit[3])

$$\frac{A^{eD}(Q^2,\nu,y)_{VA}}{Q^2} = \frac{[1-(1-y)^2]}{\left[1-y+\frac{y^2}{2(1+R)}\right]} \, a(Q^2,\nu) \,. \tag{3.2}$$

The y-dependence of the denominator is that of ordinary electroproduction; here $R = \sigma_L/\sigma_T$ is the ratio of cross sections for longitudinal and transverse virtual photons on nucleons. Thus all information on the $\epsilon_{VA}(e,q)$ is determined by measurement of

$$\frac{1}{Q^2}\frac{dA^{eD}}{dy} \approx \frac{1}{Q^2}\frac{dA^{eD}_{VA}}{dy}\bigg|_{y=0} = 2a(Q^2,\nu)\,. \tag{3.3}$$

This is again a general conclusion.

However, the question of the nonscaling behavior of A^{eD}_{VA} is model dependent, and we say nothing more in general. Somewhat general statements can be made, nevertheless, in the context of predictions of the $SU(2)\otimes U(1)$ gauge theory. The first is that in the standard model $\epsilon_{VA}(e,u)$ and $\epsilon_{VA}(e,d)$ are multiplied by an overall factor of $(1-4\sin^2\theta_W)$ and are thus expected to be quite small. The second is that J^5_μ is pure isovector. Thus, once one replaces the denominator $\langle VV\rangle + \frac{1}{9}\langle SS\rangle$ by $\frac{10}{9}\langle VV\rangle\left(1-\frac{4}{5}\delta\right) \approx \frac{10}{9}\langle VV\rangle$, the structure of the asymmetry is the same (via isospin rotation) as the asymmetry in ν-N charged current cross sections:

$$\frac{\left[1-(1-y)^2\right]}{\left[1-y+\frac{y^2}{2(1+R)}\right]}\, a(Q^2,\nu) \propto \frac{\left[\frac{d\sigma^{\nu D}_{cc}}{dQ^2 d\nu} - \frac{d\sigma^{\bar{\nu} D}_{cc}}{dQ^2 d\nu}\right]}{\left[\frac{d\sigma^{\nu D}_{cc}}{dQ^2 d\nu} + \frac{d\sigma^{\bar{\nu} D}_{cc}}{dQ^2 d\nu}\right]}\,. \tag{3.4}$$

Thus any suspected non-scaling behavior of A^{eD}_{VA}/Q^2 can be checked in principle by examination of the corresponding region of q^2 and ν in the neutrino reactions.

For some time we may neglect such niceties, inasmuch as no data yet exists which bears upon determination of $\epsilon_{VA}(e,q)$. The predictions, in the parton model limit, for the asymmetry arising from vector electron couplings is

$$\frac{A_{VA}^{eD}}{Q^2} = -\frac{3G}{10\pi\alpha\sqrt{2}}\left\{2\epsilon_{VA}(e,u) - \epsilon_{VA}(e,d)\right\}\frac{[1-(1-y)^2]}{[1+(1-y)^2]} . \tag{3.5}$$

IV. Atomic Parity-Violation: A Sketch of Phenomenology

The present experiments in bismuth[4] are sensitive to $\epsilon_{AV}(e,q)$; they determine

$$\begin{aligned} Q_W &= 2\left\{(2Z+N)\,\epsilon_{AV}(e,u) + (Z+2N)\,\epsilon_{AV}(e,d)\right\} \\ &= 584\,\epsilon_{AV}(e,u) + 670\,\epsilon_{AV}(e,d) . \end{aligned} \tag{4.1}$$

In the standard model[10]

$$Q_W = -43 - 332\sin^2\theta_W . \tag{4.2}$$

Using results from the Novosibirsk experiment gives[11]

$$Q_W = -140 \pm 40 . \tag{4.3}$$

The latest Seattle result is[11]

$$Q_W = -4 \pm 16 . \tag{4.4}$$

The Oxford result is[11]

$$Q_W = +18 \pm 32 . \tag{4.5}$$

This leads to the results

$$\epsilon_{AV}(e,u) + 1.15\,\epsilon_{AV}(e,d) \cong \begin{cases} -0.24 \pm 0.07 & \text{Novosibirsk} \\ -0.01 \pm 0.03 & \text{Seattle} \\ +0.03 \pm 0.05 & \text{Oxford} . \end{cases} \tag{4.6}$$

These are plotted in Fig. 1, along with the prediction of the standard model. Also plotted is the region which would be allowed by a 20% measurement of the deep-inelastic asymmetry at $y=0$, assuming that it agrees with the standard model. (Such a measurement does not yet exist.) One sees that unless there is severe y-dependence of the asymmetry measured thus far, the phenomenological couplings will be quite well-determined, no matter what the ultimate outcome of the Bi experiments will be.

For the vector electron couplings, the situation is more bleak. We cannot expect as accurate a measurement for the derivative of the electroproduction asymmetry with y as for the value of the asymmetry. Shown in Fig. 2 are the predictions of the Weinberg-Salam model and of the hybrid model (e_R^- in an SU(2) doublet). Also shown are the limits from a hypothetical measurement of the y-dependence with accuracy barely sufficient to rule out the hybrid-model hypothesis. Atomic physics measurements may provide help here, in particular in hydrogen and deuterium. Examination of the tabulation of Cahn and Kane[6] shows that in hydrogen only the linear combinations accessible to electroproduction asymmetry measurements are available. This occurs because the axial coupling of proton is proportional to

$$2F\epsilon_{VA}(e,u) + (F-D)\epsilon_{VA}(e,d) \approx 0.85\,\epsilon_{VA}(e,u) - 0.40\,\epsilon_{VA}(e,d) \,. \quad (4.7)$$

In deuterium, one has the opportunity of measuring the combination $\{\epsilon_{VA}(e,u) + \epsilon_{VA}(e,d)\}$. This is small in the standard model because of the factor $(1-4\sin^2\theta_W)$, and thus may not be an early candidate for a measurement.

We conclude by noting that measurement of the angular asymmetry in $e^+e^- \rightarrow q\bar{q}$ determines the parity-conserving coupling $\epsilon_{AA}(e,q)$ and is not relevant to the questions raised here.

IV. Acknowledgements

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Figure Captions

1. Allowed values of vector quark coupling constants, assuming the measured deep inelastic electron-deuteron polarized-scattering asymmetry represents its value at $y = 0$.

2. Axial vector quark coupling constants, as they might be restricted with future measurement of the y-dependence in electron-deuteron polarization asymmetry.

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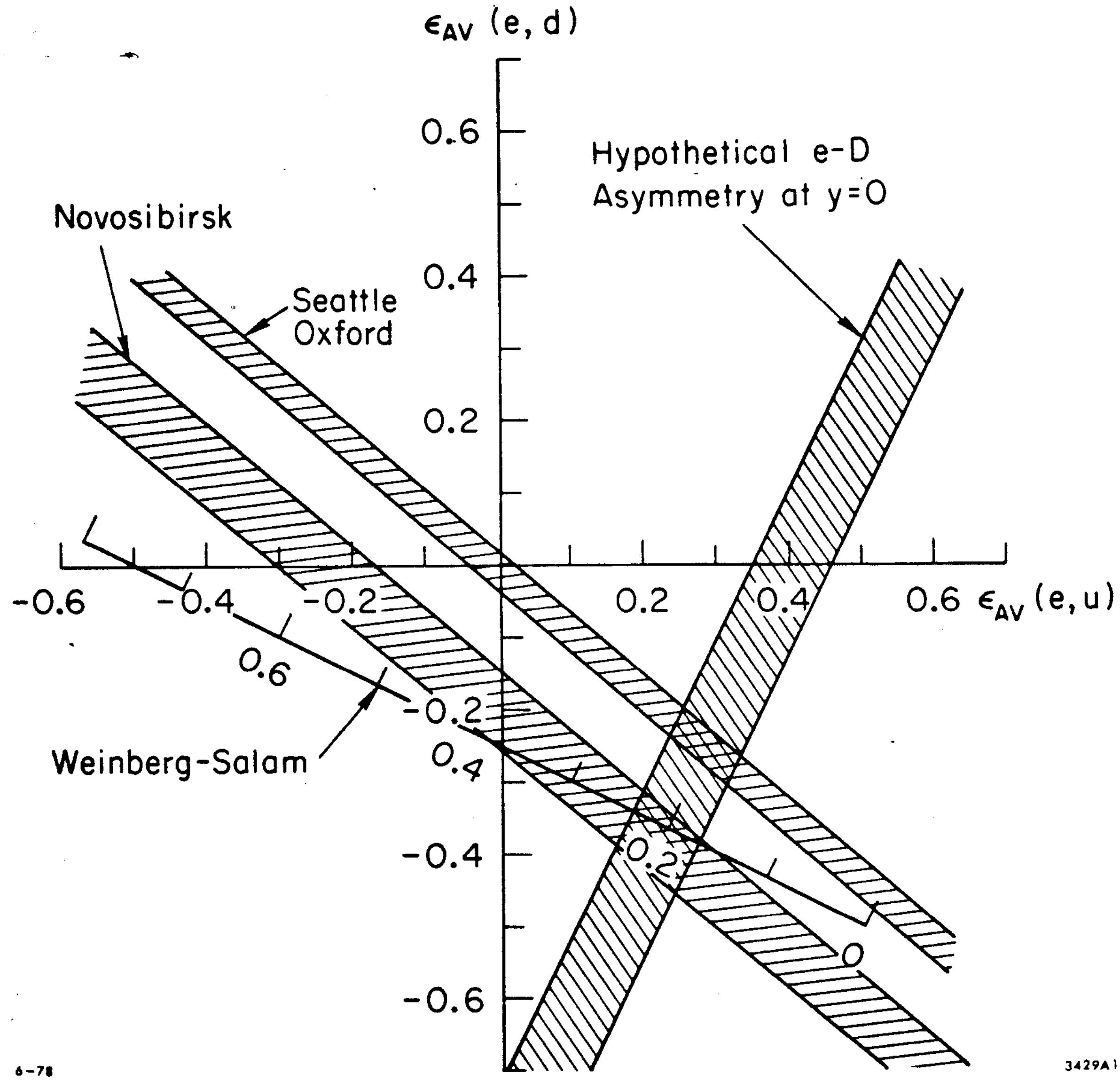
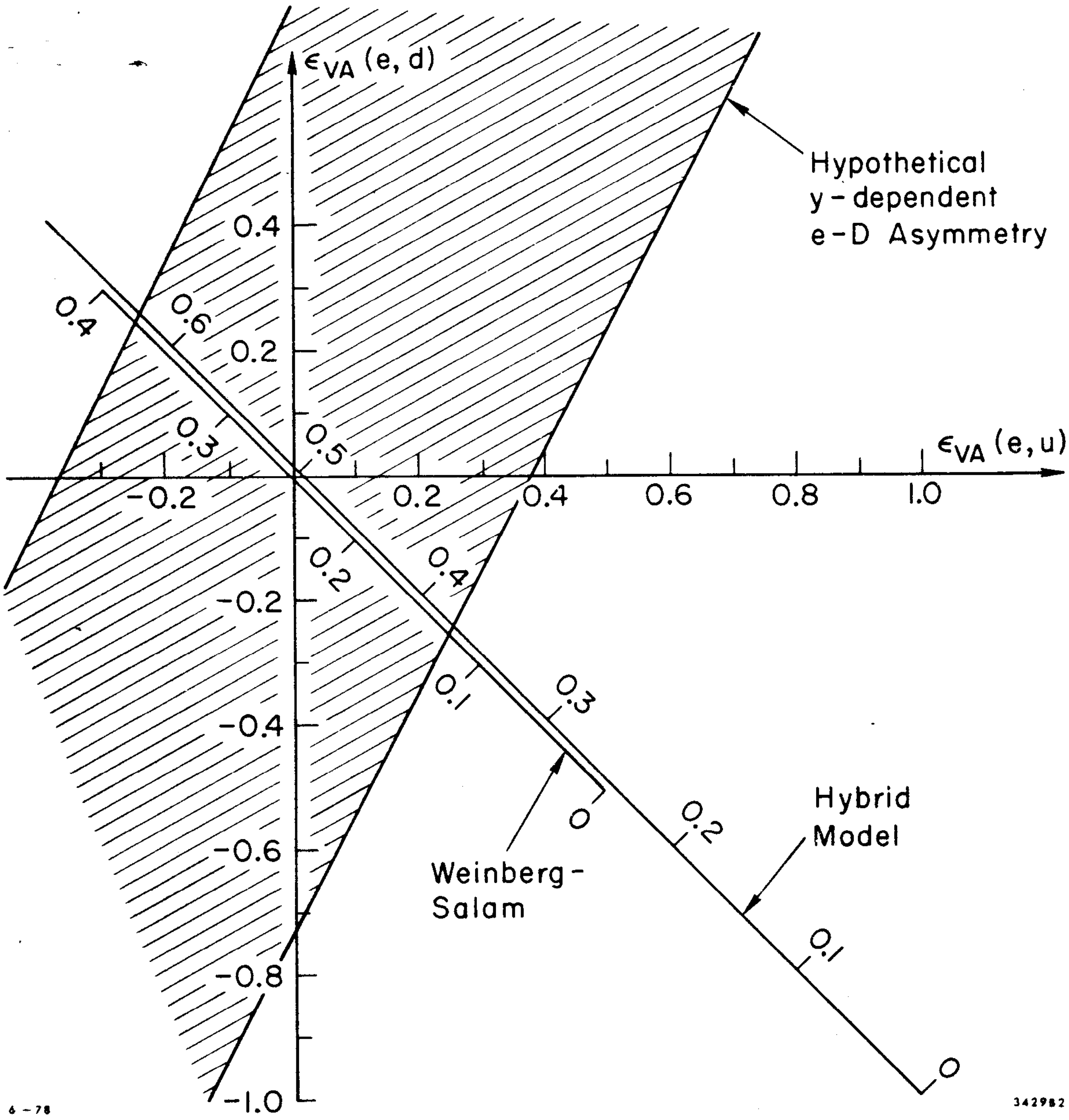


ϵAV (e, d)
0.6
0.4
0.2
-0.2
-0.4
-0.6
Hypothetical e-D
Asymmetry at y=0
Novosibirsk
Seattle
Oxford
-0.6
-0.4
-0.2
0.2
0.4
0.6 ϵAV (e,u)
0.6
0.4
0.2
0
Weinberg-Salam
6-78
3429A1

Fig. 1



ϵ_{VA} (e, d)
ϵ_{VA} (e, u)
Hypothetical
y - dependent
e-D Asymmetry
Hybrid
Model
Weinberg-
Salam
0.4
0.2
-0.2
-0.4
-0.6
-0.8
-1.0
-0.2
0.2
0.4
0.6
0.8
1.0
0.6
0.5
0.4
0.3
0.2
0.1
0
6-78
342982

Fig. 2