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#### Abstract

The asymmetries in the elastic scattering of longitudinally polarized electrons by neutrons, protons, and deuterons are computed in detail within the $S U(2) \times U(1)$ gauge theory of the weak and electromagnetic interactions. The neutron target asymmetries are several times larger, but are of the same sign as those for a proton target. Elastic polarized electron-deuteron scattering gives relatively large asymmetries of the opposite sign. At $q^{2}=1 \mathrm{GeV}^{2}$ in the standard model with $\sin ^{2} \theta_{W}=.25$, the asymmetries at high energy for proton, neutron, and deuteron targets are about $-4 \times 10^{-5},-13 \times 10^{-5}$, and $+9 \times 10^{-5}$, respectively.

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## I. INTRODUCTION

An interference between the weak and electromagnetic interactions can give rise to a difference between the cross sections for scattering of right- and left-handed electrons on either leptonic or hadronic targets. The measurement ${ }^{1}$ of deep inelastic asymmetries to the required level of accuracy increases the interest in the theoretical predictions for both elastic and inelastic electron-nucleon scattering asymmetries.

The consequences of weak-electromagnetic interference for electron scattering experiments have been explored over the last several years since gauge theories in general, and the Weinberg-Salam $\mathrm{SU}(2) \times \mathrm{U}(1)$ model ${ }^{2}$ in particular, have become central to understanding the weak interactions. The asymmetry expected in polarized electron elastic scattering has been calculated in several papers. $3,4,5$ Recently predictions for elastic electron-proton scattering, $\Delta$ electroproduction, and deep inelastic scattering were brought up-to-date in terms of gauge theories of present interest. ${ }^{6}$

However, in the case of elastic scattering Ref. 6 was restricted in that only elastic electron-proton scattering was considered, certain approximations were made which are relevant to high energy ( $\mathrm{E}_{\text {beam }} \approx 20 \mathrm{GeV}$ ) experiments done at $S L A C$, and results were presented only for $\sin ^{2} \theta_{W}=1 / 3$. In the present paper we remove these restrictions. In Section II we calculate both electron-proton and electron-neutron elastic scattering for a range of values of $\sin ^{2} \theta_{W}$. Furthermore, we calculate the terms neglected in Ref. 6 within an $S U(2) \times U(1)$ gauge theory and show their quantitative effect. While at low energy they turn out to be of considerable importance, ${ }^{7}$ we show they truly are negligible for most SLAC
energies when the right-handed electron is a singlet under the gauge group, as in the original Weinberg-Salam model. When the electron is assigned to a right-handed doublet the previously neglected terms give the whole asymmetry, and it is an order of magnitude smaller. Elastic electron-neutron scattering turns out to give an asymmetry many times bigger than that for elastic electron-proton scattering for typical values of the kinematic and $S U(2) \times U(1)$ model parameters.

With some quite general assumptions about the gauge theory transformation properties of the quarks making up the neutron and proton in the deuteron, we rederive in Section III a very simple form ${ }^{5}$ for the asymmetry in elastic electron-deuteron scattering. It is relatively large in magnitude and opposite in sign to that predicted for elastic or inelastic scattering on protons and neutrons separately. Finally in Section IV we present a discussion of our results.

## II. POLARIZED ELECTRON-NUCLEON ELASTIC SCATTERING ASYMMETRIES

As noted above, the asymmetry between the cross sections for elastic scattering of right- and left-handed electrons on nucleons expected because of the interference between exchange of a photon and a weak neutral boson, $Z^{\circ}$, has been calculated previously. ${ }^{3-6}$ Taking the couplings at the nucleon vertex to be the usual Dirac $\left(e f F_{1}^{Y}\left(q^{2}\right)\right)$ and Pauli $\left(e F_{2}^{Y}\left(q^{2}\right) / 2 M_{N}\right)$ ones for the photon, and correspondingly, $F_{1}^{Z}\left(q^{2}\right)$ and $F_{2}^{Z}\left(q^{2}\right)$ as well as $G_{A}^{Z}\left(q^{2}\right)$ (the coefficient of $\gamma_{\mu} \gamma_{5}$ ) for the $Z^{\circ}$, one finds the asymmetry: ${ }^{8,9}$

$$
\begin{gather*}
A_{e N} \rightarrow e N^{\prime}=\frac{d \sigma_{R}-d \sigma_{L}}{d \sigma_{R}+d \sigma_{L}}= \\
=\left(\frac{2 q^{2}}{e^{2 M} M_{Z}^{2}}\right)\left\{-g_{A}\left[\frac{q^{4}}{4 M_{N}^{2}}\left(F_{1}^{\gamma}+F_{2}^{\gamma}\right)\left(F_{1}^{Z}+F_{2}^{Z}\right)+\left(2 E E^{\prime}-\frac{q^{2}}{2}\right)\left(F_{1}^{\gamma} F_{1}^{Z}+\frac{q^{2}}{4 M_{N}^{2}} F_{2}^{\gamma} F_{2}^{Z}\right)\right]\right. \\
 \tag{1}\\
\times\left\{\frac{q^{4}}{4 M_{N}^{2}}\left(F_{1}^{\gamma}+F_{2}^{\gamma}\right)^{2}+\left(2 E E^{\prime}-\frac{q^{2}}{2}\right)\left[\left(F_{1}^{Y}+F_{2}^{\gamma}\right)\left(E^{2}-E^{\prime}\right)^{2}\right)\right\}
\end{gather*}
$$

Here $E$ and $E$, are the initial and final electron energies and $g_{V}$ and $g_{A}$ are the vector and axial-vector couplings of the $Z^{\circ}$ with mass $M_{Z}$ to the (assumed) point-1ike electron.

In Ref. 6 the $\operatorname{limit} q^{2} / 2 \mathrm{M}_{\mathrm{N}} \mathrm{E} \rightarrow 0$, or equivalently $\mathrm{E} \rightarrow \infty$, was taken in Eq. (1). In that limit, argued in Ref. 6 to be a good approximation in the SLAC experimental regime, Eq. (1) simplifies dramatically to ${ }^{9}$

$$
\begin{equation*}
A_{e N \rightarrow e N} \approx-\left(\frac{2 q^{2}}{e^{2} M_{Z}^{2}}\right) \frac{g_{A}\left[F_{1}^{\gamma} F_{1}^{Z}+\frac{q^{2}}{4 M_{N}^{2}} F_{2}^{\gamma} F_{2}^{Z}\right]}{\left(F_{1}^{\gamma}\right)^{2}+\frac{q^{2}}{4 M_{N}^{2}}\left(F_{2}^{\gamma}\right)^{2}} \tag{2}
\end{equation*}
$$

We now wish to examine this high energy approximation in some detail. Specifically, we investigate quantitatively how big the terms proportional to the axial-vector coupling of the nucleon can be in various cases. Of course, when $g_{A}$ of the electron vanishes, these previously neglected terms are the only contributions to the asymmetry in lowest order. To investigate this, and to calculate asymmetries with neutron as well as proton targets, we need the couplings of the $Z^{\circ}$ to the nucleon. For the sake
of completeness, we review here the usual procedure ${ }^{10}$ for obtaining these couplings.

We first recall that $F_{1}^{Z}$ and $F_{2}^{Z}$, or equivalently $G_{E}^{Z}$ and $G_{M}^{Z}$ are dependent on the particular gauge theory. Their magnitude is most directly obtained by considering the coupling of the $\gamma \gamma$ and $Z^{\circ}$ to quarks. In terms of quark fields the electromagnetic current is

$$
\frac{2 e}{3} \overline{\mathrm{u}} \gamma_{\mu} u-\frac{e}{3} \bar{d} \gamma_{\mu} d-\frac{e}{3} \bar{s} \gamma_{\mu} s+\frac{2 e}{3} \bar{c} \gamma_{\mu} c
$$

Neglecting the contribution of strange and charmed quarks to the nucleon's electromagnetic properties, we have ${ }^{11}$

$$
\begin{align*}
& \langle p| \frac{2 e}{3} \bar{u}_{\gamma_{\mu}} u-\frac{e}{3} \bar{d} \gamma_{\mu} d|p\rangle=e G_{p}^{\gamma}  \tag{3a}\\
= & \langle n| \frac{2 e}{3} \bar{d} \gamma_{\mu} d-\frac{e}{3} \bar{u}_{\gamma_{\mu}} u|n\rangle,
\end{align*}
$$

and

$$
\begin{align*}
& \langle n| \frac{2 e}{3} \bar{u} \gamma_{\mu} u-\frac{e}{3} \bar{d} \gamma_{\mu} d|n\rangle=e G_{n}^{\gamma}  \tag{3b}\\
= & \langle p| \frac{2 e}{3} \bar{d} \gamma_{\mu} d-\frac{e}{3} \bar{u} \gamma_{\mu}|p\rangle,
\end{align*}
$$

where an isospin rotation has been used to obtain the last equalities in Eqs. (3a) and (3b). Thus,

$$
\begin{equation*}
\langle p| \bar{u} \gamma_{\mu} u|p\rangle=\langle n| \bar{d}_{\gamma_{\mu}} d|n\rangle=2 G_{p}^{\gamma}+G_{n}^{Y}, \tag{4a}
\end{equation*}
$$

and

$$
\begin{equation*}
\langle\mathrm{p}| \overline{\mathrm{d}} \gamma_{\mu} \mathrm{d}|\mathrm{p}\rangle=\langle\mathrm{n}| \bar{u}_{\gamma_{\mu}} \mathrm{u}|\mathrm{n}\rangle=\mathrm{G}_{\mathrm{p}}^{\gamma}+2 \mathrm{G}_{\mathrm{n}}^{\gamma} . \tag{4b}
\end{equation*}
$$

Now in terms of right- and left-handed weak charges, $Q_{R}$ and $Q_{L}$, of the quarks, which are determined by the gauge theory model, the weak vector current is

$$
\frac{1}{2}\left(Q_{R, u}^{Z}+Q_{L, u}^{Z}\right) \bar{u}_{\gamma_{\mu}} u+\frac{1}{2}\left(Q_{R, d}^{Z}+Q_{L, d}^{Z}\right) \bar{d}_{\gamma_{\mu}}{ }^{Z}
$$

Combining this with Eq. (4) we have,

$$
\begin{align*}
G_{E, p}^{Z} & =\frac{1}{2}\left(Q_{R, u}^{Z}+Q_{L, u}^{Z}\right)\left(2 G_{E, p}^{\gamma}+G_{E, n}^{\gamma}\right)  \tag{5a}\\
& +\frac{1}{2}\left(Q_{R, d}^{Z}+Q_{L, d}^{Z}\right)\left(G_{E, p .}^{\gamma}+2 G_{E, n}^{\gamma}\right) \\
G_{M, p}^{Z} & =\frac{1}{2}\left(Q_{R, u}^{Z}+Q_{L, u}^{Z}\right)\left(2 G_{M, p}^{\gamma}+G_{M, n}^{\gamma}\right)  \tag{5b}\\
& +\frac{1}{2}\left(Q_{R, d}^{Z}+Q_{L, d}^{Z}\right)\left(G_{M, p}^{\gamma}+2 G_{M, n}^{\gamma}\right)
\end{align*}
$$

and similar equations for the vector couplings of the $Z^{\circ}$ to the neutron. The axial-vector couplings of the $Z^{\circ}$ to the nucleon are determined in a similar manner. We first recall that the isovector axial-vector current is measured in weak neutron beta decays:

$$
\begin{align*}
& \langle p| \overline{\mathrm{u}}_{\gamma_{\mu}} \gamma_{5} \mathrm{u}-\overline{\mathrm{d}} \gamma_{\mu} \gamma_{5} \mathrm{~d}|\mathrm{p}\rangle \\
= & -\langle\mathrm{n}| \overline{\mathrm{u}}_{\gamma_{\mu}} \gamma_{5} \mathrm{u}-\overline{\mathrm{d}}_{\gamma_{\mu}} \gamma_{5} \mathrm{~d}|\mathrm{n}\rangle=G_{A}\left(\mathrm{q}^{2}\right), \tag{6}
\end{align*}
$$

where $G_{A}(0)=+1.24$. The isoscalar portion of the axial-vector current is determined by demanding that the ratio of isoscalar and isovector matrix elements be the same as that for total magnetic moments: ${ }^{10}$

$$
\begin{gather*}
\langle p| \bar{u}_{\gamma_{\mu}} \gamma_{5} u+\bar{d} \gamma_{\mu} \gamma_{5} \mathrm{~d}|\mathrm{p}\rangle \\
=\frac{3\left(\mu_{p}+\mu_{n}\right)}{\mu_{i p}-\mu_{n}}\langle\mathrm{p}| \bar{u}_{\gamma_{\mu}} \gamma_{5} u-\overline{\mathrm{d}} \gamma_{\mu} \gamma_{5} \mathrm{~d}|\mathrm{p}\rangle \tag{7}
\end{gather*}
$$

where

$$
\mu_{p}=G_{M, p}^{\gamma}(0)=2.79 \text { and } \mu_{n}=G_{M, n}^{\gamma}(0)=-1.91
$$

This is supported by the F/D ratio in SU(3) being nearly the same for the weak axial-vector current and the total magnetic moment. Such an equal ity is predicted by the quark model, ${ }^{12}$ where $F / D=2 / 3$; and indeed, the factor $3\left(\mu_{p}+\mu_{n}\right) /\left(\mu_{p}-\mu_{n}\right)$ in Eq. (7) could be replaced to high accuracy by its quark model value of $3 / 5$.

Putting together the results for matrix elements of the axial-vector current yields:

$$
\begin{gather*}
\langle p| \bar{u}_{\gamma_{\mu}} \gamma_{5} u|p\rangle=\langle n| \bar{d}_{\mu} \gamma_{5} d|n\rangle \\
=G_{A}\left(q^{2}\right)\left(\frac{2 \mu_{p}+\mu_{n}}{\mu_{p}-\mu_{n}}\right) \tag{8a}
\end{gather*}
$$

and

$$
\begin{gather*}
\langle p| \bar{d} \gamma_{\mu} \gamma_{5} d|p\rangle=\langle n| \bar{u} \gamma_{\mu} \gamma_{5} u|n\rangle \\
=G_{A}\left(q^{2}\right)\left(\frac{\mu_{p}+2 \mu_{n}}{\mu_{p}-\mu_{n}}\right) \tag{8b}
\end{gather*}
$$

The axial-vector couplings of the $Z^{\circ}$ in a particular gauge theory are then related back to the quark charges: Since the axial-vector quark current is

$$
\frac{1}{2}\left(Q_{R, u}^{Z}-Q_{L, u}^{Z}\right) \bar{u} \gamma_{\mu} \gamma_{5} u+\frac{1}{2}\left(Q_{R, d}^{Z}-Q_{L, d}^{Z}\right) \bar{d} \gamma_{\mu} \gamma_{5} d
$$

we have

$$
\begin{align*}
& G_{A, p}^{Z}=\frac{1}{2}\left(Q_{R, u}^{Z}-Q_{L, u}^{Z}\right) G_{A}\left(\frac{2 \mu_{p}+\mu_{n}}{\mu_{p}-\mu_{n}}\right)  \tag{9a}\\
& \quad+\frac{1}{2}\left(Q_{R, d}^{Z}-Q_{L, d}^{Z}\right) G_{A}\left(\frac{\mu_{p}+2 \mu_{n}}{\mu_{p}-\mu_{n}}\right)
\end{align*}
$$

and

$$
\begin{align*}
& G_{A, n}^{Z}=\frac{1}{2}\left(Q_{R, u}^{Z}-Q_{L, u}^{Z}\right) G_{A}\left(\frac{\mu_{p}+2 \mu_{n}}{\mu_{p}-\mu_{n}}\right) \\
& +\frac{1}{2}\left(Q_{R, d}^{Z}-Q_{L, d}^{Z}\right) G_{A}\left(\frac{2 \mu_{p}+\mu_{n}}{\mu_{p}-\mu_{n}}\right) \tag{9b}
\end{align*}
$$

All that remains is to specify the weak quark charges. We concentrate on $\operatorname{SU}(2) \times U(1)$, where

$$
\begin{align*}
& Q_{R, u}^{Z}=\frac{e}{2 \sin \theta_{W} \cos \theta_{W}}\left(2 T_{3 R}^{u}-\frac{4}{3} \sin ^{2} \theta_{W}\right), \\
& Q_{L, u}^{Z}=\frac{e}{2 \sin \theta_{W} \cos \theta_{W}}\left(1-\frac{4}{3} \sin ^{2} \theta_{W}\right)  \tag{10}\\
& Q_{R, d}^{Z}=\frac{e}{2 \sin \theta_{W} \cos \theta_{W}}\left(2 T_{3 R}^{d}+\frac{2}{3} \sin ^{2} \theta_{W}\right), \\
& Q_{L, d}^{Z}=\frac{e}{2 \sin \theta_{W} \cos \theta_{W}}\left(-1+\frac{2}{3} \sin ^{2} \theta_{W}\right),
\end{align*}
$$

for the quarks, and

$$
\begin{align*}
& Q_{R, e}^{Z}=\frac{e}{2 \sin \theta_{W} \cos \theta_{W}}\left(2 T_{3 R}^{e}+4 \sin ^{2} \theta_{W}\right),  \tag{11}\\
& Q_{L, e}^{Z}=\frac{e}{2 \sin \theta_{W} \cos \theta_{W}}\left(-1+4 \sin ^{2} \theta_{W}\right)
\end{align*}
$$

for the electron. Here $\theta_{\mathrm{W}}$ is the Weinberg angle; recent neutrino experiments ${ }^{13}$ determine $\sin ^{2} \theta_{W}$ to be in the neighborhood of $0.25 . \mathrm{T}_{3 \mathrm{R}}^{\mathrm{i}}$ is the value of the third component of weak isospin for a right-handed fermion i, which is zero in the original Weinberg-Salam model. The most popular alternative is to put fermions in right-handed doublets, so that $T_{3 R}^{i}= \pm \frac{1}{2}$.

However, an analysis ${ }^{14}$ of a combination of neutrino induced neutral current processes (as well as results of Ref. 1) are inconsistent with eithex $\mathrm{T}_{3 \mathrm{R}}^{\mathrm{u}}=\frac{1}{2}$ or $\mathrm{T}_{3 \mathrm{R}}^{\mathrm{d}}=-\frac{1}{2}$. Henceforth we take $\mathrm{T}_{3 \mathrm{R}}^{\mathrm{u}}=\mathrm{T}_{3 \mathrm{R}}^{\mathrm{d}}=0$, but leave open the two possibilities, $T_{3 R}^{e}=0$ or $-\frac{1}{2}$.

We now employ Eqs. (5), (9), (10), and (11) to calculate the asymmetry in Eq. (1). For this purpose we assume that all the vector current form factors $G_{E}^{Y}, G_{M}^{Y}, G_{E}^{Z}, G_{M}^{Z}$, have the same dipole $q^{2}$ dependence:

$$
\begin{equation*}
G\left(q^{2}\right)=G(0) /\left[1+\frac{q^{2}}{0.71 \mathrm{GeV}^{2}}\right]^{2} \tag{12}
\end{equation*}
$$

Similarly, we take the axial-vector form factors to be of the form 10,15

$$
\begin{equation*}
\mathrm{G}_{\mathrm{A}}\left(\mathrm{q}^{2}\right)=\mathrm{G}_{\mathrm{A}}(0) /\left[1+\frac{\mathrm{q}^{2}}{0.9 \mathrm{GeV}^{2}}\right]^{2} \tag{13}
\end{equation*}
$$

For the Weinberg-Salam model with $\sin ^{2} \theta_{W}=\frac{1}{3}$ we display in Fig. 1 the asymmetry in electron-proton and electron-neutron elastic scattering when the incident beam energy is $3.23 \mathrm{GeV}, 19.38 \mathrm{GeV}$, and infinity. The first two values of beam energy are of particular relevance to SLAC, where the spin of the electron precesses by an additional $180^{\circ}$ between the linear accelerator and the end station for each 3.23 GeV of beam energy. In the limit of infinite energy Eq. (1) reduces to Eq. (2); the latter being the equation used in Ref. 6 to predict the elastic asymmetry. In fact, the curve in Fig. 1 for the elastic electron-proton scattering asymmetry at $E=\infty$ is precisely the Weinberg-Salam model curve in Fig. 3 of Ref. 6.

We see that the approximation used in Ref. 6 to compute the elastic asymmetry at high energies is very good. Only at the lowest SLAC energies does there appear to be a noticeable, and perhaps measurable, deviation. Since most of the difference between the exact Eq. (1) and
the approximate Eq. (2) comes from dropping the term proportional to $g_{V} G_{A}^{Z}$, we see that the actual value of $g_{V} G_{A}^{Z}$ has little effect on the predicted magnitude of the asymmetry in the original Weinberg-Salam model at high energies. However, at low energies, particularly below 1 GeV , the asymmetry generally does depend quite strongly on the term proportional to $g_{V} G_{A}^{Z}$, as shown in recent calculations. ${ }^{4,7}$ It is in this way that lower energy elastic scattering experiments are of special importance.

On the other hand, if the electron is in a right-handed doublet $\left(T_{3 R}^{e}=-\frac{1}{2}\right)$ then $g_{A}=0$ and the dominant contribution to the high energy elastic asymmetry vanishes. Everything now comes from the term proportional to $g_{V} G_{A}^{Z}$. Using the exact Eq. (1), the resulting elastic electronproton scattering asymmetry is a factor of about three to ten smaller than when $T_{3 R}^{e}=0$. This can be seen by direct comparison of Figs. 2 and 3 in which the elastic scattering asymmetry is computed for $\sin ^{2} \theta_{W}=0.20$, 0.25 , and 0.30 and $T_{3 R}^{e}=0$ and $-\frac{1}{2}$, respectively. 16

Figures 1, 2, and 3 also contain predictions for polarized electronneutron elastic scattering asymmetries for the same range of parameters in $S U(2) \times U(1)$ as above. The most dramatic and important difference between a neutron and a proton target is the magnitude of the predicted asymmetries. The neutron asymmetries are much larger. For $\sin ^{2} \theta_{W}=\frac{1}{3}$ and $T_{3 R}^{e}=0$ there is an order of magnitude difference at $q^{2}=1 \mathrm{GeV}^{2}$. For $\sin ^{2} \theta_{W}=\frac{1}{4}$ it is factor of about three at the same $q^{2}$. Even when $T_{3 R}^{e}=-\frac{1}{2}$ and both the neutron and proton asymmetries are much smaller in magnitude, the neutron asymmetries are still a factor of two or so bigger. This difference originates in $S U(2) \times U(1)$ primarily because the $d$ quark
has a larger vector coupling to the $Z^{\circ}$ than the $u$ quark, and shows up more dramatically in elastic than in inelastic scattering.
III. POLARIZED ELECTRON-DEUTERON ELASTIC SCATTERING ASYMMETRIES

Since elastic electron-neutron scattering will be accomplished by measuring the quasi-elastic scattering on the neutron (and proton) in deuterium, the question comes to mind as to what true elastic electrondeuteron scattering will yield. The asymmetry for polarized electron scattering on isospin zero nuclei has been considered previously. ${ }^{5}$ We review the argument briefly here.

If we neglect the strange and charmed quarks in the nucleon, and hence in the deuteron, the up and down quarks (or antiquarks) in a deuteron together have net third component of weak isospin equal to zero in $\operatorname{SU}(2) \times U(1)$. This holds for the right- and left-handed quarks separately. ${ }^{17}$ The weak charges, $Q_{R}^{Z}$ and $Q_{L} Z_{L}$, of the deuteron then only get contributions in $S U(2) \times U(1)$ from the term proportional to $Q^{\gamma} \sin ^{2} \theta_{W}$. More generally, the local current $J_{\mu}^{Z}$ to which the $Z^{\circ}$ couples, when taken between deuteron states is proportional to that for the photon:

$$
\begin{equation*}
\langle D| J_{\mu}^{Z}|D\rangle=-\frac{e \sin ^{2} \theta_{W}\langle D| J_{\mu}^{\gamma}|D\rangle}{\sin ^{2} \theta_{W} \cos \theta_{W}} \tag{14}
\end{equation*}
$$

As a result there are no axial-vector couplings of the $Z^{\circ}$ to the deuteron and the vector couplings, being proportional to the electromagnetic ones, exactly cancel between the numerator and denominator in the expression for the asymmetry. The final result ${ }^{5}$ for the elastic electron-deuteron asymmetry in $\operatorname{SU}(2) \times U(1)$ is then:

$$
\begin{equation*}
A_{e D \rightarrow e D}=+\left(\frac{\mathrm{Gq}^{2}}{2 \sqrt{2} \pi \alpha}\right)\left(1+2 \mathrm{~T}_{3 R}^{\mathrm{e}}\right)\left(2 \sin ^{2} \theta_{W}\right) \tag{15}
\end{equation*}
$$

Values of this asymmetry when $\mathrm{T}_{3 \mathrm{R}}^{\mathrm{e}}=0$ are shown in Fig. 2, along with those for elastic electron-proton and electron-neutron scattering. The magnitude is relatively large, and very importantly, positive. With $\sin ^{2} \theta_{W}<0.3$ this is the only elastic or inelastic electron scattering asymmetry in $S U(2) \times U(1)$ which is expected to be positive at high energies. Furthermore it gives a very clean measurement of $\sin ^{2} \theta_{W}$. Unfortunately, the rapid fall-off of the deuteron form factor makes it problematic as to whether this will prove to be a practical way of extracting $\sin ^{2} \theta_{W}$ with high accuracy.

## IV. DISCUSSION

We have calculated elastic electron-proton, electron-neutron, and electron-deuteron scattering asymmetries in detail within the context of the $\operatorname{SU}(2) \times U(1)$ gauge theory of weak and electromagnetic interactions. Asymmetries of the same magnitude as in deep inelastic scattering are generally found.

The terms in the elastic electron-proton (or neutron) asymmetry proportional to $G_{A}^{Z}$, the axial-vector coupling of the $Z^{\circ}$ to the nucleon, are found quantitatively to give negligible contributions at beam energies of $\sim 20 \mathrm{GeV}$. Only low energy (below a few GeV ) experiments are sensitive to such terms and can be used to determine their value in the original Weinberg-Salam model.

The asymmetries for elastic electron-neutron scattering for $\sin ^{2} \theta_{W}<0.3$ are of the same sign, but much larger in magnitude, when compared to those for electron-proton elastic scattering. Electron-deuteron scattering, however, gives asymmetries of similar magnitude but opposite sign to other predicted elastic or inelastic asymmetries within $\operatorname{SU}(2) \times U(1)$.

In contrast to deep inelastic polarized electron scattering asymmetry measurements, elastic scattering offers two advantages in testing the underłying gauge theory of weak and electromagnetic interactions. First, one does not need to depend on the applicability of the quark-parton model in general, or knowledge of quark flavor distributions in the nucleon in particular, in order to intepret the results. One only needs to relate mostly measured elastic form factors of the nucleon, often at small values of $q^{2}$, to the quark couplings of the $\gamma$ and $Z^{\circ}$. While some theoretical assumptions are necessary to carry this out, they seem relatively well founded and, importantly, different from those required to interpret deep inelastic scattering.

Second, at SLAC energies elastic scattering is very much like doing a measurement at $y=\left(E-E^{\prime}\right) / E \approx 0$ for deep inelastic scattering. In fact, for a given scattering angle as $y$ decreases one passes from the deep inelastic region, to that of resonance electroproduction, and finally to elastic scattering. We recal1 ${ }^{6}$ that in deep inelastic scattering the asymmetry is proportional to $g_{A}$ of the electron at $y=0$; if $g_{A}$ vanishes so does the asymmetry. Contrasting Figs. 2 and 3 we see a similar effect in elastic scattering; if $g_{A}$ vanishes $\left(T_{3 R}^{e}=-\frac{1}{2}\right.$, as in Fig. 3), then the asymmetry in elastic scattering at SLAC energies drops by roughly an order of magnitude. Polarized electron-nucleon elastic scattering is then an alternative, or at least complementary, method to measuring a y distribution in true deep inelastic scattering in order to determine the singlet or doublet assignment of the right-handed electron in $\operatorname{SU}(2) \times U(1)$.

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9. The $q^{2}$ dependence of the form factors in Eq. (1) and many of the following equations is suppressed for the sake of clarity.
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14. L. Abbott and R. M. Barnett, Phys. Rev. Lett. 40, 1303 (1978); L. Langacker and P. Sidhu, Phys. Lett. 74B, 233 (1978); and P. Q. Hung and J. J. Sakurai, Phys. Lett. 72B, 208 (1977).
15. The mass squared characterizing the dipole form chosen for $G_{A}\left(q^{2}\right)$ has somewhat arbitrarily been chosen as $0.9 \mathrm{GeV}^{2}$, as in Ref. 10 . Other values between the vector form factor dipole mass squared of $0.71 \mathrm{GeV}^{2}$ and $1.1 \mathrm{GeV}^{2}$ make only minor changes in our numerical results.
16. In some models where parity is conserved in lowest order, radiative corrections give rise to sizeable asymmetries. In general, such radiative corrections may give rise to non-negligible effects in the present case where the dominant contribution to the high energy elastic asymmetry vanishes in lowest order. See W. J. Marciano and A. I. Sanda, Phys. Rev. D17, 3055 (1978).
17. This is true only if the right-handed $u$ and $d$ quarks are both in doublets or both in singlets in $\operatorname{SU}(2) \times \mathbb{U}(1)$. We of course assume the left-handed $u$ and $d$ quarks share a doublet.

## FIGURE CAPTIONS

1. The asymmetry in polarized electron-proton and electron-neutron Alastic scattering for electron beam energies of $3.23 \mathrm{GeV}, 19.38 \mathrm{GeV}$, and infinity. All predictions are for the Weinbcrg-Salam model with $\sin ^{2} \theta_{W}=\frac{1}{3}$.
2. The asymmetry in polarized electron-proton, electron-neutron, and electron-deuteron elastic scattering at a beam energy of 19.38 GeV for the Weinberg-Salam model $\left(T_{3 R}^{\mathrm{e}}=0\right)$ and $\sin ^{2} \theta_{W}=0.20,0.25$, and 0.30 .
3. The asymmetry in polarized electron-proton and electron-neutron elastic scattering at a beam energy of 19.38 GeV in $\mathrm{SU}(2) \times \mathrm{U}(1)$ with the right-handed electron in a doublet $\left(T_{3 R}^{e}=-\frac{1}{2}\right)$ and $\sin ^{2} \theta_{W}=0.20$, 0.25 , and 0.30.


Fig. 1


Fig. 2 .


Fig. 3

