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THE RADIATION OF THE ELECTRONS CHANNELED BETWEEN PLANES OF A CRYSTAL*

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1. Introductory Remarks

The theoretical predictions of the new type of radiation [1] of the ultra-relativistic electron by passage through properly oriented crystals arouse a lot of interest. In particular, there is a proposal of an experiment at SLAC to detect the radiation and to measure its characteristics.

The interactions of an electron with the fields of the crystal atoms can be described by an average continuum potential [2]. For the case of the electron moving in the vicinity of a crystal plane the following form was suggested by J. Lindhard:

$$U(y) \;=\; A\left[\sqrt{y^2 + C^2a^2} - |y|\right] \; . \tag{1}$$

Here, $A = -2\pi\, Z\, e^2\, N\, d_p$, N is the number of atoms with atomic number Z per volume unit, d_p is the distance between crystal planes, y is the distance of the particle from the plane, $C = \sqrt{3}$, a is the screening length of electron-atom interaction for the Thomas-Fermi atom model. The negative sign of the constant A makes the potential U(y) attractive for electrons. In the following, we consider the case of electrons, $A < 0$ (for positrons, the sign should be reversed).

The continuum potential (1) is a fairly good approximation only for distances y bigger than the characteristic length Ca even though the energy E is very large.

$$y/Ca \;\geq\; 1 \; . \tag{2}$$

For smaller y the deflection angle in the collision with separate atoms could be bigger than the deflection angle due to collective potential (1) [2]. Strictly speaking, for $y < Ca$, either potential (1) or Thomas-

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Fermi potential for an isolated atom should be used in the motion equation depending on where the trajectory of electrons happened to go. However, due to finite values of U(y) for $y \to 0$ and relatively slow variation of U(y) with y we may assume that the small impact parameters play relatively small role in the overall radiation intensity. We shall apply the continuum potential (1) for all y.

On the other hand, y should not be too large compared to the distance d_p, between planes, since for $\left|\frac{y}{d_p}\right| > 1$ or $x_p > d_p/Ca$ $(x = y/Ca)$ the electron moves through many different planes rather than being channeled in the vicinity of only one plane. For such a motion, instead of the simple form for potential (1) one should consider the periodicity of the crystal planes with y. This condition can be formulated in the following way:

$$\dot{y}/c = \alpha < \alpha_p = \left\{(2\,U(x_p) - \varepsilon_c)/\varepsilon\right\}^{\frac{1}{2}}, \qquad (3)$$

where $\varepsilon_c = 2U(o)$ is the barrier energy for the potential (1).

Figure 1 represents the potential U/A Ca as a function of x and shows also one possible energy level in the well. The calculations of the radiation intensity fulfilled in the paper [1] (both in the classical and quantum approaches) are based on the expansion of the potential (1) into series on y/Ca. The aim of this note is to find the expression of the intensity for the motion in the potential (1). For ultra-relativistic particle the classical treatment is a very good approximation since the corresponding quantum numbers are very large.[3]

2. Intensity of Radiation

We start with the expression of the instant radiation intensity of the ultra-relativistic particle, in the electric field E [4]:

$$I_{in}(t) = \frac{2}{3}\,\frac{e^4\,E^2\,\gamma^2}{m^2c^3} \qquad (4)$$

where $\gamma = \varepsilon/mc^2$. The field E should be taken on the (classical) trajectory of the particle and hence, in general, we need the solution of the motion equation in the y direction:

$$\frac{dP_y}{dt} = eE(y)$$

$$P_y = \frac{\varepsilon}{c^2}\dot{Y}, \quad eE(y) = \partial U(y)/\partial y \tag{5}$$

However, we are interested in the intensity averaged over the period T of the particle oscillations:

$$I = \frac{1}{T}\int_0^T I_{in}\, dt \tag{6}$$

Let us now change integration over time by integration over y. Then we get

$$I = \frac{8e^2\gamma^2 A^2}{3m^2 c^2 T}\int_0^{Y_m}\left[1 - \frac{y}{\sqrt{y^2+c^2a^2}}\right]^2 \frac{dy}{\dot{y}}$$

We shall further neglect the energy change due to transverse electric field (in the same manner as it was done in [1] , ε = const). Then

$$\dot{y} = \sqrt{\frac{2c^2}{\varepsilon}\left[U(y)-U(y_m)\right]} \tag{8}$$

$$T = 4\int_0^{Y_m}\frac{dy}{\sqrt{\frac{2c^2}{\varepsilon}\,|U(y)-U(y_m)|}} \tag{9}$$

The value y_m in the above expressions is the maximum distance of the particle from the plane. Its magnitude depends upon the initial value of y_o and y'_o of the particle (also see Figure 1).

Combining now (7-9) we get:

$$I = \frac{2}{3} \frac{e^2 A^2 \gamma^2}{m^2 c^3} F(x_m), \qquad (10)$$

where $x_m = y_m/\underline{\underline{C}}a$ and

$$F(x_m) = \frac{\int_0^{x_m} \left[1 - \frac{x}{\sqrt{x^2+1}}\right]^2 \left[\sqrt{x^2+1} - \sqrt{x_m^2+1} - x + x_m\right]^{-\frac{1}{2}} dx}{\int_0^{x_m} \left[\sqrt{x^2+1} - \sqrt{x_m^2+1} - x + x_m\right]^{-\frac{1}{2}} dx} . \qquad (11)$$

For $x_m < 1 \qquad F(x_m) \simeq 1 - 4x_m/3$

Fox $x_m > 1 \qquad F(x_m) \simeq 0.3/x_m 3/2$.

As was pointed out above the region $x_m < 1$ has little physical impact and all considerations should start from the value $x_m \simeq 1$.

The period T can also be expressed in the form of:

$$T(x_m) = 4\sqrt{\frac{\varepsilon \underline{\underline{C}}a}{2c^2|A|}} \int_0^{x_m} \left[\sqrt{x^2+1} - \sqrt{x_m^2+1} - \sqrt{x_m^2+1} - x+x_m\right]^{-\frac{1}{2}} dx \qquad (12)$$

The expression (10) depends on y_m in quite a different way than the corresponding expression (1.8) in [1]. The reason for that can be clearly seen from Fig. 1: The shape of the potential well nowhere resembles the parabolic well. Figs. 2a and 2b present functions F(x) and T(x), respectively.

3. Discussion of the Spectral Characteristic of the Radiation.

Since radiation is emitted in a narrow cone of angle $\Delta\theta \sim \gamma^{-1}$ along the instant particle velocity, the frequency range $\hat{\omega}$ in which maximum numbers of quanta are emitted depends, of course, on the ratio of the angle α of the trajectory to $\Delta\theta$. This ratio can be found from (8). Since $\alpha = \dot{y}_{max}/c$, we get

$$\alpha/\Delta\theta \simeq \gamma \left[\frac{2}{\varepsilon} \left(-U(y_m)\right)\right]^{\frac{1}{2}} \tag{13}$$

$$\alpha/\Delta\theta \sim \left[2\gamma \left(-U(y_m)/mc^2\right)\right]^{\frac{1}{2}}. \tag{13'}$$

a.) Let us consider first the case where

$$\alpha/\Delta\theta \gg 1 . \tag{14}$$

Then the radiation in the given direction comes from a very small part of the trajectory parallel to this direction. We may assume that the field on this part of the trajectory is almost constant and apply all the expressions for synchrotron radiation. In particular, the main part of radiation will be emitted on the frequencies:

$$\hat{\omega}_1 \simeq \frac{|\partial U/\partial y|_{max}}{mc} \gamma^2 \tag{15}$$

$\partial u/\partial y$ as a function of y for small values of y is a rather slow function so it is of little importance at which point the derivative is evaluated. If we assume that $(\partial u/\partial y)$ should be taken at $y \simeq Ca$, then $\hat{\omega}_1 \simeq 0.3\ \gamma^2/mc$.

b.) For the opposite limit

$$\alpha/\Delta\theta \ll 1 \tag{16}$$

the radiation is gathered from all the trajectory of particle. In this case, the maximum spectra occurs at the frequencies

$$\hat{\omega}_2 \sim \gamma^2/T(y_m) , \tag{17}$$

which diverge for $y_m \to 0$. (See Fig.2b) The physical reason for that divergence is connected with limited validity of the potential (1) for very small y. It is quite reasonable to assume that the maximum frequency should be taken at $x \simeq 1$:

$$\hat{\omega}_{2max} = \gamma^2/T(1) = \frac{2|A|}{Cam} \frac{\gamma^{3/2}}{\kappa} , \tag{18}$$

where

$$\kappa = \int_0^1 \frac{dx}{\left[\sqrt{x^2+1} - x - \sqrt{2} + 1\right]^{\frac{1}{2}}} = 3.7$$

For smaller values of y/Ca as was discussed above, particle deflections will occur not on the continuum lattice field but rather on a separate atom. In this case, considered radiation will go smoothly over into the bremsstrahlung type of radiation.

3. Discussion of the Experimental Situation. Numerical Example.

Let us consider more closely the meaning of derived expressions from the point of view of the experimental possibliity to detect the channeling radiation. First of all, this type of radiation will occur with the background of bremsstrahlung. The conditions of the experiment should allow for resolution one from the other, for example, by using the difference in their spectra. That means that the value x_m for the particle should not be smaller than 1 or, in other words, the angle α of the trajectory should not be too small.

The situation will be much more clear if we consider it numerically on an example. For such an example we take the diamond crystal as a device for producing the plane channeling radiation.

Let us assume the following numbers [5]. $Z = 6$, $N = 1.1\ 10^{23} cm^{-3}$, $dp = 3.57 \times 10^{-8} cm$, $\underline{\underline{C}}a = \sqrt{3} a_0 \times 0.885 Z^{-1/3} = 0.42 \times 10^{-8} cm$ and, consequently $|A| = 2.13 \times 10^4$ MeV/cm. The barrier energy ε_c equals 1.8×10^{-4} MeV.

Let us look first at the maximum large α consistent with (3). Then x_m will be equal to 8.5 independent from γ. The value $F(x_m)$ from (10) is than 1.2×10^{-2}. Table 1 represents the characteristic quantities for different values of γ for this case. The intensity of radiation can be increased significantly by keeping α smaller. Table 1 also gives values for the case in which α is chosen in such a way that for all γ $x_m = 1$ $(F = 0.24)$.

As we see the wide beam has the disadvantage of producing relatively small amounts of radiation. Besides that the maximum of the photon spectrum is shifted toward very high frequencies. That will make it difficult to resolve the channeling radiation from the bremsstrahlung type of radiation especially for very high values of γ. If we try to reduce α more, for example to make it equal to γ^{-1}, then x_m becomes very small. For $\gamma = 10^{-4}$ $x_m = 0.33$, for $\gamma = 4 \times 10^{-4}$ $x_m = 0.075$. It would be very difficult to get convincing results in such a case.

Acknowledgements

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References

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γ	$\hat{\omega}_1$	$\hat{\omega}_2$	$x_m = 8.5$, $F = 1.2 \times 10^{-2}$			$x_m = 1$, $F = 0.24$		
			α_p	I	$\alpha_p / \Delta\theta$	α_1	I	$\alpha_1/\Delta\theta$
10^2	3.75×10^{18}	3.61×10^{19}	1.81×10^{-3}	6×10^{8}	.181	1.46×10^{-3}	1.2×10^{10}	.146
10^3	3.75×10^{20}	1.14×10^{21}	5.74×10^{-4}	6×10^{10}	.574	4.62×10^{-4}	1.2×10^{12}	.462
10^4	3.75×10^{22}	3.61×10^{22}	1.81×10^{-4}	6.10^{12}	1.81	1.46×10^{-4}	1.2×10^{14}	1.46
4.10^4	6.00×10^{23}	2.89×10^{23}	0.92×10^{-4}	9.6×10^{13}	3.68	0.73×10^{-4}	1.9×10^{15}	2.92
	sec^{-1}	sec^{-1}	rad	MeV/sec		rad	MeV/sec	

TABLE 1

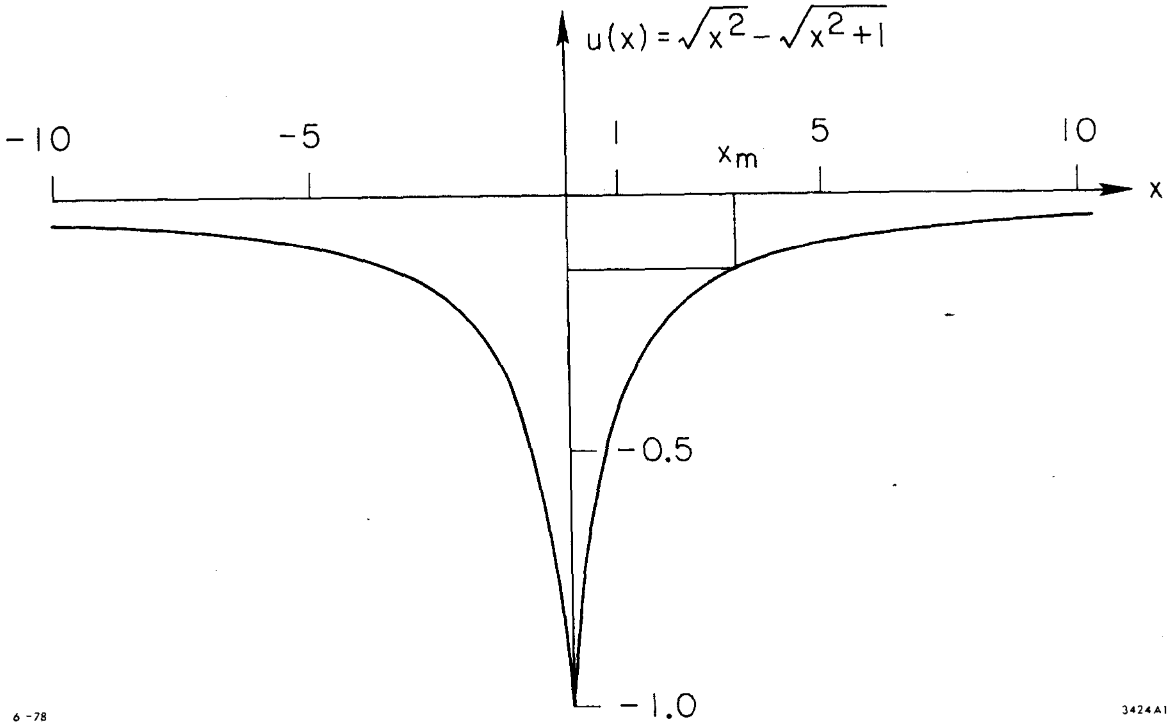

Fig. 1

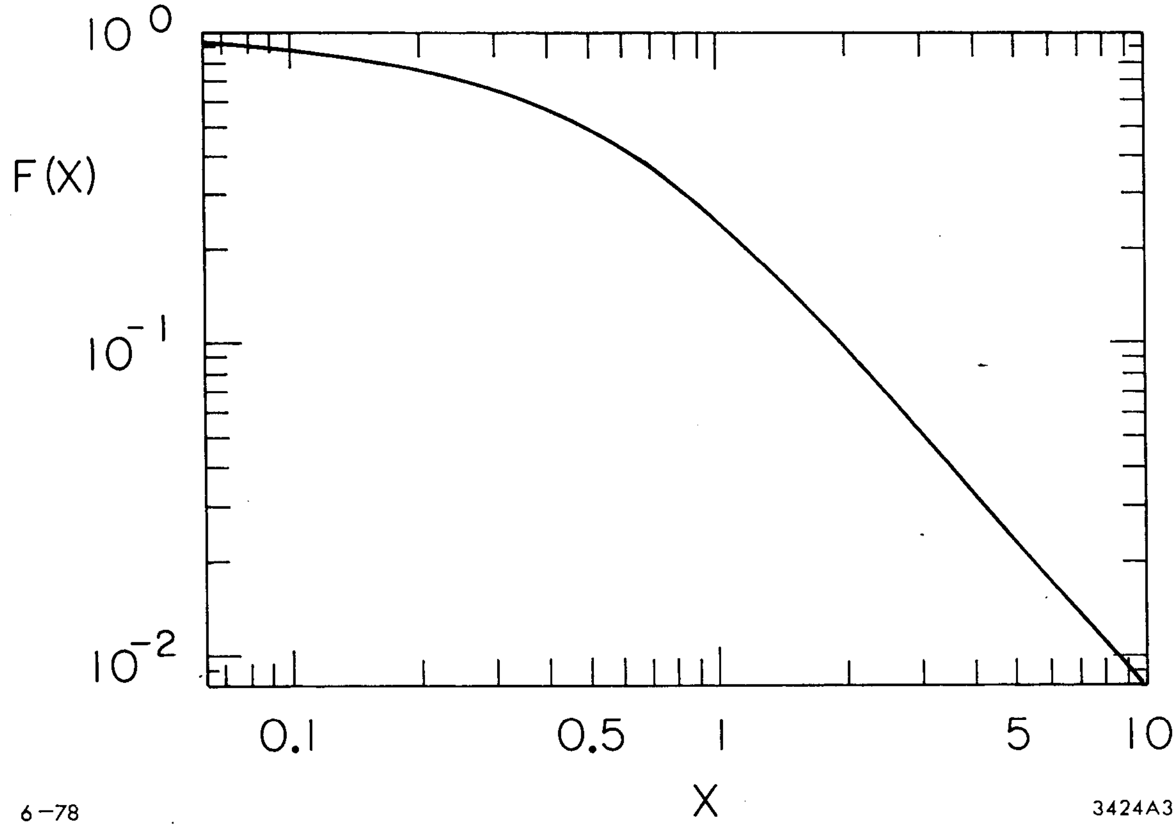

Fig. 2a

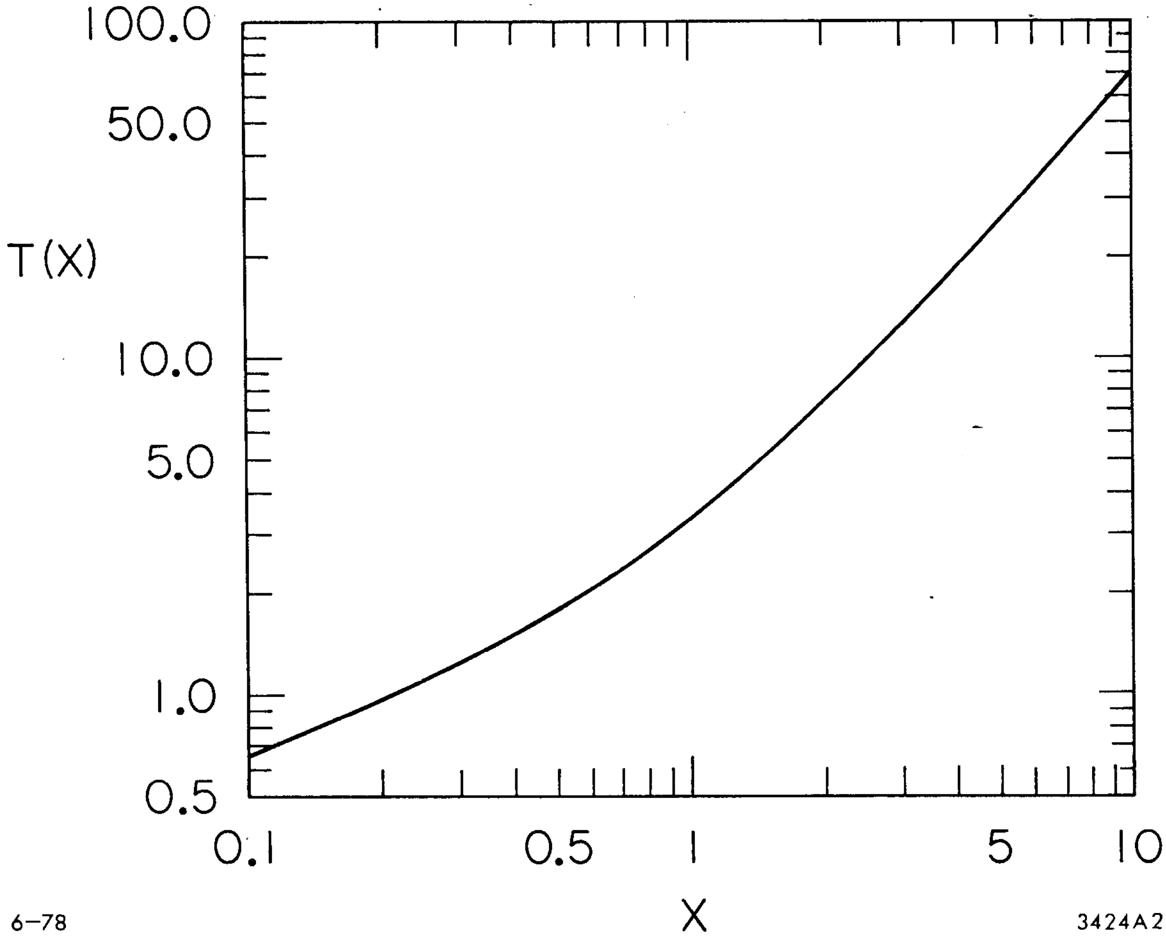

Fig. 2b