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WEAK INTERACTIONS*

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In this talk we shall look at weak interactions from a phenomenological point of view.[1] Our aim is to describe the present situation concisely, using a minimal number of theoretical hypotheses. We first discuss charged-current phenomenology, and then neutral-current phenomenology. This all can be described in terms of a global SU(2) symmetry plus an electromagnetic correction. We then introduce the intermediate-boson hypothesis and infer lower bounds on the range of the weak force. (This inference turns out to be more general than the intermediate-boson hypothesis, but that is not discussed here in detail.)

It happens that this phenomenology does not yet reconstruct all the predictions of the conventional SU(2)×U(1) gauge theory. To do that requires an additional assumption of restoration of SU(2) symmetry at asymptotic energies. Finally we comment on the connection of this work to the usual point of view.

I. Charged Currents

All data on charged-current weak processes can be summarized in terms of an effective Lagrangian

$$\mathscr{L}_{\text{eff}} = \frac{G}{\sqrt{2}} J_\mu^+ J^{\mu -} \tag{1.1}$$

where the charged current $J_\mu^\pm$ is given by

$$J_\mu^\pm = \sum_F \bar{\psi}_F \gamma_\mu (1-\gamma_5) \tau^\pm \psi_F \tag{1.2}$$

and the fermion doublets ψ_F include

$$\psi_F = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}, \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}, \begin{pmatrix} c \\ s\cos\theta_c - d\cos\theta_c \end{pmatrix} \tag{1.3}$$

Not all of these terms are fully established, although there is good evidence in τ-decay, as presented at this meeting, for the left-handed V minus A assignment. Likewise the existence of the $\begin{pmatrix} c \\ s \end{pmatrix}$ left-handed current, of approximately universal strength, follows from the combined evidence from e^+e^- annihilation into $D\bar{D}$, followed by the semileptonic decay of the D, and from ν and $\bar{\nu}$ production of opposite-sign dileptons, with accompanying K-mesons. We do not know the precise normalization of the $\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$ and $\begin{pmatrix} c \\ s \end{pmatrix}$ contributions, nor do we yet know whether the s and d have the proper Cabibbo mixture in the charm-changing charged current. Nevertheless it is reasonable to assume these currents are also of universal strength, and that the degree of freedom s′ coupled to c is orthogonal to the degree of freedom d′ coupled to u. We do make these assumptions here.

II Neutral Currents

Given the above phenomenology for charged currents, two options for neutral current processes naturally present themselves. The first ("YES") option "completes" the current-current structure exhibited in Eq. (1.1) by supposing a global SU(2) symmetry controls the form of the total effective Lagrangian.[2]

That is,

$$\mathscr{L}^{\text{"YES"}} = \frac{G}{4\sqrt{2}} \vec{J}_\mu \cdot \vec{J}^\mu \tag{2.1}$$

with

$$\vec{J}_\mu = \sum_F \bar{\psi}_F \gamma_\mu (1-\gamma_5) \vec{\tau} \psi_F \tag{2.2}$$

There appear no $\Delta S = 1$ neutral currents, because the GIM mechanism[3] applies: only the combination $\bar{s}\Gamma_\mu s + \bar{d}\Gamma_\mu d$ occurs in the neutral current, and this is invariant with respect to s-d mixing.

The second ("NO") option presumes no such intrinsic neutral current exists. In earlier times this option was prevalent because of the empirical absence of $\Delta S = 1$ neutral currents as well as the absence (in those times) of the GIM cancellation mechanism. However, even in this "NO" option, neutral currents will exist if only because of photon exchange.[4] The neutrino should possess a charge radius, i.e. its electromagnetic vertex function should not be identically zero:

$$e\Gamma_\lambda^{em}(q) \cong \bar{\nu}\gamma_\lambda\left(\frac{1-\gamma_5}{2}\right)\nu \cdot \frac{eq^2}{\Lambda^2} \tag{2.3}$$

This leads to a contact interaction between neutrino and charged matter quite analogous to the low-energy neutron-electron interaction. We easily obtain

$$\mathscr{L}_{NC}^{\text{"NO"}} = \frac{e^2}{2\Lambda^2} \bar{\nu}\gamma_\lambda(1-\gamma_5)\nu J_{em}^\lambda + \ldots \tag{2.4}$$

with J_λ^{em} the electromagnetic current-operator (at small momentum-transfer) for all charged matter, and the remaining terms (+...) describing similar contributions not involving neutrinos.

What is the right answer? Neither the "YES" answer (pure left-handed quark couplings) nor the "NO" answer (pure vector quark couplings) agrees with deep-inelastic neutrino-induced neutral-current data. Probably the right answer is the phenomenologically successful Weinberg-Salam effective Lagrangian.[5] For neutrino-induced neutral current processes it is given by

$$\mathscr{L}_{NC}^{W-S} = \frac{G}{2\sqrt{2}} \bar{\nu}_\mu\gamma_\lambda(1-\gamma_5)\nu_\mu \left\{J_3^\lambda - 4\sin^2\theta_W J_{em}^\lambda\right\} \tag{2.5}$$

However, inspection of Eqs. (2.1) and (2.4) shows that this is simply the <u>sum</u> of the "YES" and "NO" Lagrangians

$$\mathscr{L}_{NC}^{W-S} = \mathscr{L}^{\text{"YES"}} + \mathscr{L}^{\text{"NO"}} \tag{2.6}$$

provided one identifies the neutrino charge radius Λ^{-1} with the Weinberg-angle θ_W as follows

$$\left|\frac{1}{\Lambda}\right| = \left(\frac{G}{\pi\alpha\sqrt{2}}\right)^{\frac{1}{2}} \sin\theta_W \cong \frac{\sin\theta_W}{53\text{GeV}} \approx 10^{-2}\ \text{GeV}^{-1} \tag{2.7}$$

This is a rather large electromagnetic radius; from this point of view one might have a priori expected[6] $\sin^2\theta_W \sim O(\alpha) \sim$ a few %, not the observed 20-25%.

III. Intermediate-boson Hypothesis

Let us assume the intrinsic SU(2)-invariant weak interaction "YES" described by Eq. (2.1) is mediated by a triplet of intermediate bosons $W^\pm$, W_3, necessarily degenerate in mass. We define the (universal) Yukawa coupling constant g for the W such that

$$\frac{G}{\sqrt{2}} = \frac{2g^2}{m_W^2} \tag{3.1}$$

Then, just as we might imagine the neutron charge-radius to be dominated (in the dispersion-relation sense) by ρ^o and ω^o exchange, we may suppose the neutrino charge-radius to be dominated by exchange of the intermediate W_3 boson. Defining ef to be the direct coupling of W_3 to photon, the neutrino charge radius is then given by (c.f. Fig. 1d)

$$g \cdot \frac{1}{m_W^2} \cdot \frac{q^2}{m_W^2} \cdot f \cdot \frac{e^2}{q^2} = \frac{ge^2f}{m_W^4} = G\sqrt{2}\,\sin^2\theta_W \tag{3.2}$$

There is one additional effect of importance. The mixing of W_3 and photon produces a charge-renormalization and also splits the mass of the neutral boson from the $W^\pm$. The photon propagator $D(q^2) \approx e^2/q^2$ becomes, after including all proper W_3-insertions (c.f. Fig. 2).

$$D(q^2) = \frac{e_o^2}{q^2\left[1 - \dfrac{e_o^2 f^2}{m_W^2(q^2-m_W^2)}\right]} \tag{3.3}$$

This allows us to express the charge-renormalization as

$$\frac{1}{e_o^2} \equiv \frac{Z_3}{e^2} = \frac{1}{e^2} - \frac{f^2}{m_W^4} \tag{3.4}$$

Likewise, the nontrivial pole in $D(q^2)$ at $q^2 = m_Z^2$ gives the mass m_Z of the physical Z-boson

$$m_Z^2 = m_W^2 + \frac{e_o^2 f^2}{m_W^2} \tag{3.5}$$

Elimination of the coupling constants e_0, g, and f from Eqs. (3.1), (3.2), (3.4) and (3.5) leads to the results

$$m_W = \frac{37\text{ GeV}}{\sin^2\theta_W}\sqrt{1-Z_3} \tag{3.6}$$

$$\frac{m_W^2}{m_Z^2} = Z_3 \tag{3.7}$$

However, Z_3 is not yet determined.

IV. Range of the Weak Force

Equation (3.6) shows that

$$m_W \leq \frac{37 \text{ GeV}}{\sin^2\theta_W} \lesssim 150 \text{ GeV} \tag{4.1}$$

Thus the range of the weak force must be large compared to the unitarity cutoff[7] $G_F^{-\frac{1}{2}}$. The bound is in fact comparable to that expected for the unified gauge theories. This result is much more general[8] than derived in Section III. Even if the single W-exchange is replaced by exchange of a general continuum (which need not even contain discrete quanta), the same result, Eq. (4.1), can be still obtained. One writes dispersion relations for the intrinsic weak amplitude, for the neutrino charge form-factor, and for the contribution of weak quanta to vacuum polarization. The result (Eqns. (3.6) and (4.1)) then follows from application of the Schwartz-inequality to the absorptive parts of the dispersion integrals. The parameter m_W is now a general measure of the range of the weak force, and controls the dependence of G_F on momentum transfer according to the definition

$$G_F(q^2) = G_F\left(1+\frac{q^2}{m_W^2}+\ldots\right) \tag{4.2}$$

We also see from Eq. (3.6) that as m_W increases, Z_3 decreases. The quantity Z_3^{-1} measures the yield of weak quanta produced in e^+e^- annihilation. This implies a connection, again obtained via dispersion-relations and Schwartz inequalities, between the colliding-beam R_{weak}

$$R_{weak} = \frac{\sigma(e^+e^- \to \text{weak quanta})}{\frac{4}{3}\pi\alpha^2 s^{-1}} \tag{4.3}$$

and the parameters m_W^2, $\sin^2\theta_W$, G_F, etc. The resulting inequality is plotted in Fig. 3 and shows that the yield of weak quanta in e^+e^- annihilation is very large. Of course, given the intermediate boson hypothesis, this yield is dominated[9] by the resonant production of Z^0.

We must caution the reader that although m_W is bounded above, implying that the threshold for the process $e^+\nu_e \to$ weak quanta lies no higher than $m_W \lesssim 150$ GeV, we have not succeeded in making such a statement for the threshold in e^+e^- annihilation. Indeed, as Eq. (3.7) shows, we do not have a bound on m_Z, even given the intermediate-boson hypothesis.

V. Asymptotic SU(2) Symmetry

Despite the use of the intermediate-boson hypothesis, not all of the predictions of the gauge theories, in particular those for m_W and m_Z, have been recovered. What is missing is the statement of symmetry at short distances, basic to gauge theories. We may, in the phenomenological language, express this as requirements that the single-intermediate-boson-exchange dominate the weak amplitude at all energies, and that as $q^2 \to \infty$, the SU(2) symmetry of the intrinsic weak force (broken in general by the electromagnetic contribution) is restored.

For any charged elementary fermion of weak-isospin 1/2, the electromagnetic vertex function analogous to Eq. (2.3) is written

$$e\Gamma_\lambda(q) = \bar{u}\gamma_\lambda \left[\left(\frac{1-\gamma_5}{2}\right)\left\{eQ + \frac{4T_{3L}gef}{(m_W^2 - q^2)} \cdot \frac{q^2}{m_W^2}\right\} + \left(\frac{1+\gamma_5}{2}\right)\left\{eQ + \frac{4T_{3R}gef}{(m_W^2 - q^2)} \cdot \frac{q^2}{m_W^2}\right\}\right] u \qquad (5.1)$$

Writing $Q = T_{3L} + Y_L = T_{3R} + Y_R$, the condition of asymptotic symmetry is that the coefficients of T_{3L} and T_{3R} in the electromagnetic vertex operator vanish as $q^2 \to \infty$:

$$1 = \frac{4gf}{m^2_W} \qquad (5.2)$$

When this condition is combined with those already obtained, one finds

$$Z_3 = \cos^2\theta_W \qquad (5.3)$$

and consequently

$$m_W = \frac{37\ \text{GeV}}{\sin\theta_W} \qquad \frac{m_W}{m_Z} = \cos\theta_W \qquad (5.4)$$

The simple gauge-theory results are reconstructed. Assumption of pole-dominance of these weak amplitudes at all energies may be tantamount to assuming the gauge theories _in toto_,[10] although this point is not completely clear to me.

VI. Comments and Conclusions

1. The phenomenological picture of the weak effective Lagrangian as sum of an intrinsic SU(2)-invariant interaction plus electromagnetic correction is compatible with the conventional description using the gauge theories. This is seen especially clearly in a generalization of the standard model to $SU(2) \times U(1) \times G$ as constructed by Georgi and Weinberg.[11] They show that if the spontaneous symmetry breakdown is produced by Higgs bosons which transform as $(2,1) + (1,X)$ and if the neutrino is a singlet under G, the effective Lagrangian for neutrino-induced neutral currents is the same as in the standard model. In fact it can be shown[8] that the structure of this model is the same as the phenomenological picture: the weak amplitude decomposes into the two pieces, "intrinsic" and "electromagnetic", just in the way we have discussed.

2. If there does exist an alternative to the gauge theories, what might it mean? Such a question can be rephrased in terms of the analytic properties of the intrinsic weak interaction as function of squared momentum transfer q^2. In gauge theories this amplitude is dominated by poles. Pole-dominance may in fact imply the gauge theories. Alternatives (which most likely are nonrenormalizable) probably contain strong cuts as well as poles. Such a possibility could correspond to composite degrees of freedom,[12] either for intermediate bosons or for fermions, or both. But we have little of a concrete nature to offer here.

3. The general neutral-current coupling for charged as well as neutral fermions is not quite the same as for the standard model. In momentum-space the generalized effective Lagrangian for neutral-currents is, at low energies:

$$\mathscr{L}^{NC}_{eff} = \frac{G}{4\sqrt{2}}\left\{J^{\lambda}_{3}-4\sin^2\theta_W J^{\lambda}_{em}\right]\left[J^3_{\lambda}-4\sin^2\theta_W J^{em}_{\lambda}\right] + \frac{1}{2}J^{em}_{\lambda}J^{\lambda}_{em}\left\{-\frac{e^2}{q^2}+4\lambda G\sqrt{2}\sin^4\theta_W\right\} \tag{6.1}$$

The first term is the result for the standard model. Only the last term proportional to λ differs: it is parity-conserving and vanishes, given the (single) intermediate-boson hypothesis. Under general circumstances λ is nonvanishing owing to an unknown contribution from vacuum-polarization via weak quanta. However, from Schwartz inequalities it is possible to show that $\lambda \geq 0$.

4. We have not written the most general SU(2)-invariant effective Lagrangian for the intrinsic weak force. There might also be contributions from $I_W = 0$ exchange as well as from $I_W = 1$, especially were the W to be a composite of $I_W = 1/2$ constituents. At present, probably the best test for such a component comes from the deep inelastic neutral-current data. If we write for this case

$$\begin{aligned}\mathscr{L}_{NC} = \frac{G}{2\sqrt{2}}\,\bar{\nu}_\mu\gamma_\lambda(1-\gamma_5)\nu_\mu\Big\{&\left[\bar{u}\gamma^\lambda(1-\gamma_5)u-\bar{d}\gamma^\lambda(1-\gamma_5)d\right]\\ &-4\sin^2\theta_W\left[\frac{2}{3}\,\bar{u}\gamma^\lambda u-\frac{1}{3}\,\bar{d}\gamma^\lambda d\right]\\ &+\xi\left[\bar{u}\gamma^\lambda(1-\gamma_5)u+\bar{d}\gamma^\lambda(1-\gamma_5)d\right]\Big\}.\end{aligned} \tag{6.2}$$

then a crude estimate indicates that $|\xi| \leq 0.2$ is probably still allowed from experiment. It is of interest to test in general for such weak-isoscalar terms.

5. At present, the situation with regard to the atomic parity-violation experiments in Bi is unclear.[14] But even were there to be a vanishing effect, this would not affect the considerations here in a very basic way. For example, reassignment of right-handed e^- from singlet to an SU(2) doublet is sufficient to remove the problem.[15]

6. Central to the phenomenological approach presented here is the global SU(2) symmetry of the intrinsic weak interaction at low energies. From the conventional gauge-theory point of view, this symmetry occurs as a consequence of the assumption of only Higgs-doublets contributing to the intermediate-boson mass, an assumption of not an especially basic character. Perhaps the global SU(2) symmetry at low energies is a property of more fundamental origin. In any event, it would appear that there still is considerable room for alternatives to the renormalizable gauge theories of weak and electromagnetic interactions.

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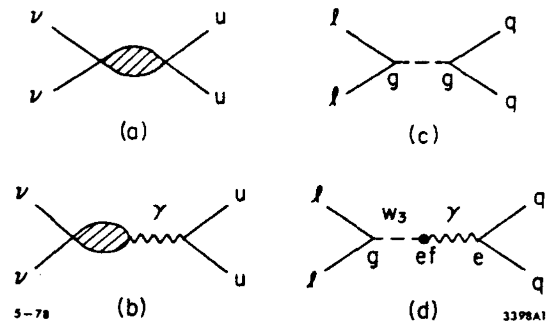

Fig. 1. Contributions to the weak interactions
(a) "YES": intrinsic SU(2) invariant weak amplitude
(b) "NO": electromagnetic contribution
(c) "YES": intermediate boson hypothesis (pole dominance)
(d) "NO": intermediate boson hypothesis (pole dominance).

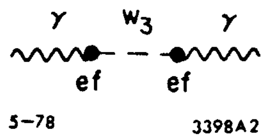

Fig. 2. Electromagnetic mixing of W_3 with photon.

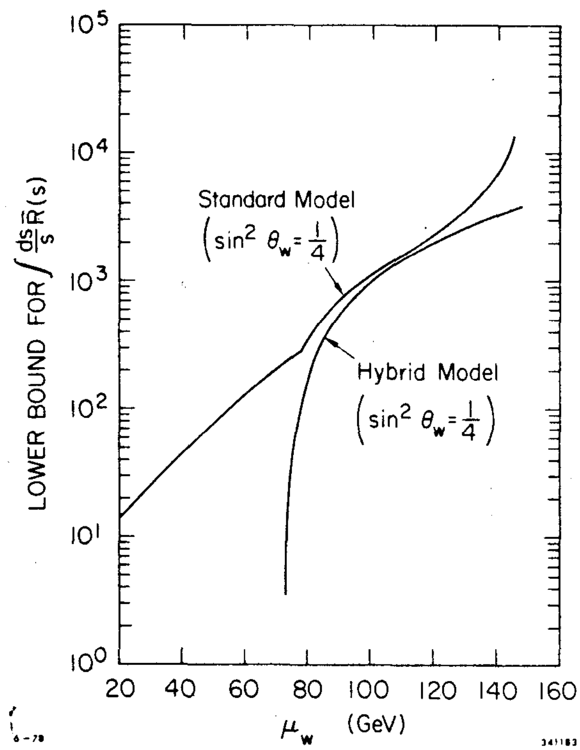

Figure 3: Lower bound for $\bar{R} = \int \frac{ds}{s} R(s)$, which measures the production of weak quanta by colliding e^+e^- beams. [Note: a similar plot given in Reference 1 is incorrect.]