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NEUTRAL-CURRENT RESULTS WITHOUT GAUGE THEORIES*

James D. Bjorken
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

Low energy weak interactions are phenomenologically described in terms of an intrinsic part possessing a global SU(2) symmetry plus an additional electromagnetic correction. This description reconstructs the Weinberg-Salam SU(2)⊗U(1) gauge theory effective Lagrangian. Use of dispersion relations and Schwartz inequalities provides a lower bound on the range of the charged-current weak force, comparable to that obtained from gauge theories. The connection with the usual gauge-theory approach, especially work of Georgi and Weinberg based on the group SU(2)⊗U(1)⊗G, is elucidated.

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I. INTRODUCTION

The past decade has been marked by spectacular progress in the development and application of quantum field theories based on principles of local gauge invariance. Perhaps the most impressive role has been played by the weak-electromagnetic unified theories based upon spontaneous symmetry breakdown. Not only did these developments to considerable extent motivate the successful search for neutral currents and for the charm quantum-number, but they have also led to a quite quantitative description of neutrino-induced neutral current phenomena. It is no wonder that nowadays it is hard to find any theoretical considerations of weak-interaction phenomena that do not presume the correctness of the gauge-theory ideology.

However, it is this very success that should force us to look with an especially critical eye at what aspects are specifically dependent upon the gauge-theory concepts, and what aspects depend upon more general considerations. For example, the existence of $\Delta S=0$ neutral currents of strength comparable to the charged currents, while a historical success of gauge theory developments, is not a strict logical consequence. It is easy in most theoretical schemes of weak interactions to include neutral currents if only for reasons of symmetry.[1] It is the striking absence of $\Delta S=1$ neutral currents which for a long time inhibited theorists from inclusion of neutral-current couplings. But the GIM cancellation mechanism[2] also solves this problem in frameworks more general than renormalizable gauge theories. (Indeed the GIM proposal preceded the development of the present formalism for calculation[3] in the renormalizable models.) On the other hand, the quantitative agreement of the standard $SU(2)\otimes U(1)$ model[4] (or the more recent $SU(2)\otimes SU(2)\otimes U(1)$ variants[5]) with data is more difficult to refute: these provide apparent objective support of weak-electromagnetic gauge theories in general and equally strong support of the specific $SU(2) \otimes U(1)$ (or $SU(2) \otimes SU(2) \otimes U(1)$) scheme. However, it seems that this evidence has not been critically examined to see whether the same <u>quantitative</u> results can be obtained in a credible, but more general framework which does not presume the renormalizable gauge-theories or even the existence of a discrete set of intermediate vector bosons. It is our purpose here to demonstrate that such a framework, albeit more phenomenological, does exist, and that all predictions of the standard model for neutrino-induced neutral currents can be obtained <u>without</u> assuming weak-electromagnetic unification, existence of intermediate bosons, or existence of a spontaneously broken local gauge symmetry.[6] What we do assume is a weak-interaction global SU(2) symmetry, universality, absence of couplings of SU(2)-singlet weak quanta with ν_μ, and a significant amount of electromagnetic symmetry-breaking,in particular, mixing. Nevertheless, as we will elaborate below, some of the qualitative results of the gauge-theories remain. For example, in this generalization the important states which mediate the ν-q weak force (which we call "weak quanta") still have J=1 (but may be a continuum), have an average mass less than 150 GeV, and must be produced copiously in e^+e^- annihilation.

In Section II we set up the general framework and deduce the bounds on the "weak cutoff". Section III is devoted to what can (and cannot) be said about production of weak quanta in colliding-beam processes. In Section IV we establish a connection of this point of view with the conventional gauge theories. Section V contains some final comments and conclusions.

II. THE GENERALIZED MODEL

We assume that in the absence of electromagnetism the S-matrix for weak lepton-lepton scattering at low energy can be obtained by an effective SU(2)-invariant interaction of the form[7]

$$-\mathscr{L}_{eff} = \frac{G}{\sqrt{2}}\left(\sum_F \vec{J}_F^{\ \mu}\right)\cdot\left(\sum_{F'} \vec{J}_{\mu F'}\right) \tag{2.1}$$

where

$$\vec{J}_F^{\ \mu} = \bar{F}\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)\vec{\tau}F \tag{2.2}$$

and the weak doublets F are

$$F = \begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L , \quad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L , \quad \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L , \quad \ldots \tag{2.3}$$

We assume that the weak couplings of hadrons can be described by the same amplitude again taken as an effective Lagrangian.[8] The sum over fermion fields F is extended to the quark doublets

$$\begin{pmatrix}u\\ d\cos\Theta_c + s\sin\Theta_c\end{pmatrix}_L , \quad \begin{pmatrix}c\\ s\cos\Theta_c - d\sin\Theta_c\end{pmatrix}_L , \quad \ldots \tag{2.4}$$

Evidently the GIM mechanism is operative, and the pure weak neutral-current interaction is

$$-\mathscr{L}_{eff} = \sqrt{2}\,G(q^2)\bar{\nu}_\mu\gamma_\lambda\left(\frac{1-\gamma_5}{2}\right)\nu_\mu \begin{bmatrix} \bar{u}\gamma^\lambda\left(\frac{1-\gamma_5}{2}\right)u - \bar{d}\gamma^\lambda\left(\frac{1-\gamma_5}{2}\right)d \\ + \bar{c}\gamma^\lambda\left(\frac{1-\gamma_5}{2}\right)c - \bar{s}\gamma^\lambda\left(\frac{1-\gamma_5}{2}\right)s \\ + \bar{\nu}_e\gamma^\lambda\left(\frac{1-\gamma_5}{2}\right)\nu_e - \bar{e}\gamma^\lambda\left(\frac{1-\gamma_5}{2}\right)e \\ + \ldots \end{bmatrix} \tag{2.5}$$

(Hereafter we shall only consider the portion of the current involving ν_μ, u and d.)

The effective Lagrangian, Eq. (2.5), does <u>not</u> agree with the standard model.[9] However it is not dissimilar; in the standard model the hadronic current contains another contribution proportional to the electromagnetic current

$$-\mathscr{L}_{eff}^{std} = \sqrt{2}\, G\bar{\nu}_\mu \gamma^\lambda \left(\frac{1-\gamma_5}{2}\right)\nu_\mu \left[\begin{array}{l} \bar{u}\gamma_\lambda\left(\frac{1-\gamma_5}{2}\right)u - \bar{d}\gamma_\lambda\left(\frac{1-\gamma_5}{2}\right)d \\ -2\sin^2\Theta_W\left\{\frac{2}{3}\,\bar{u}\gamma_\lambda u - \frac{1}{3}\bar{d}\gamma_\lambda d\right\} \\ + \ldots \end{array}\right] \qquad (2.6)$$

or schematically

$$-\mathscr{L}_{eff} = 4\sqrt{2}\, G\langle T_3\rangle_\nu \langle T_3 - \sin^2\Theta_W Q\rangle_q \qquad (2.7)$$

Furthermore the phenomenological $\langle T_3\rangle\langle T_3\rangle$ term represented in Eq. (2.6), as generated from isospin rotation of the charged current, has the same normalization as in the standard model. All that is needed for equivalence is to <u>add</u> a photon-exchange term, with inclusion of a charge-radius for the neutrino.[10] The neutrino-photon vertex is written (for squared photon momentum $q^2 \to 0$)

$$\sqrt{(2E_\nu)(2E_{\bar{\nu}})}\,\langle\nu_\mu\bar{\nu}_\mu|J_\lambda^{em}(0)|0\rangle = \bar{\nu}_\mu\gamma_\lambda\left(\frac{1-\gamma_5}{2}\right)\nu_\mu\,\frac{eq^2}{\Lambda^2} \qquad (2.8)$$

and the effective interaction is

$$-\mathscr{L}_{eff} = \bar{\nu}_\mu\gamma^\lambda\left(\frac{1-\gamma_5}{2}\right)\nu_\mu \left[\begin{array}{l} G\sqrt{2}\left\{\bar{u}\gamma_\lambda\left(\frac{1-\gamma_5}{2}\right)u - \bar{d}\gamma_\lambda\left(\frac{1-\gamma_5}{2}\right)d\right\} \\ -\frac{e^2}{\Lambda^2}\left(\frac{2}{3}\,\bar{u}\gamma_\lambda u - \frac{1}{3}\,\bar{d}\gamma_\lambda d\right) + \ldots \end{array}\right] \qquad (2.9)$$

Comparison with Eq. (2.6) gives

$$\Lambda^2 = \frac{\pi\alpha\sqrt{2}}{G\,\sin^2\Theta_W} \qquad (2.10)$$

[Notice that the sign of Λ^2 is not determined; hence neither is the sign of $x_W = \sin^2\Theta_W$.]. Putting in numbers gives

$$|\Lambda| = \frac{53\text{ GeV}}{|\sin\Theta_W|} \qquad (2.11)$$

From this estimate it would appear that there must exist structure in the weak force and/or the neutrino at a mass-scale ∿50-100 GeV, comparable to that existing for the renormalizable theories.

This can be seen by examining in more detail the structure of the contributions to the amplitudes for fermion-fermion scattering. These are shown in Fig. 1. We shall ignore the proper contribution in Fig. 1d. However all other photon-mixing contributions will be considered to all orders in e, inasmuch as we shall find that the charge-renormalization can be sizeable, i.e. much larger than $O(\alpha)$. The blobs in Fig. 1 contain intermediate states which in the gauge models are discrete intermediate-boson resonances. While not excluding that possibility, we also include the possibility that the states of weak quanta form (wholly, or in part) a continuum, and that they are a strongly interacting system. The amplitude, illustrated in Fig. 1a, which mediates the weak force may be written (assuming appropriate convergence properties) in terms of its spectral decomposition:

$$\frac{G(q^2)}{\sqrt{2}} = -\frac{h^2}{2}\int_{s_0}^{\infty}\frac{ds\; s\;\rho_W(s)}{q^2-s} \qquad (2.12)$$

An effective (phenomenological) Lagrangian describing the coupling of the weak quanta to the fermions and of all states to the photon has the form

$$-\mathscr{L}_{eff} = h \sum_F \vec{J}_F^{\mu} \cdot \vec{\mathscr{J}}_\mu + e_0 \left[\sum_F J^{\mu F}_{em} + \mathscr{J}^{\mu}_{em}\right] A_\mu \tag{2.13}$$

In this language $\vec{\mathscr{J}}^\mu$ are currents of the weak quanta. The spectral function ρ_W measures the squared matrix element of $\vec{\mathscr{J}}_\mu$ from vacuum to states of mass $\sqrt{s}$ containing weak quanta:

$$-q^2 g_{\mu\nu}\rho_W(q^2) + q_\mu q_\nu \tilde{\rho}_W(q^2) = \sum_n (2\pi)^3 \delta^4(p_n - q) <o|\mathscr{J}^3_\mu(o)|n><n|\mathscr{J}^3_\nu(o)|o> \tag{2.14}$$

The vacuum polarization bubbles in Fig. 1c can be handled in a familiar way:

$$-\mathscr{L}_{eff} = -\frac{e^2_0}{2}\left(\sum_F J^{em}_{\mu F}\right)\left(\sum_{F'} J^{\mu F'}_{em}\right) \cdot \frac{1}{q^2\left[1 - e_0^2 \int_{s_0}^{\infty} \frac{ds\ \rho_{em}(s)}{q^2 - s}\right]} \tag{2.15}$$

with

$$(-q^2 g_{\mu\nu} + q_\mu q_\nu)\rho_{em}(q^2) = \sum_n (2\pi)^3 \delta^4(p_n - q) <o|\mathscr{J}^{em}_\mu|n><n|\mathscr{J}^{em}_\nu|o> \tag{2.16}$$

The "charge-radius" contribution in Fig. 1b is given by

$$-\mathscr{L}_{eff} = e_0^2 h\left(\sum_F J^3_{\mu F}\right)\left(\sum_{F'} J^{\mu F'}_{em}\right) \int_{s_0}^{\infty} \frac{ds\rho(s)}{q^2 - s} \cdot \frac{1}{\left[1 - e_0^2 \int_{s_0}^{\infty} \frac{ds\ \rho_{em}(s)}{q^2 - s}\right]} \tag{2.17}$$

where the mixed, or charge-radius spectral function ρ is

$$(-q^2 g_{\mu\nu} + q_\mu q_\nu)\rho(q^2) = \sum_n (2\pi)^3 \delta^4(p_n - q) <o|\mathscr{J}^3_\mu|n><n|\mathscr{J}^{em}_\nu|o> \tag{2.18}$$

Notice that the amplitudes in Figs. 1b and 1c must <u>vanish</u> at q=0, i.e. the dispersion integrals over ρ must have the form

$$\Pi_{\mu\nu}(q) = (q_\mu q_\nu - g_{\mu\nu} q^2)\Pi(q^2) \tag{2.19}$$

This follows from electromagnetic gauge-invariance. However, the global SU(2) symmetry (as well as experiment) does not forbid the corresponding pure weak amplitude in Fig. 1a from having non-vanishing divergence. In this notation we may now summarize our results for the neutral-current fermion-fermion scattering amplitudes. We obtain, for scattering of one fermion F of charge Q from another of charge Q':

$$\mathcal{M}_{FF'}(q) = j_3^{F\mu} j^3_{F'\mu} h^2 \int \frac{ds\ s\rho_W(s)}{s-q^2}$$

$$+ \left[j^{em}_{F\mu} + hq^2 j^3_{F\mu} \int_{s_0}^{\infty} \frac{ds\rho(s)}{s-q^2}\right] \frac{(-e_0^2)}{q^2\left[1+e_0^2 \int_{s_0}^{\infty} \frac{ds\rho_{em}(s)}{s-q^2}\right]} \left[j^{F'\mu}_{em} + hq^2 \cdot j_3^{F'\mu} \int_{s_0}^{\infty} \frac{ds\rho(s)}{s-q^2}\right] \tag{2.20}$$

with

$$j_3^{F\mu} = \bar{F}\gamma^\mu \left(\frac{1-\gamma_5}{2}\right)\tau_3 F \quad j^{F\mu}_{em} = \bar{F}\gamma^\mu Q F \tag{2.20a}$$

In the low energy limit we find

$$\mathcal{M}^{(q)}_{FF'} = j_3^{F\mu} j^3_{F'\mu} h^2 \int_{s_0}^{\infty} ds\rho_W(s)$$

$$-\left(j_3^{F\mu} j^{em}_{F'\mu} + j^{F\mu}_{em}\ j^3_{F'\mu}\right) he_0^2 \int_{s_0}^{\infty} \frac{ds}{s}\rho(s) \cdot \frac{1}{\left[1+e_0^2 \int_{s_0}^{\infty} \frac{ds}{s}\ \rho_{em}(s)\right]} \tag{2.20b}$$

$$+ j^{F\mu}_{em}\ j^{em}_{F'\mu} \left\{-\frac{e_0^2}{q^2} \cdot \frac{1}{\left[1+e_0^2 \int_{s_0}^{\infty} \frac{ds}{s}\ \rho_{em}(s)\right]} + \frac{e_0^4 \int_{s_0}^{\infty} \frac{ds}{s^2}\ \rho_{em}(s)}{\left[1 + e_0^2 \int_{s_0}^{\infty} \frac{ds}{s}\rho_{em}(s)\right]^2}\right\}$$

from which we can identify the Fermi-constant G:

$$\sqrt{2}G = h^2 \int_{s_0}^{\infty} ds\ \rho_W(s) \tag{2.21}$$

From the electromagnetic term we have the familiar result for charge-renormalization:

$$\frac{1}{e^2} = \frac{1}{e_0^2} + \int_{s_0}^{\infty} \frac{ds}{s}\ \rho_{em}(s) \tag{2.22}$$

or

$$\int_{s_0}^{\infty} \frac{ds}{s}\ \rho_{em}(s) = \frac{1-Z_3}{4\pi\alpha} \tag{2.23}$$

where $Z_3 = e^2/e_0^2$ defines the charge-renormalization contribution coming from virtual weak quanta.

The mixing-term is related to the Weinberg angle (cf. Eq. (2.6))

$$2G\sqrt{2}\ \sin^2\Theta_W = he^2 \int_{s_0}^{\infty} \frac{ds}{s}\ \rho(s) \tag{2.24}$$

Notice that from this point of view $\sin^2\Theta_W$ <u>need</u> not be positive (or bounded), although empirically it is. Thus with these identifications, the low-energy neutral current amplitude takes the form

$$\mathcal{M}_{FF'} = G\sqrt{2}\ (j_3^{\mu F} - 2\sin^2\Theta_W j_{em}^{\mu F})(j^3_{\mu F'} - 2\sin^2\Theta_W j^{em}_{\mu F'})$$

$$+ j_{em}^{\mu F}\ j^{em}_{\mu F'} \left\{ -\frac{e^2}{q^2} + 4\lambda G\sqrt{2}\ \sin^4\Theta_W \right\}$$

with

$$\lambda = \frac{\int \frac{ds}{s^2}\ \rho_{em}(s) \int ds'\rho_W(s')}{\left[\int \frac{ds}{s}\ \rho(s)\right]^2} - 1 \geq 0$$

That λ is positive follows from a Schwartz-inequality relating the matrix-elements of currents appearing in $\rho_W(s)$, $\rho_{em}(s)$ and $\rho(s)$. The definitions in Eqs. (2.14), (2.16), and (2.19) lead directly to the inequality

$$\rho^2(s) \le \rho_W(s)\rho_{em}(s) \tag{2.25}$$

which in turn leads to

$$\left[\int_{s_0}^{\infty} \frac{ds}{s}\, \rho(s)\right]^2 \le \int_{s_0}^{\infty} \frac{ds}{s}\, \rho_W(s) \int_{s_0}^{\infty} \frac{ds'}{s'}\, \rho_{em}(s')$$

and

$$\left[\int_{s_0}^{\infty} \frac{ds}{s}\, \rho(s)\right]^2 \le \int_{s_0}^{\infty} ds\rho_W(s) \int_{s_0}^{\infty} \frac{ds'}{s'^2}\, \rho_{em}(s')\Bigg] \tag{2.26}$$

thus demonstrating the positivity of λ.

Interesting bounds follow from Eqs. (2.26). Defining

$$\mu^2_W = \frac{\int_{s_0}^{\infty} ds\; \rho_W(s)}{\int_{s_0}^{\infty} \frac{ds}{s}\; \rho_W(s)} \tag{2.27}$$

as the mean mass which mediates the charged current weak processes, we then find from Eqs. (2.21), (2.23), and (2.24) that the inequality in Eq. (2.26) becomes

$$\frac{\sqrt{2}\, G \sin^4\Theta_W}{\pi\alpha} \le \frac{(1-Z_3)}{\mu_W^2} \tag{2.28}$$

or

$$\mu_W \le (37.4\ \text{GeV})\sqrt{\frac{1-Z_3}{\sin^2\Theta_W}} \tag{2.29}$$

Notice that in the standard $SU(2) \otimes U(1)$ scheme

$$\mu_W = m_W$$
$$Z_3 = \cos^2\Theta_W \qquad (2.30)$$

and Eq. (2.29) reduces to the familiar formula for the intermediate-boson mass m_W.

Equation (2.29) has important consequences. First of all, for $\sin^2\Theta_W \geq 0.25$ one has, even for $Z_3=0$,

$$\mu_W \leq 150 \text{ GeV} \qquad (2.31)$$

Furthermore, notice that μ_W measures directly the deviation of charged current amplitudes from the value given in the Fermi theory. From Eqs. (2.12), (2.21) and (2.27)

$$G(q^2) = G\left(1 + \frac{q^2}{\mu_W^2} + \ldots\right) \qquad (2.32)$$

Hence μ_W is what is <u>defined</u> in charged-current phenomenlogy as the "intermediate-boson mass". Here we obtain a strict upper limit on its magnitude. Furthermore, this upper bound is only attained at the price of setting $Z_3=0$, an assumption which implies a very large vacuum polarization contribution. This in turn implies a large colliding-beam cross section for production of weak quanta. Indeed, with the experimental limit[11] (from charged-current neutrino reactions) of

$$\mu_W \gtrsim 30 \text{ GeV} \qquad (2.33)$$

we already find

$$1-Z_3 \geq 0.04 \qquad (2.34)$$

which is a significant contribution to charge-renormalization relative to standard vacuum polarization insertions. Recall those are given by

$$\delta Z_3 = \frac{\alpha}{3\pi}\int_0^{s_{max}} \frac{ds}{s}R(s) + \frac{\alpha}{3\pi}\left[\log\frac{s_{max}}{m_e^2} + \log\frac{s_{max}}{m_\mu^2}\right] \tag{2.35}$$

with R the famous ratio measured in colliding-beam reactions. The sum of all contributions thus far measured (up to $\sqrt{s_{max}} \sim 7$ GeV) gives only $\delta Z_3 \sim 0.03$.

Given that the "standard" assignment of right-handed electron as a weak SU(2) singlet is correct, we can easily relate Z_3 to colliding-beam cross sections. For e^+e^- collisions for which e^- is right-handed (and e^+ is unpolarized), the contribution to

$$R_R(s) = \frac{\sigma_R(e^+e^- \to X)}{\left[\frac{4\pi\alpha^2}{3s}\right]} \tag{2.36}$$

from the states X containing weak quanta is

$$R_R(s) = \frac{12\pi^2\rho_{em}(s)}{e^4\left|\frac{1}{e_0^2} + \int \frac{ds'\rho_{em}(s')}{s'-s+i\varepsilon}\right|^2} \tag{2.37}$$

Now consider the function

$$d_R(s) = \frac{1}{\frac{1}{e_0^2} + \int_{s_0}^{\infty} \frac{ds'\rho_{em}(s')}{s'-s}} \tag{2.38}$$

It is essentially a J=1 helicity amplitude for electron-positron scattering. It is analytic in the cut s plane[12] and satisfies a once-subtracted dispersion relation

$$d_R(s) = d_R(0) - \frac{s}{\pi}\int_{s_0}^{\infty} \frac{ds' \; \mathrm{Im}\, d_R(s')}{s'(s'-s)} \qquad (2.39)$$

$$= e^2 - s\int_{s_0}^{\infty} \frac{ds' \; \rho_{em}(s')\,|d_R(s')|^2}{s'(s'-s)}$$

As $s \to \infty$, $d_R(s) \to e_0^2$ and

$$e_0^2 - e^2 = \int_{s_0}^{\infty} \frac{ds'}{s'}\, \rho_{em}(s') \frac{1}{\left|\frac{1}{e_0^2} + \int_{s_0}^{\infty} \frac{ds''\rho_{em}(s'')}{s''-s'}\right|^2} \qquad (2.40)$$

$$= \frac{e^4}{12\pi^2}\int_{s_0}^{\infty} \frac{ds'}{s'}\, R_R(s')$$

In other words, we retain the familiar relation

$$Z_3^{-1} - 1 = \frac{\alpha}{3\pi}\int \frac{ds'}{s'}\, R_R(s') \qquad (2.41)$$

because we have assumed the right-handed electron does not have any intrinsic weak couplings. Using the inequality in Eq. (2.29), we obtain our main result

$$\bar{R}_R \equiv \int_{s_0}^{\infty} \frac{ds}{s}\, R_R(s) \geq \frac{3\pi}{\alpha}\left\{\left[1 - \frac{\mu_W^2 \sin^4\Theta_W}{(37.4\ \mathrm{GeV})^2}\right]^{-1} - 1\right\} \qquad (2.42)$$

Again the bound becomes an equality if one includes only the contribution of a "standard" Z^o resonance in $R_R(s)$. Using the Breit-Wigner formula, one finds the result for Z_0 in the standard model to be[13]

$$\int \frac{ds}{s}\, R_R(s)\Big|_{Z\ \mathrm{resonance}} = \frac{18\pi}{\alpha^2}\, \frac{\Gamma(Z\to e_R^- e^+)}{m} = \frac{3\pi}{\alpha}\tan^2\Theta_W \qquad (2.43)$$

If one inserts $\mu_W^2 = m_W^2$ into Eq. (2.42) one gets precisely the same result.

Thus we see that the main qualitative conclusions of gauge theories follow: the low energy form of the charged-current weak interactions must break down in the energy region $\sqrt{Q^2} < 150$ GeV; furthermore this breakdown must lead to large e^+e^- cross sections characterized by

$$\bar{R} = \tfrac{1}{2}(\bar{R}_L + \bar{R}_R) >> 10 \tag{2.44}$$

In the gauge theories this large e^+e^- cross section is concentrated in a discrete Z_0 resonance (or set of resonances). However, in the generalizations we consider, this need not occur. But the above result does depend on the assignment of e_R^- as a weak singlet. To remove this dependence, we now consider $R_L(s)$. Here the situation is complicated by the presence of the intrinsic weak interaction terms. However, we may still use similar techniques by defining $d_L(s)$ in a way parallel to that used for $d_R(s)$. The quantities $d_L(s)$ and $d_R(s)$ are essentially the appropriate helicity amplitudes $e^{i\delta}\sin\delta$ for e^-e^+ scattering in the J=1 state. We define, utilizing Eq. (2.20) and normalizing in accordance with Eq. (2.38),

$$\frac{d_L(s)}{s} = -h^2\int_{s_0}^{\infty}\frac{ds'\,s'\rho_W(s')}{s'-s} + \frac{e_0^2\left[1+hs\displaystyle\int_{s_0}^{\infty}\frac{ds'\rho(s')}{s'-s}\right]^2}{s\left[1+e_0^2\displaystyle\int_{s_0}^{\infty}\frac{ds'\rho_{em}(s')}{s'-s}\right]} \tag{2.45}$$

As before, $d_L(s)$ satisfies a once-subtracted dispersion relation:

$$d_L(s) = d_L(o) - \frac{s}{\pi}\int_{s_0}^{\infty}\frac{ds'\,\mathrm{Im}\,d_L(s')}{s'(s'-s)} \tag{2.46}$$

Because Im $d_L(s)$, by unitarity, is related only to $\sigma_L(e^+e^- \rightarrow$ weak quanta) we must arrive at the same result, Eq. (2.40), as for $d_R(s)$:

$$d_L(s) = d_L(o) - \frac{se^4}{12\pi^2} \int_{s_0}^{\infty} \frac{ds' \; R_L(s')}{s'(s'-s)} \tag{2.47}$$

Letting $s \rightarrow \infty$ we again have

$$d_L(\infty) = e^2 \left[1 + \frac{\alpha}{3\pi} \int_{s_0}^{\infty} \frac{ds'}{s'} R_L(s') \right] \tag{2.48}$$

We can read off $d_L(\infty)$ from Eq. (2.45):

$$d_L(\infty) = h^2 \int_{s_0}^{\infty} ds' s' \rho_W(s') + e_0^2 \left[1 - h \int_{s_0}^{\infty} ds' \rho(s') \right]^2 \tag{2.49}$$

We now define the pure number

$$\Lambda = h \int ds' \; \rho(s') \tag{2.50}$$

which measures the amount and nature of proper-vertex-function renormalization. From Eq. (2.25) one finds the Schwartz-inequality

$$h^2 \int ds' s' \rho_W(s') \geq \frac{\Lambda^2}{\int \frac{ds'}{s'} \rho_{em}(s')} = \frac{e^2 \Lambda^2}{1-Z_3} \tag{2.51}$$

and thus, from Eqs. (2.47), (2.49), and (2.51),

$$\frac{d_L(\infty) - d_L(o)}{e^2} = \frac{\alpha}{3\pi} \int \frac{ds}{s} R_L(s) \equiv \bar{R}_L \geq \frac{\Lambda^2}{1-Z_3} + \frac{(1-\Lambda)^2}{Z_3} - 1 = \frac{(\Lambda - 1 + Z_3)^2}{Z_3(1-Z_3)}$$

(2.52

Without some control over Λ and/or Z_3, this gives no useful bound. We may do somewhat better by first defining the dispersion Δs in the spectrum of charged weak quanta as follows:

$$(\Delta s)^2 \equiv \frac{\displaystyle\int_{s_0}^{\infty} \frac{ds}{s}(s-\mu_W^2)^2 \rho_W(s)}{\displaystyle\int_{s_0}^{\infty} \frac{ds}{s}\, \rho_W(s)} \qquad (2.53)$$

Here μ_W^2 is defined by Eq. (2.27). It follows that

$$h^2\int ds's'\rho_W(s') = h^2\mu_W^2\left[1 + \frac{(\Delta s)^2}{\mu_W^4}\right]\int ds'\rho_W(s')$$
$$= G\mu_W^2\,\sqrt{2}\left[1 + \frac{(\Delta s)^2}{\mu_W^4}\right] \qquad (2.54)$$

We now may return to Eq. (2.49) and use Eq. (2.54) instead of the inequality, Eq. (2.51). Then Eq. (2.52) is replaced by

$$\frac{\alpha}{3\pi}\,\bar{R}_L \geq \frac{G\mu_W^2\sqrt{2}}{e^2}\left[1 + \frac{(\Delta s)^2}{\mu_W^4}\right] + \frac{(1-\Lambda)^2}{Z_3} - 1 \geq \left(\frac{\mu_W}{75\ \text{GeV}}\right)^2 - 1 \qquad (2.55)$$

Putting this together with Eq. (2.42) for $\bar{R}_R$ gives, for the standard model,

$$\bar{R} \equiv \int \frac{ds}{s}\, R(s) \geq \frac{3\pi}{2\alpha}\left(\left[\left\{1 - \frac{\mu_W^2 \sin^4\Theta_W}{(37\ \text{GeV})^2}\right\}^{-1} - 1\right] + \left[\left(\frac{\mu_W}{75\ \text{GeV}}\right)^2 - 1\right]\Theta(\mu_W - 75\ \text{GeV})\right) \qquad (2.56)$$

For the alternative ("hybrid") assignment of e_R^- to an SU(2) doublet, we obtain

$$\bar{R} \geq \frac{3\pi}{\alpha}\left[\left(\frac{\mu_W}{75\ \text{GeV}}\right)^2 - 1\right]\Theta(\mu_W - 75\ \text{GeV}) \qquad (2.57)$$

These are plotted in Fig. 2. For the hybrid model, we in general lose a useful bound if μ_W is too light. However, we have only considered general Λ and Z_3. There may in fact be some reason to prefer specific values for Λ, in particular $\Lambda=0$ or $\Lambda=1/2$. Recall that Λ determines the structure of the proper vertex function at asymptotically large momentum transfer. As we shall elaborate upon in Section IV, in the case of SU(2)⊗U(1) gauge-theories the photon at short distances no longer couples to the charge of the fermion but rather to the hypercharge. This is implied by the fact that at short distances the original unbroken SU(2) symmetry is restored. Inspection of Eq. (2.20) shows that in order to have the photon-exchange contribution be SU(2) invariant as $q^2 \to \infty$, we must have the proper vertex be SU(2) invariant. But in that limit, for a left-handed fermion,

$$\Gamma_\mu \xrightarrow[q^2 \to \infty]{} \bar{F}\gamma_\mu\left(\frac{1-\gamma_5}{2}\right) \left[Q - \Lambda\ \tau_3\right] F \quad . \tag{2.58}$$

With

$$Q = \frac{\tau_3}{2} + Y \tag{2.59}$$

we see that asymptotic SU(2) symmetry implies $\Lambda=1/2$. This is a requirement one might impose even in a context more general than that of the gauge theories.

On the other hand, it is folklore in conventional electrodynamics that if the bare charges of sources are equal so also are the physical charges. If the weak interaction is "softer" than electromagnetic and remains non-unified, we might expect that as $q^2 \to \infty$ the proper vertex function be proportional to the charge of the source; this is clearly effected if $\Lambda=0$.

For these cases, the bounds are stronger and easily obtainable from Eq. (2.55). For the standard assignment of e_R^- as an SU(2) singlet

$$\frac{\alpha}{3\pi}\,\bar{R}_L \geq \left(\frac{\mu_W}{75\text{ GeV}}\right)^2 + \left[1 - \frac{\mu_W^2\sin^4\Theta_W}{(37\text{ GeV})^2}\right]^{-1} - 1 \qquad (\Lambda=0)$$

$$\frac{\alpha}{3\pi}\,\bar{R}_L \geq \left(\frac{\mu_W}{75\text{ GeV}}\right)^2 + \frac{1}{4}\left[1 - \frac{\mu_W^2\sin^4\Theta_W}{(37\text{ GeV})^2}\right]^{-1} - 1 \qquad (\Lambda=\tfrac{1}{2}) \tag{2.60}$$

This leads to stronger bounds on $\bar{R}$ than in general, as follows:

$$\bar{R} = \int \frac{ds}{s}\,R(s) \geq \frac{3\pi}{\alpha}\left\{\frac{1}{2}\left(\frac{\mu_W}{75\text{ GeV}}\right)^2 + \left[1 - \frac{\mu_W^2\sin^4\Theta_W}{(37\text{ GeV})^2}\right]^{-1} - 1\right\} \tag{2.61}$$

(standard model, $\Lambda=0$)

$$\bar{R} \geq \frac{3\pi}{\alpha}\left\{\left(\frac{\mu_W}{75\text{ GeV}}\right)^2 + \left[1 - \frac{\mu_W^2\sin^4\Theta_W}{(37\text{ GeV})^2}\right]^{-1} - 1\right\} \tag{2.62}$$

(hybrid model, $\Lambda=0$)

$$\bar{R} \geq \begin{cases} \dfrac{3\pi}{\alpha}\left\{\dfrac{1}{2}\left(\dfrac{\mu_W}{75\text{ GeV}}\right)^2 + \dfrac{5}{8}\left[1 - \dfrac{\mu_W^2\sin^4\Theta_W}{(37\text{ GeV})^2}\right]^{-1} - 1\right\} & \mu_W > 75\text{ GeV} \\[2ex] \dfrac{3\pi}{2\alpha}\left\{\left[1 - \dfrac{\mu_W^2\sin^4\Theta_W}{(37\text{ GeV})^2}\right]^{-1} - 1\right\} & \mu_W < 75\text{ GeV} \end{cases} \tag{2.63}$$

(standard model, $\Lambda=\frac{1}{2}$)

$$\bar{R} \geq \frac{3\pi}{\alpha}\left\{\left(\frac{\mu_W}{75\text{ GeV}}\right)^2 + \frac{1}{4}\left[1 - \frac{\mu_W^2\sin^4\Theta_W}{(37\text{ GeV})^2}\right]^{-1} - 1\right\} \tag{2.64}$$

(hybrid model, $\Lambda=\frac{1}{2}$)

III. THRESHOLD BOUNDS IN COLLIDING-BEAM REACTIONS

While we have put an upper bound on the energy at which charged-current weak interactions exhibit structure, it does not automatically follow that the same energy should apply to the e^+e^- cross section. It is in fact true for the vacuum polarization. To see this, we return to Eq. (2.25), writing it as

$$\frac{\rho^2(s)}{s^2} \leq \rho_W(s)\,\frac{\rho_{em}(s)}{s^2} \tag{3.1}$$

Therefore

$$\left[\int_{s_0}^{\infty} \frac{ds}{s}\,\rho(s)\right]^2 \leq \int_{s_0}^{\infty} ds\rho_W(s) \int_{s_0}^{\infty} \frac{ds'}{s'^2}\,\rho_{em}(s') \tag{3.2}$$

We define

$$\mu_{em}^2 = \frac{\int \frac{ds}{s}\,\rho_{em}(s)}{\int \frac{ds}{s^2}\,\rho_{em}(s)} \tag{3.3}$$

and, using the same information as leading to Eq. (2.29), find the same result:

$$\mu_{em}^2 \leq \frac{\pi\alpha(1-Z_3)}{G\sqrt{2}\,\sin^4\Theta_W} \tag{3.4}$$

However, this does not imply the same mass-scale for the important physical thresholds in e^+e^- collisions. We may more closely relate the result in Eq. (3.4) to observables by considering the electron-positron amplitude $d_R(s)$ defined in Eq. (2.38). Its derivative at s=0 is

$$d_R'(o) = \frac{d}{ds}\,d_R(s)\Big|_{s=0} = -e^4\int_{s_0}^{\infty} \frac{ds'}{s'^2}\,\rho_{em}(s') = -\frac{e^2(1-Z_3)}{\mu_{em}^2} \tag{3.5}$$

On the other hand, from the dispersion relation for $d_R(s)$, Eq. (2.39), we have

$$d'_R(o) = -\int_{s_0}^{\infty} \frac{ds'}{s'^2}\, \rho_{em}(s')\,|d_R(s')|^2 = \frac{-e^4}{12\pi^2}\int_{s_0}^{\infty} \frac{ds'}{s'^2}\, R_R(s') \qquad (3.6)$$

Upon using the expression for Z_3 in terms of R_R, Eq. (2.41), we can solve for μ^2_{em}:

$$\mu^2_{em} = Z_3 \frac{\int_{s_0}^{\infty} \frac{ds}{s} R_R(s)}{\int_{s_0}^{\infty} \frac{ds}{s^2} R_R(s)} = \frac{\int_{s_0}^{\infty} \frac{ds}{s} R_R(s)}{\int_{s_0}^{\infty} \frac{ds}{s^2} R_R(s) \left[1+ \frac{\alpha}{3\pi} \int_{s_0}^{\infty} \frac{ds}{s} R_R(s)\right]} \qquad (3.7)$$

Denote the threshold for production of weak quanta in e^+e^- annihilation by s_0^{em}. Then, using Eq. (3.7) and (3.4)

$$Z_3\, s_0^{em} \leq \mu^2_{em} \leq \frac{\pi\alpha(1-Z_3)}{G\sqrt{2}\, \sin^4\Theta_W} \qquad (3.8)$$

and

$$s_0^{em} \leq \frac{\pi\alpha(Z_3^{-1}-1)}{G\sqrt{2}\, \sin^4\Theta_W} = \frac{\alpha^2}{3G\sqrt{2}\, \sin^4\Theta_W} \int_{s_0} \frac{ds'}{s'} R_R(s')$$

$$\qquad (3.9)$$

$$= \frac{(37.4\ \text{GeV})^2}{\sin^4\Theta_W}\,(Z_3^{-1}-1)$$

We again see that in the standard SU(2)$\otimes$U(1) gauge theory, where $Z_3=\cos^2\Theta_W$,

$$\sqrt{s_0^{em}} \leq \frac{37.4\ \text{GeV}}{\sin\Theta_W\, \cos\Theta_W} = m_Z \qquad (3.10)$$

and the inequality becomes an equality. In general, putting in numbers gives (for the standard model only)

$$\sqrt{s_0^{em}} \le \frac{1.04\ \text{GeV}}{\sin^2\Theta_W} \left[\int_{s_0}^{\infty} \frac{ds'}{s'} R_R(s')\right]^{\frac{1}{2}} \le \frac{1.5\ \text{GeV}}{\sin^2\Theta_W} \left[\int_{s_0}^{\infty} \frac{ds}{s} R(s)\right]^{\frac{1}{2}} \qquad (3.11)$$

Unlike the case for μ_{em}^2, we have not been able to bound $\sqrt{s_0^{em}}$ from above.

Of course, if the spectral function ρ has only cuts and no poles, the threshold must lie no higher than μ_W. The only problem occurs from a possible upward level shift of the lowest lying discrete resonance, because of the electromagnetic mixing.

IV. CONNECTIONS WITH GAUGE THEORY RESULTS[14]

The considerations in the previous section have a great deal of similarity to recent work in the renormalizable gauge theories. It has been shown[5] that generalizations beyond the original SU(2)⊗ U(1) model are possible, without losing the original predictions for neutrino-induced neutral current processes (but not for neutral current processes in general). The most general such study has been given by Georgi and Weinberg[15], who have replaced SU(2)⊗ U(1) with SU(2)⊗ U(1)⊗ G, given a specific form of spontaneous symmetry breakdown. In this section we shall review their work in a way which helps to expose the similarities of their argumentation with the arguments in the previous section.

We begin, following Georgi and Weinberg, by considering the mass-matrix of the neutral bosons W_i which mix with the U(1) generator W_0. The basis used will be appropriate to the unbroken theory, with index 0 reserved for the U(1) generator, index 1 for the neutral SU(2) generator, and all indices greater than 1 for the generators of G.

The main assumptions needed in the theorem of Georgi and Weinberg are

(i) the (real symmetric) mass-matrix μ^2_{ij} resulting from spontaneous symmetry breakdown does not mix the neutral SU(2) generator with the generators of G. This occurs if the Higgs-structure is (2,1)+(1,X).

$$\mu^2_{1j} = \mu^2_{j1} = 0 \qquad j \geq 2 \tag{4.1}$$

(ii) There exists one zero eigenvalue of μ^2_{ij} with eigenvector (the photon) p_j, which is known

$$\mu^2_{ij}\, p_j = 0 \tag{4.2}$$

(iii) The photon is mixed with the U(1) generator

$$p_0 \equiv \sqrt{Z} \neq 0 \tag{4.3}$$

Then without loss of generality, the mass-matrix of generators of G may be assumed to have been diagonalized, and the problem reduces to the study of the mass matrix for which the only off-diagonal elements are in the first row and first column; in other words

$$\mu^2_{0i} = \mu^2_{i0} \neq 0 \text{ (in general)} \tag{4.4}$$

but

$$\mu^2_{ij} = \mu^2_i \,\delta_{ij} \quad i,j \geq 1 \tag{4.5}$$

This is in fact the classic problem of mixing of a single state (i=0) with a "continuum" ({i>0}).[16] It is therefore possible to construct (in the original basis) the propagator-matrix of the gauge bosons, and directly verify the results proved much more efficiently by Georgi and Weinberg. However the additional information we obtain will facilitate the comparison with the work in the previous section.

The main problem is to construct the matrix $T(s)=(s-\mu^2)^{-1}$. As usual, we divide μ^2 into a diagonal part and a perturbation

$$\mu^2 = \hat{\mu}^2 + \lambda \tag{4.6}$$

with

$$(\hat{\mu}^2)_{ij} \equiv \mu^2_i \,\delta_{ij} \qquad i=0,1,\ldots N$$

$$\lambda_{ij} = \lambda_{ji} \equiv \begin{cases} \lambda_j & i=0 \quad j\neq 0 \\ \lambda_i & j=0 \quad i\neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.7}$$

and write (with $T_0=(s-\hat{\mu}^2)^{-1}$)

$$T(s) = T_0(s)+T_0(s)\lambda T_0(s)+T_0\lambda T_0\lambda T_0+\ldots \tag{4.8}$$

It is sufficient to find the matrix element T_{00}. The structure of the perturbation matrix shows that for $i,j \geq 1$

$$T_{0i} = T_{i0} = \frac{\lambda_i}{(s-\mu_i^2)} T_{00}$$

$$T_{ij} = \frac{\lambda_i}{(s-\mu_i^2)} T_{00} \frac{\lambda_j}{(s-\mu_j^2)} + \frac{\delta_{ij}}{(s-\mu_i^2)} \tag{4.9}$$

Furthermore, construction of T_{00} is not hard because the off-diagonal interaction λ, iterated twice, brings one back to the "ground state" $i=0$. (It is a standard "bubble-sum.")

$$\begin{aligned} T_{00} &= \frac{1}{(s-\mu_0^2)} + \frac{1}{(s-\mu_0^2)}\left[\sum_{i=1}^{N} \frac{\lambda_i^2}{(s-\mu_i^2)}\right]\frac{1}{(s-\mu_0^2)} + \ldots \\ &= \frac{1}{\left[s-\mu_0^2 - \sum_{i=1}^{N} \frac{\lambda_i^2}{(s-\mu_i^2)}\right]} \end{aligned} \tag{4.10}$$

Next we use the assumption that there exists a massless state, which must contribute a pole in T_{00}. Thus

$$\mu_0^2 = \sum_{i=1}^{N} \frac{\lambda_i^2}{\mu_i^2} \tag{4.11}$$

and

$$T_{00} = \frac{1}{s\left[1 - \sum_{i=1}^{N} \frac{\lambda_i^2}{\mu_i^2(s-\mu_i^2)}\right]} \longrightarrow \frac{Z}{s} \text{ as } s\to 0 \tag{4.12}$$

with

$$Z^{-1} = \left[1 + \sum_{i=1}^{N} \frac{\lambda_i^2}{\mu_i^4}\right] \tag{4.13}$$

Inasmuch as

$$T_{ij} \longrightarrow \frac{p_i p_j}{s} \quad (i,j=0,\ldots N) \text{ as } s\to 0 \tag{4.14}$$

we can read off the photon eigenvector to be

$$p_0 = \sqrt{Z} \qquad p_i = -\frac{\lambda_i}{\mu_i^2}\sqrt{Z} \qquad (i\geq 1) \tag{4.15}$$

We can now construct the (neutral-current) amplitude for scattering of a fermion F from another fermion F'. Let the couplings of F and F' to W_i be g_i and g_i', respectively. Then omitting spinors, etc., the amplitude has the structure

$$\begin{aligned} \mathcal{M}_{FF'} &= \sum_{i,j=0}^{N} g_i g_j' \, T_{ij} \\ &= \left[g_0 + \sum_{i=1}^{N} \frac{g_i \lambda_i}{(s-\mu_i^2)}\right] \cdot \left[g_0' + \sum_{j=1}^{N} \frac{g_j \lambda_j}{(s-\mu_j^2)}\right] \frac{1}{s\left[1 - \sum_{i=1}^{N} \frac{\lambda_i^2}{\mu_i^2(s-\mu_i^2)}\right]} \\ &\quad + \sum_{i=1}^{N} \frac{g_i g_i'}{(s-\mu_i^2)} \end{aligned} \tag{4.16}$$

The various factors have a direct diagrammatic interpretation. The first two are the unrenormalized proper electromagnetic vertices of F and F'. The third factor is the unrenormalized photon propagator. The final term on the right is the intrinsic weak interaction. It is straightforward to now "renormalize" the various factors:

Complete Photon Propagator

$$\frac{1}{s\left[1-\sum_{i=1}^{N}\frac{\lambda_i^2}{\mu_i^2(s-\mu_i^2)}\right]}=\frac{Z}{s\left[1-Zs\sum_{i=1}^{N}\frac{\lambda_i^2}{\mu_i^4(s-\mu_i^2)}\right]} \tag{4.17}$$

$$=\frac{Z}{s\left\{1-s\sum_{i=1}^{N}\frac{p_i^2}{(s-\mu_i^2)}\right\}}$$

Fermion Proper Vertex:

$$g_0+\sum_{i=1}^{N}\frac{g_i\lambda_i}{(s-\mu_i^2)}=Z^{-\frac{1}{2}}\left\{\sum_{i=0}^{N}g_ip_i+s\sum_{i=1}^{N}\frac{g_ip_i}{(\mu_i^2-s)}\right\} \tag{4.18}$$

Evidently the charge e_F of the fermion F is given by

$$e_F=\sum_{i=0}^{N}g_ip_i\equiv e\left[T_3^{(F)}+Y^{(F)}\right] \tag{4.19}$$

and

$$\mathcal{M}_{FF'}=\left[e_F+s\sum_{i=1}^{N}\frac{g_ip_i}{(\mu_i^2-s)}\right]\cdot\left[e_{F'}+s\sum_{i=1}^{N}\frac{g_i'p_i}{(\mu_i^2-s)}\right]\frac{1}{s\left[1+s\sum_{i=1}^{N}\frac{p_i^2}{(\mu_i^2-s)}\right]}$$

$$+\sum_{i=1}^{N}\frac{g_ig_i'}{(s-\mu_i^2)} \tag{4.20}$$

It is clear from the structure of Eq. (4.20) that the "unperturbed" poles at $s=\mu_i^2$ do not appear in the full amplitude $\mathcal{M}$, but only the poles in T_{00} associated with the true eigenstates of the mass matrix.

The theorem of Georgi and Weinberg now follows by inspection. If there exists a fermion F (such as ν_μ) which is neutral and a singlet with respect to G, then e=0 and $g_i=0$ for $i \geq 2$. Then as $s \to 0$

$$\mathcal{M}_{FF'} \longrightarrow \frac{g_1 p_1}{\mu_1^2} e_{F'} - \frac{g_1 g_1'}{\mu_1^2} = - \frac{g_1}{\mu_1^2} (g_1' - e_{F'} p_1) \qquad (4.21)$$

Since the only term on the left-hand side of (4.19) which is T_3-dependent is $g_1 p_1$, we have

$$g_1 \doteq \frac{e\, T_3^{(F)}}{p_1} \qquad g_1' = \frac{e\, T_3^{(F')}}{p_1} \qquad (4.22)$$

and thus

$$\mathcal{M} \propto T_3^{(F)} \cdot \frac{g_1^2}{\mu_1^2} \cdot (T_3^{(F')} - p_1^2 Q_{F'}) \qquad (4.23)$$

with the $T_3 \cdot T_3'$ term unchanged in strength from the unbroken theory,[17] and with $\sin\Theta_W$ identified with p_1.

The connections with the previous section are now also evident. We see that the full amplitude is merely the sum of the "unperturbed" or intrinsic weak coupling associated with exchange of the unmixed bosons of $SU(2) \otimes G$, plus a photon-exchange piece which contains nontrivial form factors and vacuum polarization contributions. As $s \to \infty$, the "photon-exchange" term turns into the U(1)-boson exchange

$$\mathcal{M}_{FF'} \longrightarrow \frac{g_0 g_0'}{s} + \sum_{i=1}^{N} \frac{g_i g_i'}{s} \qquad (4.24)$$

Thus the photon couples at short distances to weak hypercharge.

As mentioned already in Section II, this property is novel and need not occur in the nonunified general description we outlined, where we might expect bare charge to be proportional to physical charge. The distinction is cast in objective terms in the value of $\Lambda=\int ds'\ \rho(s')$ introduced in Section II. $\Lambda=1/2$ is required by the $SU(2)\otimes U(1)\otimes G$ gauge theories, or generalizations which produce SU(2) symmetry at short distances. The condition $\Lambda=0$ might correspond to the nonunified version. An objective distinction (for the case of e_R^- a weak singlet) occurs already in colliding beam reactions, where one has, as $s \to \infty$.

$$\frac{\sigma(e_R^- e^+ \to \mu_R^- \mu^+)}{\sigma(e_R^- e^+ \to \mu_L^- \mu^+)} \longrightarrow \begin{cases} 4 & \Lambda = 1/2,\ SU(2)\otimes U(1)\otimes G \text{ gauge theories} \\ 1 & \Lambda = 0;\ \text{nonunification} \end{cases} \tag{4.25}$$

Before concluding this section we mention that Georgi and Weinberg demonstrated that at least one neutral gauge boson must have mass no larger than the $m_Z=(37.4\ \text{GeV})(\cos\Theta_W \sin\Theta_W)^{-1}$ of the standard model. This follows directly from the structure and positivity properties of T_{00}. However, we have not succeeded in generalizing this result (c.f. Section III).

V. FINAL COMMENTS AND CONCLUSIONS

We reiterate the main results:

1. A weak global SU(2) symmetry supplemented by a universality hypothesis and by an electromagnetic mixing contribution suffices to describe all neutrino-induced neutral current phenomena in a way identical to the standard model.

2. The amount of electromagnetic mixing is measured by $\sin^2\Theta_W$ and is very large, leading to a contribution to the charge renormalization constant of order unity.

3. The Fermi structure of the weak interaction must break down at characteristic masses $\lesssim$ 150 GeV, just as in the renormalizable theories.

4. One can relate the breakdown of the Fermi theory for charged-currents to the behavior of colliding beam cross sections. For a given value μ_W of what the charged-current phenomenologist calls m_W (the charged intermediate-boson mass), one can bound the contribution of production of weak-interaction quanta to the colliding-beam R in the manner shown in Fig. 2. For a reasonable range of μ_W, the lower bound on $\int_{s_0}^{\infty} \frac{ds}{s} R(s)$ is $\geq$ 100 to 1000, given the "standard" assignment of e_R^- as an SU(2) singlet.

5. This picture is quite compatible with renormalizable gauge theories. However, it does not require them.

Within this generalization, what kind of concrete alternatives exist to the usual renormalizable theories? Two general classes can be envisaged. The first puts unexpected structure into the intermediate bosons themselves but leaves the coupling to fermions predominantly via a local current.[18] Then the colliding-beam cross section (or for that matter quark-antiquark cross section) into weak quanta might have a structure similar (but on a much larger scale) to charm production: some narrow resonances followed at higher energies by a continuum of W-W pairs or pairs of constituents of W's (if that concept makes sense).

A second, more extreme alternative puts structure into everything. Not only would there be t-channel contributions to fermion-fermion scattering, but also possible s and/or u-channel pieces. If this were somehow to be the case, it might be necessary to assume that the dominant part of the amplitude be helicity conserving in order to understand the vector nature of weak processes at low energies ($s,t \ll 10^4$ GeV^2). Then the principal additional terms in the effective Lagrangian would contain operators with derivative couplings (e.g. $\bar{\psi}\gamma_\mu \partial_\nu \psi$) corresponding to higher-spin exchanges. The relative importance of such terms would increase with increasing energy. A phenomenology of such terms has already been given.[8]

We have no specific model to offer for either alternative. And, despite the foregoing arguments that credible generalizations of renormalizable gauge theories do exist, it is unlikely that either the arguments or the alternatives we have sketched have enough force to induce many theorists to abandon gauge theories. There are strong, albeit mostly subjective, reasons favoring the gauge-theory approach, some of which we list below.

(i) An underlying local gauge principle: Both quantum electrodynamics and general relativity have this feature. Maybe all successful theories do.

(ii) Renormalizability: In my opinion this is only a subjective criterion. However, in the generalized context, the smallness of the proper-diagram radiative corrections in Fig. 1e, despite the rather large effective photon coupling, may be considered an objective argument in favor of the renormalizable models, for which such terms are demonstrably small.

(iii) Intermediate Vector Boson Hypothesis: This general hypothesis (based, to be sure, on objective evidence) argues in favor of existence of J=1 bosons to mediate the weak interactions. While these objects might be composites and/or strongly coupled to each other, it is not too big a step to make them gauge particles.

(iv) Universality: That the coupling of W to the fermion currents is universal is a consequence of the gauge-theories. However, a hypothesis of invariance of the low energy effective Lagrangian under permutation among fermion types[8] suffices to obtain this result.

(v) Unification: While the alternatives we describe indicate there is much less evidence for unification of the forces than we would prefer to have, it is true that even the generalization we discussed predicts the intrinsic masses of weak quanta to be of the same size as in the gauge theories. Therefore the coupling strengths associated with the weak force are of the same order as the electromagnetic. This is certainly supportive of the unification idea. But it is conceivable that even if the unification occurs it occurs in a nonrenormalizable way.

(vi) Origin of Gauge-Boson and Fermion Mass: In principle the renormalizable gauge theories answer this question in terms of the Higgs mechanism. However, the lack of concrete success in understanding the pattern of fermion masses and mixings raises suspicions that something basic is missing from the existing picture.

But whatever the theoretical pros and cons, it is experiment that must be the arbiter. The next steps are clear: the form of the low-energy weak Lagrangian must be further tested. Searches for the W and Z will determine whether they are "classical." Likewise searches for the Higgs sector will become increasingly important. Nevertheless it might take a long time to distinguish between renormalizable gauge theories and their alternatives.

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13. For example, B. Ioffe and V. Khoze, Leningrad preprint 76-274 (1976).

14. The work in this section was carried out in collaboration with Marvin Weinstein, to whom I express my gratitude.

15. H. Georgi and S. Weinberg, Phys. Rev. D17, 275 (1978).

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17. Provided $\mu_1^2 = \mu_W^2$, i.e. only Higgs doublets contribute to the mass of Z (in the absence of U(1) mixing).

18. See for example, H. Terazawa, Phys. Rev. D7, 3663 (1973); D16, 2373 (1977).

19. N. Calder, "The Key to the Universe," p. 164, Jolly and Barber Ltd., Rugby, England (1977).

FIGURE CAPTIONS

1. Amplitudes for fermion-fermion scattering.

   a) Intrinsic weak interaction.

   b) Charge-radius contribution.

   c) Vacuum polarization contributions.

   d) Proper electromagnetic correction to intrinsic weak interaction (which we neglect.)

2. Lower bound for $\bar{R}=\int \frac{ds}{s} R$, which measures the production of weak quanta by colliding e^+e^- beams. The formulae are Eqs. (2.56) and (2.57). The range of the weak force, μ_W, is given in Eq. (2.32). [Note: a similar plot given in Reference 6 is incorrect.]

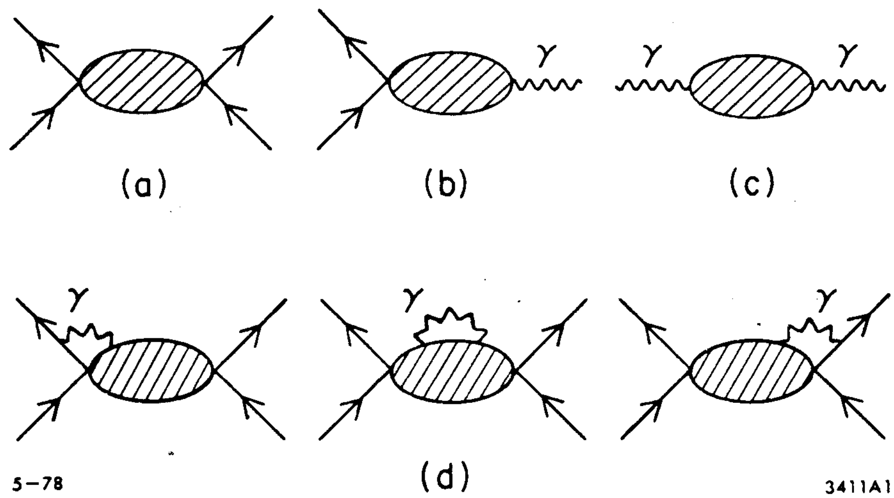
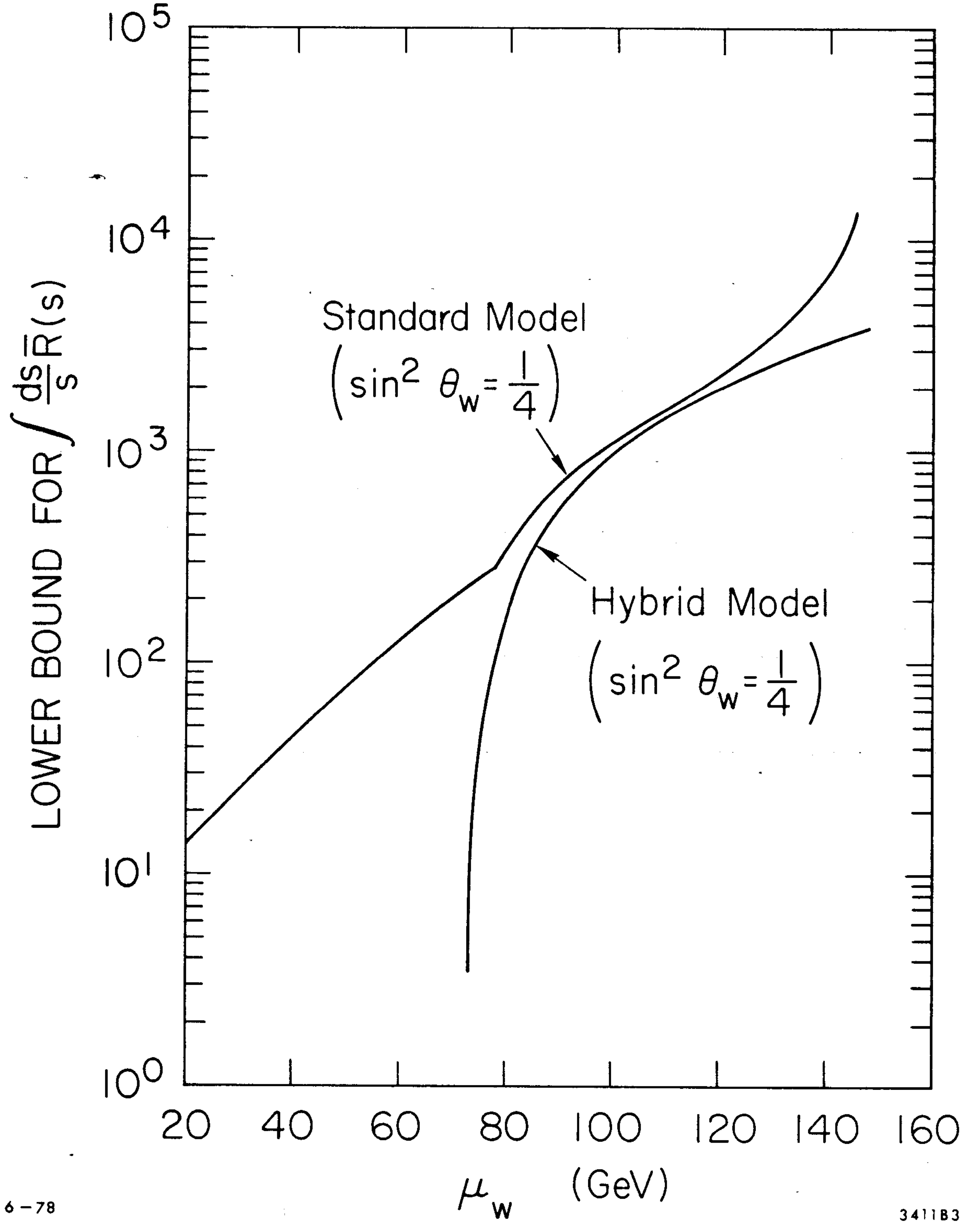


Fig. 1



10^5
10^4
10^3
10^2
10^1
10^0
LOWER BOUND FOR $\int \frac{ds}{s} \bar{R}(s)$
Standard Model
$\left(\sin^2 \theta_w = \frac{1}{4}\right)$
Hybrid Model
$\left(\sin^2 \theta_w = \frac{1}{4}\right)$
20 40 60 80 100 120 140 160
μ_w (GeV)
6-78
3411B3

Fig. 2