SLAC-PUB-2098
March 1978
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HIGH ENERGY TOTAL CROSS SECTION AND ANGULAR DISTRIBUTION (ASYMMETRY) IN $\bar{e}e \to \bar{\mu}\mu, \bar{q}q$ IN MODELS OF THE SALAM-J. C. WARD AND WEINBERG TYPE IN THE TREE APPROXIMATION*

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ABSTRACT

Explicit formulas are given for the high-energy total cross section and angular distribution (asymmetry) in $\bar{e}e \to \bar{\mu}\mu$, $\bar{q}q$ in models of the Salam-J. C. Ward and Weinberg type in the tree approximation. As expected, the pure weak terms cannot be neglected as the center of momentum energy approaches the mass of the neutral heavy vector boson Z.

(Submitted to Phys. Rev. D)

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*Work supported by The Department of Energy.

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It is of some interest to know the explicit c. m. energy dependence of the various cross sections in e^+e^- annihilation as predicted by models of the Salam-J. C. Ward and Weinberg[1] (S-W-W) type, with little prejudice as to the fundamental hadronic sector, i.e., as to the quark sector $\{q_i\}$. Thus we shall record $d\sigma\left(e^+e^- \to \bar{q}_i\, q_i\right)/d\Omega$ with the lone assumption that the q_i's come in (right) left-handed (singlets) doublets with non-zero mass and with electric charges Q_i and $Q_i - 1$ for the upper and lower members of a doublet, respectively. (As usual, the left-handed parts of ν_μ and μ form a doublet.) The kinematics is summarized in Fig. 1. As usual, the center of momentum scattering angle $\theta_{c.m.}$ is the angle between the center of momentum e^- 3-momentum and the center of momentum produced fermion (as opposed to anti-fermion) 3-momentum, as illustrated in the figure, i.e., $d\Omega$ is associated with the direction of the produced fermion relative to the incoming e^- direction. We shall imagine $S \equiv (p^- + p^+)^2$ to be large enough that all masses m_e, m_μ, m_{q_i} may be ignored, where m_a is the mass of a.

Let σ^Q be the invariant cross section $\sigma(e^+e^- \to \bar{f}f)$ for the fermion f of charge Q whose left-handed part forms the upper member ($I_3^W = +1/2$ member, where I^W is weak isospin) of a doublet and σ^{Q-1} be the analogous quantity for the lower member ($I_3^W = -1/2$ member). Then, by the standard methods we have from Fig. 1 (θ_W is Weinberg's angle and should not be confused with $\theta_{c.m.}$ in the following formulae)

$$\frac{d\sigma^Q}{d\Omega} = \left\{\frac{Q^2\alpha^2}{4S} + \frac{\alpha^2 S\left[(\frac{1}{2} - 2Q\sin^2\theta_W)^2 + \frac{1}{4}\right]}{64\sin^4\theta_W\cos^4\theta_W(S-M_Z^2)^2}\left[(\frac{1}{2} - 2\sin^2\theta_W)^2 + \frac{1}{4}\right]\right.$$

$$\left. + \frac{Q\alpha^2(\frac{1}{2} - 2\sin^2\theta_W)(\frac{1}{2} - 2Q\sin^2\theta_W)}{8\sin^2\theta_W\cos^2\theta_W(S-M_Z^2)}\right\}(1+\cos^2\theta_{c.m.})$$

$$- \left\{\frac{\alpha^2 S(-\frac{1}{2} + 2Q\sin^2\theta_W)(\frac{1}{2} - 2\sin^2\theta_W)}{32\sin^4\theta_W\cos^4\theta_W(S-M_Z^2)^2} - \frac{Q\alpha^2}{16\sin^2\theta_W\cos^2\theta_W(S-M_Z^2)}\right\}\cos\theta_{c.m.}, \tag{1}$$

$$\frac{d\sigma^{Q-1}}{d\Omega} = \left\{\frac{(Q-1)^2\alpha^2}{4S} + \frac{\alpha^2\left[(\frac{1}{2} + 2(Q-1)\sin^2\theta_W)^2 + \frac{1}{4}\right]S}{64\sin^4\theta_W\cos^4\theta_W(S-M_Z^2)^2}\left[(\frac{1}{2} - 2\sin^2\theta_W)^2 + \frac{1}{4}\right]\right.$$

$$\left. - \frac{(Q-1)\alpha^2(\frac{1}{2} - 2\sin^2\theta_W)(\frac{1}{2} + 2(Q-1)\sin^2\theta_W)}{8\sin^2\theta_W\cos^2\theta_W(S-M_Z^2)}\right\}(1+\cos^2\theta_{c.m.})$$

$$- \left\{\frac{\alpha^2(-\frac{1}{2} - 2(Q-1)\sin^2\theta_W)(\frac{1}{2} - 2\sin^2\theta_W)S}{32\sin^4\theta_W\cos^4\theta_W(S-M_Z^2)^2} + \frac{(Q-1)\alpha^2}{16\sin^2\theta_W\cos^2\theta_W(S-M_Z^2)}\right\}\cos\theta_{c.m.} \tag{2}$$

Here, $\alpha = e^2/4\pi$, where e is the electron (as opposed to the positron) charge and G/4 is the Z-fermion-anti-fermion axial vector coupling strength, so that

$$G\sin\theta_W\cos\theta_W \equiv e\ . \tag{3}$$

The mass of Z may be understood to have a negative imaginary part, in which case

$$(S-M_Z^2)^{-2} \rightarrow |S-M_Z^2|^{-2} \tag{4}$$

and

$$\frac{1}{S-M_Z^2} \rightarrow \mathrm{Re}\ \frac{1}{S-M_Z^2} \tag{5}$$

Since we expect $\left|(\mathrm{Im}\ M_Z^2)/M_Z^2\right| \sim \alpha$, we shall usually ignore $\mathrm{Im}M_Z^2$. (But, remember (4) and (5) in what follows!)

Of some interest is the limit of Steinberger et al.,[2]

$$\sin^2\theta_W \rightarrow 1/4 \quad . \tag{6}$$

In this case we have

$$\frac{d\sigma^Q}{d\Omega} = \left\{\frac{Q^2\alpha^2}{4S} + \frac{\alpha^2\left[(Q-1)^2+1\right]S}{36(S-M_Z^2)^2}\right\}(1+\cos^2\theta_{c.m.})$$

$$+ \frac{Q\alpha^2\cos\theta_{c.m.}}{3(S-M_Z^2)} \quad , \tag{7}$$

$$\frac{d\sigma^{Q-1}}{d\Omega} = \left\{\frac{(Q-1)^2\alpha^2}{4S} + \frac{\alpha^2\left[Q^2+1\right]S}{36(S-M_Z^2)^2}\right\}(1+\cos^2\theta_{c.m.})$$

$$- \frac{(Q-1)\alpha^2\cos\theta_{c.m.}}{3(S-M_Z^2)} \quad . \tag{8}$$

More specifically, since the left-handed parts of ν_μ and μ form a doublet in the S-W-W model with Q = 0, we have

$$\frac{d\sigma}{d\Omega}(e^+e^- \to \mu^+\mu^-) \equiv \frac{d\sigma^{(\mu)}}{d\Omega} = \left.\frac{d\sigma^{Q-1}}{d\Omega}\right|_{Q=0}$$

$$= \left\{\frac{\alpha^2}{4S} + \frac{\alpha^2\left[\left(\frac{1}{2} - 2\sin^2\theta_W\right)^2 + 1/4\right]^2 S}{64\sin^4\theta_W\cos^4\theta_W(S-M_Z^2)^2}\right.$$

$$\left. + \frac{\alpha^2(1/2 - 2\sin^2\theta_W)^2}{8\sin^2\theta_W\cos^2\theta_W(S-M_Z^2)}\right\}(1 + \cos^2\theta_{c.m.})$$

$$+ \left\{\frac{-\alpha^2\left(\frac{1}{2} - 2\sin^2\theta_W\right)^2 S}{32\sin^4\theta_W\cos^4\theta_W(S-M_Z^2)^2}\right.$$

$$\left. + \frac{\alpha^2}{16\sin^2{}_W\cos^2\theta_W(S-M_Z^2)}\right\}\cos\theta_{c.m.} \qquad (9)$$

so that

$$\sigma^{(\mu)} \equiv \int d\Omega\, \frac{d\sigma^{(\mu)}}{d\Omega}$$

$$= \frac{4\pi\alpha^2}{3S} + \frac{\pi\alpha^2\left[\left(\frac{1}{2} - 2\sin^2\theta_W\right)^2 + 1/4\right]^2 S}{12\sin^4\theta_W\cos^4\theta_W(S-M_Z^2)^2}$$

$$+ \frac{2\pi\alpha^2(1/2-2\sin^2\theta_W)^2}{3\sin^2\theta_W\cos^2\theta_W(S-M_Z^2)} \quad . \qquad (10)$$

Similarly,

$$\sigma^Q \equiv \int d\Omega\, \frac{d\sigma^Q}{d\Omega} = \frac{4\pi Q^2\alpha^2}{3S} + \frac{\pi\alpha^2\left[\left(\frac{1}{2} - 2Q\sin^2\theta_W\right)^2 + 1/4\right]}{12\sin^4\theta_W\cos^4\theta_W}$$

$$\times \frac{\left[(1/2-2\sin^2\theta_W)^2 + 1/4\right] S}{(S-M_Z^2)^2} + \frac{2\pi Q\alpha^2\left(\frac{1}{2} - 2\sin^2\theta_W\right)\left(\frac{1}{2} - 2Q\sin^2\theta_W\right)}{3\sin^2\theta_W\cos^2\theta_W(S-M_Z^2)} \qquad (11)$$

$$\sigma^{Q-1} = \frac{4\pi(Q-1)^2\alpha^2}{3S} + \frac{\pi\alpha^2\left[(\frac{1}{2}+2(Q-1)\sin^2\theta_W)^2+\frac{1}{4}\right]\left[(\frac{1}{2}-2\sin^2\theta_W)^2+\frac{1}{4}\right]S}{12\sin^4\theta_W\cos^4\theta_W(S-M_Z^2)^2}$$

$$-\frac{2\pi(Q-1)\alpha^2(\frac{1}{2}-2\sin^2\theta_W)(\frac{1}{2}+2(Q-1)\sin^2\theta_W)}{3\sin^2\theta_W\cos^2\theta_W(S-M_Z^2)} \quad . \tag{12}$$

Thus, for $S \to M_Z^2$,

$$R \equiv \frac{\sum_{q_i}\sigma(e^+e^- \to q_i q_i)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$\to \sum_{\substack{\text{quark}\\ \text{doublets}}} \frac{((\frac{1}{2}-2Q_i\sin^2\theta_W)^2+1/4)+((\frac{1}{2}+2(Q_i-1)\sin^2\theta_W)^2+1/4)}{((1/2-2\sin^2\theta_W)^2+1/4)}$$

$$= \frac{N_D(1-2\sin^2\theta_W)+4R_{QED}\sin^4\theta_W}{(1/2-2\sin^2\theta_W+4\sin^4\theta_W)} \quad .$$

$$\xrightarrow[\sin^2\theta_W \to 1/4]{} 2N_D + R_{QED} \quad , \tag{13}$$

where

N_D is the number of quark doublets

and

$$R_{QED} \equiv \sum_{\substack{\text{quark}\\ \text{doublets}}} (Q_i^2 + (Q_i-1)^2) = \sum_{\text{quarks}} Q_i^2 \quad .$$

To repeat, we ignore $\mathrm{Im}M_Z^2$. (But, see the discussion immediately following (17).)

Finally, defining the asymmetry a^Q by

$$\sigma^Q_{\substack{+\\(-)}} \equiv \int_{\cos\theta_{\mathrm{c.m.}} \substack{>\\(<)} 0} \frac{d\sigma^Q}{d\Omega}$$

$$a^Q \equiv \frac{\sigma^Q_+ - \sigma^Q_-}{\sigma^Q_+ + \sigma^Q_-} \tag{15}$$

we have

$$a^Q = \left\{ \frac{(\frac{1}{2} - 2Q\sin^2\theta_W)(\frac{1}{2} - 2\sin^2\theta_W)\, S}{16\sin^4\theta_W\cos^4\theta_W(S-M_Z^2)^2} + \frac{Q}{8\sin^2\theta_W\cos^2\theta_W(S-M_Z^2)} \right\}$$

$$\Bigg/ \left\{ \frac{4Q^2}{3S} + \frac{\left[(\frac{1}{2} - 2Q\sin^2\theta_W)^2 + 1/4\right]\left[(\frac{1}{2} - 2\sin^2\theta_W)^2 + 1/4\right] S}{12\sin^4\theta_W\cos^4\theta_W(S-M_Z^2)^2} \right.$$

$$\left. + \frac{2Q(\frac{1}{2} - 2\sin^2\theta_W)(\frac{1}{2} - 2Q\sin^2\theta_W)}{3\sin^2\theta_W\cos^2\theta_W(S-M_Z^2)} \right\}$$

$$\underset{S \to M_Z^2}{\longrightarrow} \left\{ \frac{3}{4} \frac{(\frac{1}{2} - 2Q\sin^2\theta_W)(\frac{1}{2} - 2\sin^2\theta_W)}{\left[(\frac{1}{2} - 2Q\sin^2\theta_W)^2 + \frac{1}{4}\right]\left[(\frac{1}{2} - 2\sin^2\theta_W)^2 + \frac{1}{4}\right]} \right.$$

$$\left. + \frac{3}{2} \frac{Q\sin^2\theta_W\cos^2\theta_W\left[(S-M_Z^2)/S\right]}{\left[(\frac{1}{2} - 2Q\sin^2\theta_W)^2 + \frac{1}{4}\right]\left[(\frac{1}{2} - 2\sin^2{}_W)^2 + \frac{1}{4}\right]} \right\}, \tag{16}$$

$$a^{Q-1} = \left\{ \frac{(\frac{1}{2} + 2(Q-1)\sin^2\theta_W)(\frac{1}{2} - 2\sin^2\theta_W)\ S}{16\cos^4\theta_W \sin^4\theta_W (S-M_Z^2)^2} \right.$$

$$\left. - \frac{(Q-1)}{8\sin^2\theta_W\cos^2\theta_W(S-M_Z^2)} \right\} \Big/ \left\{ \frac{4(Q-1)^2}{3S} \right.$$

$$+ \frac{\left[(\frac{1}{2} + 2(Q-1)\sin^2\theta_W)^2 + \frac{1}{4}\right]\left[(\frac{1}{2} - 2\sin^2\theta_W)^2 + \frac{1}{4}\right] S}{12\sin^4\theta_W\cos^4\theta_W(S-M_Z^2)^2}$$

$$\left. - \frac{2(Q-1)(\frac{1}{2} - 2\sin^2\theta_W)(\frac{1}{2} + 2(Q-1)\sin^2\theta_W)}{3\sin^2\theta_W\cos^2\theta_W(S-M_Z^2)} \right\}$$

$$\xrightarrow[S \to M_Z^2]{} \left\{ \frac{3}{4} \frac{(\frac{1}{2} + 2(Q-1)\sin^2\theta_W)\ (\frac{1}{2} - 2\sin^2\theta_W)}{\left[(\frac{1}{2} + 2(Q-1)\sin^2\theta_W)^2 + \frac{1}{4}\right]\left[(\frac{1}{2} - 2\sin^2\theta_W)^2 + \frac{1}{4}\right]} \right.$$

$$\left. - \frac{3}{2} \frac{(Q-1)\sin^2\theta_W\cos^2\theta_W\ \left[(S-M_Z^2)/S\right]}{\left[(\frac{1}{2} + 2(Q-1)\sin^2\theta_W)^2 + \frac{1}{4}\right]\left[(\frac{1}{2} - 2\sin^2\theta_W)^2 + \frac{1}{4}\right]} \right\}. \tag{17}$$

(In the last line of (16) and of (17), (4) and (5) give $((S-M_Z^2)/S) \to ((S-\mathrm{Re}M_Z^2)/S)$.)

In (13), (16), and (17) for $S \to M_Z^2$, we have used the fact that, even though $(\mathrm{Im}M_Z^2)/M_Z^2 \sim \alpha$, with $\sin^2\theta_W \simeq .26$, $\frac{1}{2} - 2\sin^2\theta_W \sim -.02$ and $\sin^2\theta_W\cos^2\theta_W \sim \frac{3}{16}$ so that, for $|Q| \leq 1$, for $S \to M_Z^2$, in $\sigma_+^Q + \sigma_-^Q$

(a) $\frac{4}{3}\frac{Q^2}{S} \sim \frac{4}{3S}$

(b) $$\frac{\left[(\frac{1}{2} - 2Q\sin^2\theta_W)^2 + \frac{1}{4}\right]\left[(\frac{1}{2} - 2\sin^2\theta_W)^2 + \frac{1}{4}\right] S}{12\sin^4\theta_W\cos^4\theta_W\ |S-M_Z^2|^2} \sim \frac{1/16}{12(3/16)^2\alpha^2 S}$$

$$= \frac{4}{27\alpha^2\ S} ,$$

$$\text{(c)} \quad \mathrm{Re}\ \frac{2Q\left(\frac{1}{2}-2\sin^2\theta_W\right)\left(\frac{1}{2}-2Q\sin^2\theta_W\right)}{3\sin^2\theta_W\cos^2\theta_W(S-M_Z^2)} \sim \frac{2}{3}\,\frac{(-.02)}{(3/16)}\,\frac{(S-\mathrm{Re}M_Z^2)((1-Q)/2)\,Q}{\left[(S-\mathrm{Re}M_Z^2)^2+\alpha^2S^2\right]}$$

$$\rightarrow 0$$

$$S \rightarrow \mathrm{Re}M_Z^2\ , \tag{18}$$

i.e., (b) dominates near resonance. (The same argument works for $\sigma_+^{Q-1} + \sigma_-^{Q-1}$.) Note that, for $S \rightarrow \mathrm{Re}M_Z^2$, where $E^2 \equiv S$,

$$\frac{1}{S-\mathrm{Re}M_Z^2 - i\mathrm{Im}M_Z^2} = \frac{1}{E^2-\mathrm{Re}M_Z^2 - i\mathrm{Im}M_Z^2}$$

$$\cong \frac{1}{E+\sqrt{\mathrm{Re}M_Z^2}}\ \frac{1}{(E-\sqrt{\mathrm{Re}M_Z^2})-i\mathrm{Im}M_Z^2/(E+\sqrt{\mathrm{Re}M_Z^2})}$$

$$\longrightarrow -\ \Gamma/2 \doteq (\mathrm{Im}M_Z^2)/(E+\sqrt{\mathrm{Re}M_Z^2}) \simeq (\mathrm{Im}M_Z^2)/2\,\mathrm{Re}M_Z$$

$$\longrightarrow \quad (\mathrm{Re}M_Z)\,\Gamma \doteq -\ \mathrm{Im}M_Z^2\ , \tag{19}$$

where Γ is the familiar Breit-Wigner width of Z.

About R, a^Q, a^{Q-1}, etc., it should be noticed that the pure weak term $\propto S/|S-M_Z^2|^2$ cannot be neglected as one approaches $S = \mathrm{Re}M_Z^2$. The results above appear to be in general agreement with the more model independent results of Ref. 3, for example.

ACKNOWLEDGEMENT

This paper was written at the suggestion of Professor A. Odian. The author appreciates Professor Odian's comments.

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FIGURE CAPTION

1. Tree approximation for $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \bar{q}q$ in models of the Salam–J. C. Ward and Weinberg type. As usual, $p^- \cdot t_1 = S/4 - \sqrt{S/4 - m_e^2}\,\sqrt{S/4 - m_f^2}\cos\theta_{c.m.} \to (S/4)\,(1 - \cos\theta_{c.m.})$, where $m_f = m_\mu$, m_{q_i} in (a), (b) respectively, and $S = (p^- + p^+)^2$. Higgs scalar exchange is taken to be negligible.

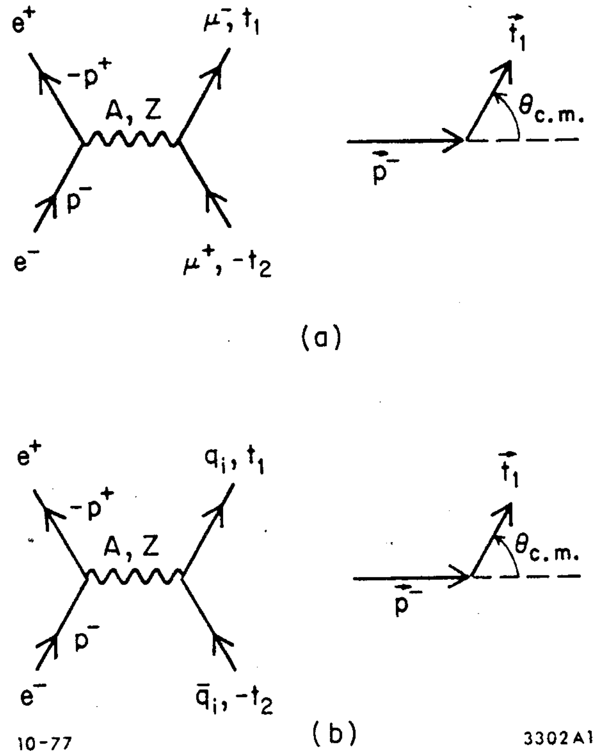
e+
−p+
A, Z
μ−, t1
p−
e−
μ+, −t2
t1
θc.m.
p−
(a)
e+
−p+
A, Z
qi, t1
p−
e−
q̄i, −t2
t1
θc.m.
p−
(b)
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Fig. 1