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NONLEPTONIC CHARMED MESON DECAYS\*

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ABSTRACT

In this paper we compute and interrelate various non-leptonic charmed meson decay matrix elements. With the assumption that the decay Hamiltonian is in the  $\Delta I = 1$  form, numerous decay rates are interrelated. The predictions are compared to the experiment whenever sufficient data is available. In other cases we compare our results to those of Quigg and Rosner. To estimate the value of the  $D \rightarrow \bar{K}\pi\pi$  decay amplitudes we make use of PCAC, and we attribute the enhancement of these amplitudes relative to  $D \rightarrow \bar{K}\pi$  amplitudes to the final state interactions. We suggest an approximate treatment of the problem and show that, even though the parameter dependence is unavoidable, it is possible to obtain the desired enhancement. Some numerical and graphical results are presented.

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## I. INTRODUCTION

### A. A Brief Review of the Current Experimental Status

The presently known charmed particles  $D^0$  and  $D^+$  have been discovered by the SLAC-LBL group as peaks in  $\bar{K}\pi$ ,  $\bar{K}\pi\pi$ ,  $\bar{K}\pi\pi\pi$ , invariant masses coming from  $e^+e^-$  annihilation.<sup>1</sup> Evidence for the charmed-strange meson  $F^+$  is accumulating, but it is not yet conclusive.<sup>2</sup>

Experimentally  $D^0 \rightarrow K^- \pi^+$ ,  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$ ,  $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ ,  $D^+ \rightarrow \bar{K}^0 \pi^+$ ,  $D^+ \rightarrow K^- \pi^+ \pi^+$  are the only established decay modes of  $D^0$  and  $D^+$ .<sup>3</sup> It is also known that the leptonic and semileptonic decays occur about 10 to 20% of the time, but the exact decay modes have not yet been determined.<sup>4</sup>

In an analysis carried out by the SLAC-LBL group consistency of the data with the 0-spin assignment is reported.<sup>5</sup> The same analysis also confirmed the 1-spin assignment for the  $D^{0*}$  and  $D^{+*}$ . The study of the Dalitz plot by the same group has shown that the spin-parity of the final state in  $D^+ \rightarrow K^- \pi^+ \pi^+$  decay is not compatible with  $J^P = 1^-$ , or  $2^+$ . On the other hand, clearly, three pseudo-scalars cannot be in  $J^P = 0^+$ . Since  $D^+ \rightarrow \bar{K}^0 \pi^+$  decay has a final state of spin-parity  $0^+$ ,  $1^-$ ,  $2^+$ , ... it is obvious that unless  $D$  is of spin 3 or higher, the existence of both modes  $D^+ \rightarrow \bar{K}^0 \pi^+$ ,  $D^+ \rightarrow K^- \pi^+ \pi^+$  implies parity violation, and establishes that the decay must be weak. The corresponding analysis of the Dalitz plot for the  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$  decay has not yet been done, but the most reasonable assumption that  $D^+$  and  $D^0$  are iso-doublets would suffice to extend the same conclusion to  $D^0$  decays as well.

The question of  $\bar{D}^0 - D^0$  mixing has not been settled yet. If one assumes that the fourth flavor carried by  $D^0$  and  $D^+$  is the same flavor originally conjectured by Bjorken and Glashow, one expects that  $D^+ \rightarrow K^- \pi^+ \pi^+$  should go but

$D^+ \rightarrow K^+ \pi^+ \pi^-$  must be forbidden.<sup>6</sup> Furthermore the decays that do not involve a  $\bar{K}$  must be suppressed by the Cabibbo angle. The experimental evidence supports both of these predictions, and limits the possible weak  $D^0 - \bar{D}^0$  mixing to be small; however, the data is not very accurate and conclusive yet.<sup>2</sup>

The analysis of the Dalitz plots has also established that the  $\bar{K}^* \pi$ , or  $\bar{K} \rho$  contributions to the  $\bar{K} \pi \pi$  final state are negligible.<sup>7</sup> In the Dalitz plots no other intermediate resonant states are observed. The present data are not sufficient to determine the variation in the Dalitz plot accurately, but the obvious feature is that such a variation must be fairly smooth.

Even though the parity violation is well established, the present data, including the leptonic or semileptonic decays, is insufficient to determine the form of the interaction. It is consistent with V-A, but not good enough to rule out other possibilities.

The final piece of experimental evidence relevant to our paper is the enhancement of the three and four body non-leptonic final states over the two body non-leptonic final states. As given by G. J. Feldman, the relative decay rates are listed in Table 1.<sup>8</sup>

## B. The Method

In this paper we will attempt to interrelate various non-leptonic low multiplicity decays of the charmed mesons.

Because the Dalitz plots seem to be fairly smooth, the simple assumption that the decay matrix element is constant over the Dalitz plot is naturally the first thing to try. This matrix element was first estimated by Gaillard, Lee, and Rosner,<sup>9</sup> J. D. Jackson,<sup>10</sup> and Quigg and Rosner,<sup>11</sup> based on a simple statistical model. As the mass of the D particle approaches infinity, and as the multiplicity of the final state grows ( $N > 4$ ), their approach may be

the best approximation. Nevertheless since the D mass is not all that high, their predictions for low multiplicities ( $N \leq 4$ ) may not be correct. In this paper we will follow an alternate method for the two and three body final states using the standard LSZ reduction technique and the hypothesis of Partially Conserved Axial Current (PCAC).

One may object that the success of the PCAC hypothesis is limited to the low energy phenomena, and yet in the decay of D particles the final products are fairly energetic. However, PCAC need not be interpreted as an approximation to physics for zero momentum. Alternately, we can assume that PCAC is an exact operator equation between the pseudoscalar field and the divergence of the axial current. In this case, all the results obtained by the application of this hypothesis to the standard reduction of scalar matrix elements must be interpreted as exact results. However, the actual computation can only be carried out for zero momentum, which lies outside the physical region. We will call these zero momentum points the PCAC points. The task is then to extrapolate the matrix element computed at the PCAC points to the physical region. There is no proven and universally accepted way of doing this, and without approximations and plausibility arguments the task may be impossible. For low energy phenomena, of course, the matrix element has to be extrapolated to a small nearby region for which a simple linear approximation may work very well, as was the case with  $K \rightarrow \pi\pi\pi$  decays.<sup>12</sup>

Such a linear approximation is again the first thing to try even though for the decay of D particles it is not expected to produce correct results considering the range of extrapolation. The actual calculations showed that the result is about four times smaller in amplitude than the experimental value. To extrapolate the matrix element to the physical region more

realistically the final state interactions must be included. As will be shown in Section II it is possible to employ the final state interactions to obtain the desired enhancement of the matrix element over the physical region without introducing any resonant intermediate states, which seem to be absent experimentally.

In this paper actual numerical results will be obtained for various decays, and wherever the experimental data is available the comparison will be made. Due to the lack of sufficient data for most of the decays that has one or more  $\pi^0$  in the final state, the comparison with the experiment will not be possible for most of the predicted decay modes. For these cases we will compare our results to that of Quigg and Rosner. For several particular decay ratios our prediction will be different by more than 100%.

At the end we provide two appendices. The first one is for the computation of the matrix elements in detail, the next one is for the computation of the Omnés functions relevant to our problem.<sup>13</sup> Most of the numerical part of the work for this paper was done on the IBM/370 computer at SLAC. The numerical integrations were carried out by the use of the Fortran integration routine SHEP.

## II. MATRIX ELEMENTS FOR D AND F DECAYS

### A. The General Assumptions

In the rest of this paper we will assume that the charm selection rule  $\Delta S = \Delta C$  holds for all weak charm decays, and  $\bar{D}^0 - D^0$  mixing is small or vanishing. As mentioned in the introduction, the experimental evidence for this assumption is good but not conclusive. We will further assume that  $D^0$  and  $D^+$  form an isospin doublet and both are assigned  $J^P = 0^-$  spin-parity. We will also assume that

$$\left[ Q_a + Q_a^5, H_w(0) \right] = 0 \quad (1)$$

where  $H_w$  is the weak Hamiltonian density, and  $Q_a$  and  $Q_a^5$  are the weak scalar and pseudo-scalar charges associated with the vector and axial currents respectively. The subscript "a" is the group index. For most purposes this assumption is equivalent to the V-A form of the interaction Hamiltonian.

The PCAC hypothesis will be stated as

$$\partial_\mu A_a^\mu(x) = f_{(a)} m_{(a)}^2 \phi_{(a)}(x) \quad (2)$$

where  $\mu$  is the Lorentz index,  $A_a^\mu$  the axial vector current, and the pseudo-scalar field, where  $m_{(a)}$  is its mass, and  $f_{(a)}$  is the decay constant defined by

$$\left( 0 | A_a^\mu(0) | \phi_b(p) \right) = i \delta_{ab} p^\mu f_{(b)}$$

We will also assume that the final pseudo-scalars in the D decays are all in relative  $L = 0$  states of orbital angular momentum. Due to the angular momentum barrier, we expect this assumption to be true in the majority of the decays. The lack of  $\bar{K}^*\pi$  and  $\bar{K}0$  final states supports our assumption. We must mention that there is evidence that  $D \rightarrow \bar{K}0\pi$  decay may be the dominant

$\overline{K}\pi\pi\pi$  source.<sup>7</sup> The application of the standard reduction technique and PCAC will then imply the existence of  $D \rightarrow \overline{K}\rho$  decay, which is very suppressed. A possible explanation is to assume that  $\rho\pi$  in the final state come from the decay of a more massive particle. Experimentally  $A_2$  is ruled out to a good confidence level, but  $A_1$  and others may account for the data.<sup>7</sup> We emphasize, however, that the s-wave approximation is probably the weakest of all our assumptions. It is conceivable that some unknown dynamical mechanism may enhance the higher partial amplitudes.

With the above assumption then, whenever applicable we will symmetrize the final state with respect to all the pions in it, since the pions must be in relative  $I = 0, 2$  isospin states and not in  $I = 1$ . Some of our results will crucially depend on this symmetrization, and if for some reason the p-waves are enhanced, some of the relative decay rates we obtained may change drastically.

#### B. Reduction of the 3-Body Matrix Element

Define

$$\mathcal{M}^{ij1}(k_1, k_2, k_3; q) \equiv \left( \pi^i(k_1) \pi^j(k_2) \pi^1(k_3) \left| H_w(0) \right| D(p) \right)$$

where  $i, j, 1$  are group (SU(3)) indices, and  $q = p - k_1 \leftarrow -k_2 \leftarrow -k_3$ , which we allow to be non-zero until the end. The standard LSZ reduction yields

$$\mathcal{M}^{ij1}(k_1, k_2, k_3; q) = -i(k_3^2 - m_{(1)}^2) \int d^4x e^{ik_3 \cdot x} (\pi^i(k_1) \pi^j(k_2) \left| T(\phi_1(x) H_w(0)) \right| D(p))$$

which with the help of PCAC becomes

$$\begin{aligned} \mathcal{M}^{ij1}(k_1, k_2, k_3; q) = & \frac{i(k_3^2 - m_{(1)}^2)}{f_{(1)} m_{(1)}^2} \left\{ (\pi^i(k_1) \pi^j(k_2) \left| [Q_1^5(0), H_w(0)] \right| D(p)) \right. \\ & + ik_3^\mu \int d^4x e^{ik_3 \cdot x} (\pi^i(k_1) \pi^j(k_2) \left| T(A_\mu^1(x) H_w(0)) \right| D(p)) \\ & \left. + \text{Possible Schwinger Terms...} \right\} \end{aligned}$$

The reason for writing this equation in full is to emphasize that we consider it exact. Then we take the  $k_3 \rightarrow 0$  limit and use Eq. (1) to get

$$\mathcal{M}^{ij1}(k_1, k_2, 0; q) = \frac{i}{f(1)} (\pi^i(k_1) \pi^j(k_2) | [Q^1, H_w(0)] | D(p)) \quad (3)$$

The individual matrix elements for each one of the possible five decays of  $D^+$  and  $D^0$  are given by Eq. (A1) in Appendix A.

Because we assume that the pions are in  $I = 0, 2$  iso-spin states, we have to symmetrize the matrix element with respect to the final pions. Then we define the symmetrized matrix elements  $\mathcal{N}$ :

$$\mathcal{N}(k_1, k_2, k_3; q) = \begin{cases} \mathcal{M}(k_1, k_2, k_3; q) & \text{if the pions are identical.} \\ (\mathcal{M}(k_1, k_2, k_3; q) + \mathcal{M}(k_1, k_2, k_3; q)) / \sqrt{2} & \text{otherwise.} \end{cases}$$

Since the pions are treated symmetrically, taking both of them out of the matrix element and letting their momenta go to zero will not produce any non-trivial relations because we can take  $m_\pi^2 \approx 0$ . This point will be clearly explained in the next section. Again a full account of the PCAC relations when both pions are taken out of the matrix element are given in Appendix A.

Even though the kaon PCAC relations are not considered as reliable as those of pions, the case for which the kaon is taken out of the matrix element is also worked out in Appendix A. At the end, we will make use of these as well as the pion PCAC relations.

The equations (A2) and (A4) in Appendix A are particularly important.

They state that

$$\mathcal{N}^{-++}(k_1, k_2, 0; q) = -\sqrt{2} \mathcal{N}^{0+-}(k_1, k_2, 0; q) \quad (4)$$

If we make the roughest assumption now, that the value of the matrix element computed at the pion PCAC points is approximately equal to the value of the



matrix element over the actual Dalitz plot, using Eq. (4) we reach the following two conclusions, both of which are in disagreement with experiment:

The first prediction is that the  $D^+ \rightarrow K^- \pi^+ \pi^+$  rate must be twice as large as that of  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$ . The experimental number is clearly not 2 but about  $0.85 \pm 0.30$ .<sup>8</sup> Secondly we predict that the ratio of the  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$  rate to that of  $D^0 \rightarrow K^- \pi^+$  must be less than 0.4 compared to the experimental value  $1.75 \pm 0.70$ .<sup>8</sup> It is clear, therefore, that some mechanism of enhancement is needed.

The simplest way of achieving such an enhancement is by means of one or more intermediate resonant states. Experimental evidence, however, clearly negates this possibility. On the other hand, the final state interactions can conceivably produce such an enhancement without going through a resonance. Unfortunately, there is no unique, relativistic formulation of the final state interactions for three particles. We will assume the following simple approximate form for the matrix element

$$\mathcal{N}(k_1, k_2, k_3 ; q) \approx R_{\bar{K}\pi}^{\pi}(s_{12}, t_3) + R_{\bar{K}\pi}^{\pi}(s_{13}, t_2) + R_{\pi\pi}^{\bar{K}}(s_{23}, t_1) \quad (5)$$

where the R amplitudes will be separated to their isospin parts  $R_{I_1}^{I_2}$ ,  $I_1$  being the total isospin of lower two particles and  $I_2$  being the isospin of the  $\bar{K}\pi\pi$  system, and  $s_{ij} = (k_i + k_j)^2$  for  $i \neq j$ , and  $t_i = q \cdot k_i$ ,  $i, j = 1, 2, 3$ .

Each amplitude R has the form

$$R_{I_1}^{I_2}(s, t) = N_{I_1}^{I_2}(s, t) / D_{I_1}(s) \quad (6)$$

where the numerator is presumably a smooth function of s and t, and the denominator is the corresponding Omnés function. (See Appendix B.) This form of the amplitude is not strictly correct. Above all, it violates

unitarity. However, in various situations it provides a fair approximation to the actual scattering amplitude, and it is the only form that is easy to manipulate without trying to solve several coupled integro-differential equations.<sup>13</sup> In the actual application to our problem we will approximate the numerator as a simple linear polynomial, and hope that it provides a reasonable approximation over the energy range we are concerned with. The coefficients of the linear polynomials for each different amplitude will be treated as unknowns to be determined by the value of the amplitude computed at PCAC points.

Unfortunately, as it stands, the problem cannot be solved without additional assumptions. Assuming that the Hamiltonian takes the most general form introduces too many amplitudes (hence too many coefficients to be determined in the numerator). At this point we found it impossible to continue without introducing the extra assumption that the Hamiltonian is mainly of  $\Delta I = 1$ ,  $\Delta I_3 = 1$  type; the  $\Delta I = 2$ ,  $\Delta I_3 = 1$  part of it is either very small or vanishing.

This is not an "ad hoc" assumption at all. It can be justified by the current-current form of the Hamiltonian.

If the Hamiltonian density is of the current-current form, the  $\Delta I = 2$  part of the Hamiltonian vanishes identically. The current-current form of the Hamiltonian may not be strictly correct; nevertheless, in a lot of cases we know that it provides a good approximation. We also have to emphasize that the  $\Delta I = 2$  part of the Hamiltonian may be negligible even if the form of the Hamiltonian is not current-current type. PCAC itself is very suggestive for the  $\Delta I = 1$  assumption. (See Appendix A.)

We use the  $\Delta I = 1$  hypothesis to write

$$[I^+, H_w(0)] = 0 \quad (7)$$

By manipulating the isospin raising and lowering operators we get

$$\begin{aligned} (\bar{K}^0 \pi^+ |_{H_w} | D^+) &= -(\bar{K}^0 \pi^+ | [H_w, I^+] | D^0) + (K^- \pi^+ |_{H_w} | D^0) \\ &+ \sqrt{2} (\bar{K}^0 \pi^0 |_{H_w} | D^0) \end{aligned}$$

which implies then

$$T^{0+} - T^{-+} - \sqrt{2} T^{00} = 0$$

where the notation refers to the charges of the final particles starting with the charge of  $\bar{K}$ . (See Eq. (A1) in Appendix A.)

Having limited the form of the amplitude we can proceed immediately to compute the  $D \rightarrow \bar{K} \pi \pi$  matrix elements. However, we would like to postpone this for a while and study the immediate consequences of the  $\Delta I = 1$  assumption. As we will see, this one turns out to be a very powerful assumption in its predictive power.

### C. Immediate Consequences of $\Delta I = 1$ Assumption

In the following section we treat  $(K^-, \bar{K}^0)$  and  $(D^0, D^+)$  as isospin anti-doublets rather than doublets. This way our phase convention matches that of standard  $SU(N)$ . The same convention also requires  $(\pi^-, \pi^0, \pi^+)$  be an isotriplet with the following signs for the raising and lowering operators:  $I^- |\pi^+\rangle = -\sqrt{2} |\pi^0\rangle$ ,  $I^- |\pi^0\rangle = \sqrt{2} |\pi^-\rangle$ ,  $I^+ |\pi^-\rangle = \sqrt{2} |\pi^0\rangle$ , and  $I^+ |\pi^0\rangle = -\sqrt{2} |\pi^+\rangle$ . For the group  $SU(2)$  alone, admittedly, this is a highly unorthodox convention because a doublet is unitarily equivalent to an anti-doublet; however, it is easier to manipulate when extensions to  $SU(3)$  and  $SU(4)$  are made, for which  $N$ -plets and anti- $N$ -plets are no longer equivalent.

i) Starting from

$$(\bar{K}^0 \pi^+ \pi^+ |_{H_w} | I^+ | D^+) = 0$$

and manipulating the  $I^\pm$  operators and using Eq. (7) we get

$$0 = -\sqrt{2} \left\{ (\bar{K}^0 \pi^+ \pi^0 | H_w | D^+) + (\bar{K}^0 \pi^0 \pi^+ | H_w | D^+) \right\} - (K^- \pi^+ \pi^+ | H_w | D^+)$$

which implies

$$\mathcal{N}^{0+0}(k_1, k_2, k_3; q) = -\frac{1}{2} \mathcal{N}^{-++}(k_1, k_2, k_3; q) \quad (9)$$

where the amplitude is symmetrized with respect to both the pions and labeled by the charges of the final particles starting from  $\bar{K}$ .

The manipulation of  $I^\pm$  operators with other amplitudes immediately produces two more similar relations

$$\mathcal{N}^{-+0}(k_1, k_2, k_3; q) = \frac{1}{2} \mathcal{N}^{-++}(k_1, k_2, k_3; q) \quad (10)$$

$$\mathcal{N}^{000}(k_1, k_2, k_3; q) = \frac{1}{2} \left\{ \sqrt{2} \mathcal{N}^{0+-}(k_1, k_2, k_3; q) - \mathcal{N}^{-++}(k_1, k_2, k_3; q) \right\} \quad (11)$$

We no longer have five different amplitudes; we can express the three amplitudes containing a final  $\pi^0$  in terms of the other two.

ii) We can use the same method for the four particle final states as well. There are seven  $D \rightarrow \bar{K} \pi \pi \pi$  decays, five of which can be eliminated in terms of only two of them. The results are

$$\mathcal{N}^{-++0} = 0$$

$$\mathcal{N}^{0+00} = \frac{1}{2} \mathcal{N}^{0++-}$$

$$\mathcal{N}^{-+00} = \frac{1}{2} \mathcal{N}^{-++-}$$

$$\mathcal{N}^{0+-0} = \frac{1}{2} (\mathcal{N}^{0++-} - \mathcal{N}^{-++-})$$

$$\mathcal{N}^{0000} = \frac{1}{2} \sqrt{\frac{3}{2}} (\mathcal{N}^{0++-} - \mathcal{N}^{-++-})$$

where again the labels in the amplitudes refer to the charges of the final particles starting from  $\bar{K}$ .

iii) The application of this assumption and technique to the  $D \rightarrow \bar{K}\eta\pi$  decays gives

$$\mathcal{N}_{(\eta)}^{0+} - \mathcal{N}_{(\eta)}^{-+} = \sqrt{2} \mathcal{N}_{(\eta)}^{00} \quad (13)$$

with obvious notation.

iv) Similar to the case of  $D \rightarrow \bar{K}\pi\pi$  decays, the application of the  $\Delta I = 1$  hypothesis eliminates the three of the five  $D \rightarrow \bar{K}\eta\pi\pi$  amplitudes in terms of two of them

$$\begin{aligned} \mathcal{N}_{(\eta)}^{0+0} &= -\frac{1}{2} \mathcal{N}_{(\eta)}^{-++} \\ \mathcal{N}_{(\eta)}^{-+0} &= \frac{1}{2} \mathcal{N}_{(\eta)}^{-++} \\ \mathcal{N}_{(\eta)}^{000} &= \frac{1}{2} (\sqrt{2} \mathcal{N}_{(\eta)}^{0+-} - \mathcal{N}_{(\eta)}^{-++}) \end{aligned} \quad (14)$$

v)  $F^+ \rightarrow \bar{K}(\pi\pi)$  decays are Cabibbo suppressed, and we do not discuss them here.  $F^+ \rightarrow \pi^+\pi^0$  is not allowed by isospin. So we start with  $F \rightarrow \pi\pi\pi$  for the F decays. In this case, starting from

$$(\pi^+\pi^+\pi^0 | H I^+ | F^+) = 0$$

we obtain

$$\mathcal{N}_{F^+ \rightarrow \pi^+\pi^+\pi^-} = 2 \mathcal{N}_{F^+ \rightarrow \pi^+\pi^0\pi^0} \quad (15)$$

The two  $F \rightarrow \pi\pi\pi\pi$  decays can be related similarly

$$\sqrt{3} \mathcal{N}_{F^+ \rightarrow \pi^+\pi^0\pi^0\pi^0} = \mathcal{N}_{F^+ \rightarrow \pi^+\pi^+\pi^0\pi^-} = 0 \quad (16)$$

where to get the correct factor, symmetrization with respect to all pions was carried out. Note that the above decays have to vanish as s-wave amplitudes, but they can proceed through p-wave or higher channels.

vi) For the three possible  $F \rightarrow K \bar{K} \pi$  we find the relation

$$(K^0 \bar{K}^0 \pi^+ | H_W | F^+) = (K^+ K^- \pi^+ | H_W | F^+) + \sqrt{2} (K^+ \bar{K}^0 \pi^0 | H_W | F^+) \quad (17)$$

and for the five allowed  $F \rightarrow K \bar{K} \pi \pi$  decays we get

$$\begin{aligned} \mathcal{N}^{00+0} &= -\frac{1}{2} \mathcal{N}^{0-++} \\ \mathcal{N}^{+-+0} &= \frac{1}{2} \mathcal{N}^{0-++} \\ \mathcal{N}^{+000} &= \frac{1}{2} (\sqrt{2} \mathcal{N}^{+0+-} - \mathcal{N}^{0-++}) \end{aligned} \quad (18)$$

where now, the labels of the amplitudes refer to the charges starting from K, then followed by  $\bar{K}$  and pions.

vii) The two possible  $F \rightarrow \eta \pi \pi \pi$  decays are of course related the same way as the  $F \rightarrow \pi \pi \pi$  decays:

$$\mathcal{N}_{F^+ \rightarrow \eta \pi^+ \pi^+ \pi^-} = 2 \mathcal{N}_{F^+ \rightarrow \eta \pi^+ \pi^0 \pi^0} \quad (19)$$

At present, experimental data is not available to check most of these predictions. Various decay ratios are summarized in Table 3.

We can compare some of our predictions to those of Quigg and Rosner based on the statistical model.<sup>11</sup>

i) We predict that the  $D^+ \rightarrow K^- \pi^+ \pi^+$  rate will be four times larger than the  $D^+ \rightarrow \bar{K}^0 \pi^+ \pi^0$  rate. Quigg and Rosner predict a ratio of about 65%.

ii) We predict that the  $D^0 \rightarrow K^- \pi^+ \pi^0$  rate will be 4 times smaller than the  $D^+ \rightarrow K^- \pi^+ \pi^+$  rate. Experimental branching ratio for the  $D^0 \rightarrow K^- \pi^+ \pi^0$  decay is too high [ $\%(12 \pm 6)$ ], but it is based on little data.<sup>3</sup> Considering the poor statistics and the fact that it is only two standard deviations away from anything else, we consider the possibility that, with better statistics, a lower rate for this decay will be established in the future experiments.

iii) We predict that the  $D^+ \rightarrow \bar{K}^- \pi^+ \pi^+ \pi^0$  decay rate must be much smaller than other  $D \rightarrow \bar{K} \pi \pi \pi$  decays. This ratio in Quigg and Rosner's paper is in the order of one. Also, similar to the previous case we estimate the rate for  $D^+ \rightarrow \bar{K}^0 \pi^+ \pi^+ \pi^-$  to be four times as large as that of  $D^+ \rightarrow \bar{K}^0 \pi^+ \pi^0 \pi^0$ , where their prediction states a 1.3 times larger rate for the  $D^+ \rightarrow \bar{K}^0 \pi^+ \pi^+ \pi^-$  decay.

iv) In the case of  $F^+$  decays, the ratio of the rates of  $F^+ \rightarrow \pi^+ \pi^+ \pi^-$  and  $F^+ \rightarrow \pi^+ \pi^0 \pi^0$  decays is 4, while Quigg and Rosner predict a number like 1.6. For the  $F^+ \rightarrow \pi^+ \pi^+ \pi^0 \pi^-$ , and  $F^+ \rightarrow \pi^+ \pi^0 \pi^0 \pi^0$  decays we predict a vanishing rate, but their predictions for these channels are comparable to the other decay modes of  $F^+$ .

Clearly then,  $F \rightarrow \pi \pi \pi \pi$  and  $D^+ \rightarrow \bar{K}^- \pi^+ \pi^+ \pi^0$  decays provide a very good test to see if and how much the p-wave and higher partial amplitudes are suppressed. A vanishing or small rate would support the s-wave dominance, while a p-wave enhancement would allow these modes to have comparable rates to the other modes of decay.

#### D. $D \rightarrow \bar{K} \pi \pi$ Matrix Elements and the Enhancement Over the Dalitz Plot

The  $\Delta I = 1$  hypothesis eliminated three of the  $D \rightarrow \bar{K} \pi \pi$  amplitudes that have one or more  $\pi^0$  in the final state in terms of the experimentally observed decays  $D^+ \rightarrow \bar{K}^- \pi^+ \pi^+$  and  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$ . Therefore in this section we only worry about these two modes.

Eq. (5) approximates the amplitude as the sum of three terms. Diagrammatically this can be represented as in Figure 1. Now we can neglect the  $\Delta I = 2$  contribution to the amplitudes, and separate the proper isospin parts to obtain

$$\begin{aligned} \mathcal{A}^{-++}(k_1, k_2, k_3; q) = & -\sqrt{\frac{2}{3}} \left( \bar{R}_{1/2}^{-3/2}(s_{12}, t_3) + \bar{R}_{1/2}^{-3/2}(s_{13}, t_2) \right) - \frac{2}{\sqrt{5}} \bar{R}_2^{-3/2}(s_{23}, t_1) \\ & - \sqrt{\frac{2}{15}} \left( \bar{R}_{3/2}^{-3/2}(s_{12}, t_3) + \bar{R}_{3/2}^{-3/2}(s_{13}, t_2) \right) \end{aligned} \quad (20)$$

$$\begin{aligned}
 \mathcal{N}^{0+-}(k_1, k_2, k_3; q) &= \frac{1}{3} \left( R_{1/2}^{3/2}(s_{12}, t_3) + R_{1/2}^{3/2}(s_{13}, t_2) \right) + \sqrt{\frac{2}{15}} R_2^{3/2}(s_{23}, t_1) \\
 &+ \frac{\sqrt{2}}{3} \left( R_{1/2}^{1/2}(s_{12}, t_3) + R_{1/2}^{1/2}(s_{13}, t_2) \right) + \sqrt{\frac{2}{3}} R_0^{1/2}(s_{23}, t_1) \\
 &+ \frac{2}{3} \left( R_{3/2}^{1/2}(s_{12}, t_3) + R_{3/2}^{1/2}(s_{13}, t_2) \right) + \frac{1}{\sqrt{45}} \left( R_{3/2}^{3/2}(s_{12}, t_3) \right. \\
 &\left. + R_{3/2}^{3/2}(s_{13}, t_2) \right) \tag{21}
 \end{aligned}$$

For the notation see Eq. (5) and (6). Note that  $\bar{R}_{1/2}^{-3/2} \neq R_{1/2}^{3/2}$ ,  $\bar{R}_{3/2}^{-3/2} \neq R_{3/2}^{3/2}$ ,  $\bar{R}_{3/2}^{-2} \neq R_{3/2}^2$ , but of course they are related. To find the relation, imagine two (imaginary) intermediate states Y and U with  $I = 3/2$  and  $I = 1/2$  respectively, as shown diagrammatically in Figure 2. We emphasize that we do not identify Y or U with any physical states. This must be considered no more than a gimmick to make it easy to picture the situation. Then we observe that

$$\begin{aligned}
 (Y^+ | H_w | D^+) &= -(Y^+ | H_w I^+ | D^0) \\
 &= -\sqrt{3} (Y^0 | H_w | D^0)
 \end{aligned}$$

which gives us the desired relations

$$\bar{R}_{3/2}^{-3/2}(s, t) = -\sqrt{3} R_{3/2}^{3/2}(s, t) \tag{22}$$

$$\bar{R}_{1/2}^{-3/2}(s, t) = -\sqrt{3} R_{1/2}^{3/2}(s, t) \tag{23}$$

$$\bar{R}_2^{-3/2}(s, t) = -\sqrt{3} R_2^{3/2}(s, t) \tag{24}$$

Now we are ready to impose the PCAC relations:

We substitute Eqs. (20) through (24) into Eq. (4), and equate the parts with the same denominator (since the denominator is a known but an arbitrary function of  $s_{ij}$ , only those parts of the amplitude with the same denominator,



i.e., the same Omnés function, can add up to zero for all possible values of  $s_{ij}$ ), and obtain

$$R_{3/2}^{3/2}(s_{12}, 0) = -\frac{\sqrt{5}}{2} R_{3/2}^{1/2}(s_{12}, 0) \quad (25)$$

$$R_{1/2}^{3/2}(s_{12}, 0) = -\frac{1}{2\sqrt{2}} R_{1/2}^{1/2}(s_{12}, 0) \quad (26)$$

$$R_o^{1/2}(m_\pi^2, 0) = -\frac{4}{\sqrt{5}} R_2^{3/2}(m_\pi^2, 0) \quad (27)$$

We will only be interested in the matrix elements when the four momentum is conserved, that is  $q = 0$ . Substituting Eqs. (25) and (26) into (20) and (21) we obtain

$$\begin{aligned} \mathcal{N}^{-++}(k_1, k_2, k_3; 0) &= \sqrt{2} (R_{1/2}^{3/2}(s_{12}, 0) + R_{1/2}^{3/2}(s_{13}, 0) + 2\sqrt{\frac{3}{5}} R_2^{3/2}(s_{23}, 0) \\ &\quad + \sqrt{\frac{2}{5}} (R_{3/2}^{3/2}(s_{12}, 0) + R_{3/2}^{3/2}(s_{13}, 0))) \end{aligned} \quad (28)$$

$$\begin{aligned} \mathcal{N}^{o+-}(k_1, k_2, k_3; 0) &= - (R_{1/2}^{3/2}(s_{12}, 0) + R_{1/2}^{3/2}(s_{13}, 0)) + \sqrt{\frac{2}{15}} R_2^{3/2}(s_{23}, 0) \\ &\quad - \frac{1}{\sqrt{5}} (R_{3/2}^{3/2}(s_{12}, 0) + R_{3/2}^{3/2}(s_{13}, 0)) + \sqrt{\frac{2}{3}} R_o^{1/2}(s_{23}, 0) \end{aligned} \quad (29)$$

Now note that the two PCAC conditions (A2) and (A4) in Appendix A are not independent, and they produce a single constraint

$$\begin{aligned} -\sqrt{\frac{5}{2}} \frac{i}{f_\pi} T^{oo} &= (R_{3/2}^{3/2}(m_D^2, 0) + R_{3/2}^{3/2}(m_K^2, 0)) + \sqrt{6} R_2^{3/2}(m_\pi^2, 0) \\ &\quad + \sqrt{5} (R_{1/2}^{3/2}(m_D^2, 0) + R_{1/2}^{3/2}(m_K^2, 0)) \end{aligned} \quad (30)$$

Also note that the equation (A8) derived in the Appendix A implies

$$R_o^{1/2}(0, 0) = -\frac{4}{\sqrt{5}} R_2^{3/2}(0, 0)$$

which we do not consider any different from Eq. (27). Our accuracy in this problem is far worse than  $m_\pi^2/m_D^2$  and we have to take  $m_\pi^2 \approx 0$ . Therefore the PCAC relation when both pions are taken out of the matrix element simultaneously does not give us an extra constraint. Next we use the PCAC relations (A11) and (A12) derived for the kaons in the Appendix A to get

$$R_{3/2}^{3/2}(m_K^2, 0) + \sqrt{5} R_{1/2}^{3/2}(m_K^2, 0) - \frac{1}{\sqrt{6}} R_2^{3/2}(m_D^2, 0) - \sqrt{\frac{5}{6}} R_0^{1/2}(m_D^2, 0) = -\frac{\sqrt{5}}{2} \frac{i}{f_K} T^{-+} \quad (31)$$

$$R_{3/2}^{3/2}(m_K^2, 0) + \sqrt{5} R_{1/2}^{3/2}(m_K^2, 0) + \frac{1}{\sqrt{6}} R_2^{3/2}(m_D^2, 0) - \frac{1}{2} \sqrt{\frac{5}{6}} R^{1/2}(m_D^2, 0) = \frac{1}{2} \sqrt{\frac{5}{2}} \frac{i}{f_K} T^{00} \quad (32)$$

As promised earlier we approximate the numerator function by a linear polynomial

$$\mathcal{N}_{I_1}^{I_2}(s, 0) \approx \left( a_{I_1}^{I_2} s + c_{I_1}^{I_2} \right)$$

and for the individual amplitudes we write

$$R_{I_1}^{I_2}(s, 0) \approx \frac{a_{I_1}^{I_2} s + c_{I_1}^{I_2}}{D_{I_1}(s)}$$

where we suppressed the upper index, for we no longer need it. This way we see that we have 8 coefficients to determine:  $a_0, c_0, a_{1/2}, c_{1/2}, a_{3/2}, c_{3/2}, a_2, c_2$ , but we have only four equations (Eqs. (27), (30), (31), (32)).

Unfortunately the problem is underdetermined.

Since  $I = 3/2$  phase shift is approximately zero,  $D_{3/2}(s) \approx 1$ , and  $R_{3/2}^{3/2}(s, 0)$  amplitude does not get enhanced. Also note that the coefficient of  $a_{3/2}$  and  $c_{3/2}$  relative to  $a_{1/2}$  and  $c_{1/2}$  is  $1/\sqrt{5}$ , which makes it a little less important. Encouraged by these observations, we will approximate

$$R_{3/2}^{3/2}(s, 0) \approx 0$$

This assumption, even though very plausible, is somewhat "ad hoc," for we have not been able to justify it to our satisfaction.

This way, though, we are able to eliminate the variables  $a_{3/2}$  and  $c_{3/2}$ . Four of the remaining six variables can be eliminated in terms of any two of them. We choose to express everything in terms of  $c_0$  and  $c_2$ . First we define the following quantities for shorthand:

$$\begin{aligned} \gamma &= T^{0+}/T^{-+}, \quad \eta \equiv \frac{D_0(0)}{D_2(0)} \frac{D_2(m_D^2)}{D_0(m_D^2)} \\ \epsilon &= \left[ \frac{1}{4} \sqrt{\frac{15}{2}} (1 + \gamma) D_2(m_D^2) + \eta (c_2 - c_0) \right] / (1 - \eta) \\ \beta &\equiv \sqrt{\frac{6}{5}} \left[ \epsilon + c_2 (1 - m_\pi^2/m_D^2) \right] / D_2(m_\pi^2) \\ \alpha &\equiv \frac{1}{2} + \frac{1}{\sqrt{30}} \left[ \epsilon (4\eta - 1) + 4\eta (c_2 - c_0) \right] / D_2(m_D^2) \end{aligned}$$

Then we can write

$$\begin{aligned} -if_{K^0} R_{1/2}^{1/2}(s, 0) &= -\frac{4}{\sqrt{5}} \frac{D_0(0)}{D_2(0)} \frac{c_2 + \epsilon - s c_0/m_D^2}{D_0(s)} T^{-+} \\ -if_{K^2} R_{3/2}^{3/2}(s, 0) &= \frac{C_2 (1 - s/m_D^2) + \epsilon}{D_2(s)} T^{-+} \\ if_{K^1} R_{1/2}^{3/2}(s, 0) &= \frac{T^{-+}}{D_{1/2}(s)} \left\{ \alpha D_{1/2}(m_K^2) + \right. \\ &\left. \frac{m_K^2 - s}{2} \frac{2}{m_D^2 - m_K^2} \left[ \alpha (D_{1/2}(m_K^2) + D_{1/2}(m_D^2)) - D_{1/2}(m_D^2) \left( \beta - \frac{1}{2} \frac{f_K}{f_\pi} (1 - \gamma) \right) \right] \right\} \end{aligned}$$

Clearly  $C_0$  and  $C_2$ , the two free parameters not yet fixed, will take values in the neighborhood of  $D_0(m_D^2)$  and  $D_2(m_D^2)$  respectively. Unfortunately, we do

not have any more constraints to determine them any better. Instead we chose to vary the values of  $C_0$  and  $C_2$  around the expected numbers and see how the results depend on it.

As explained in Appendix B, the  $I = 0$   $\pi\pi$  phase shift seems to approach  $2\pi$  and the corresponding function  $D_0(s)$  is determined up to another free parameter. By the help of the computer we studied the behavior of the final answer on these three parameters. We found that the answer does depend on the parameters, and this dependence is rather sensitive. Variations by a factor of 4 in the rate occur as the parameters are allowed to vary over a reasonable range of different values. However, we were encouraged that by choosing the best value we could find for the variables we obtained

$$\frac{\Gamma(D^+ \rightarrow K^- \pi^+ \pi^+)}{\Gamma(D^0 \rightarrow K^- \pi^+)} \approx 1.38, \quad \frac{\Gamma(D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-)}{\Gamma(D^0 \rightarrow K^- \pi^+)} \approx 2.12$$

with the following values

$$C_2 \approx -0.4$$

$$C_0 \approx 0.1$$

and the Omnés functions shown in Figures 3, 4, and 6. The ratios of the rates for other decays are summarized in Table 2.

Certainly this answer is much more accurate than we have any right to expect. The weakest thing about it is, of course, its dependence on the choice of the parameters. The underdetermined nature of the problem has forced us to guess the value of the parameters, and we have shown that there exist values for which the right enhancement is produced and the ratio of the three body rates to that of two body is increased by a factor of four to six compared to the constant approximation. We admit that this is no

proof that the enhancement mentioned is actually produced by the mechanism we suggested. A similar analysis of the other charmed decays, especially that of  $F^+$ , may provide examples that are not over determined (or just depending on one parameter only). After such analysis is completed and compared to the forthcoming data on  $F^+$ , we hope a better understanding of the problem can be achieved.

### III. CONCLUSION

We conclude that the  $\Delta I = 1$  assumption is a powerful one, and its predictions are summarized in Table 3. The preliminary results based on little data indicate a possible disagreement. The experimental rate for the  $D^0 \rightarrow K^- \pi^+ \pi^0$  seems to be too large compared to both our prediction and that of Reference 1.<sup>3</sup> In any case, more data and better statistics are needed to establish this disagreement, if it exists. Such a disagreement may jeopardize the  $\Delta I = 1$  hypothesis. However, a more likely explanation for a possible disagreement would be the dynamical (by some yet unknown reason) enhancement of the p-wave amplitudes. We must also mention here that the current preliminary experimental rate is not only too high for our s-wave amplitudes, but also for the statistical model prediction by Quigg and Rosner, in which the p-wave amplitudes are certainly not suppressed.

The question of the three body final state enhancement by means of the final state interactions may take a lot longer to settle. Certainly, if the  $\Delta I = 1$  assumption is supported by the data, the manipulation of the final state interactions becomes easier, and there is hope that  $F^+$  decays may help our understanding. If, however, the  $\Delta I = 1$  assumption turns out to be incorrect, the enhancement by means of the final state interactions is still possible and may still be correct; nevertheless, the problem contains very many parameters in this case, and the issue becomes hopelessly complicated and too parameter-dependent for us to deal with.

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APPENDIX A

The PCAC Relations for  $D \rightarrow \bar{K}\pi\pi$  Decays

a) The Pion PCAC Relations

Using Eq. (3) we obtain for all  $D \rightarrow \bar{K}\pi\pi$  decays

$$i\sqrt{2} f_{\pi} \mathcal{M}^{-++}(k_1, k_2, 0; q) = -T^{-+}(k_1, k_2; q) + T^{0+}(k_1, k_2; q)$$

$$\mathcal{M}^{-++}(k_1, 0, k_3; q) = \mathcal{M}^{-++}(k_1, k_3, 0; q)$$

$$i\sqrt{2} f_{\pi} \mathcal{M}^{0+0}(k_1, k_2, 0; q) = -\sqrt{2} T^{0+}(k_1, k_2; q)$$

$$i\sqrt{2} f_{\pi} \mathcal{M}^{0+0}(k_1, 0, k_3; q) = \sqrt{2} T^{0+}(k_1, k_3; q) - T^{00}(k_1, k_3; q)$$

$$i\sqrt{2} f_{\pi} \mathcal{M}^{0+-}(k_1, k_2, 0; q) = -T^{0+}(k_1, k_2; q) + T^{-+}(k_1, k_2; q) + \sqrt{2} T^{00}(k_1, k_2; q)$$

$$i\sqrt{2} f_{\pi} \mathcal{M}^{0+-}(k_1, 0, k_3; q) = -\sqrt{2} T^{00}(k_1, k_3; q) \tag{A1}$$

$$i\sqrt{2} f_{\pi} \mathcal{M}^{-+0}(k_1, k_2, 0; q) = -\sqrt{2} T^{-+}(k_1, k_2; q)$$

$$i\sqrt{2} f_{\pi} \mathcal{M}^{-+0}(k_1, 0, k_3; q) = \sqrt{2} T^{-+}(k_1, k_3; q) + T^{00}(k_1, k_3; q)$$

$$i\sqrt{2} f_{\pi} \mathcal{M}^{000}(k_1, k_2, 0; q) = -\sqrt{2} T^{00}(k_1, k_2; q)$$

$$\mathcal{M}^{000}(k_1, 0, k_3; q) = \mathcal{M}^{000}(k_1, k_3; q)$$

where

$$T^{-+}(k_1, k_2; q) \equiv \left( \bar{K}^{-}(k_1) \pi^{+}(k_2) \left| H_w(0) \right| D^0(p) \right)$$

$$T^{00}(k_1, k_2; q) \equiv \left( \bar{K}^0(k_1) \pi^0(k_2) \left| H_w(0) \right| D^0(p) \right)$$

$$T^{0+}(k_1, k_2; q) \equiv \left( \bar{K}^0(k_1) \pi^{+}(k_2) \left| H_w(0) \right| D^{+}(p) \right)$$



Now we symmetrize these amplitudes with respect to the pions

$$\mathcal{N}^{-++}(k_1, k_2, k_3; q) = \mathcal{M}^{-++}(k_1, k_2, k_3; q)$$

$$\sqrt{2} \mathcal{N}^{0+0}(k_1, k_2, k_3; q) = \mathcal{M}^{0+0}(k_1, k_2, k_3; q) + \mathcal{M}^{0+0}(k_1, k_3, k_2; q)$$

$$\sqrt{2} \mathcal{N}^{0+-}(k_1, k_2, k_3; q) = \mathcal{M}^{0+-}(k_1, k_2, k_3; q) + \mathcal{M}^{0+-}(k_1, k_3, k_2; q)$$

$$\sqrt{2} \mathcal{N}^{-+0}(k_1, k_2, k_3; q) = \mathcal{M}^{-+0}(k_1, k_2, k_3; q) + \mathcal{M}^{-+0}(k_1, k_3, k_2; q)$$

$$\mathcal{N}^{000}(k_1, k_2, k_3; q) = \mathcal{M}^{000}(k_1, k_2, k_3; q)$$

Then the PCAC relations can be written as

$$i\sqrt{2} f_\pi \mathcal{N}^{-++}(k_1, k_2, 0; q) = -T^{-+}(k_1, k_2; q) + T^{0+}(k_1, k_2; q) \quad (\text{A2})$$

$$i\sqrt{2} f_\pi \mathcal{N}^{0+0}(k_1, k_2, 0; q) = -\frac{1}{\sqrt{2}} T^{00}(k_1, k_2; q) \quad (\text{A3})$$

$$i\sqrt{2} f_\pi \mathcal{N}^{0+-}(k_1, k_2, 0; q) = \frac{1}{\sqrt{2}} \left( T^{-+}(k_1, k_2; q) - T^{0+}(k_1, k_2; q) \right) \quad (\text{A4})$$

$$i\sqrt{2} f_\pi \mathcal{N}^{-+0}(k_1, k_2, 0; q) = \frac{1}{\sqrt{2}} T^{00}(k_1, k_2; q) \quad (\text{A5})$$

$$i\sqrt{2} f_\pi \mathcal{N}^{000}(k_1, k_2, 0; q) = -\sqrt{2} T^{00}(k_1, k_2; q) \quad (\text{A6})$$

b) The PCAC Relations for Both Pions Taken Out Simultaneously

The simultaneous treatment of both pions is more complicated than the previous case. The general framework can be found in Reference 14 or in more detail in Reference 15. We will merely state the general result applied to the decays of D mesons

$$\begin{aligned}
\mathcal{M}^{ij}(k_1, 0, 0; q) = & -\frac{i}{2f_{(i)}f_{(j)}} \left\{ \left( \overline{K}(k_1) \left| \left[ Q^j, \left[ Q^i, H_w \right] \right] \right|_{D(p)} \right) \right. \\
& + \left( \overline{K}(k_1) \left| \left[ Q^i, \left[ Q^j, H_w \right] \right] \right|_{D(p)} \right) \\
& - f_{(i)} m_\pi^2 \int d^4x \left( \overline{K}(k_1) \left| T \left( \sigma^{ji}(x) H_w(0) \right) \right|_{D(p)} \right) \\
& \left. - f_{(j)} m_\pi^2 \int d^4x \left( \overline{K}(k_1) \left| T \left( \sigma^{ij}(x) H_w(0) \right) \right|_{D(p)} \right) \right\}
\end{aligned}$$

We will neglect the  $\sigma$  terms since they are proportional to the squared pion mass. Then for individual amplitudes we get

$$f_\pi^2 \mathcal{M}^{-++}(k_1, 0, 0; q) = \left( \overline{K}^0(k_1) \left| H_w(0) \right|_{D^0(p)} \right)$$

$$f_\pi^2 \mathcal{M}^{o+o}(k_1, 0, 0; q) = -\frac{1}{2\sqrt{2}} \left( \overline{K}^0(k_1) \left| H_w(0) \right|_{D^0(p)} \right)$$

$$f_\pi^2 \mathcal{M}^{o+-}(k_1, 0, 0; q) = -\frac{1}{2} \left( \overline{K}^0(k_1) \left| H_w(0) \right|_{D^0(p)} \right)$$

$$f_\pi^2 \mathcal{M}^{-+o}(k_1, 0, 0; q) = \frac{1}{2\sqrt{2}} \left( \overline{K}^0(k_1) \left| H_w(0) \right|_{D^0(p)} \right)$$

$$f_\pi^2 \mathcal{M}^{ooo}(k_1, 0, 0; q) = -\left( \overline{K}^0(k_1) \left| H_w(0) \right|_{D^0(p)} \right)$$

After symmetrization this can be written as

$$\mathcal{N}^{o+o}(k_1, 0, 0; q) = -\frac{1}{2} \mathcal{N}^{-++}(k_1, 0, 0; q) \quad (\text{A7})$$

$$\mathcal{N}^{o+-}(k_1, 0, 0; q) = -\frac{1}{\sqrt{2}} \mathcal{N}^{-++}(k_1, 0, 0; q) \quad (\text{A8})$$

$$\mathcal{N}^{-+o}(k_1, 0, 0; q) = \frac{1}{2} \mathcal{N}^{-++}(k_1, 0, 0; q) \quad (\text{A9})$$

$$\mathcal{N}^{ooo}(k_1, 0, 0; q) = -\mathcal{N}^{-++}(k_1, 0, 0; q) \quad (\text{A10})$$

As we see here, PCAC strongly suggests the  $\Delta I = 1$  hypothesis. Actually with the  $\Delta I = 1$  assumption, Eqs. (A7) and (A9) become trivial.

c) Kaon PCAC Relations

As in section (a), we immediately obtain

$$\text{if}_K \mathcal{N}^{\circ+-}(0, k_2, k_3; q) = -\frac{1}{2} \left( T^{-+}(k_2, k_3; q) + T^{-+}(k_3, k_2; q) \right) \quad (\text{A11})$$

$$\text{if}_K \mathcal{N}^{\circ\circ\circ}(0, k_2, k_3; q) = \frac{1}{2} \left( T^{\circ\circ}(k_2, k_3; q) + T^{\circ\circ}(k_3, k_2; q) \right) \quad (\text{A12})$$

$$\text{if}_K \mathcal{N}^{-++}(0, k_2, k_3; q) = -\frac{1}{\sqrt{2}} \left( T^{\circ+}(k_2, k_3; q) + T^{\circ+}(k_3, k_2; q) \right) \quad (\text{A13})$$

Note that only two of these equations are independent. When substituted into Eqs. (28) and (29), they yield

$$T^{-+} - T^{\circ+} + \sqrt{2} T^{\circ\circ} = 0$$

which is identical to Eq. (8).

APPENDIX B

Final State Interactions and Omnés Functions

For a good review of this subject and an extensive list of references, see J. L. Basdevant.<sup>16</sup>

a) Omnés Functions

Suppose we would like to find an analytic function  $A(s)$  in the complex  $s$ -plane cut along the real axis from  $s_0$  to  $+\infty$ , such that

i) The phase of  $A(s + i\epsilon)$  is a known function  $\delta(s)$  as  $\epsilon \rightarrow 0^+$  along the cut,

ii)  $A(s)$  is real on an open interval on the real axis that lies to the left of the point  $s_0$ ,

iii)  $A(s_0) = 1$

iv)  $\lim_{s \rightarrow \infty} |A(s)| = 1$ .

For real  $s$ , it turns out that the solution can be written as

$$A(s) = \frac{Q_m(s)}{|s-s_0|^m} e^{i\delta(s)\phi(s-s_0)} \exp \left\{ \frac{1}{\pi} \lim_{\epsilon \rightarrow 0^+} \int_{s_0}^{\infty} (\delta(s') - m\pi) \frac{(s'-s)ds'}{(s'-s)^2 + \epsilon^2} \right\}$$

where  $Q_m(s)$  is a polynomial of degree  $m \in \mathbb{N}$  with unit leading coefficient. Other coefficients in  $Q_m(s)$  must be fixed such that  $A(s_0) = 1$ . For analyticity we also have to have  $m = \lim_{s \rightarrow \infty} \delta(s)/\pi$ . Without this condition the problem cannot be solved.

We will define

$$D(s) = 1/A(s)$$

For K scattering  $\lim_{s \rightarrow \infty} \delta^{1/2}(s) = \pi$ , hence  $D_{1/2}(s)$  is fully determined.

Same is true for  $I = 2\pi$  scattering and  $D_2(s)$ . For  $I = 0\pi$  scattering,

however,  $\lim_{s \rightarrow \infty} \delta^0(s) = 2\pi$  and  $Q_2(s)$  is not uniquely fixed by the requirement

$A(s_0) = 1$ . The  $I = 3/2$  part of  $K\pi$  scattering is trivial since  $\delta^{3/2}(s) \approx 0$ , hence  $D_{3/2}(s) \approx 1$ . In all of these applications  $s_0$  is the invariant mass square at the threshold.

b) Elastic and Inelastic Scattering:

Consider the  $D \rightarrow \bar{K}\pi$ ,  $D \rightarrow \pi\pi$ , and  $D \rightarrow K\bar{K}$  amplitudes. Separating the isospin parts and using unitarity we immediately obtain for the  $I = 1/2$  and  $I = 3/2$   $D \rightarrow \bar{K}\pi$  decay amplitudes

$$T_{1/2} \cong \left| T_{1/2} \right| \exp(i\delta^{1/2}(s))$$

$$T_{3/2} \cong \left| T_{3/2} \right| \exp(i\delta^{3/2}(s))$$

Therefore  $T_{1/2}$  and  $T_{3/2}$  are given by the inverse of the corresponding Omnés functions.  $I = 2$   $D \rightarrow \pi\pi$  gives similarly

$$T_2 \cong |T_2| \exp(i\delta^2(s))$$

However,  $I = 0$   $\pi\pi$  or  $K\bar{K}$  final state is fairly inelastic, especially around  $S^*$ . The transition  $\pi\pi \rightarrow K\bar{K}$  can proceed strongly. In this case the transition amplitude is

$$u \cong \sqrt{1-\eta^2} \exp \left[ i(\delta_{\pi\pi}^0 + \delta_{K\bar{K}}^0) \right]$$

Writing

$$T_0^{\pi\pi} = |T_0| e^{i\phi}$$

$$T_0^{K\bar{K}} = |T_0'| e^{i\phi'}$$

we obtain

$$\left( \eta - e^{2i(\phi - \delta_{\pi\pi}^0)} \right) \left( \eta - e^{2i(\phi' - \delta_{K\bar{K}}^0)} \right) = 1 - \eta^2$$

which has solutions

$$\phi = \delta_{\pi\pi}^0 + \frac{1}{2} \text{arc cos}(\eta)$$

$$\phi' = \delta_{\overline{K}K}^0 + \frac{1}{2} \text{arc cos}(\eta)$$

Therefore even in the inelastic case it is possible to define a modified Omnés function by redefining the phase shift. For our approximate treatment of the  $D \rightarrow \overline{K}\pi\pi$  decay the usage of this effective phase shift will suffice, and we will not attempt to treat the inelastic problem with any more rigor than this.

c) Three Body Final States

The general treatment is very difficult, and it is never used in this paper. The treatment of the case where particles interact two at a time, but never all three together, can be reduced to the linear approximation in terms of two particle amplitudes we used.

d) Actual Omnés Functions for  $I = 1/2 K\pi$ ,  $I = 2\pi\pi$ , and  $I = 0\pi\pi$   
Scattering

Using the known phase shifts and the  $\pi\pi$  inelasticity for  $I = 0$ , we produced the graphs given in Figures 3 through 7 by means of numerical integration. The best values for the enhancement of  $D \rightarrow \overline{K}\pi\pi$  decays involve Figure 6 for  $I = 0\pi\pi$  scattering. In Figures 9 through 20 we plotted the variation of the full matrix element on the selected axes shown in Figure 9 to exhibit the enhancement without any resonant intermediate states.

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which has solutions

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$$\phi' = \delta_{KK}^0 + \frac{1}{2} \arccos(\eta)$$

Therefore even in the inelastic case it is possible to define a modified Omnés function by redefining the phase shift. For our approximate treatment of the  $D \rightarrow \bar{K}\pi\pi$  decay the usage of this effective phase shift will suffice, and we will not attempt to treat the inelastic problem with any more rigor than this.

#### c) Three Body Final States

The general treatment is very difficult, and it is never used in this paper. The treatment of the case where particles interact two at a time, but never all three together, can be reduced to the linear approximation in terms of two particle amplitudes we used.

#### d) Actual Omnés Functions for $I = 1/2 K\pi$ , $I = 2\pi\pi$ , and $I = 0\pi\pi$

##### Scattering

Using the known phase shifts and the  $\pi\pi$  inelasticity for  $I = 0$ , we produced the graphs given in Figures 3 through 7 by means of numerical integration. The best values for the enhancement of  $D \rightarrow \bar{K}\pi\pi$  decays involve Figure 6 for  $I = 0\pi\pi$  scattering. In Figures 9 through 20 we plotted the variation of the full matrix element on the selected axes shown in Figure 9 to exhibit the enhancement without any resonant intermediate states.



Table 1

$\sigma \cdot B$  in nb for Various D Decay Modes at Three Values of  $E_{c.m.}$

Mode	$E_{c.m.}$ (GeV)		
	3.774	4.028	4.414
$D^0 \rightarrow \bar{K}^+ \pi^-$	$0.27 \pm 0.05$	$0.57 \pm 0.11$	$0.30 \pm 0.09$
$\bar{K}^0 \pi^+ \pi^- + c.c.$	$0.44 \pm 0.11$	$1.09 \pm 0.30$	$0.91 \pm 0.34$
$\bar{K}^+ \pi^- \pi^+ \pi^-$	$0.34 \pm 0.09$	$0.83 \pm 0.27$	$0.91 \pm 0.39$
$\pi^+ \pi^-$	--	$<0.04$	--
$K^+ K^-$	--	$<0.04$	--
Total $D^0$ observed modes	$1.15 \pm 0.15$	$2.49 \pm 0.42$	$2.42 \pm 0.53$
$D^+ \rightarrow \bar{K}^0 \pi^+ + c.c.$	$0.15 \pm 0.05$	$<0.18$	--
$\bar{K}^+ \pi^- \pi^+$	$0.34 \pm 0.05$	$0.40 \pm 0.10$	$0.33 \pm 0.12$
$\pi^+ \pi^- \pi^+$	--	0.03	--

Mode	Branching Fraction (%)
$D^0 \rightarrow \bar{K}^- \pi^+$	$2.2 \pm 0.6$
$\bar{K}^0 \pi^+ \pi^-$	$3.5 \pm 1.1$
$\bar{K}^- \pi^+ \pi^- \pi^+$	$2.7 \pm 0.9$
$D^+ \rightarrow \bar{K}^0 \pi^+$	$1.5 \pm 0.6$
$\bar{K}^- \pi^+ \pi^+$	$3.5 \pm 0.9$

Table 2

The Predicted Rates for  $D \rightarrow \bar{K}\pi\pi$  Decays with  $C_2 \approx -0.4$ ,  $C_0 \approx 0.1$ , and the Omnés Functions Given in Figures 3, 4, and 6.

All rates are normalized to the  $D^0 \rightarrow \bar{K}^- \pi^+$  rate.

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Decay Rate For	Computed Relative Rate
$(D^0 \rightarrow \bar{K}^- \pi^+)$	1
$(D^+ \rightarrow \bar{K}^- \pi^+ \pi^+)$	1.38
$(D^+ \rightarrow \bar{K}^0 \pi^+ \pi^0)$	0.35
$(D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-)$	2.12
$(D^0 \rightarrow \bar{K}^- \pi^+ \pi^0)$	0.35
$(D^0 \rightarrow \bar{K}^0 \pi^0 \pi^0)$	0.59

---

Table 3

The Constraints on Various Amplitudes Imposed by  $\Delta I = 1$  hypothesis

Decay Mode	Decay Rate
$D^0 \rightarrow K^- \pi^+$	$ a_1 ^2$
$D^+ \rightarrow \bar{K}^0 \pi^+$	$ b_1 ^2$
$D^0 \rightarrow \bar{K}^0 \pi^0$	$ a_1 - b_1 ^2/2$
$D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$	$ a_2 ^2$
$D^+ \rightarrow K^- \pi^+ \pi^+$	$ b_2 ^2$
$D^+ \rightarrow \bar{K}^0 \pi^+ \pi^0$	$ b_2 ^2/4$
$D^0 \rightarrow K^- \pi^+ \pi^0$	$ b_2 ^2/4$
$D^0 \rightarrow \bar{K}^0 \pi^0 \pi^0$	$ \sqrt{2} a_2 - b_2 ^2/4$
$D^0 \rightarrow K^- \pi^+ \eta$	$ a_3 ^2$
$D^+ \rightarrow \bar{K}^0 \pi^+ \eta$	$ b_3 ^2$
$D^0 \rightarrow \bar{K}^0 \pi^0 \eta$	$ a_3 - b_3 ^2/2$
$D^+ \rightarrow K^- \pi^+ \pi^+ \pi^0$	0
$D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$	$ a_4 ^2$
$D^+ \rightarrow \bar{K}^0 \pi^+ \pi^+ \pi^-$	$ b_4 ^2$
$D^+ \rightarrow \bar{K}^0 \pi^+ \pi^0 \pi^0$	$ b_4 ^2/4$
$D^0 \rightarrow K^- \pi^+ \pi^0 \pi^0$	$ b_4 ^2/4$

Table 3 (cont.)

Decay Mode	Decay Rate
$D^0 \rightarrow \bar{K}^0 \pi^+ \pi^- \pi^0$	$ a_4 - b_4 ^2/4$
$D^0 \rightarrow \bar{K}^0 \pi^0 \pi^0 \pi^0$	$3 a_4 - b_4 ^2/8$
$F^+ \rightarrow \pi^+ \pi^0$	0
$F^+ \rightarrow \pi^+ \pi^+ \pi^-$	$ c_1 ^2$
$F^+ \rightarrow \pi^+ \pi^0 \pi^0$	$ c_1 ^2/4$
$F^+ \rightarrow K^+ K^- \pi^+$	$ c_2 ^2$
$F^+ \rightarrow K^0 \bar{K}^0 \pi^+$	$ d_2 ^2$
$F^+ \rightarrow K^+ \bar{K}^0 \pi^0$	$ c_2 - d_2 ^2/2$
$F^+ \rightarrow \pi^+ \pi^+ \pi^0 \pi^-$	0
$F^+ \rightarrow \pi^+ \pi^0 \pi^0 \pi^0$	0
$F^+ \rightarrow \pi^+ \pi^+ \pi^- \eta$	$ c_3 ^2$
$F^+ \rightarrow \pi^+ \pi^0 \pi^0 \eta$	$ c_3 ^2/4$
$F^+ \rightarrow K^+ \bar{K}^0 \pi^+ \pi^-$	$ c_4 ^2$
$F^+ \rightarrow K^0 \bar{K}^- \pi^+ \pi^+$	$ d_4 ^2$
$F^+ \rightarrow K^+ \bar{K}^- \pi^+ \pi^0$	$ d_4 ^2/4$
$F^+ \rightarrow K^0 \bar{K}^0 \pi^+ \pi^0$	$ d_4 ^2/4$
$F^+ \rightarrow K^+ \bar{K}^0 \pi^0 \pi^0$	$ \sqrt{2} c_4 - d_4 ^2/4$

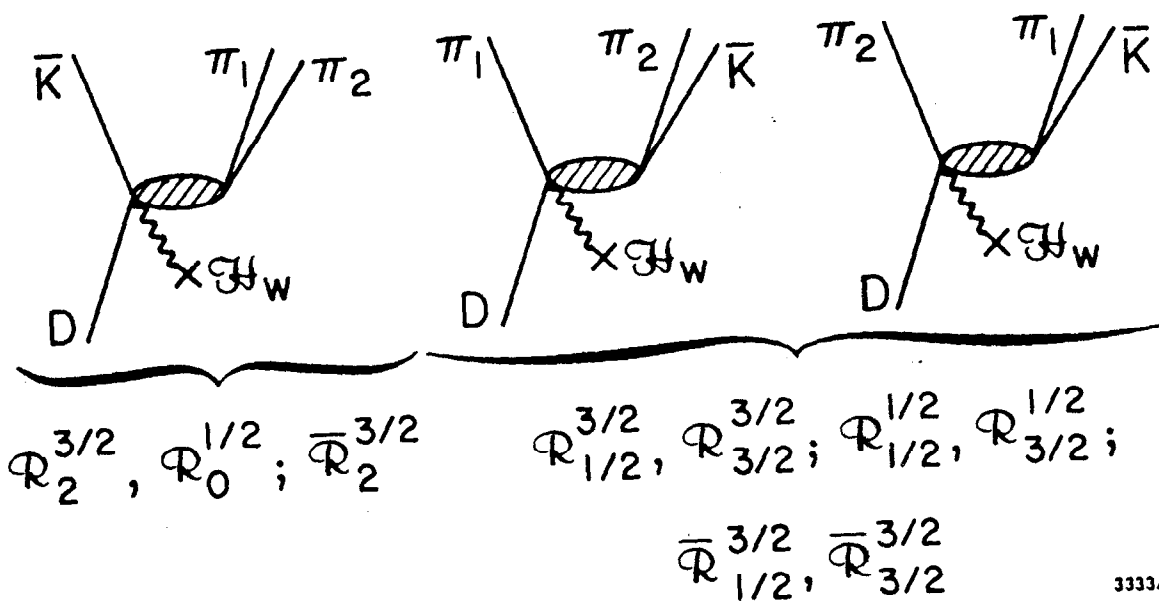
FIGURE CAPTIONS

1. Approximate form of the three body amplitude assuming that the final particles interact two at a time, but not all three together.
2. Equations (22) through (24) can be deduced by assuming the reaction proceeds through the imaginary intermediate states Y and U.
3. Numerically computed Omnés function for  $I=1/2$   $K\pi$  scattering. The solid, dashed, and dotted curves correspond to  $\text{Re}(1/D(s))$ ,  $\text{Im}(1/D(s))$ , and  $1/|D(s)|$  respectively.
4. Numerically computed Omnés function for  $I=2$   $\pi\pi$  scattering. The solid, dashed, and dotted curves correspond to  $\text{Re}(1/D(s))$ ,  $\text{Im}(1/D(s))$ , and  $1/|D(s)|$  respectively.
5. Numerically computed Omnés function for  $I=0$   $\pi\pi$  scattering. The solid, dashed, and dotted curves correspond to  $\text{Re}(1/D(s))$ ,  $\text{Im}(1/D(s))$ , and  $1/|D(s)|$  respectively. The coefficient of  $s$  in  $Q_2(s)$  is  $A$ .
6. Numerically computed Omnés function for  $I=0$   $\pi\pi$  scattering. The solid, dashed, and dotted curves correspond to  $\text{Re}(1/D(s))$ ,  $\text{Im}(1/D(s))$ , and  $1/|D(s)|$  respectively. The coefficient of  $s$  in  $Q_2(s)$  is  $A$ .
7. Numerically computed Omnés function for  $I=0$   $\pi\pi$  scattering. The solid, dashed, and dotted curves correspond to  $\text{Re}(1/D(s))$ ,  $\text{Im}(1/D(s))$ , and  $1/|D(s)|$  respectively. The coefficient of  $s$  in  $Q_2(s)$  is  $A$ .
8. The allowed energy region for the  $D \rightarrow \bar{K} \pi\pi$  decay, where the axes correspond to the energies of the final pions. In the following figures the matrix elements are plotted along the  $z_1$  and  $z_2$  axes shown in this figure to demonstrate that the enhancement is achieved without any resonant intermediate states.
9. The matrix element for  $D^+ \rightarrow K^- \pi^+ \pi^+$  decay plotted along the  $z_1$  axis. The solid curve is for the real part, the dashed curve is for the imaginary part, and the dotted curve is for the absolute value.

10. The matrix element for  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$  decay plotted along the z1 axis. The solid curve is for the real part, the dashed curve is for the imaginary part, and the dotted curve is for the absolute value.
11. The matrix element for  $D^+ \rightarrow K^- \pi^+ \pi^+$  decay plotted along the z2 axis. The solid curve is for the real part, the dashed curve is for the imaginary part, and the dotted curve is for the absolute value.
12. The matrix element for  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$  decay plotted along the z2 axis. The solid curve is for the real part, the dashed curve is for the imaginary part, and the dotted curve is for the absolute value.
13. The matrix element for  $D^+ \rightarrow K^- \pi^+ \pi^+$  decay plotted along the z1 axis. The solid curve is for the real part, the dashed curve is for the imaginary part, and the dotted curve is for the absolute value.
14. The matrix element for  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$  decay plotted along the z1 axis. The solid curve is for the real part, the dashed curve is for the imaginary part, and the dotted curve is for the absolute value.
15. The matrix element for  $D^+ \rightarrow K^- \pi^+ \pi^+$  decay plotted along the z2 axis. The solid curve is for the real part, the dashed curve is for the imaginary part, and the dotted curve is for the absolute value.
16. The matrix element for  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$  decay plotted along the z2 axis. The solid curve is for the real part, the dashed curve is for the imaginary part, and the dotted curve is for the absolute value.
17. The matrix element for  $D^+ \rightarrow K^- \pi^+ \pi^+$  decay plotted along the z1 axis. The solid curve is for the real part, the dashed curve is for the imaginary part, and the dotted curve is for the absolute value.
18. The matrix element for  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$  decay plotted along the z1 axis. The solid curve is for the real part, the dashed curve is for the imaginary part, and the dotted curve is for the absolute value.

19. The matrix element for  $D^+ \rightarrow K^- \pi^+ \pi^+$  decay plotted along the  $z_2$  axis. The solid curve is for the real part, the dashed curve is for the imaginary part, and the dotted curve is for the absolute value.
20. The matrix element for  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$  decay plotted along the  $z_2$  axis. The solid curve is for the real part, the dashed curve is for the imaginary part, and the dotted curve is for the absolute value.

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Fig. 1



$I = 3/2: Y^{--}, Y^-, Y^0, Y^+$   
 $I = 1/2: U^-, U^0$

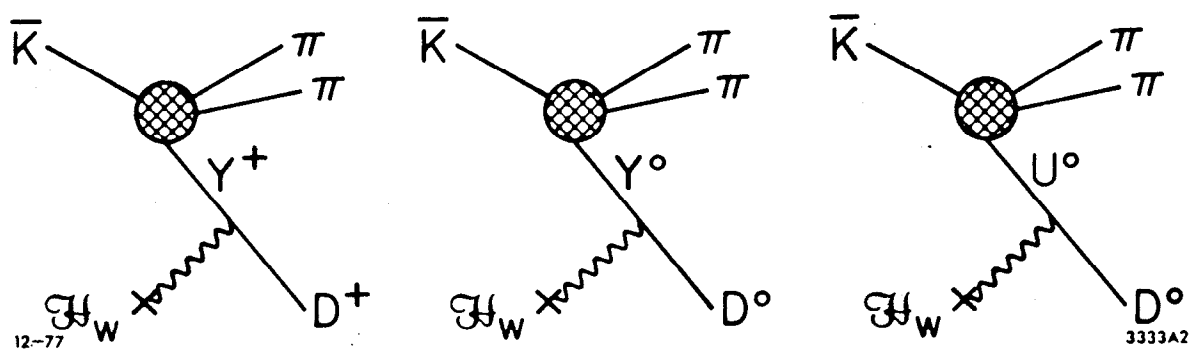


Fig. 2

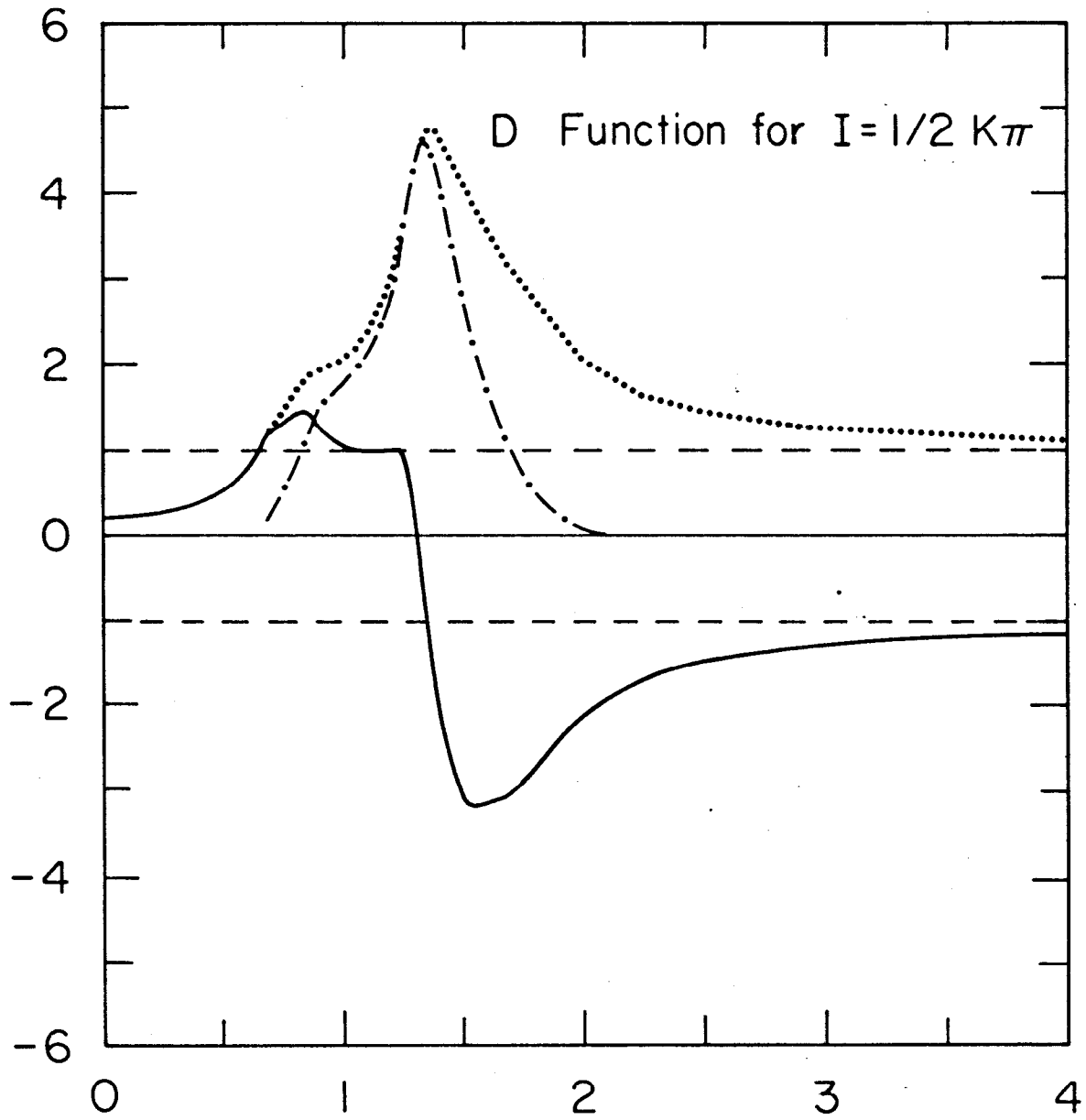


Fig. 3

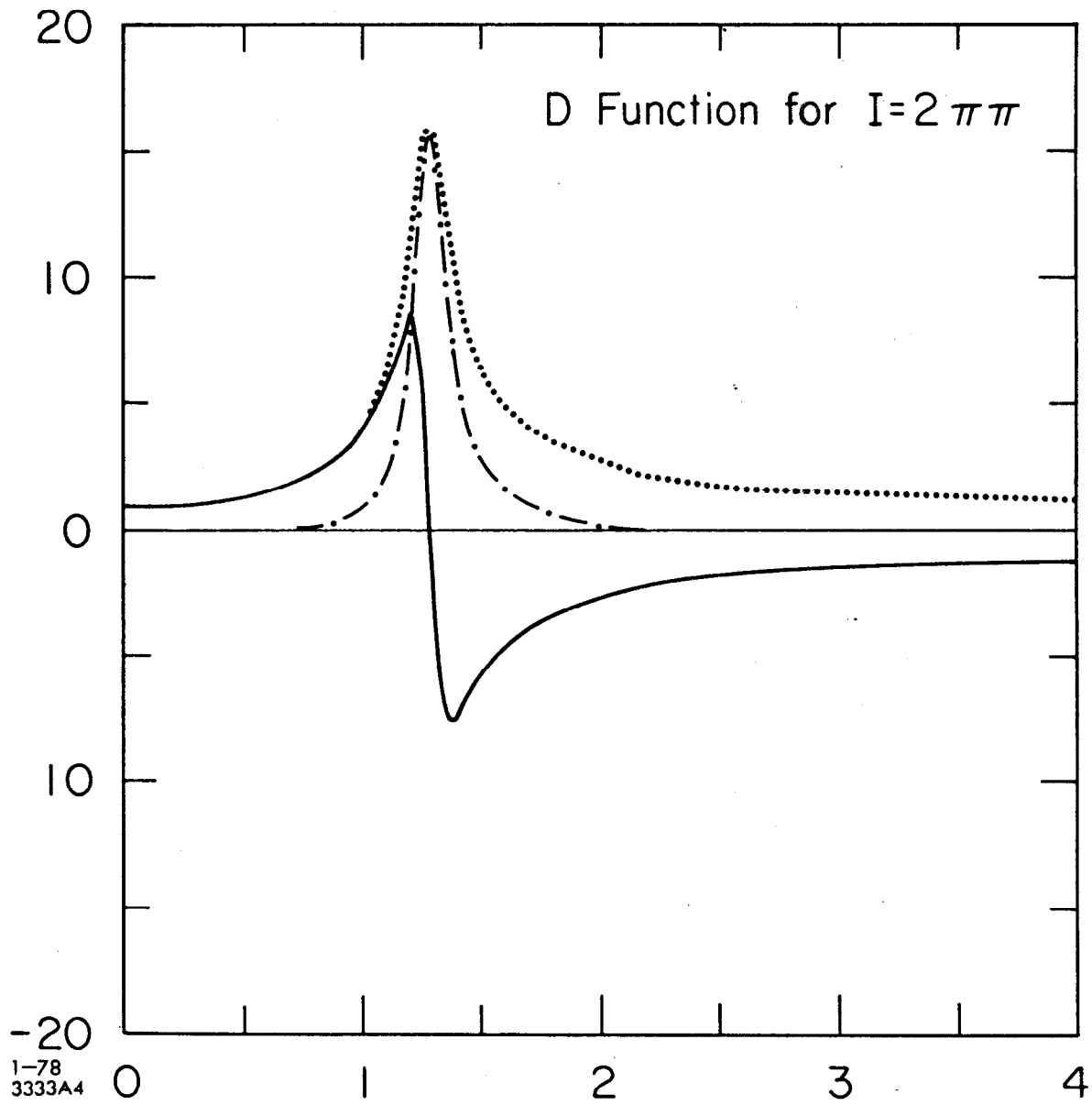
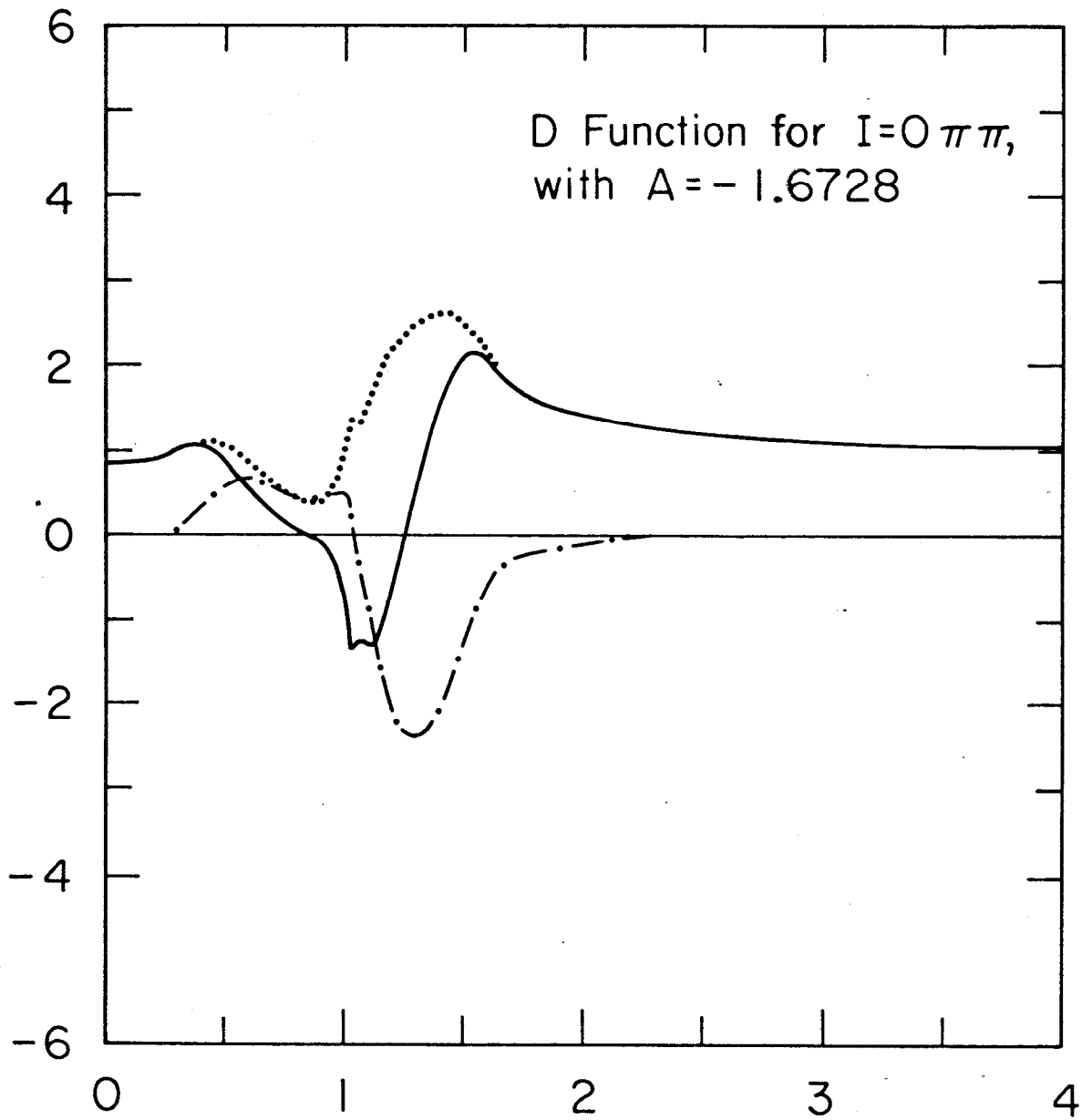


Fig. 4



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Fig. 5

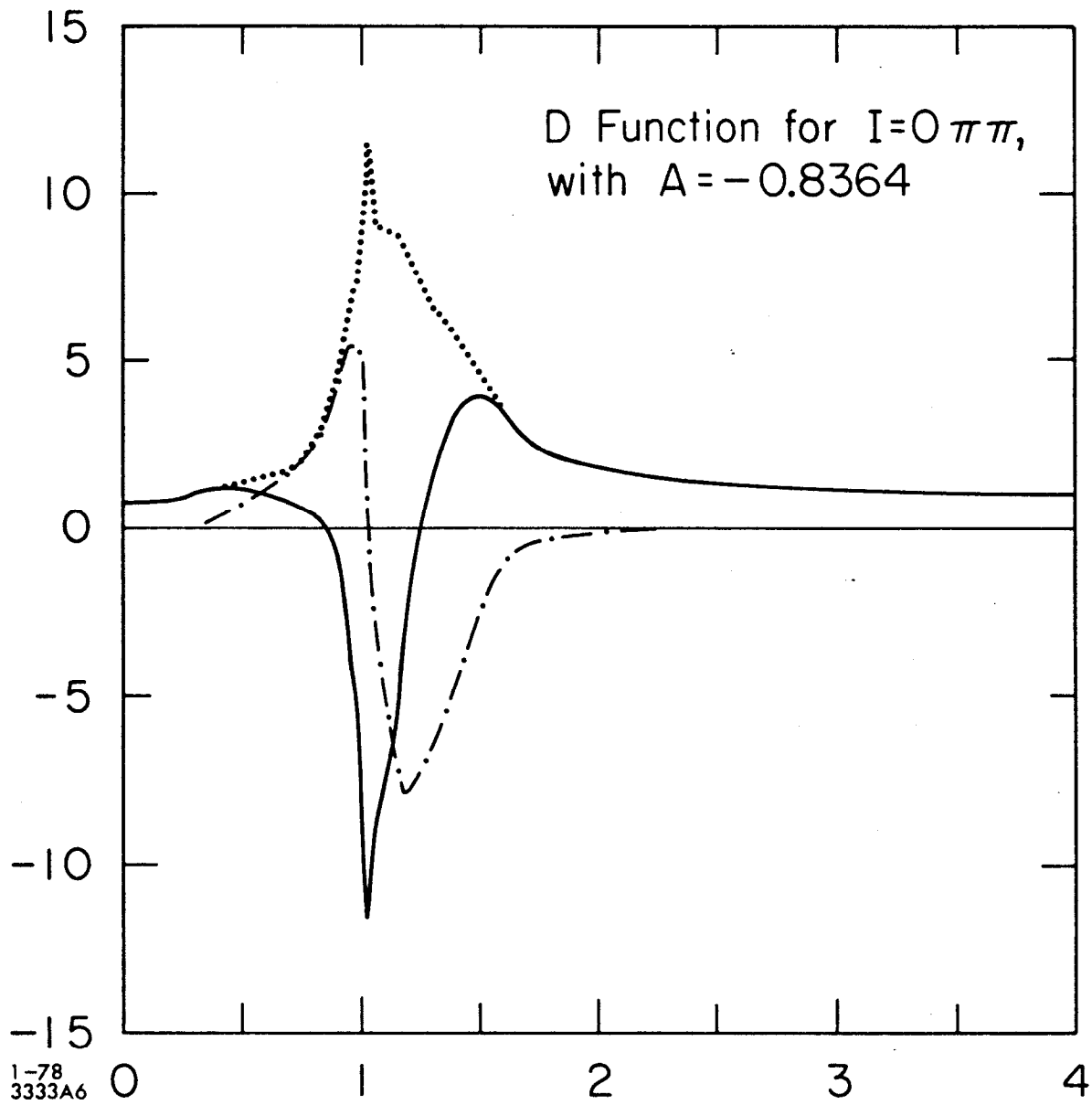


Fig. 6

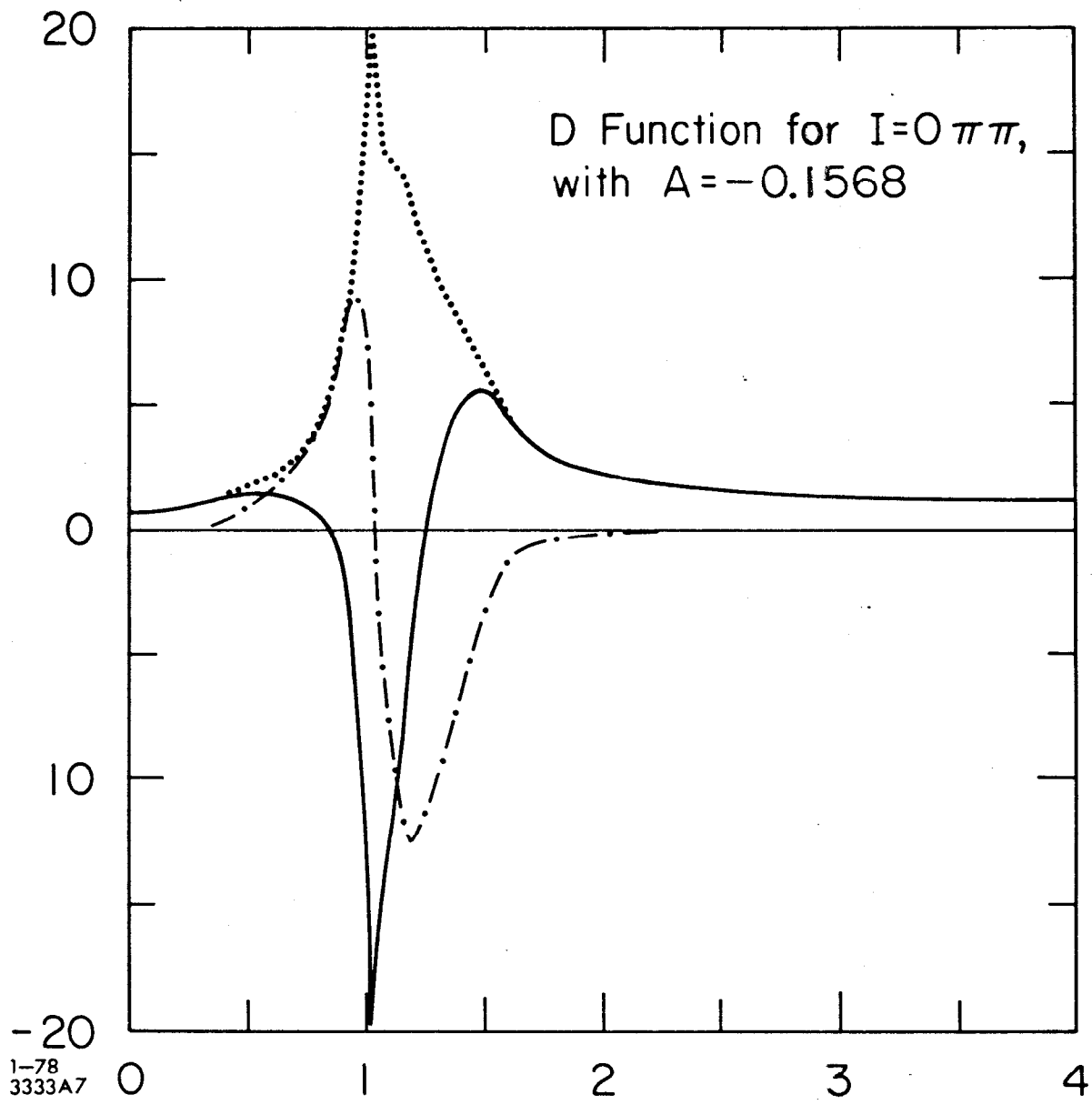
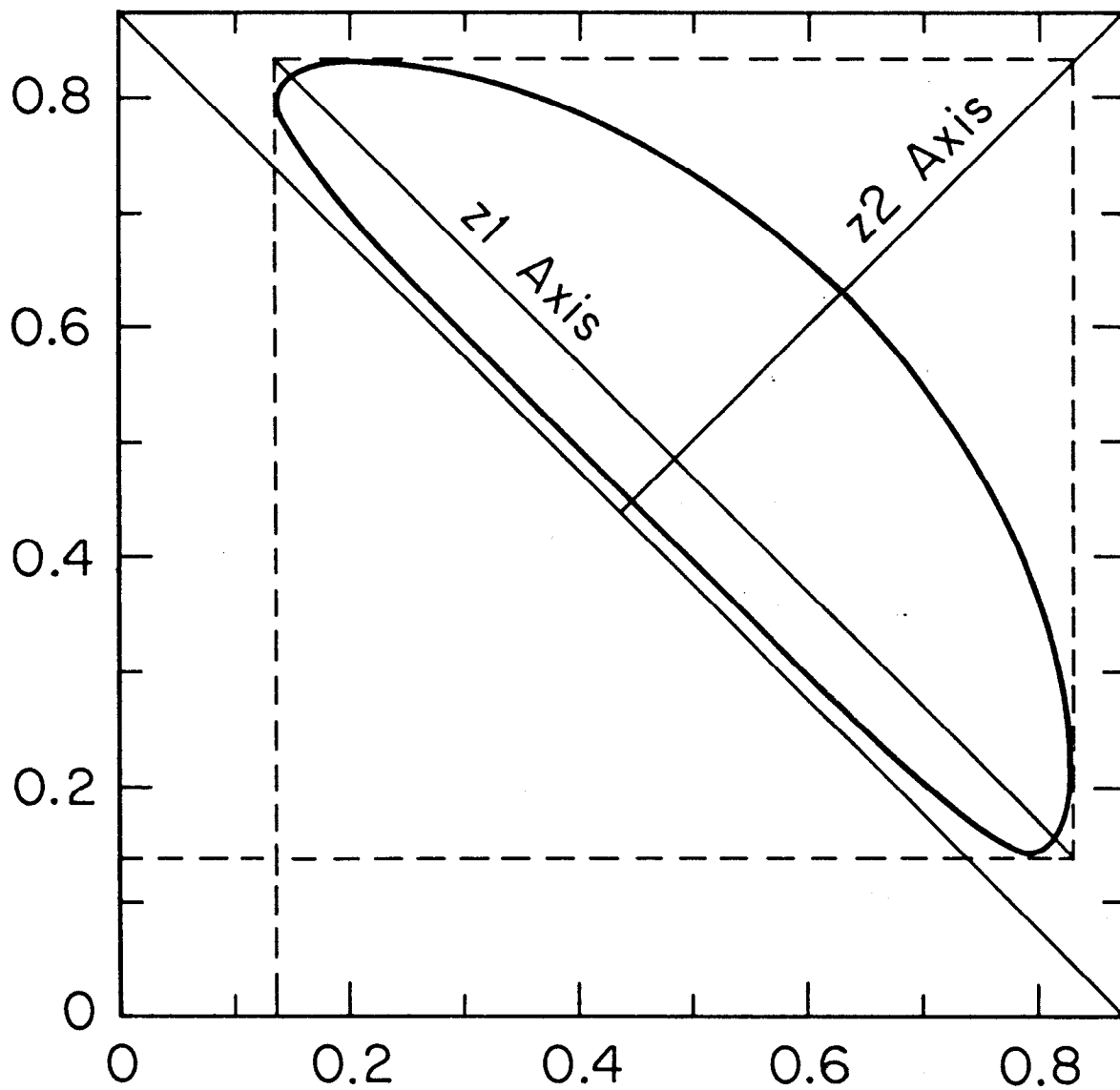


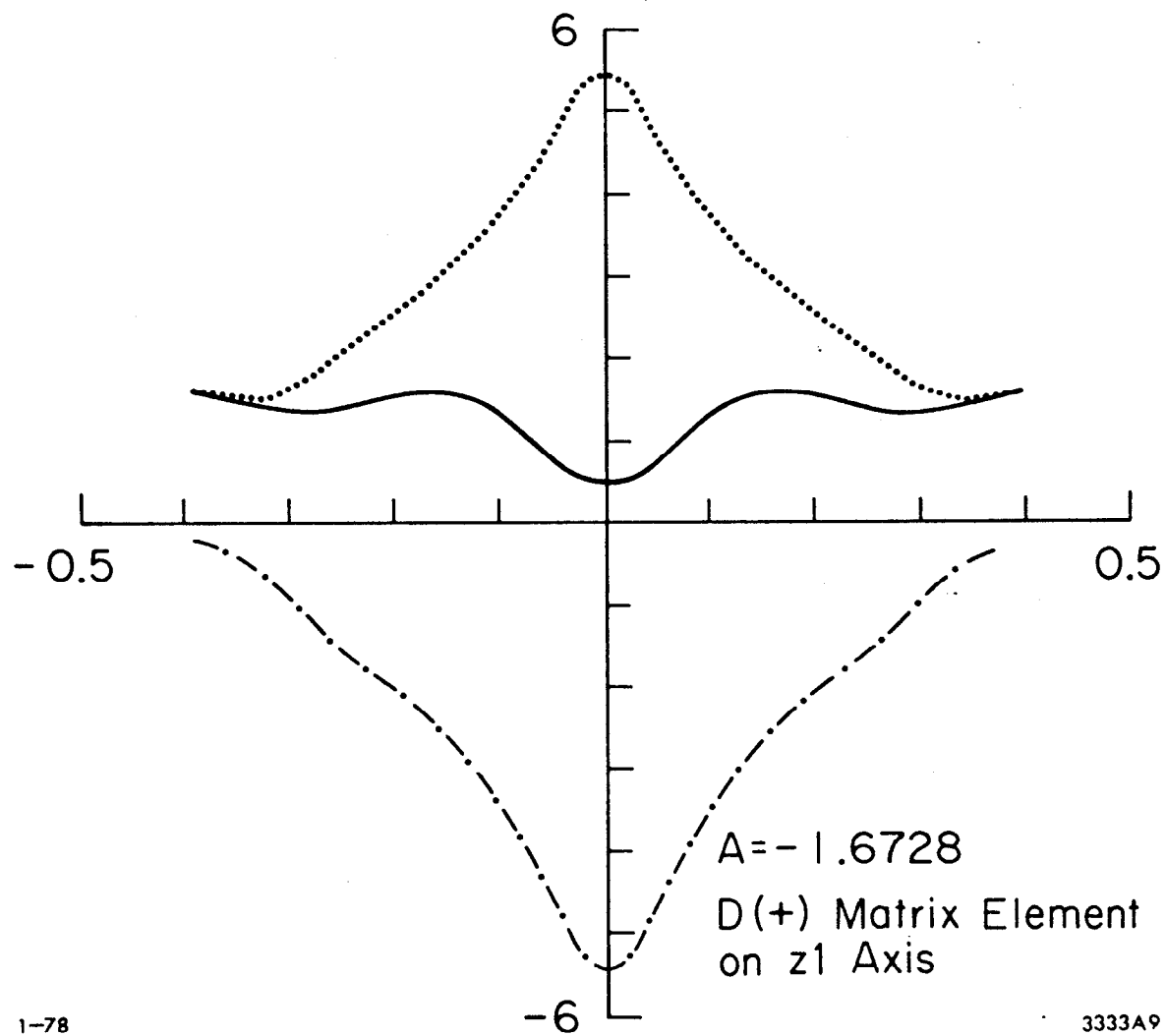
Fig. 7



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Fig. 8

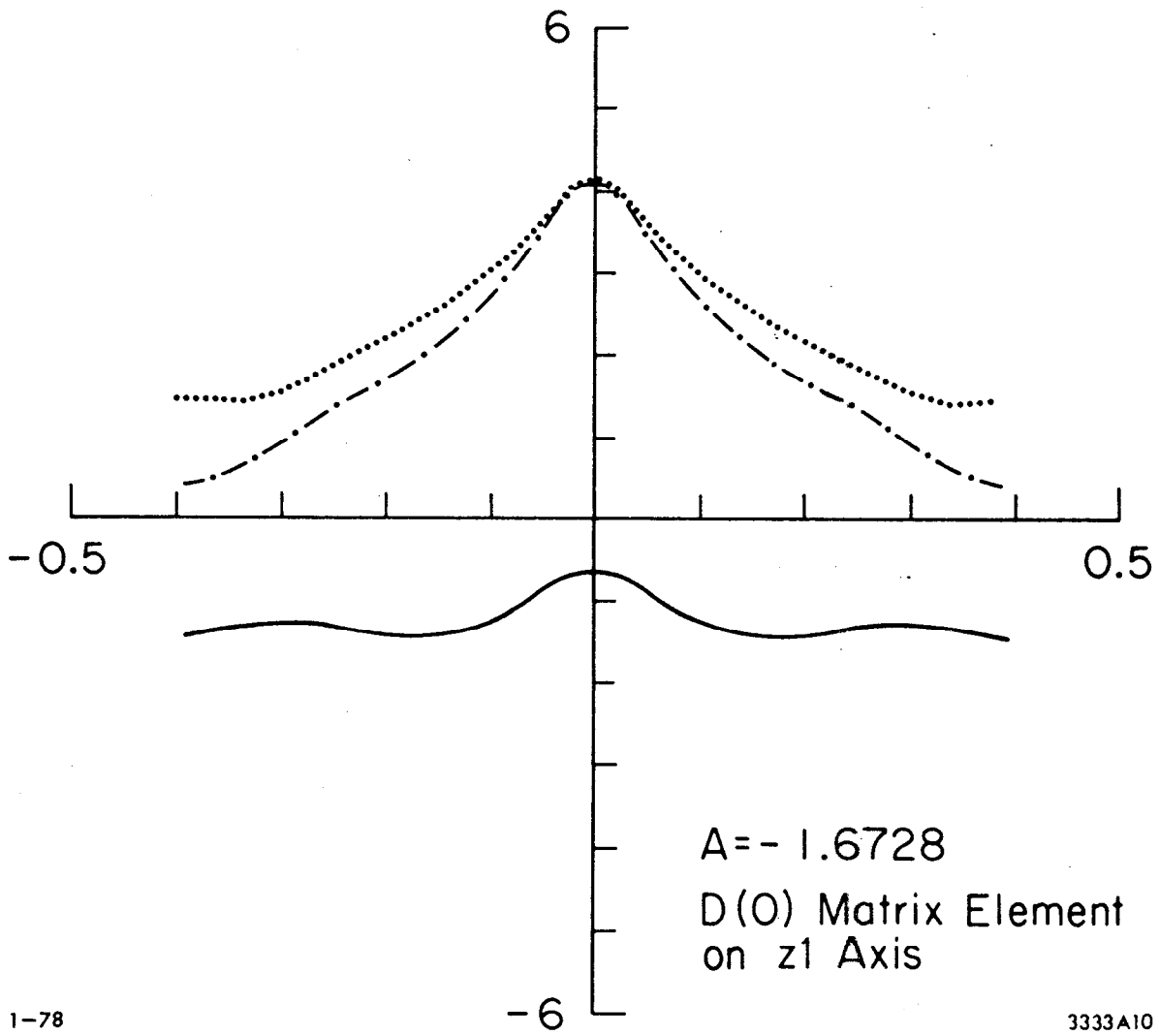


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Fig. 9





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Fig. 10

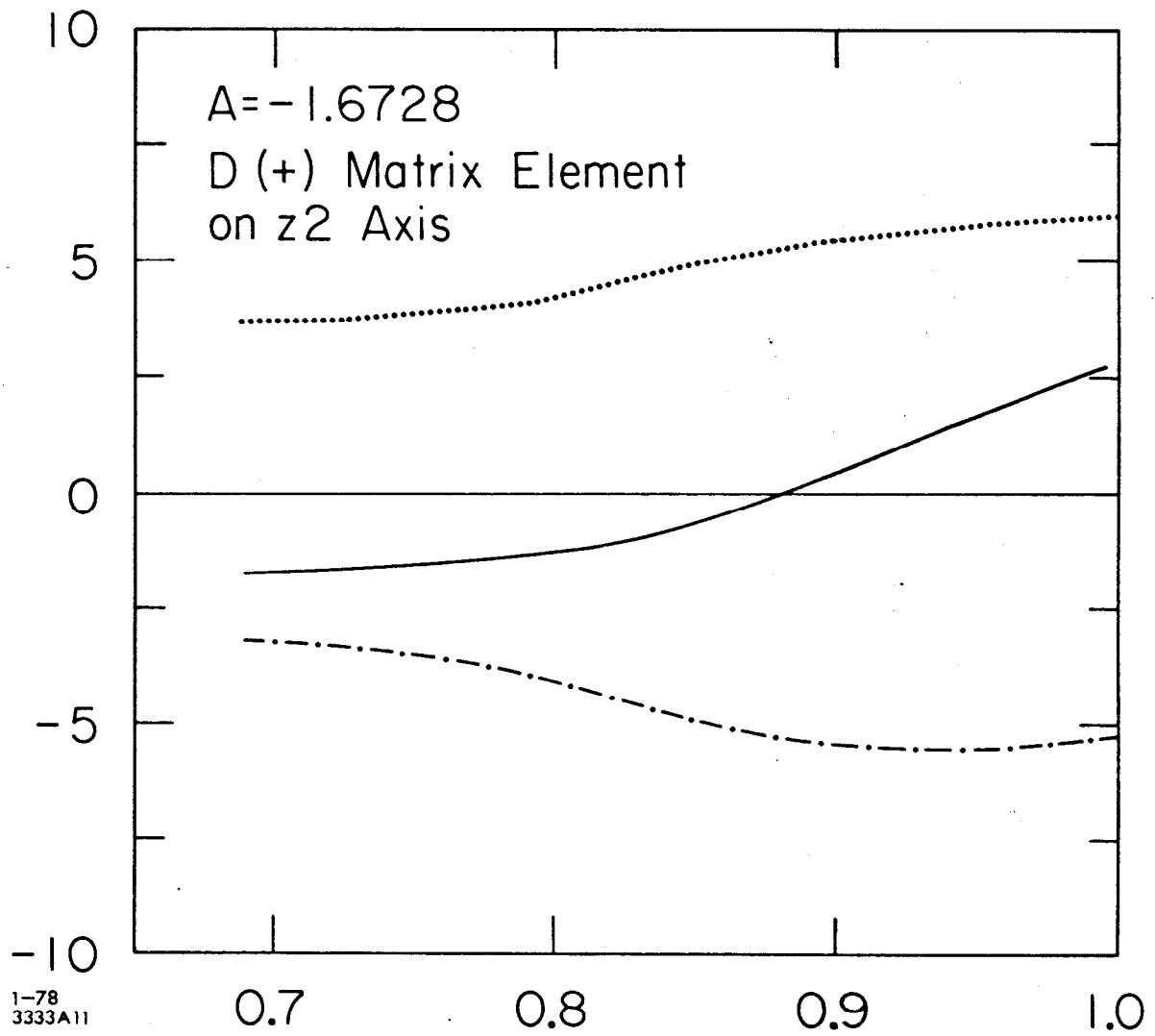


Fig. 11

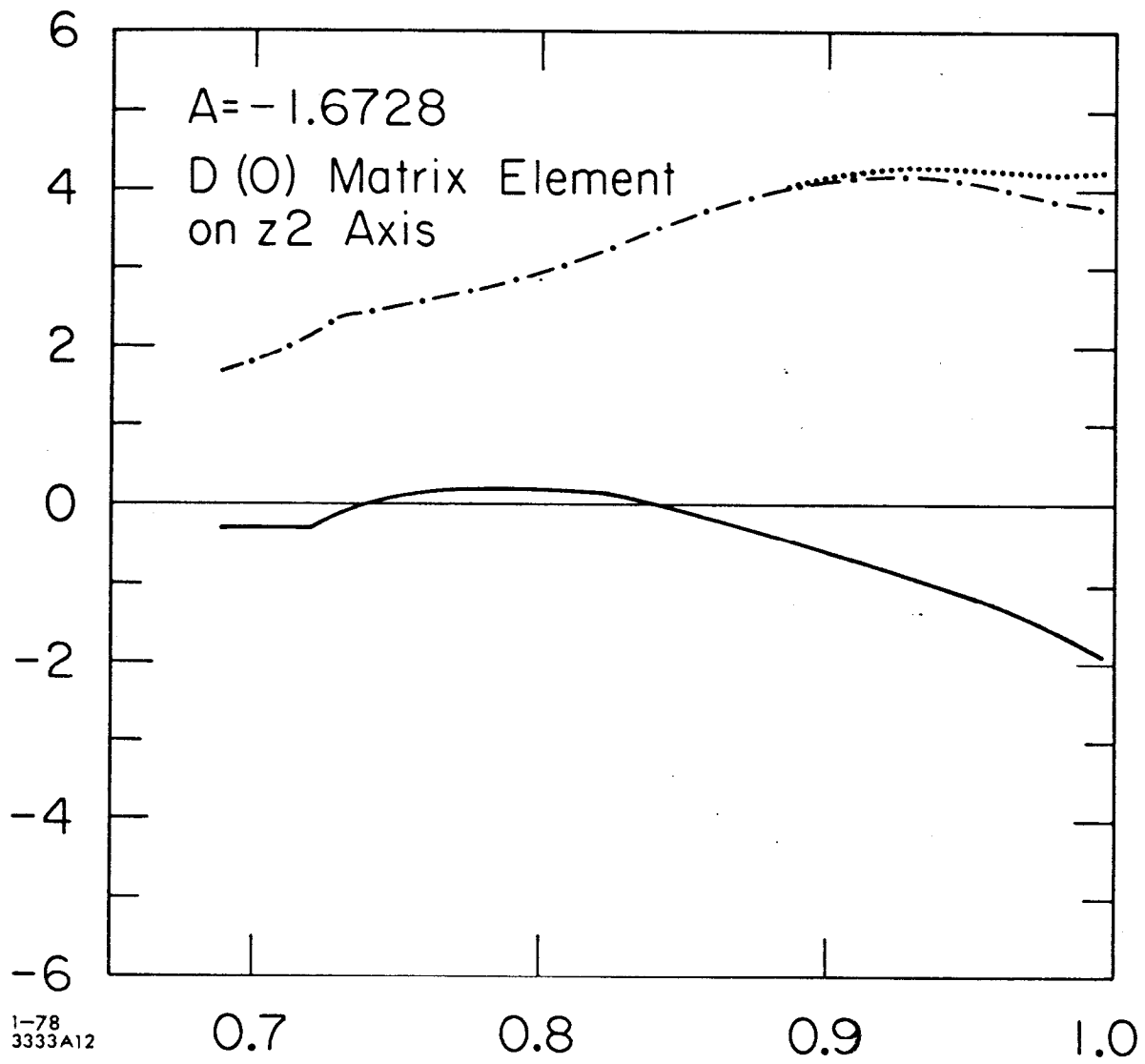


Fig. 12

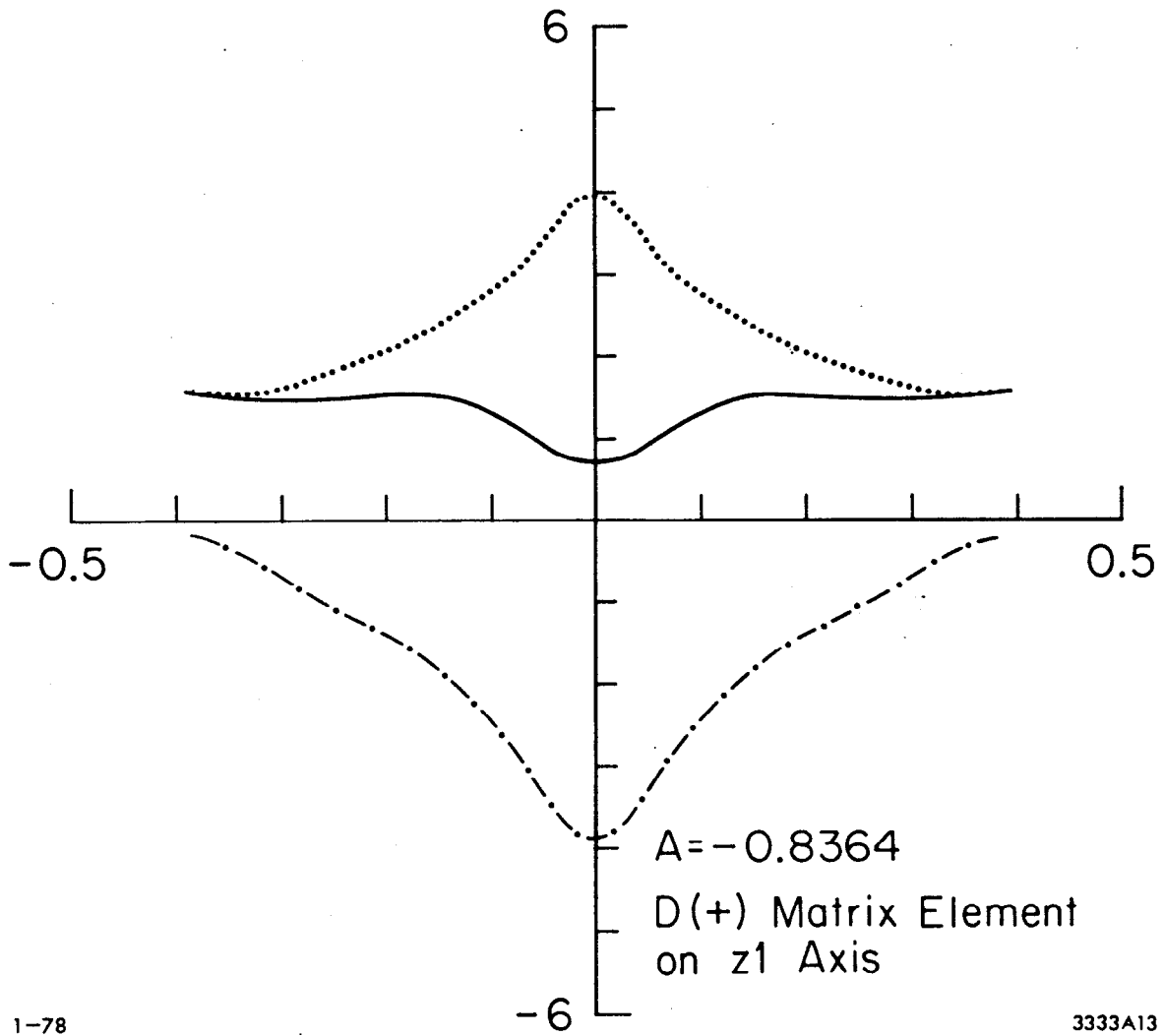
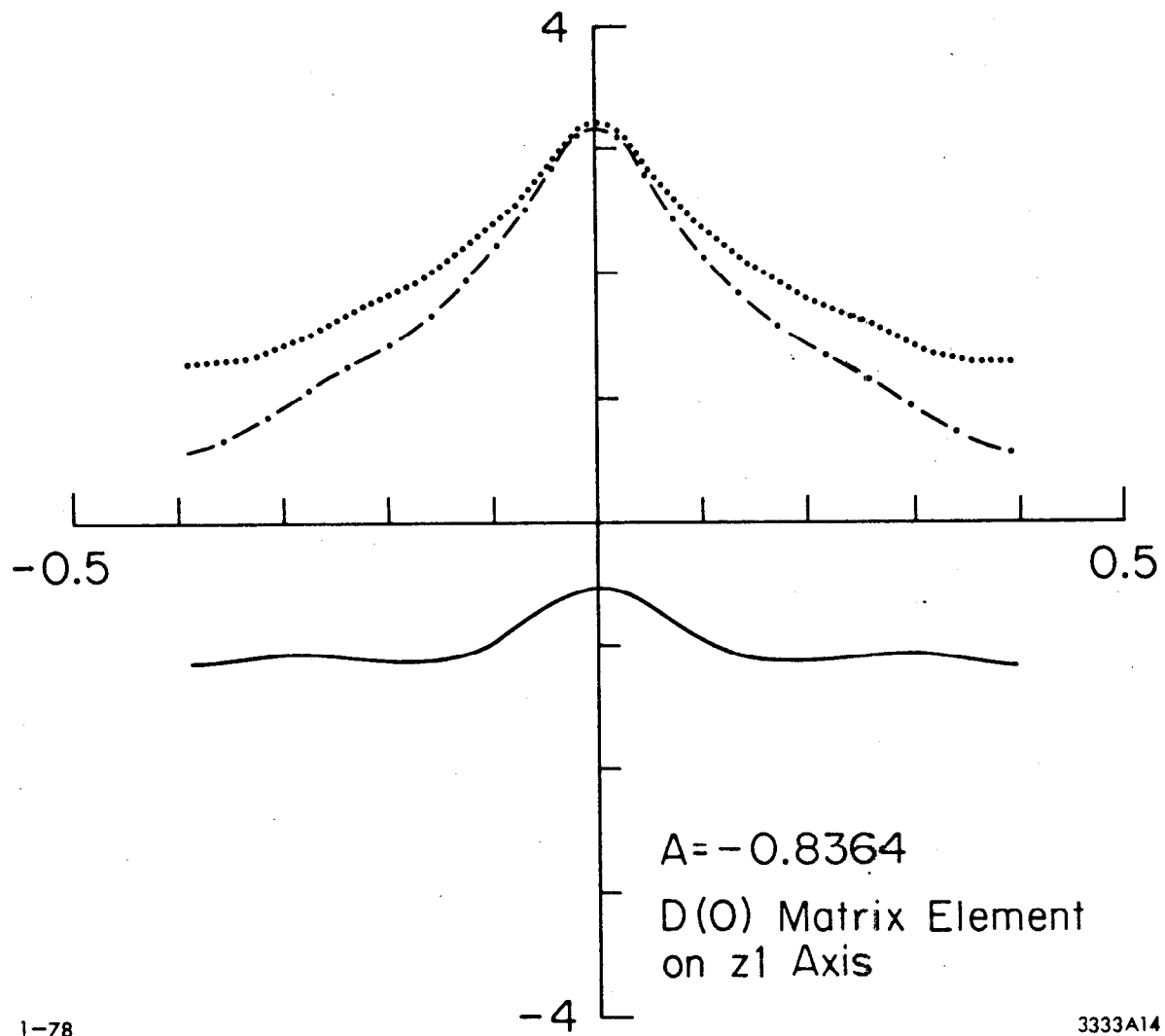


Fig. 13



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Fig. 14

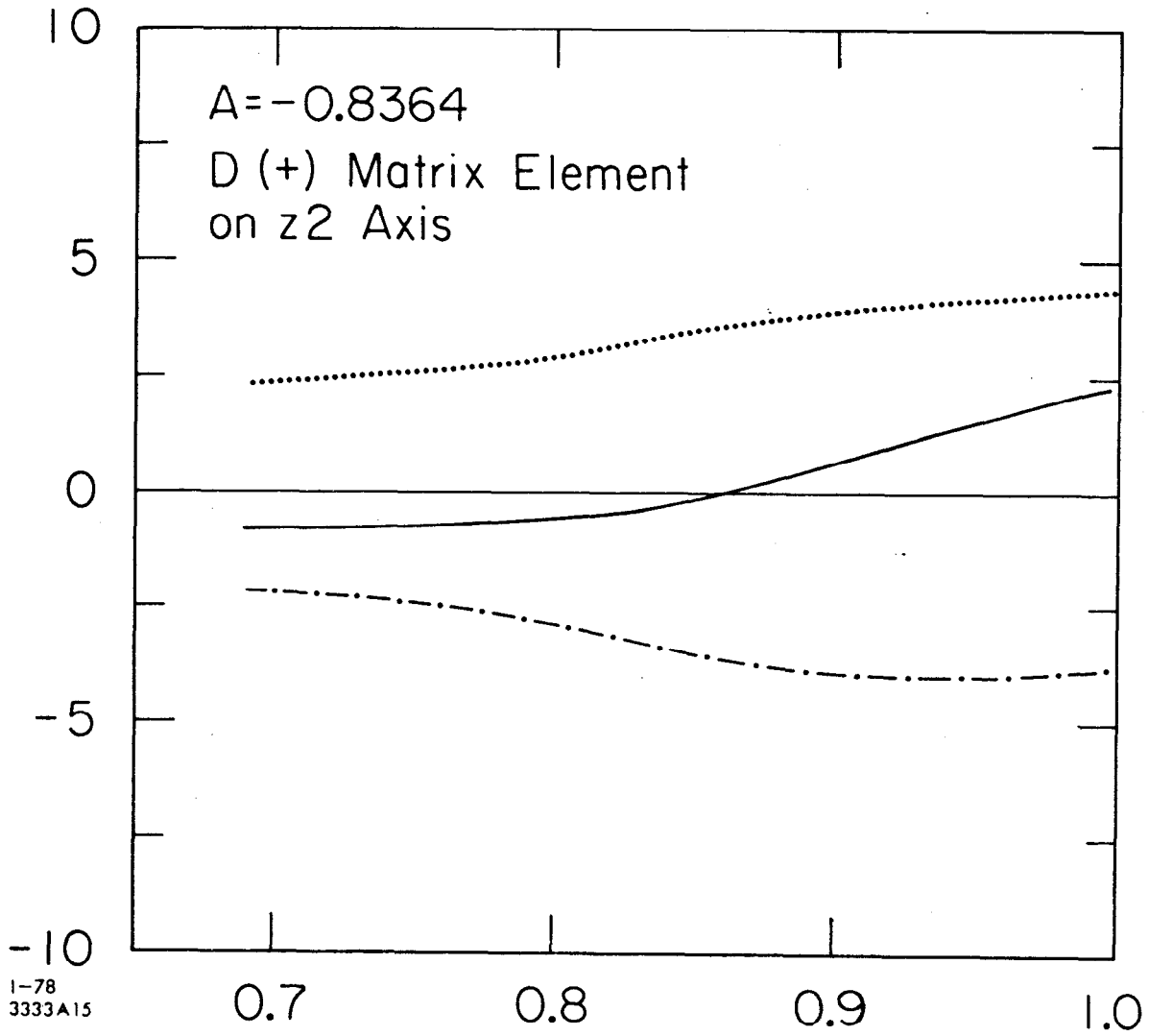


Fig. 15

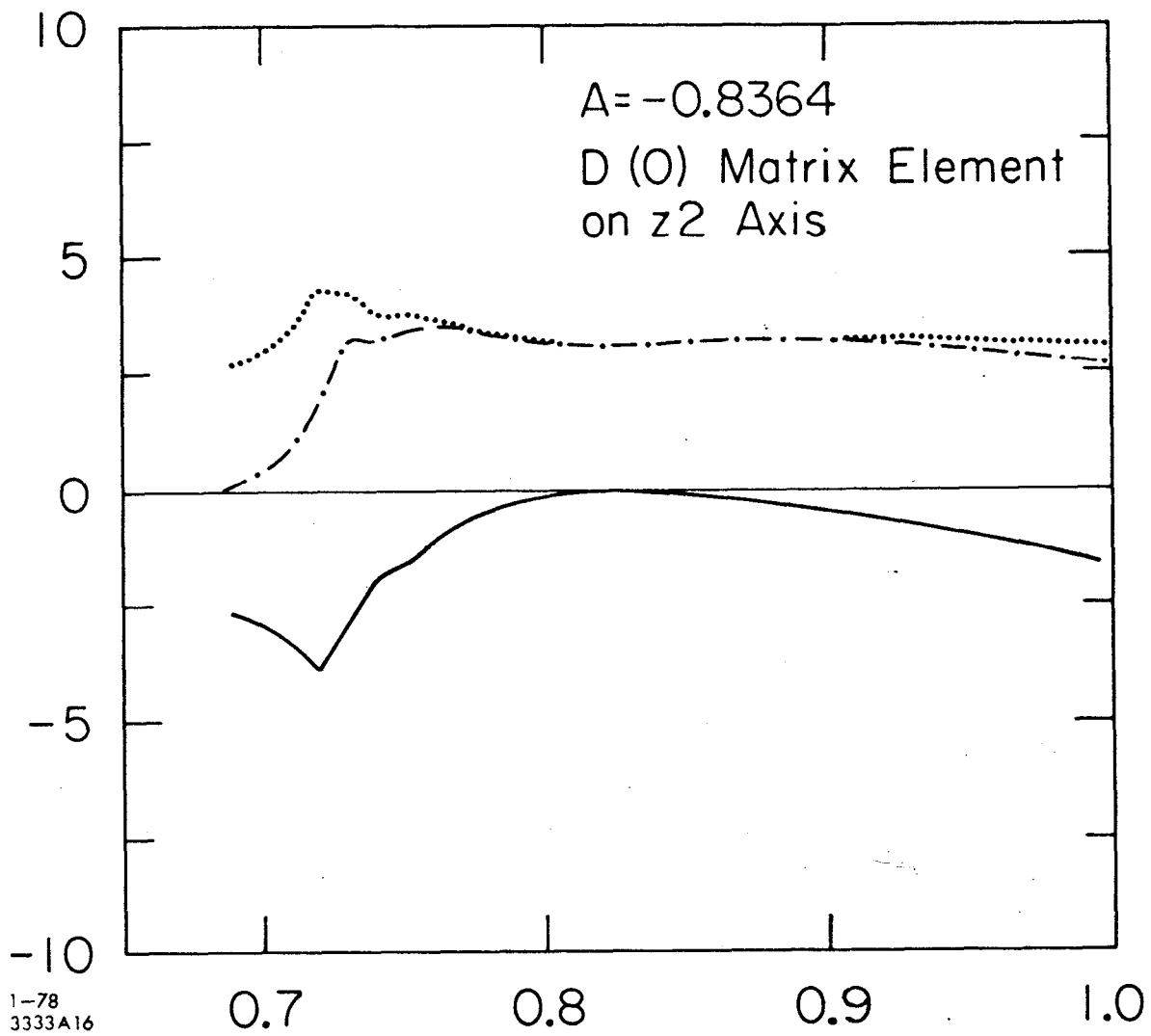
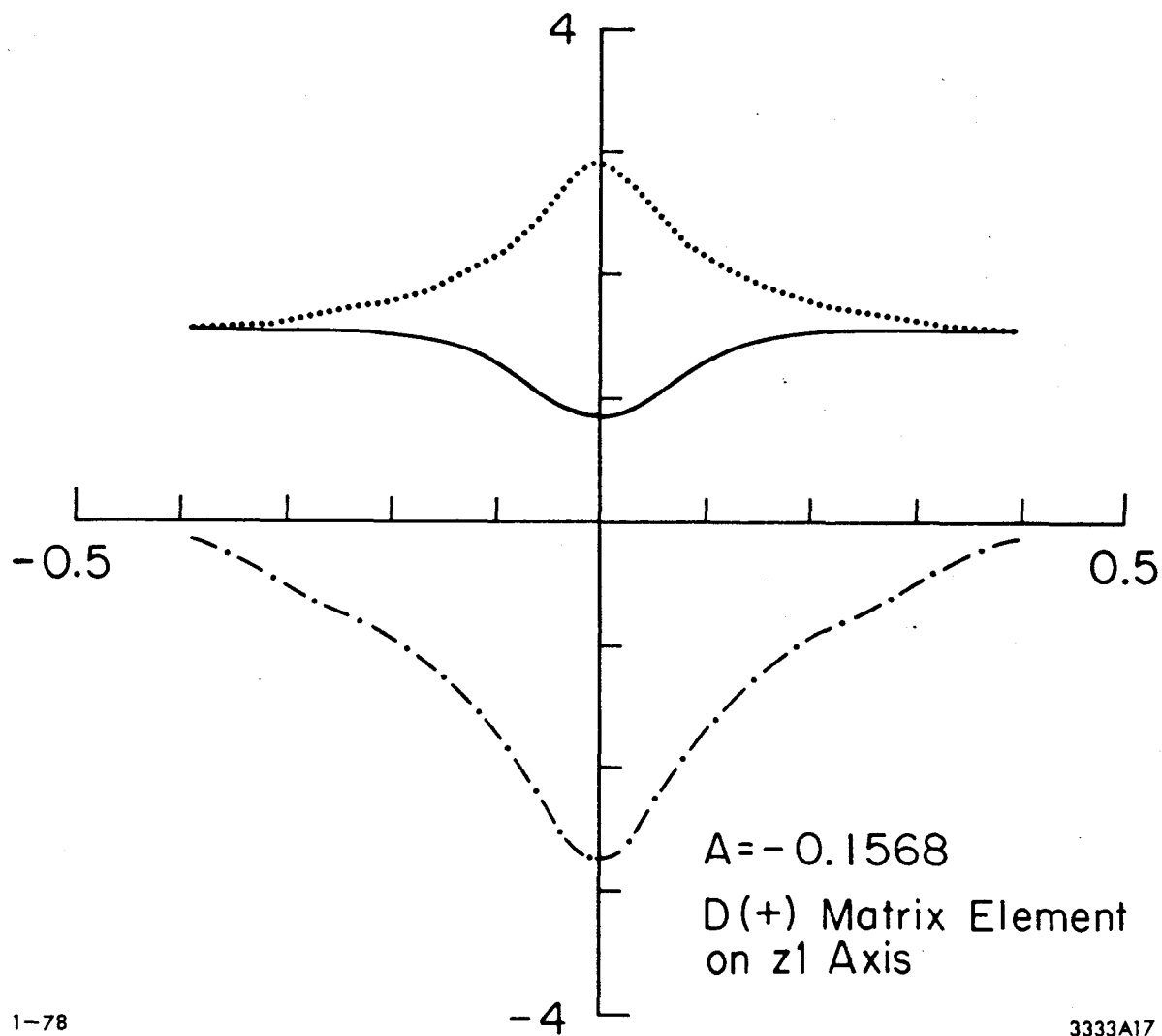


Fig. 16



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Fig. 17



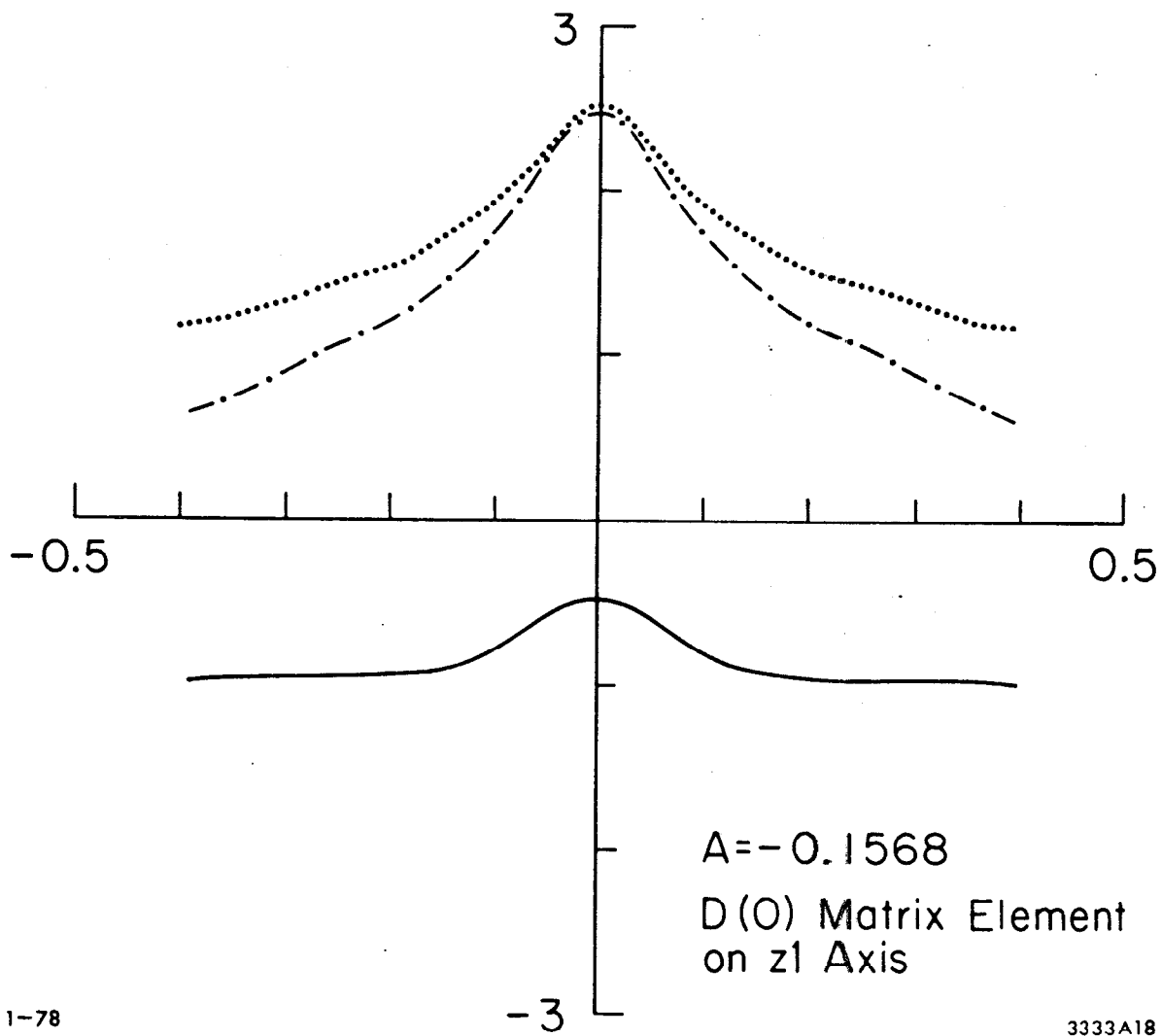


Fig. 18

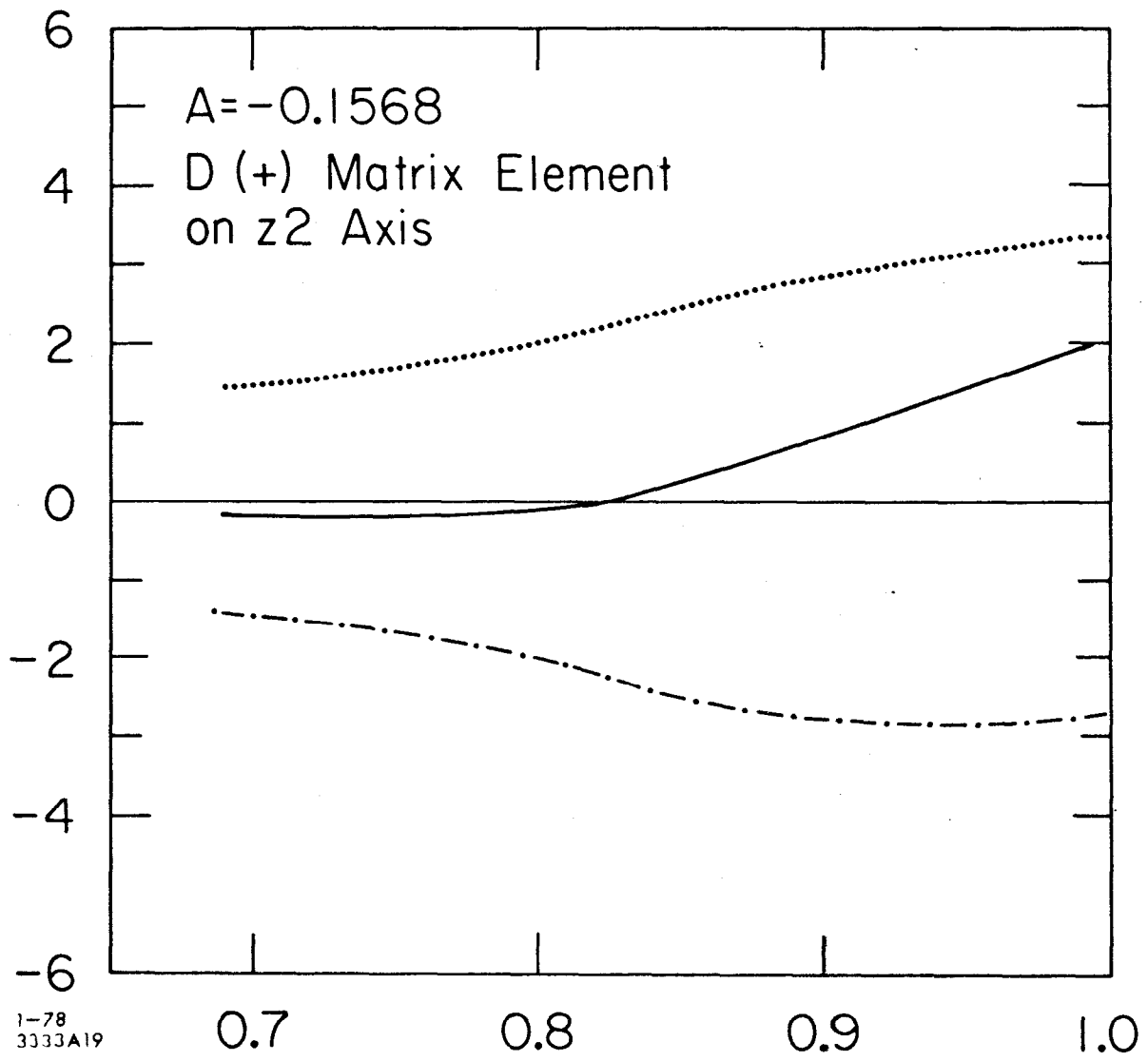


Fig 19

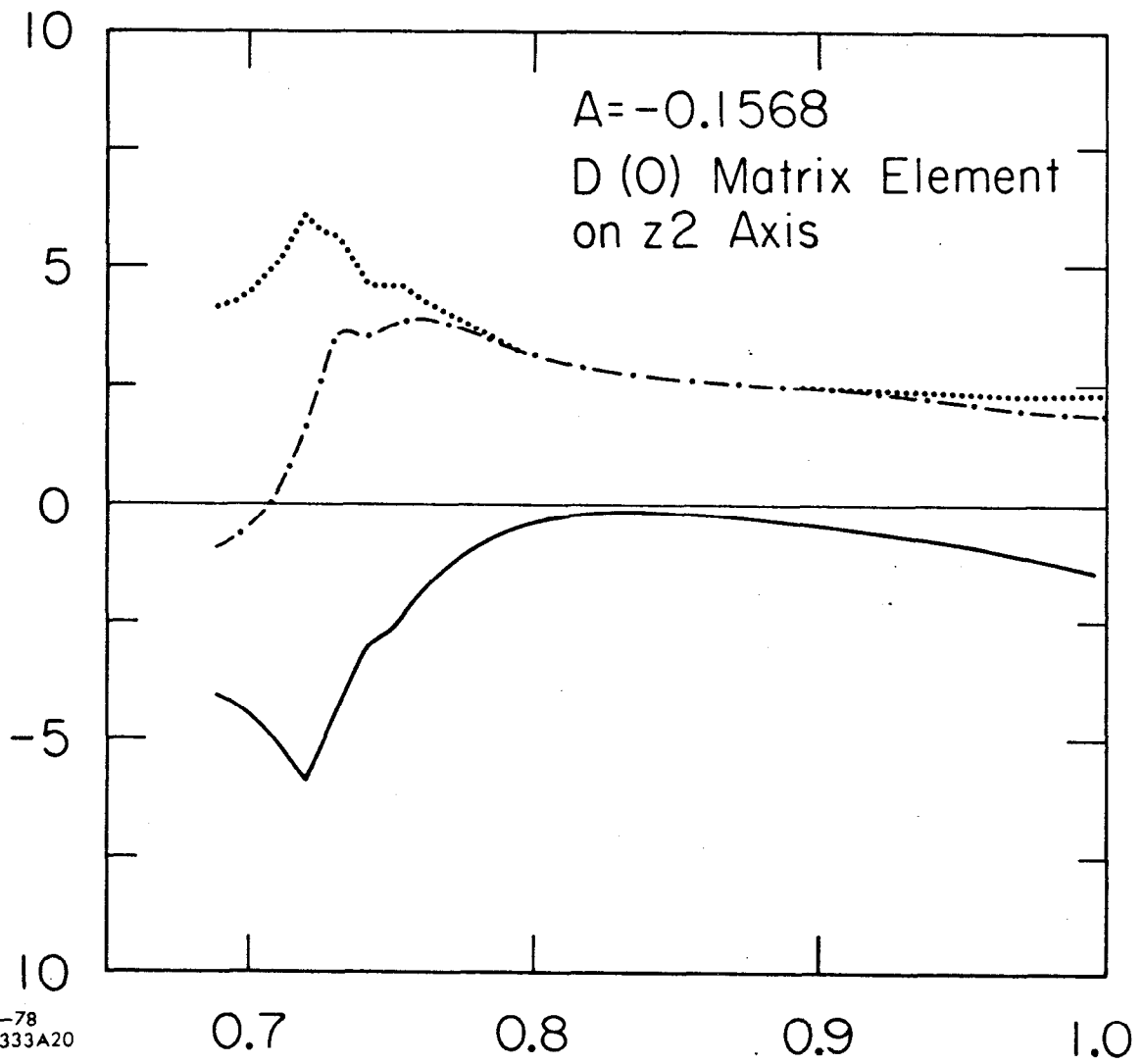


Fig. 20