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RADIAL EXCITATIONS OF HADRONIC BAGS*

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ABSTRACT

We investigate the spectrum of radial excitations of the bag model. Breathing excitations of the surface of the bag couple to the radially-excited states of quarks in the bag, resulting in a spectrum of states which interpolates between the energy levels of the fixed-cavity approximation. We discuss this effect in detail for a bag containing bosons. We apply our results to fermionic systems and find that the radial excitations of baryons contain an NP11(1410) Roper resonance candidate as a natural consequence of the effects of breathing modes.

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I. INTRODUCTION

It is generally believed that hadrons consist of confined colored quarks and vector gluons. The bag model[1] is an explicitly relativistic confined quark model which in its static cavity approximation[2] has had great success in reproducing the spectrum and other properties of the light hadrons.[3] In this approximation the quarks are treated as modes of a static spherical cavity which interact among themselves only via the exchange of massless vector gluons, the radius of the cavity being chosen to minimize the energy of the state.

The cavity approximation to the bag has been applied to the orbitally-excited states of baryons[4], where it was found to give rise to too many states. In an SU(6) of flavor and spin, the P-wave states of the cavity form both a (56) and a (70). This is in poor agreement with experiment, as all of the negative-parity baryon resonances below about 1800 MeV can be accommodated in a single (70).

However, the cavity approximation to the bag model is just that — an approximation. In a more general formulation of the bag model one would find that the motion of the boundary is coupled to the motion of the quarks, and that as the quarks move, so does the surface of the bag. One can allow the surface of the bag to undergo small P-wave deformations away from a static equilibrium shape and solve the resulting coupled system. This calculation has been carried out in Ref. (5) for bosonic systems and in Ref. (6) for fermionic bags. Then one finds that some excited states of a fixed cavity are in fact translation modes of the deformed cavity - and are hence spurious states in the spectrum of mass eigenvalues of the system. In the baryon spectrum, these states form a $L = 1$ (56). When they are projected out of the allowed Hilbert space of states, a spectrum of P-wave states is obtained which is in much better qualitative agreement with experiment.[7]

In this note we will use the techniques just outlined to investigate the lowest radially excited states of the bag model. Our results are superficially similar to the case of orbital excitations but have important and interesting differences. We find that if we allow the surface of the bag to undergo small radial "breathing" oscillations about an equilibrium spherical shape, these modes will couple to states where the fields inside the bag are excited, through the field boundary conditions. Diagonalizing the Hamiltonian for these systems, one finds a tower of excitations, the lowest one of which is lower in energy than the lowest excited cavity eigenstate. In the P-wave case this mode is a translation mode of zero energy (measured with respect to the ground state). In the radial excitation case, this mode cannot have zero energy: that would correspond to the ground state blowing up like a balloon, manifestly at odds with energy conservation. The state is, however, pushed to fairly low energy, as the system "relaxes" by exciting surface modes.

This energy shift has important phenomenological consequences for baryon spectroscopy. The Roper resonance NP11(1470) is a strong candidate for a radial recurrence of the nucleon, as indicated by photoproduction and Melosh-type analyses.[8] The cavity approximation to the bag predicts both SU(6) $(56, 0^+)$ and $(70, 0^+)$ radial excitations, both centered at around 1700 MeV. The resulting nucleon states are much too heavy to be good candidates for the Roper, and the ordering of states is generally in poor agreement with experiment.

The inclusion of surface fluctuations dramatically alters this picture. The surface modes can only couple to states in the same flavor-spin multiplets as the ground state. Hence they can affect only the (56). They reduce its energy by about 200 MeV with respect to the (70) while leaving the latter unchanged — yielding an NP11 resonance at about 1410 MeV together with the rest of its multiplet slightly higher in energy, and a slightly heavier still (70). The resulting picture is in good qualitative and fair quantitative agreement with experiment.

We will begin the actual calculations by considering the problem of radial excitations of a boson bag. We have not been able to compute the energy shift for a fermion bag, but we can investigate the phenomenology of that system with our bosonic results. The qualitative form of the results will be the same in either case: allowing the surface to move softens the spectrum of some of the excited states of the cavity. We will then present the details of our phenomenological investigation of baryon spectroscopy.

II. RADIAL EXCITATIONS OF BOSONIC SYSTEMS

A general treatment of the motion of the bag in the limit of small boundary oscillations is presented in Refs. 5-6. We indicate here only the most relevant steps for the derivation of the spectral equation governing the radial, spherically symmetric oscillations of a bosonic bag.

The static cavity is conveniently parametrized in terms of a pair of conjugate variables: Q, the total charge of the system, and θ, the phase of the bosonic field $\phi(r,t)$ inside the bag. Precisely, $\phi(r,t)$ is expanded as

$$\phi\,(r,t) = \frac{e^{-i\theta(t)}}{r}\left(\Phi_1(t)\sqrt{2}\,\sin\frac{\pi r}{R} + \sum_{n\geq 2}\Phi_n(t)\sqrt{2}\,\sin\frac{n\pi r}{R}\right) \tag{2.1}$$

where $\Phi_1(t)$ is real, $\Phi_n(t)$, $n \geq 2$, generally complex, and R is the radius of the bag. In the static cavity solution

$$\Phi_1(t) = \text{constant} = \frac{1}{2\sqrt{2\pi}}\sqrt{Q}$$
$$R = \text{constant} = \frac{1}{\sqrt{2}}\,B^{-1/4}\,Q^{1/4} \tag{2.2}$$

B being the bag constant. To study the small oscillations of the system one replaces $\Phi_1(t)$ and R(t) with new coordinates

$$\begin{aligned} \phi_1(t) &= \Phi_1(t) - \frac{1}{2\sqrt{2}}\frac{\sqrt{Q}}{\pi} \\ r(t) &= R(t) - \frac{1}{\sqrt{2}} B^{-1/4} Q^{1/4} \end{aligned} \tag{2.3}$$

and expands then the Hamilton H up to second order in all canonical variables except the charge Q, which is considered the large variable of the expansion. (θ is a cyclic variable and does not appear in H.)

In this way one finds

$$\begin{aligned} H = \frac{4\sqrt{2}}{3}\pi B^{1/4} Q^{3/4} + \sqrt{2}\,\pi\, B^{1/4} Q^{-1/4} \times \\ \times \left\{ 4r^2\sqrt{\frac{B}{Q}} + \sum_{n=1}^{\infty} \frac{n}{2}\left(p_{n+}^2 + q_{n+}^2\right) + \right. \\ \left. + \sum_{n=1}^{\infty} \frac{n}{2}\left(p_{n-}^2 + q_{n-}^2\right) \right\} \end{aligned} \tag{2.4}$$

H appears as the Hamiltonian of a collection of oscillators, with momenta $p_{n\pm}$ and coordinates $x_{n\pm}$. These variables are related by a linear canonical transformation to the field inside the bag and its conjugate momentum. The excitation of an (n, +) oscillator corresponds to the promotion of a quantum of the field to a higher radial mode inside the fixed cavity, whereas the excitation of an (n, -) oscillator represents the creation of a pair of quanta of opposite charge, so as to leave the total charge Q of the system unchanged.

As noticed in Ref. 5, the system is a constrained system, and the r variable in H cannot be considered as an ordinary coordinate. Because the boundary of the bag carries no kinetic energy, the momentum p conjugate to r is related to the field momentum by an equation of constraint, which in the limit of small oscillations linearizes to

$$P \equiv B^{-1/4} Q^{1/4} p - \sum_{n=1}^{\infty} 2\sqrt{\frac{n+1}{n}} (-)^n p_{n+} - \sum_{n=2}^{\infty} 2\sqrt{\frac{n+1}{n}} p_{n-} = 0 \qquad (2.5)$$

The dynamical effect of the constraint is most easily found as follows. By a linear canonical transformation one takes P itself as a new momentum variable. This requires a change in the canonical coordinates

$$x_{n\pm} \to \hat{x}_{n\pm} = x_{n\pm} + \frac{2\sqrt{n\pm 1}}{n} (-)^n 2B^{1/4} Q^{-1/4} \qquad (2.6)$$

and the Hamiltonian becomes

$$H = \frac{4\sqrt{2}}{3} B^{1/4} Q^{3/4} + \sqrt{2}\,\pi B^{1/4} Q^{-1/4} \times$$

$$\times \left\{ \sum_{n=1}^{\infty} \frac{n}{2} \left(p_{n+}^2 + q_{n+}^2 \right) + \sum_{n=2}^{\infty} \frac{n}{2} \left(p_{n-}^2 + q_{n-}^2 \right) \right.$$

$$-2\, r\, B^{1/4} Q^{-1/4} \left[\sum_{n=1}^{N} \sqrt{n+1}\, (-)^n \hat{x}_{n+} + \sum_{n=2}^{N} \sqrt{n-1}\, (-)^n \hat{x}_{n-} \right] \qquad (2.7)$$

$$\left. + 4 r^2 \sqrt{B/Q} \left(N+1 \right) \right\}$$

where a cut-off has been temporarily introduced to remove a formal divergence. The constraint $P = 0$ implies the consistency condition

$$\left\{ H, P \right\} = 0 \qquad (2.8)$$

which gives

$$B^{1/4} Q^{-1/4} r = \frac{\sum_{n=1}^{N} \sqrt{n+1}\,(-)^n \hat{x}_{n+} + \sum_{n=2}^{N} \sqrt{n-1}\,(-)^n \hat{x}_{n-}}{4 \; (N+1)} \qquad (2.9)$$

Substituting back into H one finally finds

$$H = \frac{4\sqrt{2}\,\pi}{3} B^{1/4} Q^{1/4} + \sqrt{2}\,\pi B^{1/4} Q^{-1/4} \times$$

$$\times \left\{ \sum_{n=1}^{N} \frac{n}{2} \left(p_{n+}^2 + q_{n+}^2 \right) + \sum_{n=2}^{N} \frac{n}{2} \left(p_{n-}^2 + q_{n-}^2 \right) \right. \qquad (2.10)$$

$$- \left. \frac{\left(\sum_{n=1}^{N} \sqrt{n+1}\,(-)^n \hat{x}_{n+} + \sum_{n=2}^{N} \sqrt{n-1}\,(-)^n \hat{x}_{n-} \right)^2}{4 \quad (N+1)} \right\}$$

It is apparent that the net effect of the motion of the boundary is to produce a collective coupling among the modes of the field.

H is diagonalized easily by looking for solutions to the equations of motion of the form

$$\hat{x}_{n\pm} = C_{n\pm} e^{-i\omega t} \tag{2.11}$$

The equation $\ddot{\hat{x}}_{n\pm} = \{H, \{H, x_{n\pm}\}\}$ gives immediately

$$-\omega^2 C_{n\pm} = -n^2 C_{n\pm} + \frac{n\sqrt{n\pm 1}\,(-)^n}{2\ (N+1)}\ C \tag{2.12}$$

with

$$C = \sum_{n=1}^{N} \sqrt{n+1}\,(-)^n\ C_{n+} + \sum_{n=2}^{N} \sqrt{n-1}\,(-)^n\ C_{n-} \tag{2.13}$$

If $C \neq 0$, evaluating $C_{n\pm}$ from Eq. (2.12) and inserting it into Eq. (2.13), one finds the spectral equation

$$1 = \sum_{n=1}^{N} \frac{1}{n^2 - \omega^2} - N \tag{2.14}$$

Notice that the cut-off can be removed, and Eq. (2.14), in the limit $N \to \infty$, becomes simply

$$-\pi\,\omega\ \cot(\pi\omega) = 1, \tag{2.15}$$

which is solved by a sequence of eigenfrequencies ω_n, $n = 1, 2, \ldots$ approaching $n - \frac{1}{2}$ as $n \to \infty$. Numerically one finds

$$\omega_1' \approx .64577\ldots$$

$$\omega_2' \approx 1.56391\ldots$$

$$\omega_3' \approx 2.53968\ldots$$

Also, Eq. (2.12) can be solved with $\omega_n'' = n$, $C = 0$. We find therefore that in the limit of small radial oscillations, the bag is still described by a collection of oscillators, but with characteristic frequencies given by the sequences ω_n' and ω_n''.

The problem of analyzing the breathing modes of a bag containing fermions is much more difficult than the bosonic case, for several reasons. Recall that fermion fields obey linear (n_μ is an inwardly-directed normal to the bag's surface)

$$i n_\mu \gamma^\mu \psi = \psi \tag{2.16}$$

and quadratic

$$n_\mu \partial^\mu \left(\overline{\psi} \psi \right) = 2 B \tag{2.17}$$

boundary conditions at the surface of the bag. The linear boundary condition involves velocities and therefore cannot be imposed as a holonomic constraint on the degrees of freedom of the system. Moreover, the quadratic boundary condition relates the velocity of displacement of the surface of the bag to the gradients of the fields. But because Eq. (2.17) contains the velocities (and not the accelerations) of the boundary coordinates, these cannot be taken as independent canonical variables and should be expressed in terms of the field degrees of freedom. Finally, fermions simply do not have any "large" quantum numbers which one can use to characterize a zero-order solution.

These facts introduce complexities which we have not been able to master satisfactorially. We expect, however, that the physical effects of the motion of the boundary of a fermionic system will be analogous to those which occur in the bosonic case. This expectation is supported by the P- wave analyses of bosonic and fermionic systems of Ref. (5-6). Therefore we will proceed, adapting to the more realistic system of quarks and gluons our calculations in the bosonic sector.

III. BARYONIC PHENOMENOLOGY

The lowest radial excitations of the bag are those with two quarks in the ground $(1S_{1/2})$ state and one quark excited to a $(2S_{1/2})$ cavity eigenmode. Their wave functions are easily constructed by coupling together three-quark flavor, spin, and space wave functions to form totally symmetric combinations. Explicit forms of the wave functions may be found in Ref. 4.[9] These radial excitations form a (56) and a (70) in the SU(6) of flavor and spin.

In the absence of gluon interactions, states of nonstrange quarks are good SU(6) eigenstates. The wave function of any quark in an SU(6) eigenstate is a linear combination of $(1S_{1/2})$ and $(2S_{1/2})$. However, these states are not eigenstates of the Hamiltonian if one (or two) of the quarks are strange, since for any $n, \omega_n(\text{strange}) \neq \omega_n(\text{non-strange})$. Λ and Σ radial recurrences are mixtures of pure (56) and (70) states in which the strange quark lies completely in a 1S or a 2S cavity eigenstate.

Gluon-exchange contributions to the Hamiltonian are shown in Fig. 1. They are two kinds: direct (Fig. 1a) and exchange (Fig. 1b). contributions. The exchange contributions in the $(1S_{1/2})^2(1P_{1/2})$ calculation were found to be small and we expect them to be negligible here too. Therefore we keep only the direct interaction terms and write

$$\Delta E_g = \frac{8}{3}\,\frac{\alpha_c}{R} \sum_{i>j} \vec{\sigma}_i \cdot \vec{\sigma}_j \; M_{ij}\,(m_i R, m_j R) \qquad (3.1)$$

where α_c is the color coupling constant, x_i/R is the eigenvalue of momentum of quark i and M_{ij} is given by Ref. 3. Gluon matrix elements between the various eigenstates may be obtained from Ref. 4.

The diagonalization of the Hamiltonian which includes quark kinetic energies, gluon corrections, and the zero-point energy $-Z_0/R$ is shown in Fig. 2. The four parameters of the theory: B the bag const [illegible] parameters of the theory, B the bag constant, Z_0, α_c, and m_s the strange quark mass, are fixed by the fit to the light hadrons of Ref. 3. As Bowler and Hey[10] have noted, the NP11 states are poor candidates for the Roper resonance: they are too heavy and too nearly degenerate. Moreover, the physical states are

$$
\begin{aligned}
\mathrm{NP11}\ (1543) &= \frac{1}{\sqrt{2}}\left(|56\rangle - |70\rangle\right) \\
\mathrm{NP11}\ (1646) &= \frac{1}{\sqrt{2}}\left(|56\rangle + |70\rangle\right)
\end{aligned}
\tag{3.2}
$$

the heavier one having a vanishing photoproduction matrix element.[10] As both the lighter experimentally-observed NP11 states, the N(1470) and N(1780), are seen in photoproduction experiments,[11] the NP11 bag states do not seem to be good candidates for the states of experiments.

However, this is not the whole story. In analogy with the boson calculation, we expect that the coupling of quark radial excitations to surface breathing modes will lower the energy of radially-excited states by an amount Δ. Since the breathing-mode states are excitations of the surface of the bag, and do not involve the quarks' flavors or spins, their SU(6) structure is a (56). Thus, they can couple only to the radially-excited (56) and not to the (70). We are led naturally to the result that the radially-excited (56) should be driven down in energy, while the (70) should remain centered at about 1700 MeV.

We include this effect in our phenomenology by adding a new term to the effective Hamiltonian:

$$
\begin{aligned}
\langle 56|\, H_{\mathrm{breathe}}\, |56\rangle &= -\Delta/R \\
\langle 70|\, H_{\mathrm{breathe}}\, |70\rangle &= 0 \\
\langle 70|\, H_{\mathrm{breathe}}\, |56\rangle &= 0.
\end{aligned}
\tag{3.3}
$$

(Taking Δ to be a constant is equivalent to ignoring quark mass effects in the derivation of Section II.) We may take Δ from the boson calculation

$$\Delta = .354\,\pi \simeq 1.11 \tag{3.4}$$

which is remarkably close to the value which would be obtained if it were fit to the mass of the Roper resonance:

$$\Delta \simeq 1.05.$$

A calculation of the spectrum of radial recurrences in the bag with the former value for Δ is shown in Fig. 3. We also show the positions of presently-accepted experimentally observed positive parity baryon resonances for which our states are candidates.[11] Table I gives the mixing angles of our states with respect to the SU(6) eigenstates of Ref. 4.

We see that the fit is in good qualitative agreement with experiment. Our Roper resonance has a mass of 1410 MeV. The ordering of states is in agreement with observation; however, our higher NP11 and both Δ states are a bit too light. The lightest ΛP11 and ΣP11 are nearly pure (56). The heavier Λ's and Σ's are mixtures of SU(3) multiplets, due to the effects of the quarks' kinetic energies, as we have explained above. The heaviest experimentally seen states of Fig. 3, in particular the Δ P31(1910), may be members of different, heavier SU(6) multiplets. That being the case, an additional, lighter Δ P31 is required of experiment by the model.

Our solution to the radial recurrence problem is still incomplete, since we have not yet computed Δ for fermions. Nevertheless, the near equality of calculated and observed Roper masses is a strong encouragement that the general program is a success.

We see that, as in the case of the P-wave baryon resonances, the inclusion of boundary fluctuations has resulted in a marked improvement in bag spectroscopic calculations over the cavity approximation. We may draw two conclusions from our analysis. First, the static cavity approximation is a poor representation both of the spectrum of excited states of the bag model and of the spectrum of excited states of baryons. Allowing the shape of the confining region to fluctuate results in a new calculational approximation whose spectroscopy is in much better agreement with observation. Second, we expect that the sort of effects we have described here will be found in any system in which the confining mechanism has dynamical degrees of freedom. The spectrum of states of systems in which the confining degrees of freedom are excited may be quite different from the spectrum which arises only from the excitations of the confined quarks.

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TABLE I

Mixing matrices between fluctuating-surface bag eigenstates and SU(6) eigenstates, with $\Delta = 0.354\pi$. SU(6) eigenstates are further labelled by an SU(3) quantum number.

$$NP13(1756) = |70, \underline{8}\rangle$$

$$\begin{matrix} NP11(1603) \\ NP11(1410) \end{matrix} = \begin{bmatrix} .09 & .996 \\ .996 & -.09 \end{bmatrix} \begin{matrix} |70, \underline{8}\rangle \\ |56, \underline{8}\rangle \end{matrix}$$

$$\Delta P33(1572) = |56, \underline{10}\rangle$$

$$\Delta P31(1652) = |70, \underline{10}\rangle$$

$$\Delta P03(1910) = |70, \underline{8}\rangle$$

$$\begin{matrix} \Lambda P01(1796) \\ \Lambda P01(1694) \\ \Lambda P01(1537) \end{matrix} = \begin{bmatrix} .17 & .83 & -.53 \\ .27 & .48 & .84 \\ .95 & -.29 & .14 \end{bmatrix} \begin{matrix} |56, \underline{8}\rangle \\ |70, \underline{1}\rangle \\ |70, \underline{8}\rangle \end{matrix}$$

$$\begin{matrix} \Sigma P13(1870) \\ \Sigma P13(1685) \end{matrix} = \begin{bmatrix} .25 & -.97 \\ -.97 & -.25 \end{bmatrix} \begin{matrix} |56, \underline{10}\rangle \\ |70, \underline{8}\rangle \end{matrix}$$

$$\begin{matrix} \Sigma P11(1788) \\ \Sigma P11(1739) \\ \Sigma P11(1579) \end{matrix} = \begin{bmatrix} .20 & -.94 & .28 \\ .11 & -.26 & -.96 \\ .97 & .23 & .05 \end{bmatrix} \begin{matrix} |56, \underline{8}\rangle \\ |70, \underline{10}\rangle \\ |70, \underline{8}\rangle \end{matrix}$$

FIGURE CAPTIONS

1. One gluon exchange contributions to the energy of $(1S_{1/2})^2(2S_{1/2})$ cavity eigenstates. (a) Direct interaction. (b) Exchange interaction.

2. Spectrum of $(1S_{1/2})^2(2S_{1/2})$ bag states in the static cavity approximation. States are labelled by their angular momentum and flavor.

3. Spectrum of $(1S_{1/2})^2(2S_{1/2})$ bag states with the inclusion of boundary fluctuations. States are labelled by their angular momentum and flavor. Shaded regions indicate the positions of established baryon resonances as given by Ref. 11; the number of asterisks indicates the trustworthiness of the state's existence, as quoted by that reference.

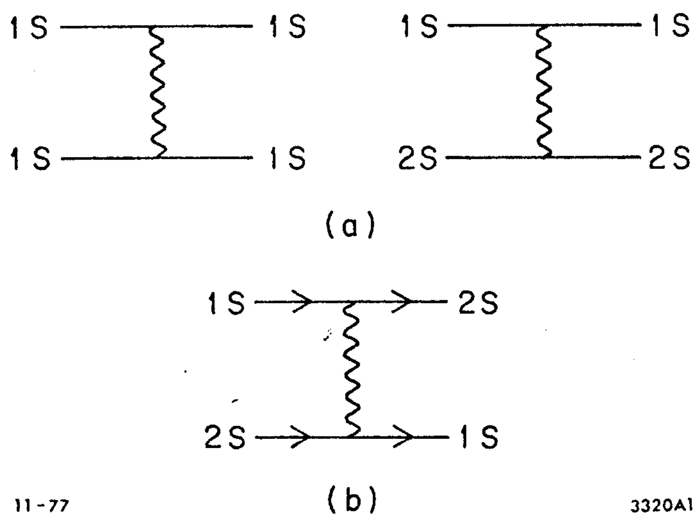
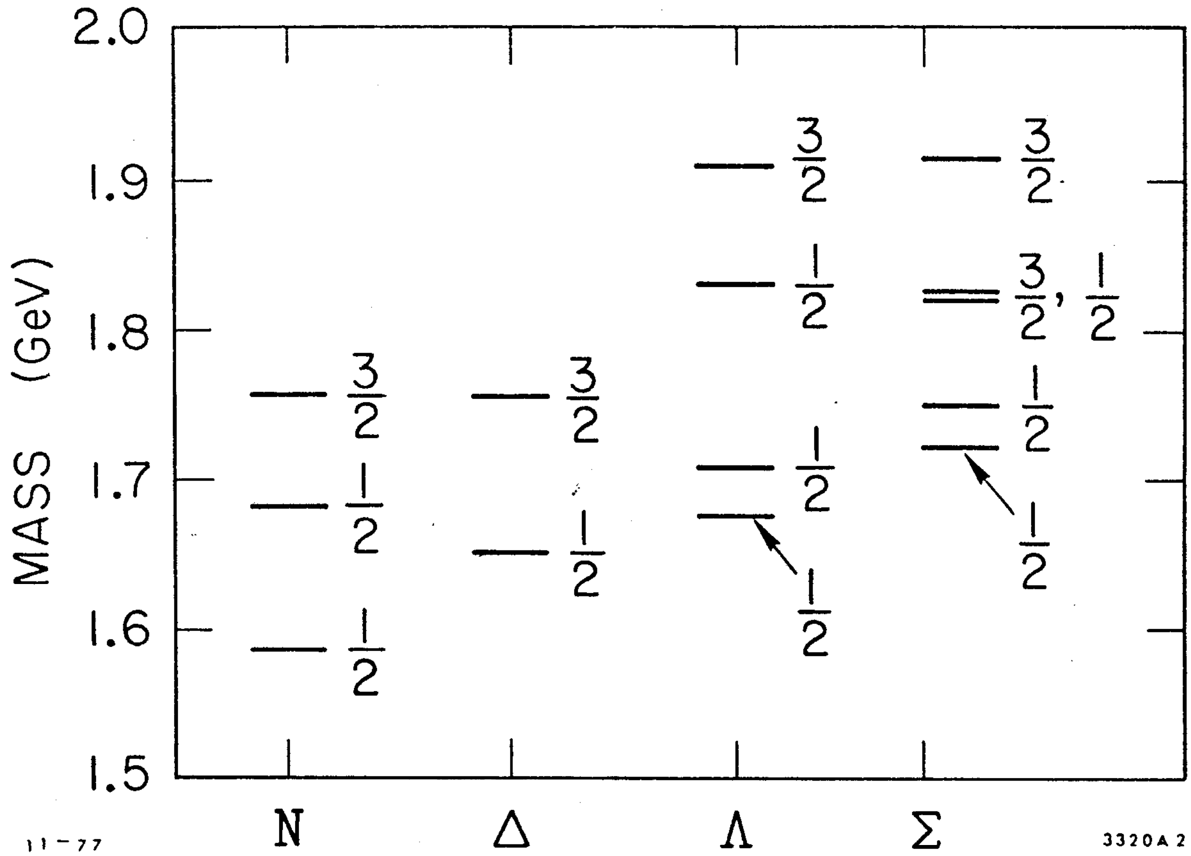
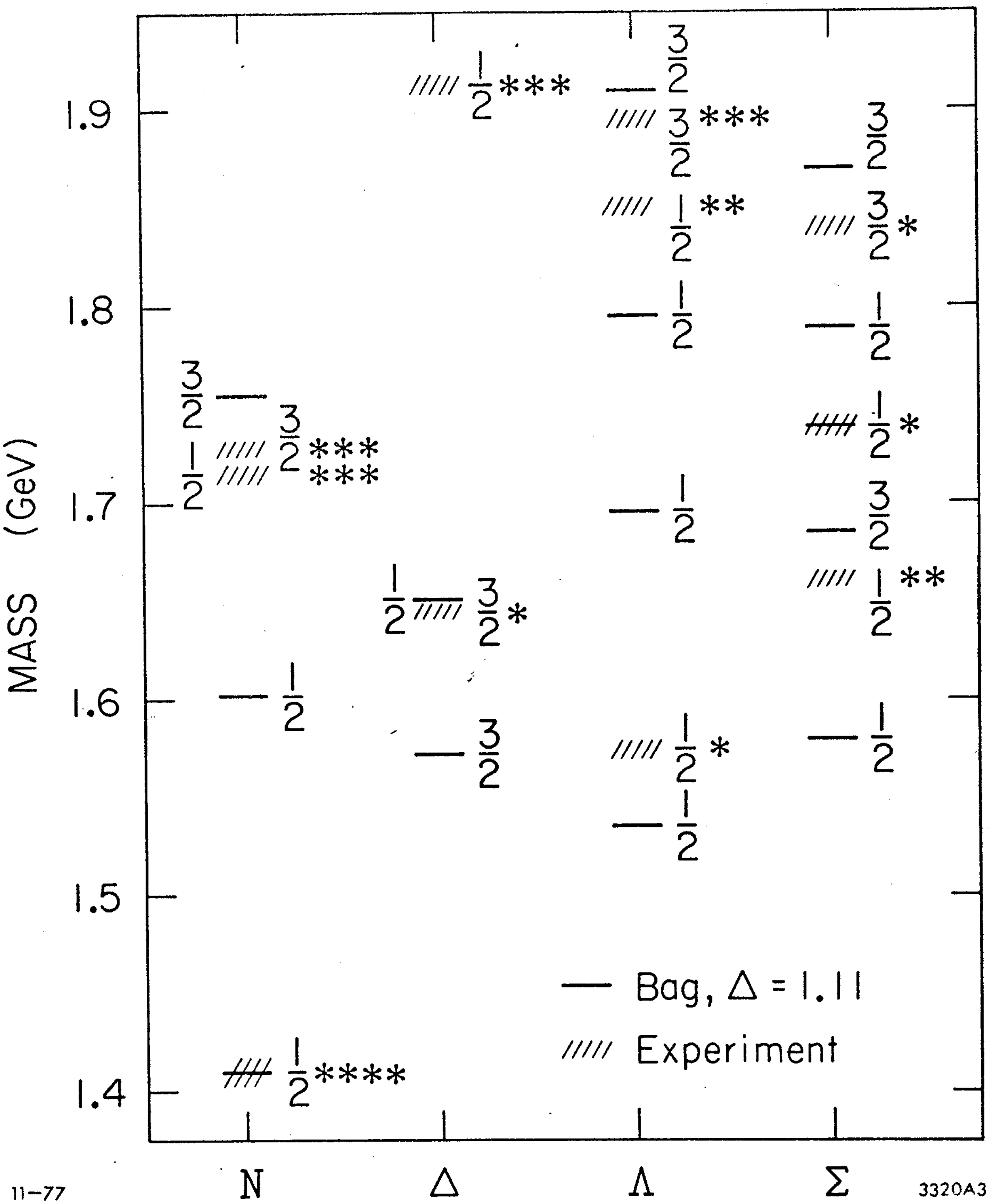


Fig. 1



2.0
1.9
1.8
1.7
1.6
1.5
MASS (GeV)
3/2
1/2
1/2
3/2
1/2
3/2
1/2
1/2
1/2
3/2
3/2, 1/2
1/2
1/2
N
Δ
Λ
Σ
11-77
3320A2

Fig. 2



MASS (GeV)
1.9
1.8
1.7
1.6
1.5
1.4
N
Δ
Λ
Σ
Bag, Δ = 1.11
Experiment
11-77
3320A3

Fig. 3