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ALTERNATIVES TO GAUGE THEORIES

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To a remarkable degree, gauge principles now permeate the description of almost all of the basic interactions of elementary particles. The proceedings of this Conference make it rather clear why this is so, and why it is hard to find any theorist (including myself) working actively and continuously on theories which lead in a direction contradictory to that of the gauge theories.

This state of affairs is a great tribute to those who created and developed this impressively rich, beautiful, and promising theoretical structure - and this tribute manifestly includes Ben Lee, whom we honor here.

But I think this situation has its dangers. Even the greatest creative endeavors should be made in the face of a strong critical background, in order to be certain that we really proceed in the right direction. To be sure, the absence of criticism of gauge-theory ideology these days is quite understandable. To work on something else is to become a bit of a social outcast, and that is something the younger (untenured) generation may choose not to face. Also, ever since we were taught how to calculate with non-Abelian gauge theories, there have been a lot of calculations - especially for weak processes - to be carried through. Indeed this calculability is a hallmark of the weak-electromagnetic theories: an experimentally accessible observable may be calculated in a definite way. It may be calculated not only in one definite way, but many definite ways with many different definite answers, depending on one's starting point. It is no wonder it is a popular

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business. Thus far, the alternatives to gauge theories offer nothing much to calculate, but only the supposition that the situation is too complicated to anticipate and to calculate precisely.

In any case it is clearly important to examine the whole gauge-theory enterprise in the most hardheaded and pragmatic way possible: is it conceivable that we can get along without it? And, while raising this question here might seem to be disrespectful to Ben Lee's contributions, this is just not the case. Ben appreciated the need to ask such questions, and in fact was responsible for this subject being included in the program.

What are the arguments for gauge theories? I divide them into two categories: subjective and objective. The subjective arguments are those which persuade even in the absence of data. Objective arguments are those which only persuade in the presence of data. I do not mean to imply that "subjective" and "objective" are euphemisms for "bad" and "good". For me physics without subjective argument isn't worth doing. But however one chooses to weigh subjective and objective arguments, I think that one must make a clear distinction between them. This will be what we try to do here. We shall first try to catalogue objective and subjective arguments for quantum chromodynamics (QCD), and then do the same for weak-electromagnetic gauge theories. Gravity, supergravity, gauge supersymmetry, and superunified theories will not be discussed at all.

I. Arguments for QCD

We begin with objective arguments.

1. Asymptotic Freedom

If one insists upon describing quark interactions by a renormalizable local field theory, then the existence of approximate scaling-behavior is an argument for QCD, as opposed to other renormalizable theories with anomalous dimensions, where scaling-violations might be expected to be bigger. Also, the correct values of the cross section for $e^+e^- \rightarrow$ hadrons, both below and above charm threshold, are rather successfully predicted in QCD but not in other theories. Finally, the scale-breaking effects in electroproduction, while still interpretable in several ways, are in the direction anticipated by QCD.[1]

2. Chiral Structure

The successes of current-algebra, PCAC, and the $(3,\bar{3})\oplus(\bar{3},3)$ structure of chiral $SU(3) \otimes SU(3)$ symmetry breaking are naturally contained within the QCD Lagrangian. Even the potential trouble with the U(1) problem[2] (there should be a Goldstone mode with mass comparable to the pion) seems to be solved via the rich vacuum structure of QCD.[3]

3. Spectroscopy

Color singlets lie lowest in naive calculations with QCD potentials. Also the splitting of Δ from N can be accounted for by the spin-spin force generated by colored gluon exchange.[4]

4. Charmonium Spectroscopy

There is a certain level of qualitative and semiquantitative agreement with what is expected from QCD, perhaps including some insight into the Zweig-rule. But the hyperfine splittings and some properties of the radiative transitions are not in good condition, so that they tend to cancel out the successes.[5]

None of the above arguments appear to me as especially strong. And I doubt that they are what really motivate theorists to take QCD seriously. Rather there exist strong subjective arguments:

(1) Existence of the color quantum number. I list this here because, while baryon spectroscopy, the $\pi^\circ \rightarrow \gamma\gamma$ decay rate, and the e^+e^- total cross section do provide objective evidence, it is only evidence for *global* color symmetry, and

not a local gauge principle. One would like to see evidence of the existence (as partons) of the gauge-gluons. But that is still for the future.[1]

(2) Existence of a local gauge-principle is a hypothesis that flows naturally from the previous item. It is a property in common with general covariance and electromagnetic gauge invariance and goes very deep. Perhaps all successful field theories will be gauge theories, and perhaps the key to unification lies in extension of gauge principles.

The idea, embodied in the gauge principle, that existence of color implies existence of strong interaction is also a unifying principle which helps to synthesize lepton-world with quark-world: existence of color is the only feature which distinguishes lepton from quark. But could one accomplish the same end without a gauge principle?

(3) Infrared slavery: Pure QCD is a very sick theory at low momenta and energies. This sickness is turned into radiant health by arguing that this is the key to the problem of apparent confinement of quarks and gluons. There is some support for this from the beautiful work of Wilson[6] and others on lattice QCD. But nothing yet exists that is clearly applicable to the real world.

(4) Renormalizability: We reserve comment on this until we discuss weak-electromagnetic gauge theories.

(5) Dynamics of high energy deep-inelastic processes: The evolution of hadron final states in, say, the process

$$e^+e^- \rightarrow q\bar{q} \rightarrow \text{hadrons}$$

from initial quark pair to final hadron system appears to demand the existence of long-range correlation in rapidity. This in turn suggests the J=1 gluons of QCD as the source of this correlation.[7]

II. Alternatives to QCD

A serious alternative to QCD is the integer-charge (Han-Nambu) version of the quark model, as pursued by Pati and Salam. If color is an exact (ungauged?) symmetry in such a theory, and if only color-singlet states exist at present energies, then the present-day phenomenology is isomorphic to the fractionally charged quark model.[8] On the other hand, if one tries to liberate color at present energies it appears that it can only be done (if at all) within the context of a gauge theory.[9] Therefore that option is not an alternative to gauge theories.

Another alternative is the string model of hadrons. Someday it might be preempted by QCD, but that has not yet happened. So far it does not fit into the mold of standard renormalizable local field-theories. Indeed alternatives may well need to have this property; otherwise the scaling behavior of deep inelastic processes tends to force one back to asymptotically free theory, i.e., QCD.

Could quarks be composites? We generally think that (current) quarks are so fundamental that they should be described in a simple way via a simple and beautiful Lagrangian. Not too long ago there existed such hopes for nucleons and their interactions. And even now a rather pretty - but nonrenormalizable - effective Lagrangian does not do badly in describing low energy interactions of mesons and nucleons. But as energies increase, that description evaporates. Maybe history can repeat.

Continued proliferation in the number of quark flavors might make this notion more attractive, just as nucleon compositeness (or at least "democracy") became increasingly attractive as the continuing discoveries of hadron resonances fattened the Particle Data Group compliations. However, compositeness of quarks is not as easy a notion as compositeness of nucleons. Once one disregards the color assignment, quarks and leptons have remarkably similar properties: the same kind of weak and electromagnetic interactions, and most likely a common origin for their bare mass. It is not much easier to contemplate internal structure for the quark than to contemplate internal structure for the neutrino.

The present situation in strong interactions has been summarized by saying that, while there is little objective evidence in favor of QCD, any known alternative has much less to offer. In other words, QCD is the only known field theory of strong interactions that has a chance of being right.

III. Arguments for Weak-Electromagnetic Gauge Theories

The situation for the weak-electromagnetic interactions is quite different from QCD. Not only are there good subjective arguments for the gauge-theory description, but there is also apparent objective evidence in the remarkable agreement of a great deal of data with the simplest version. Let us begin by listing the subjective arguments:

1. Existence of a gauge-principle: The comments made with regard to QCD again may apply.
2. Unification with electromagnetism: How can one object to that?
3. Renormalizability: Much is made of renormalizability as a requirement or criterion for choosing theories. I have never understood this. If we were to be consistent, we might be obligated to throw out general relativity. It is clear that even if somebody showed that general relatively was incurably nonrenormalizable, the theory would not be abandoned.

I am also unimpressed with related subjective criteria imposed upon weak-electromagnetic gauge theories, such as "anomaly-cancellation" or "naturalness". The anomaly plague only affects observables at a level of precision comparable to that reached in g-2 experiments. Is it obvious the correct theory attains such perfection?. The naturalness concept[10] presupposes that the parameters determining the quark and lepton masses and mixings are random variables, and only theories that respect observed properties (such as small strength of $\Delta S = 2$ nonleptonic transitions) for all values of the parameters are to be acceptable. This would imply the masses themselves are random variables. But why, then, the very large ratios such as m_u/m_c? Why the curious degeneracies such as $m_\tau \sim m_c$ and $m_\mu \sim m_s$? It seems that one problem of masses is not that they are randomly distributed, but that they show organization: patterns do exist, but we do not know how to explain them.

I therefore prefer to avoid such criteria, criteria which threaten to exclude the one Correct Theory everyone is searching for. If such a theory really exists, I would expect that when it is found, we will not need such criteria: we will know it is correct.

There is one cogent reason for demanding renormalizability. To presume it, along with QCD, is to presume that we can have a closed, essentially complete description of particle phenomena at all distances down to the Planck distance $\sim 10^{-33}$ cm. If we really can extrapolate our present concepts 18 orders of magnitude we have some reason to ask for perfection. Nevertheless there does remain the possibility that the push from 10^{-15} cm toward 10^{-33} cm may reveal just as much richness as we have found in going from 10^{-8} cm to 10^{-15} cm. If there is such richness we need not demand that the present theoretical description be a closed one. And if it is not closed, then the criterion of renormalizability need not apply.

4. Existence of a mechanism for generation of intermediate-boson and fermion mass: The Higgs-mechanism provides a beautiful way to render an intractable theory into a manageable and calculable form, as well as providing insight into how quark and lepton masses, and perhaps even the Cabibbo-angle and CP violation, might be generated. But when the Higgs sector of a typical theory is studied

in detail, it is rare that output exceeds input. The elegance of the typical fermion and gauge-boson Lagrangian is only surpassed by the clumsiness of the Lagrangian for its Higgs sector. Despite all the effort that has gone into this question, there does not yet exist a detailed picture of mass-generation that persuades.

In addition to subjective arguments there exist objective arguments for the weak-electromagnetic gauge theories. Some of these I consider specious; others however are very impressive.

(1) Existence of neutral currents: Despite the fact that historically the gauge theories were of crucial importance in stimulating the search for $\Delta S = 0$ neutral-currents, this does not imply a logical connection. If only for reasons of symmetry[11], it is easy to motivate the incorporation of $\Delta S = 0$ neutral currents (of strength comparable to charged currents) into theories of weak interactions. It was the absence of $|\Delta S| = 1$ neutral currents which inhibited theorists from doing this. The solution to this problem lay in charm and the GIM mechanism,[12] which leads to the next argument for gauge theories:

(2) Existence of charm and the GIM mechanism: Again the development of gauge theories,[13,14] together with the discovery of neutral currents, was instrumental in stimulating the search for charm.[15] The GIM mechanism neatly solves, within the gauge theory framework, the problem of $\Delta S = 1$ neutral currents. But again it also appears to solve the problem in a more general context. In fact the GIM argument preceded the demonstration that the gauge-theories are renormalizable. We shall also give an example of a more general scheme for which the GIM mechanism applies.

(3) The intermediate boson hypothesis: This suggests introduction of gauge-bosons. It is also the case that at one time the prominence of the ρ- and ω-exchange contribution to the nucleon-nucleon force invited the hypothesis that they

be gauge particles. One may also obtain the V & A structure of the weak-interaction effective Lagrangian by assumptions of chirality conservation.

(4) University of strength of the weak couplings: This is also a predictable consequence of the gauge-theory approach. But similar conclusions have been reached from a phenomenological starting point, assuming only symmetry of the effective Lagrangian under permutation of various fermion degrees of freedom.[16]

Despite the caveats, I would agree that items 3 and 4 do represent objective arguments in favor of gauge theories, although not completely compelling ones. The really strong one is the last:

(5) $SU(2) \otimes U(1)$ agrees with experiment: To the extent that this is true, this not only argues for gauge-theories in general, but also for the specific model at hand - along with those variants which do not affect the predictions for neutrino-induced neutral currents. [Those variants include putting e_R^- into a doublet, as well as making the $SU(2)_L \otimes SU(2)_R \otimes U(1)$ or $SU(2) \otimes U(1) \otimes G$ extensions discussed[17] recently.]

I tried to assess how much objective evidence for the standard model is provided by existing experiments. A good starting point is the data for deep inelastic neutral currents, i. e., the ratios R and $\bar{R}$ of neutral-current to charged-current cross sections. The standard one-parameter model predicts R, given $\bar{R}$, and the predictions work very well. As argued above, even in alternatives to gauge theories we could reasonably expect R and $\bar{R}$ to be nonvanishing and of order 1. To crudely assess the significance of the success of $SU(2) \otimes U(1)$, assume that in a random model $R \leq 1$ and $\bar{R} \leq 1$. What fraction of this piece of R-$\bar{R}$ space does the $SU(2) \otimes U(1)$ prediction cover? Strictly a set of measure zero. But, if for no reason other than theoretical uncertainty, a certain percentage deviation of R and $\bar{R}$ from the prediction should be allowed. Here we take it as ±15%. In Fig. 1 is plotted the range of values of R and $\bar{R}$ allowed for the SU(2)$\otimes$U(1) model.[18] The fraction of the space which is acceptable is only about 8%. This is fairly significant. But if one allows other one-parameter gauge theories,[18] then the space quickly begins to fill up (Fig. 2). And just allowing m_Z/m_W to vary from its standard value by a factor κ, with $0.7 \leq \kappa \leq 1.4$, is sufficient (Fig. 3) to fill up half the space. Thus the quantitative success is as much a success of standard SU(2)$\otimes$U(1) as it is a success of general gauge-theory ideology.

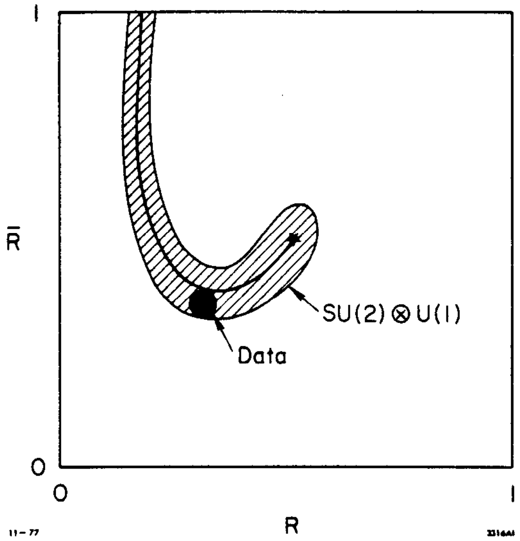

Fig. 1. Region of R-$\bar{R}$ space allowed by the standard SU(2)$\otimes$U(1) model: $R = \dfrac{\sigma(\nu_\mu N \to \nu_\mu \text{ hadrons})}{\sigma(\nu_\mu N \to \mu^- \text{ hadrons})}$ and $\bar{R} = \dfrac{\sigma(\bar{\nu}_\mu N \to \bar{\nu}_\mu \text{ hadrons})}{\sigma(\bar{\nu}_\mu N \to \mu^+ \text{ hadrons})}$. (The curve is for the most naive quark model and is only schematic.)

This success is further sharpened by the restrictions on the isospin and chiral structure of the neutral-current couplings. In Fig. 4 is shown an analysis of Hung and Sakurai,[19] based on studies by Sehgal[20] of the charge-ratio of leading pions produced in neutral-current processes.[21] The solutions in the first and third quadrants for the neutral-current coupling constants[22] $\epsilon_L(u)$ and $\epsilon_L(d)$ are unacceptable when one includes the information from elastic νp and $\bar{\nu} p$ measurements. The significance of agreement is better than 20%, so that the combined data argue for SU(2)$\otimes$U(1) at better than the 2% level.

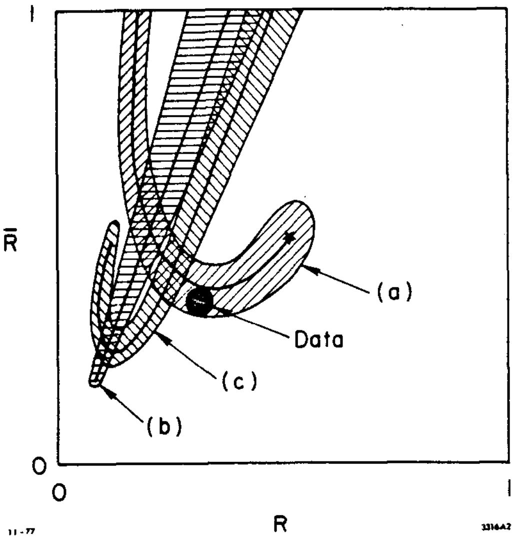

Fig. 2. Region of R-$\bar{R}$ space allowed by various models: (a) standard model, (b) vector-like model, and (c) 5-quark model.

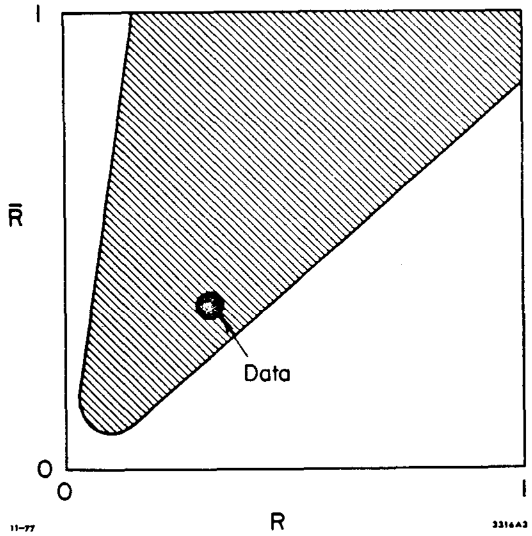

Fig. 3. Region of R-$\bar{R}$ space allowed in the standard model if one varies $\kappa = (m_Z/m_W)\cos\theta_W$ from ~0.7 to ~1.4.

Sehgal's analysis depends on both the parton model and the assumption that Gargamelle energies are high enough to use that model. So one should still be very careful. But let us cast such doubts to the wind and accept that the agreement of data with the simplest SU(2)$\otimes$U(1) model is, by any reasonable objective criterion, quite significant, and the chance that it is accidental is at the few percent level or less. This would seem to imply impeccable objective evidence for the weak-electromagnetic gauge theories.

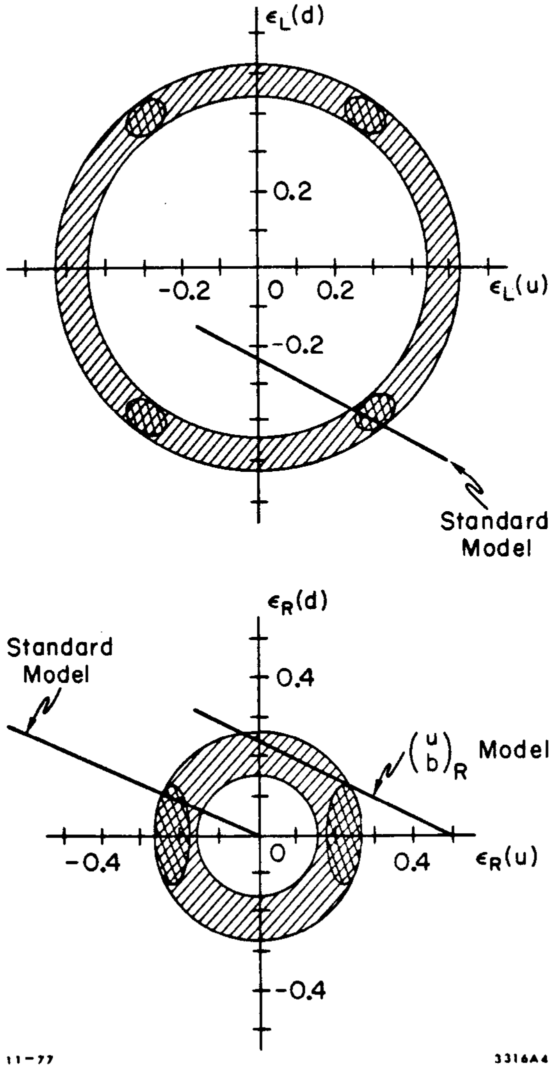

Fig. 4. Region of neutral-current coupling-constant space allowed by deep-inelastic data (hatched) and pion charge-ratio data (cross-hatched). $\epsilon_L(u)$ measures the coupling of ν_μ to a left-handed up-quark; the other ϵ's are defined similarly. The upper right and lower left quadrants in $\epsilon_L(u)$-$\epsilon_L(d)$ space are disfavored by elastic neutral current data.

But before drawing such a conclusion it is necessary to examine whether the Weinberg-Salam effective Lagrangian may be obtained in alternative ways. We now turn to that question.

IV. Alternatives to Weak-Electromagnetic Gauge Theories

Let us accept that the Weinberg-Salam effective Lagrangian for neutrino-induced neutral currents is indeed correct. We shall now argue that it can be obtained in a more general context. To motivate this, we revise history and suppose that both charm and strangeness (including the GIM mechanism) were discovered earliest, that $\Delta S = 0$ neutral-current processes were measured later on, and that the development of renormalizable gauge-theories came latest.

In the first stage, the charged-current effective Lagrangian (with Cabibbo-mixing) would have been determined. And a large number of theorists would find it irresistible not to "complete" the Lagrangian[11] by adding neutral currents in such a way as to give the weak interaction a global SU(2) symmetry:

$$-\mathscr{L}_{\text{eff}} = \frac{G}{\sqrt{2}} \vec{J}_\lambda \cdot \vec{J}^\lambda \tag{4.1}$$

where

$$\vec{J}_\lambda = \sum_i \bar{\psi}_i \gamma_\lambda \left(\frac{1-\gamma_5}{2} \right) \vec{\tau} \psi_i \tag{4.2}$$

with, as usual, the weak doublets

$$\psi_i = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}, \begin{pmatrix} c \\ s\cos\theta_c - d\sin\theta_c \end{pmatrix}, \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \cdots \tag{4.3}$$

The GIM mechanism is operative, and the neutral current is strangeness conserving. It has the form

$$-\mathscr{L}^{NC}_{\text{eff}} = \sqrt{2}\, G\, \bar{\nu}_\mu \gamma^\lambda \left(\frac{1-\gamma_5}{2} \right) \nu_\mu J_\lambda^{\,3} \tag{4.4}$$

with

$$J_\lambda^3 = \bar{u}\gamma_\lambda\left(\frac{1-\gamma_5}{2}\right)u - \bar{d}\gamma_\lambda\left(\frac{1-\gamma_5}{2}\right)d + \cdots \tag{4.5}$$

Skeptics, who might refuse to admit such neutral currents without any objective evidence for them, would still expect an effective neutral-current generated by photon-exchange.[23] The electrically neutral neutrino could have a charge-radius and couple to photon via a vertex function

$$e\Gamma_\lambda(q) = e\bar{\nu}_\mu\,\gamma_\lambda\left(\frac{1-\gamma_5}{2}\right)\nu_\mu\,\frac{q^2}{\Lambda^2} \tag{4.6}$$

This leads to a contact-interaction (similar to the low energy neutron-electron interaction) with charged particles of the form

$$-\mathcal{L}_{\rm eff}^{\rm NC} = \frac{-e^2}{\Lambda^2}\,\bar{\nu}_\mu\,\gamma_\lambda\left(\frac{1-\gamma_5}{2}\right)\nu_\mu\,J^\lambda_{\rm em} + \cdots \tag{4.7}$$

with

$$J^\lambda_{\rm (em)} = \frac{2}{3}\,\bar{u}\gamma^\lambda u - \frac{1}{3}\,\bar{d}\gamma^\lambda d + \cdots \tag{4.8}$$

What is the verdict of the neutral current experiments? It is simple: one just <u>adds</u> the two contributions, Eqs. (4.4) and (4.7), and the result is precisely the Weinberg-Salam Lagrangian. The single parameter $\sin\theta_W$ of the Weinberg-Salam Lagrangian is equivalent to the single parameter Λ^{-1} (neutrino charge-radius); the connection is

$$2\sqrt{2}\,G_F\,\sin^2\theta_W = \frac{e^2}{\Lambda^2} \tag{4.9}$$

or

$$|\Lambda| = \frac{53\ \text{GeV}}{\sin\theta_W} \lesssim 100\ \text{GeV} \tag{4.10}$$

So this is the alternative point of view: It is simple enough to be taught to children,[24] and it accounts for all the data on neutrino-induced neutral currents. The assumptions used include:

(a) Existence of a global weak SU(2) symmetry

(b) Existence of a "large" photon exchange contribution

(c) Absence of large electromagnetic corrections to the intrinsic weak couplings via proper diagrams as in Fig. 5. The reason for including this point is that, in this general context, the bare photon coupling might be much larger than e, even at TeV energy scales. (See Section 5.)

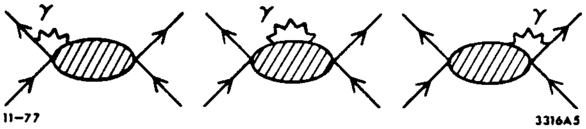

Fig. 5. Proper electromagnetic corrections to intrinsic weak amplitudes, which we neglect.

Assumptions <u>not</u> necessary to make include:

(a) Existence of a local gauge principle

(b) Renormalizability

(c) Intermediate boson hypothesis

(d) Unification of the weak and electromagnetic force

(e) Mass generation via symmetry breakdown of Higgs-bosons.

While I am sure this description must have occurred to others in the past it surprises me that it seems not to be generally known. The brilliance of the gauge theories has blinded us from seeing the simplest of alternatives.

Let us return to our revisionist version of history. The large[25] value for the neutrino charge radius Λ^{-1} (or alternatively $\sin\theta_W$) already implies a "weak cut-off" $\lesssim$ 150 GeV, as we shall describe in the next section. Thus if one assumes existence of $W^{\pm}$, its coupling constant to fermions is necessarily comparable to e. Thus strong motivation for weak-electromagnetic unification would exist, even in the absence of the modern formalism. Development of the modern theory would in turn take us to the present situation, with no doubt the same great enthusiasm for the gauge theories. After all, they provide a comprehensive and calculable framework encompassing all the experimental results and predicting many more. And that is something the simple description does not do.

However, given such a historical background, it seems to me that a critic in search of objective evidence would demand a crucial test to distinguish the gauge theories from other possible options. The simple general description of weak interactions in terms of a global SU(2) symmetry plus an electromagnetic contribution has its own credibility even in the absence of an explicit, fully calculable theory to support it. Nor is it in contradiction with the $SU(2)\otimes U(1)$ gauge theory (or its variants), as we shall elaborate in the next section. The gauge theories are a specific realization of the general picture. But I know no argument why they should be the only possible realization.[26]

The fact that $\sin^2\theta_W$ is positive[27] and of order unity can be considered as objective evidence for the gauge-theories. Beyond that, the crucial tests lie in the future: existence of W and Z with the predicted masses and couplings, evidence for the existence of the Higgs-sector, and (ultimately) evidence that at center-of-mass energies in excess of m_W and m_Z, the weak cross sections are as small as predicted by the renormalizable theories.

V. Additional Consequences

The alternative viewpoint we have sketched does have a few quite strong and specific consequences. One is that the weak cutoff, defined in a way appropriate for charged-current neutrino reactions, must be less than 37 $\mathrm{GeV}/\sin^2\theta_W$.

Another is that the yield of neutral weak quanta from e^+e^- colliding beams must be large, as large as in the standard theory[28,29] (where it is concentrated in the Z^0 resonance).

To obtain these results, we only assume that the weak force is generated by some kind of weak quanta (with total $J = 1$) which are exchanged in the t-channel. The weak quanta may be one or more discrete intermediate bosons, or a more complicated continuum, or both. The weak amplitudes become nontrivial functions of t, analytic in the cut t-plane. The various options are illustrated in Fig. 6. For the standard $SU(2) \otimes U(1)$ model the amplitude is dominated by a single pole, with contributions from cuts suppressed by a factor of α. An amplitude with

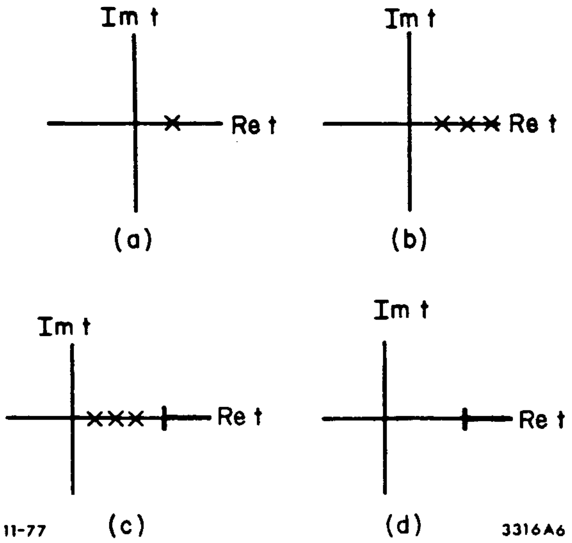

Fig. 6. Options for the analytic properties of the intrinsic weak t-channel amplitude:
(a) Single pole; negligible cut. This is the case of the $SU(2) \otimes U(1)$ gauge theory.
(b) Several poles; negligible cut. This could occur for complicated gauge theories.
(c) Some poles + non-negligible cut. This could occur were W's composites of something else, or if they were to interact strongly with each other.
(d) Only cuts. If the weak amplitude has important s and u channel structure, then there could also be a left-hand cut as well. We here disregard this possibility.

poles and cuts of comparable strength could arise were the intermediate bosons composites of something else,[30] or were they to interact strongly with each other.[31] Amplitudes with only cuts might arise were there s, t, and u-channel contributions.[32]

The amplitudes we need consider are illustrated in Fig. 7. Figure 7a shows the intrinsic weak interaction $T_W(t)$. In Fig. 7b, the neutrino charge form factor T(t) is a weak-electromagnetic interference term. In addition, it is important to include the contribution $T_{em}(t)$ to the electromagnetic vacuum-polarization coming from the photonic coupling of neutral weak quanta. The absorptive parts ρ_W, ρ, and ρ_{em} of these amplitudes must satisfy a Schwartz-inequality

$$\rho_W(t)\ \rho_{em}(t) \geq \rho^2(t) \tag{4.11}$$

and thus the electromagnetic contribution is bounded from below. Considerable manipulation of the above inequality leads to a lower bound on the production of weak quanta in e^+e^- colliding beams[34]

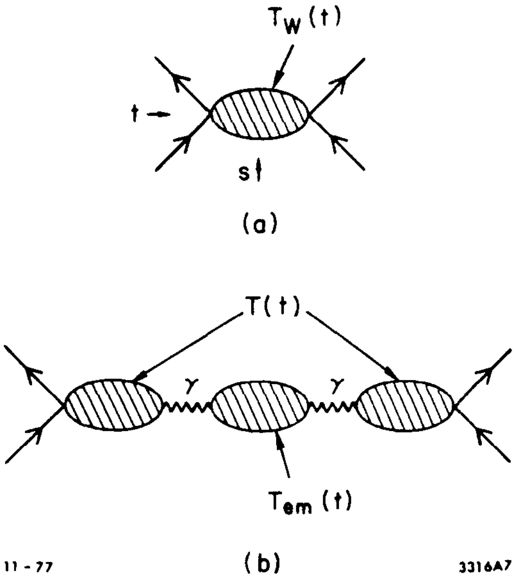

Fig. 7. Schematic of contributions to the neutral current amplitude: (a) Intrinsic weak interaction, and (b) photon exchange, including proper-vertex insertions and photon vacuum polarization contributions.

$$\overline{R} = \int \frac{ds}{s} \; R(s) \geq \frac{3\pi}{2\alpha} \left\{ \left[1 - \left(\frac{37\ \text{GeV}}{m_W \sin^2\theta_W} \right)^{-2} \right]^{-1} - 1 \right\} \left\{ 1 + \left(\frac{1}{2\sin^2\theta_W} - 1 \right)^2 \right\} \tag{4.12}$$

where

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{weak quanta})}{4\alpha^2/3\pi s} \tag{4.13}$$

The parameter m_W is not necessarily an intermediate-boson mass, but is what charged-current neutrino experimentalists call an intermediate-boson mass. It is defined in terms of the t-dependence of the Fermi-coupling:

$$G_F(t) = G_F \left(1 + \frac{t}{m_W^2} + \ldots \right) \tag{4.14}$$

or

$$\left. \frac{d\sigma}{dQ^2 d\nu} \right|_{\substack{\text{charged}\\ \text{current}}} = \left. \frac{d\sigma}{dQ^2 d\nu} \right|_{\text{"scaling"}} \cdot \left(1 - \frac{Q^2}{m_W^2} + \ldots \right)^2 \tag{4.15}$$

Eq. (4.12) is plotted in Fig. 8. One sees that the bound

$$m_W < \frac{37\ \text{GeV}}{\sin^2\theta_W} \tag{4.16}$$

is associated with $\overline{R} \rightarrow \infty$, i.e., with the electromagnetic charge renormalization $Z_3^{-1} \rightarrow \infty$.

The threshold for electromagnetic production of weak quanta need not be the same as m_W (unless the T(t) have no poles). What can be said in general is that the threshold "m_Z" is bounded above as follows:

$$\text{"}m_Z\text{"} \leq \frac{1.5\ \text{GeV}}{\sin^2\theta_W} \sqrt{\overline{R}} \tag{4.17}$$

In other words, it's worth the waiting: For larger "m_Z" one has a larger yield $\overline{R}$.

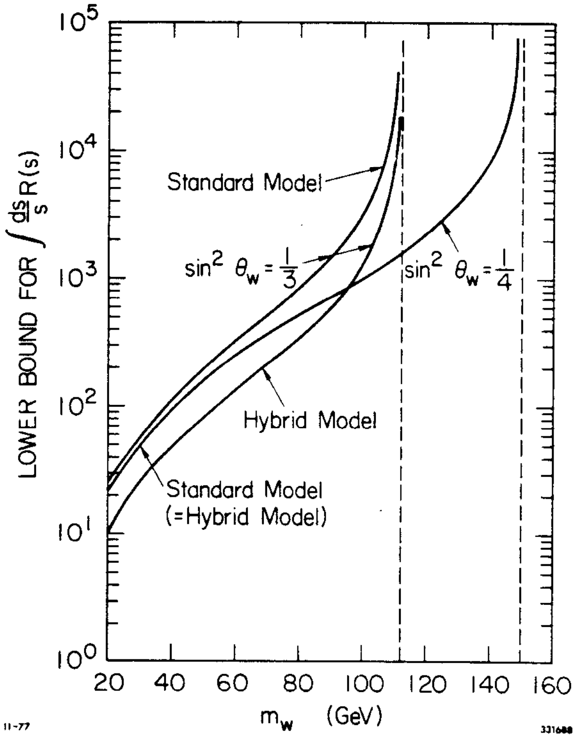

Fig. 8. Lower bound for $\bar{R} = \int \frac{ds}{s} R$, which measures the production of weak quanta by colliding e^+e^- beams. The parameter m_W is defined in Eqs. (4.14) and (4.15); it is the m_W measured in deep-inelastic charged-current neutrino reactions.

It is also enlightening - and simple - to reconstruct the predictions of the standard model. This is accomplished by assuming single-pole dominance for the amplitudes T_W, T, and T_{em}. Let g be the coupling of W's to fermions and $e_0 f$ the direct coupling of W_3 bare photon. The effective neutral current amplitude $\mathcal{M}$ is (including a charge-radius term for the quarks or other fermions):

$$\mathcal{M} = \bar{\nu}_\mu \gamma_\lambda \left(\frac{1-\gamma_5}{2}\right) \nu_\mu \left\{ \frac{g^2}{(m_W^2 - t)} J_3^\lambda \right. \qquad (4.18)$$

$$\left. + \frac{gft}{m_W^2 (m_W^2 - t)} \cdot \frac{(-e_o^2)}{t\left[1 + \frac{e_o^2 f^2}{m_W^2 (m_W^2 - t)}\right]} \cdot \left[J_{em}^\lambda + \frac{gft}{m_W^2 (m_W^2 - t)} J_3^\lambda\right] \right\}$$

Identification with the Weinberg-Salam Lagrangian, as t → 0, gives two conditions:

$$\sqrt{2}\ G = \frac{g^2}{m_W^2}$$

$$2\sqrt{2}\ G\sin^2\theta_W = \frac{gfe^2}{m_W^4} \tag{4.19}$$

Definition of the physical charge e and the mass of the Z^o (the latter from the pole in the photon propagator) gives two more:

$$\frac{1}{e^2} - \frac{1}{e_o^2} \equiv \frac{1-Z_3}{e^2} = \frac{f^2}{m_W^4}$$

$$m_Z^2 = m_W^2 + \frac{e_o^2 f^2}{m_W^2} \tag{4.20}$$

Elimination of f and g then gives two equations for m_W and m_Z

$$m_W^2 = \frac{\pi\alpha(1-Z_3)}{2G\sin^4\theta_W} = \left(\frac{37\ \text{GeV}}{\sin^2\theta_W}\right)^2 (1-Z_3)$$

$$m_Z^2 = m_W^2\, Z_3^{-1} \tag{4.21}$$

With only this information we cannot determine m_W and m_Z. What is missing? It is a statement of unification: At short distances (t → ∞), the photon couples only to the hypercharge, not to T_3. If this is the case, the SU(2) symmetry is restored at short distances and the photon (as we have defined it) is transmogrified into the hypercharge generator of unbroken SU(2)⊗U(1). The implication for the amplitude $\mathcal{M}$ is that the proper vertex of the quark or other fermion, as t → ∞ , should not depend on J_3^λ. Letting $J_\lambda^{em} = \frac{1}{2} J_\lambda^3 + Y_\lambda$, with Y_λ a hypercharge current which is SU(2) singlet, we find from Eq. (4.18) that we must have

$$\frac{gft}{m_W^2\,(m_W^2 - t)} \to -\frac{1}{2} \qquad (t \to \infty) \tag{4.22}$$

or

$$2\,\frac{gf}{m_W^2} = 1 \tag{4.23}$$

This leads via Eq. (4.19) to

$$m_W = \left(\frac{37\ \text{GeV}}{\sin\theta_W}\right)^2 \tag{4.24}$$

and

$$m_Z^2 = m_W^2 \sec^2\theta_W \tag{4.25}$$

What do we see from this exercise? It is that the issue of weak-electromagnetic unification is connected with the as-yet-untested short-distance behavior of the theory. Furthermore the predicted W and Z masses (especially m_Z) appear to be sensitive to this hypothesis of unification.

We also see that the simplest SU(2)⊗U(1) gauge theory fits naturally within the general picture we have discussed. This is in fact true for the known generalizations which preserve the Weinberg-Salam effective Lagrangian for neutrino-induced neutral currents, such as the ambidextrous SU(2)⊗SU(2)⊗U(1) models. The most general such version, based on SU(2)⊗U(1)⊗G, has been discussed by Georgi and Weinberg.[35] It turns out that the neutral-current amplitude in their model breaks up quite naturally into the two pieces (intrinsic SU(2)-invariant weka amplitude and photon-exchange contribution) we discussed in the general context.[36] We shall not go into this in detail.[37] However, Georgi and Weinberg also showed that at least one neutral gauge boson can be no more massive than the standard Z^o: $m_Z \leq m_W \sec\theta_W = (37\ \text{GeV}) \csc\theta_W \sec\theta_W$. With the single-pole approximation for the intrinsic weak amplitude, along with the "unification condition," Eq. (4.22), their result can, not surprisingly, be recovered. However, thus far my attempts to derive this important result from a more general starting point have not been successful, even upon assuming an asymptotic SU(2) symmetry at short distances.

VI. Conclusions

Quantum chromodynamics and the weak-electromagnetic gauge theories provide by far the most profound and promising description of particle interactions we have. Nevertheless, there is as yet not much objective evidence in support of quantum chromodynamics. For the weak interactions, the gauge theories have played a crucial role in guiding weak-interaction experimentation and indeed in correctly predicting many results for neutral-current experiments. However, we have seen that these results are obtainable in terms of a simple and credible framework which, while more phenomenological, is more general than that of the gauge theories. Therefore in assessing the objective evidence for weak-electromagnetic gauge theories I think one should examine those features of the theory which go beyond the level of low-energy phenomenological four-fermion interaction. These include existence of intermediate bosons, weak-electromagnetic unification, renormalizability, and spontaneous symmetry breakdown via the Higgs mechanism. The universal current-current structure of the low-energy effective Lagrangian does provide objective evidence for intermediate bosons. And the large value of $\sin^2\theta_W$ and the consequent low value for "weak cutoff" ($\lesssim$ 150 GeV) provides some objective evidence for weak-electromagnetic unification. But beyond this I find little if any objective evidence for unified, renormalizable gauge theories of weak and electromagnetic interactions.

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36. This was taught to me by Marvin Weinstein, to whom I express my gratitude. It provided important motivation for the entire study.

37. This is carried out in Ref. 33.