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HADRON SPECTROSCOPY

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I. INTRODUCTION: THE QUARK MODEL

Hadron Spectroscopy is a subject which is interesting in its own right. The question of how matter is organized certainly falls within the bounds of the subject matter of physics and is therefore something we as physicists want to understand. With hundreds of hadronic states known to exist we have a substantial problem on our hands. Aside from that, it must be admitted that some people enjoy botany--working out a classification scheme and finding a specimen of some rare species can be a lot of fun.

In past years, another reason often given[1] for studying hadron spectroscopy was that it reflects the symmetries of strong interactions. In other words, a symmetry group of the strong interactions, if realized in the conventional way, results in hadrons falling into mass degenerate multiplets which correspond to irreducible representations of the symmetry group. If the symmetry was only approximate, there could be breaking of the mass degeneracy, but still recognizable multiplets.

However, much of our present interest in hadron spectroscopy stems from another source, the evidence for hadronic substructure. Hadron spectroscopy, plus a great deal of other evidence points toward such a substructure, and in particular to a quark basis for hadronic matter. By interpreting hadron spectroscopy we can deduce some of the properties of the quarks.

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Furthermore, in the absence of free quarks, it is through hadron spectroscopy that we can learn about the dynamics of hadron constituents, and in particular about the quark-quark forces. Included in this is the nature of quark confinement and the kind of force law involved, as well as finer details such as the "residual forces" responsible for mass splittings, the relativistic or non-relativistic character of the situation, etc.

In the following lectures we shall pursue the subject of hadron spectroscopy from what has now become the conventional, quark model point of view. Much of the discussion of the "new" particles will be found in the lectures of M. Perl[2] which complement these.

We start by outlining the basic components of the picture of hadron spectroscopy we will use. This is widely, but not universally, shared by most particle physicists.

Quarks

Having already cited quarks as the building blocks of hadrons, let us review the evidence briefly:

(1) Deep Inelastic eN, μN, νN, and $\bar{\nu}$N Scattering.

The magnitude of the cross section, scaling behavior, and the relationship of structure functions observed in deep inelastic scattering indicate that the nucleon has point, spin 1/2 constituents with which the weak or electromagnetic current interacts.[3] Further, the amount of scattering depends on whether the target is a neutron or proton and on the spin orientation of the proton,[4] so that the constituents which are related to the nucleons isospin or spin are also what is "seen" by the weak or electromagnetic currents.

(2) Electron-Positron Annihilation

The ratio R of the cross section for $e^+e^- \to$ hadrons to that for $e^+e^- \to \mu^+\mu^-$ is a (different) constant both below and above charm threshold, as it should be if the basic process were production of a pair of point particles, followed by their eventual materialization as hadrons.[5] In fact, the part of R due to charmed meson production at SPEAR agrees with what is expected from the basic process of production of a pair of charmed quarks.[6] Furthermore, the observation of back-to-back jets at SPEAR yields the additional information that their angular distribution is characteristic of production of a pair of spin 1/2 particles.[5]

(3) Hadron Spectroscopy.

With a few possible exceptions,[7] the hundreds of hadrons we now know are understood as quark-anti-quark bound states (mesons) or three quark bound states (baryons). An enormous simplification has taken place and is part of the standard "lore". Now one often forgets that something as basic as the ordering of spins and parities of states ($0^-, 1^-$ as lowest mass mesons and $1/2^{+}$, $3/2^+$ as lowest mass baryons is trivially understood in the quark model but is otherwise quite mysterious.

(4) Weak and Electromagnetic Current Matrix Elements.

The quark model gives us a quantitative understanding of both the magnetic moments and magnetic transition moments between the ground state baryons, as we will see in detail later. When formulated in the general framework of the transformation from current to constituent quarks,[8] one can discuss the photon transition amplitudes from the nucleon to excited nucleon resonances. When a few reduced matrix elements are fixed in terms of known amplitudes, one gets correct predictions for the signs and magnitudes of a fair number of other amplitudes.[1] Further, if one is willing to use PCAC to relate matrix elements of the axial-vector current to pion amplitudes, then a similar theory of pionic transitions ensues. Again the signs and magnitudes of many amplitudes are correctly given. It would seem very unlikely that all this is an accident.

(5) High Transverse Momentum Phenomena

It seems very plausible that high transverse momentum hadron production in hadron-hadron collisions has its origin in "hard scattering" of constituents of the hadrons.[9] When compared to (1)-(4) above the connection to quarks is much less direct, and certainly not unique. But the similarities to hadron production in deep inelastic scattering and electron-positron annihilation, especially the production of jets in each case, are quite striking. Although it is much harder to get precision information on quarks in this case, this is an important area of research exactly because it may give us information on quark dynamics in a different setting.[10]

Color

Quarks are thought to carry a strong interaction charge called color. There are three such colors, which we take as red, yellow, and blue. Present experimental evidence for the need for color comes from three sources:

(1) The rate for $\pi^o \rightarrow \gamma\gamma$. The amplitude for $\pi^o \rightarrow \gamma\gamma$, when related to that for $\partial_\mu A_\mu \rightarrow \gamma\gamma$ by PCAC, has a magnitude and sign given by the triangle graph (with a closed fermion loop) anomaly[11] in the coupling of two vector currents to an axial-vector current. Without color, one gets the wrong rate. With it, the amplitude is increased by a factor of three and the rate by a factor 9. It then agrees with experiment.[12]

(2) The ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$. Color increases the predicted cross section (on the basis of the quark model) by a factor of three. This is needed to get even rough agreement with experiment both below and above charm threshold.[5]

(3) The Baryon Wave Function. The wave function for fermions should be totally antisymmetric. If the three quarks in a baryon are a singlet with respect to color (see below), the color part of the wave function is antisymmetric. Thus

the remainder (spin, space and quark type or flavor) must be symmetric. This is indeed the case from the experimental spectrum and in particular is true for the ground state which has a symmetrical spatial wave function combined with one symmetrical in spin and flavor.

Each of these experimental pieces of evidence for color needs some theoretical analysis to deduce the appropriateness of the concept of color, but they only involve "counting" the color quantum numbers. There are other, non-experimental, reasons for color, which have a much less solid basis in concrete facts. They all involve using color as a non-Abelian charge in a gauge field theory context. Nevertheless, they are important and have much to do with the overwhelming acceptance of the idea that colored quarks and gluons are the basis of all strong interactions.

(1) Quantum Chromodynamics (QCD).

The theory of quarks coupled via the color "charge" to gauge vector bosons (gluons) is often referred to as QCD. The color gauge group is SU(3) and there is an octet of gluons. It is a non-trivial point that QCD is the only known field theory (and a non-Abelian gauge theory at that) which has a change of being the correct one for strong interactions.

(2) Asymptotic Freedom

Under certain conditions a non-Abelian gauge theory like QCD has the property of asymptotic freedom:[13] the effective coupling constant vanishes logarithmically at small distances, i.e. at large four-momentum squared. This allows one to "understand" the scaling behavior (characteristic of free field theory) observed in deep inelastic scattering. Even more, the theory predicts that scaling is not exact and the (small) predicted logarithmic breaking is consistent with what is seen in recent experiments.[3]

(5) Infrared Slavery

The increase in the coupling constant as one goes to larger distances inspires the hope that the color forces become infinite for very large separations and quarks (and other objects with color) are confined. Up to now this has not been shown rigorously, but there are some suggestive model calculations of how it might come about.[14]

(4) Okubo-Zweig-Iizuka[15] Rule Violation

Certain strong interaction decays where none of the final hadrons contain the quarks of the initial hadron, are very much suppressed in rate. This is particularly well exemplified in the case of the "new" particles. A theory of these processes involving intermediate gluons leads to a systematics of the mass and spin-parity dependence of the degree of suppression.[16]

(5) Spin-Dependent Quark-Quark Forces

Such forces result in hadron states with the same quark content but different relative spin orientations being split in mass. From the experimental observations it seems that the force between quark and quark in a baryon must have the same sign as between quark and antiquark in a meson. Exchange of a neutral vector meson without color does not yield such a result, but exchange of gluons coupled to color does if within color singlet mesons and baryons. While single gluon exchange does not have to be the origin of all such forces, it still is to be desired that the lowest order effect have at least the right sign.

(6) Dynamical Gluons

Sum rules for deep inelastic scattering indicate that quarks do not carry all the momentum or energy of the nucleon.[3] If the remainder is assigned to the gluons they should manifest themselves in a variety of ways by interacting with quarks and other gluons to produce hadrons in hadron-hadron collisions, to produce gluon jets in e^+e^- collisions, etc.[17]

Confinement

As we have already indicated above in our discussion of "infrared slavery," color is central to another aspect of quarks, that of confinement. We will take as a principle, perhaps derivable at a later time from QCD, that color is confined, i.e. only color singlet states can be seen. Then both quarks and gluons are not found among the asymptotic states of the theory. Bound states which are colorless can be and are seen: they are the hadrons.

The form of the effective color confining potential is not known for sure. Some arguments[14] in QCD and the string model suggest that the effective potential is linear, $V(r) = kr$, so that the force, $-dV/dr = -k$ is a constant. It then takes infinite energy to move a quark infinitely far away, as expected for a confining potential. Estimates of the constant, k, principally from fitting charmonium spectroscopy[18] suggest that $k \approx 0.2\ \text{GeV}^2$ = 17 metric tons × (the acceleration of gravity).

Flavor

In addition to carrying color, quarks are distinguished from one another by their "flavor." At present we know of four flavors for quarks: up, down, strange, and charm. A fifth flavor (at least) is strongly suspected on the basis of the recently discovered Υ enhancement[19] at ~ 9.5 GeV in the muon pair spectrum produced in proton-nucleon collisions. A particle data group type summary of the quark flavors is given in Table I.

The masses given in Table I of course cannot have the usual meaning since we do not see the quarks as free particles. They are so-called "constituent masses" and occur as parameters with the dimensions of mass in certain equations. They are different than current quark masses which occur in other equations. Any meaning to be attached to them is only within these equations, if then.

TABLE I

Quark Flavors à la the Particle Data Group

Quark	J^P	Mass (MeV)	Q/e	Baryon No.	Strangeness	Charm
u	$1/2^+$	~350	2/3	1/3	0	0
d	$1/2^+$	~350	-1/3	1/3	0	0
s	$1/2^+$	~500	-1/3	1/3	-1	0
c	$1/2^+$	~1650	2/3	1/3	0	1
?	$1/2^+$	~5000	?	1/3	0	0

The values of Q/e are most easily obtained by noting that baryons contain three quarks. Then the Δ^{++}, Δ^{+}, Δ^{o}, Δ^{-} charge states of the 3-3 resonance yield the u and d quark charges, while the Σ^{*+}, Σ^{*o}, Σ^{*-} or Ω^- force the charge on the s quark to be -1/3. In the case of the charmed quark, the best present evidence for Q/e = 2/3 comes from the charges of the D^o and D^+, the (non-strange) mesons containing a charmed quark. That it is a charmed quark and not antiquark in the D^o and D^+ follows from assuming that c → s in weak decays (so that the states with a charmed quark decay into final hadrons with strangeness -1).

There is other confirmatory evidence for all these charge assignments, such as the size of the change in R in e^+e^- annihilation on crossing the appropriate threshold, the size of the electromagnetic coupling of the vector mesons, etc.

We now review briefly the states composed of quarks which we do observe, the hadrons. The simplest possible hadrons are made of a quark and an antiquark forming a meson, or three quarks, forming a baryon. All other combinations of one, two, or three quarks and/or antiquarks have a net color (are not singlets under color SU(3)) and are forbidden by the principle of color confinement.

Mesons

In the case of mesons it is simple to see that the color wave function

$$\left(\bar{q}_{1R}\, q_{2R} + \bar{q}_{1B} q_{2B} + \bar{q}_{1Y} q_{2Y}\right)\Big/\sqrt{3}$$

is a normalized color singlet for any antiquark $(\bar{q}_1)$ -quark (q_2) bound state.

The quark and antiquark spin may be combined to form a total quark spin, S, which is either 0 or 1. When coupled with the relative internal orbital angular momentum, L, we can form a total meson angular momentum, $\vec{J} = \vec{L} + \vec{S}$.

To complete the meson wave function we can choose any of the four (or more ?) flavors (u, d, s, c) for the quark and any of the four flavors for the antiquark. Thus there are 16 possible flavor possibilities for each value of L, S, and J. A meson wave function in the quark model then can be written in factorized form as

$$\Psi_{\text{color}}\,(\text{singlet}) \times \Psi_{\text{flavor}} \times \left[\Psi_{\text{spin}}\,(S=0,1) \times \Psi_{\text{orbital}}\,(L=0,1,\ldots)\right]_{\text{total } J} \, .$$

Since the quark and antiquark have opposite intrinsic parity, the overall parity of such a mesonic system is $P = (-1)^{L+1}$. For charge self-conjugate meson states, i.e. composed of $\bar{u}u$, $\bar{d}d$, $\bar{s}s$, $\bar{c}c$ or a linear combination, the charge conjugation quantum number is $C = (-1)^{L + (S+1) + 1} = (-1)^{L+S}$. Thus the L = O, charge self-conjugate mesons, with S = O and 1 have $J^{PC} = O^{-+}$ and 1^{--}, respectively.

Baryons

For baryons the situation is a little more complicated. The normalized color singlet state with quarks $q_1 q_2 q_3$ is

$$\frac{1}{\sqrt{3!}}\begin{vmatrix} q_{1R} & q_{2R} & q_{3R} \\ q_{1B} & q_{2B} & q_{3B} \\ q_{1Y} & q_{2Y} & q_{3Y} \end{vmatrix}$$

as is easily seen by noting that a transformation induced by an element of the color SU(3) group multiplies the matrix in the determinant above by another matrix of determinant unity.

The total quark spin, S, may be 1/2 or 3/2. This is to be combined with the net internal orbital angular momentum L, to form the total baryon angular momentum, $\vec{J} = \vec{L} + \vec{S}$. The internal orbital angular momentum can be constructed in several ways, but most simply one may take $\vec{\ell}_{12}$ as the orbital angular momentum between quarks 1 and 2 and add to it $\vec{\ell}_3$, the orbital angular momentum of the third quark relative to the center of mass of the first two, to form $\vec{L} = \vec{\ell}_{12} + \vec{\ell}_3$.

The flavor wave function is also a bit more complicated than for mesons because not all flavor states are allowed for a given L and S due to Fermi statistics. With a color singlet wave function which is antisymmetric, the remainder of the baryon wave function must be symmetric. We will discuss the detailed implications of this later.

The parity of a baryon state defined as above is $P = (-1)^{\ell_{12} + \ell_3}$. In particular, the ground state, with all relative orbital angular momenta zero, has positive parity.

Exotics

The meson and baryon states we have discussed so far are the conventional ones of the quark model and involve the minimum number of quarks and/or antiquarks which can form a color singlet. We might well define a manifest exotic as a state with quantum numbers such that it cannot be made out of quark-antiquark in the case of a meson and three quarks in the case of a baryon.

Traditionally, one breaks up exotics into two categories. Exotics of the first kind, or "flavor exotics," are states in SU(2), SU(3),... representations not found when hadrons are formed as described above. Examples include doubly charged mesons, a baryon with positive strangeness, a meson with two units of charm, etc.

Exotics of the second kind are sometimes called "CP exotics." These are specifically mesons with parity $P = (-1)^J$ which have CP = -1 or a meson with $J^{PC}=0^{--}$. Neither of these can be formed from a quark and antiquark. A particular example of such an exotic is a vector meson with even charge conjugation.

In models which have a mechanism for forming exotic states, very often there are hadrons which do not have manifestly exotic quantum numbers themselves, but which have a quark content such that they have exotic relatives. These states are sometimes called "crypto-exotics." It is convenient to extend our definition of an exotic to include them. From here on an exotic is a meson which is not a quark-antiquark state or a baryon which is not three quarks. To use this definition we of course imply that we can tell what quarks are inside a given hadron!

There are many examples of predictions of such exotic states:

(1) $q\bar{q}q\bar{q}$ mesons and $qqq\,q\bar{q}$ baryons as in bag model calculations;[20]

(2) $(c\bar{q})(\bar{c}q)$ bound states of two charmed mesons to form "molecular" charmonium;[21]

(3) Baryonium;[22]

(4) Mesons composed of $\bar{q}q$ in a color octet state coupled to a gluon;[23]

(5) Quarkless states composed of gluons alone or "glueballs;"[24]

(6) States where the energy-momentum and perhaps spin are carried by fields other than the quarks, such as a neutral "soul;"[25]

(7) String excitations in a model of quark binding through a field theoretic string. String excitations may also be coupled to the quark orbital angular momentum to produce a fairly complicated spectroscopy.[26]

In fact, it is difficult to avoid exotic states with any real dynamics in a field theoretic framework. For no matter whether we confine quarks with gluons, with strings, or with some other fields, in a true field theory the binding field will have dynamical degrees of freedom of its own. Then, in addition to the quarks, there will be other fields which carry energy and momentum--which have their own spectrum of excitations and can "slosh" around inside the hadron relative to the quarks. The coupling of these excitations to the quark excitations in general gives rise to extra, sometimes manifestly exotic, states in the hadronic spectrum in addition to the ones usually expected. It is thus not a question so much of whether exotic states exist at all: almost any theory of hadrons worthy of the name predicts them at some mass. The important question is quantitative: at what mass and with exactly what quantum numbers do they occur?

II. RADIAL AND ORBITAL EXCITATIONS: THE "OLD" MESONS

As indicated in the previous section any flavor of quark can be combined with any flavor of antiquark to form a possible flavor state for a meson. With four flavors of quarks the possible states are then:

$\bar{u}u$	$\bar{u}d$	$\bar{u}s$	$\bar{u}c$
$\bar{d}u$	$\bar{d}d$	$\bar{d}s$	$\bar{d}c$
$\bar{s}u$	$\bar{s}d$	$\bar{s}s$	$\bar{s}c$
$\bar{c}u$	$\bar{c}d$	$\bar{c}s$	$\bar{c}c$

These sixteen flavor possibilities are available, indeed they are compulsory for each value of L, S, and J. With a fifth quark, there are 25 such states; with a sixth, 36.

If the u and d quarks are degenerate in mass and have the same strong interactions, e.g. through gluon exchanges, then there is an SU(2) symmetry of strong interactions, usually called isotopic spin invariance. Similarly, to the extent that the u, d, and s quarks may be regarded as degenerate and have the same strong interactions one has SU(3) symmetry. The 16 states shown above may be split up into multiplets corresponding to irreducible representations of these symmetry groups as shown in Table II.

TABLE II

SU(2) and SU(3) Multiplets for Mesons

Quark Flavor State			Isospin (SU(2) Representation)	SU(3) Representation
$\bar{d}u$	$(\bar{u}u - \bar{d}d)/\sqrt{2}$	$\bar{u}d$	1	
$\bar{s}u$	$\bar{s}d$		1/2	8
	$\bar{d}s$	$\bar{u}s$	1/2	
	$(\bar{u}u + \bar{d}d - 2\bar{s}s)/\sqrt{6}$		0	
$\bar{d}c$	$\bar{u}c$		1/2	$\bar{3}$
$\bar{s}c$			0	
	$\bar{c}u$	$\bar{c}d$	1/2	3
		$\bar{c}s$	0	
	$(\bar{u}u + \bar{d}d + \bar{s}s)/\sqrt{3}$		0	1
	$\bar{c}c$		0	1

In the limit where all four quarks are degenerate, one would have an SU(4) flavor symmetry. The SU(3) 8, $\bar{3}$, 3, and the combination of singlets, $(\bar{u}u + \bar{d}d + \bar{s}s - 3\bar{c}c)/\sqrt{12}$ then form a 15 dimensional representation of SU(4), with the orthogonal state, $(\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c)/\sqrt{4}$, remaining as a singlet of SU(4).

One might ask why one now bothers to study the "old" meson spectroscopy since charmonium (the set of $\bar{c}c$ states) serves as such a clear example of the meson ground state and excited levels, and "when you've seen one quark flavor combination, you've seen them all." The answer, first of all, is that is is precisely by comparing the ground state and excited levels for different flavors, that we learn that they are very similar. Secondly, there are differences in the detailed level structure and these reflect critically on the dynamics between quarks. Thirdly, certain L, S states are much more accessible and hence better studied, for the "old" mesons (e.g., high orbital angular momentum excitations), while other types of states are clearer experimentally for charmonium or charm.

For the L = 0 ground state there are sixteen $J^P = 0^-$ (S = 0) and sixteen 1^- (S = 1) flavor combinations. It now appears that all these mesons have been found experimentally. They are discussed in the lectures of M. Perl.[2] We proceed then to discuss the radial and orbital excitations, particularly of the "old" mesons.

We define a meson radial excitation as a state which has all the same quantum numbers, including internal quark L and S, as another $\bar{q}_1 q_2$ state at lower mass. The idea as well as the name for such states is borrowed from non-relativistic potential theory. There, in a potential of sufficient strength, one finds a series of such levels, each successive radial excitation having another node in its radial wave function. Familiar examples of such a situation occur for the Coulomb, harmonic oscillator and linear potentials.

Suppose such a higher mass pseudoscalar or vector meson is discovered; is it necessarily a radial excitation of the ground state? For a $J^P = 0^-$ state the answer is yes; one can only make a pseudoscalar out of a quark and antiquark if $L = S = 0$. Thus all quantum numbers including L and S are the same as that for the ground state pseudoscalar. For a $J^P = 1^-$ state, this is not necessarily so. Both internal $L = 0$, $S = 1$ and $L = 2$, $S = 1$ can result in $J^P = 1^-$ states and only the first case meets our definition of a radial excitation of the ground state. Furthermore, the closeness in mass of $L = 0$ radial excitations and $L = 2$ states in linear and harmonic potentials makes mixing between the corresponding $J^P = 1^-$ states very likely.

Barring such complete mixing, how can we tell the $L = 0$ from $L = 2$ vector mesons? First, if a pseudoscalar partner is found nearby in mass, we know it must be a radial excitation, and hence also the vector meson. Second, if we have enough confidence in our knowledge of the potential binding the quark and antiquark together, then we can calculate the mass predicted for a given state and expect experiment to agree. Along the same lines, if we know experimentally the mass of expected nearby states, it may be possible to associate a new state with $L = 0$ or $L = 2$ depending on its mass. Third, in a nonrelativistic picture $\Gamma(V^o \to e^+e^-) \propto |f(r = 0)|^2$, the square of the spatial wave function at the origin. This vanishes for $L = 2$ in the nonrelativistic approximation. For charmed quarks at least, even after relativistic corrections, the $L = 2$ vector mesons should have a very much smaller leptonic width than those with $L = 0$. Last, in a theory of pionic decays based on the quark model, the relative signs of various vector meson decay amplitudes are different depending on whether $L = 0$ or 2. For example, the amplitudes for $\rho' \to \pi\omega$ vs. $\rho' \to \pi\pi$ have a different relative sign[27] if the ρ' is a quark-antiquark state with $L = 2$ rather than $L = 0$. Similar considerations led to the establishment[28] of a $J^P = 3/2$, $I = 3/2$ pion-nucleon resonance at ~1700 MeV as a radial excitation of the $\Delta(1232)$ rather than an $L = 2$ baryon state.

The most persuasive evidence for a sequence of mesonic radial excitations comes from charmonium. There we have[29] the $\psi \equiv \psi$ (3095) and its radial excitation $\psi' \equiv \psi(3684)$. The new state,[30] $\psi(3772)$, on the basis of its leptonic width and agreement with potential model calculations is most likely an $L = 2$ level, though with some mixture of the $L = 0$ radial excitation, ψ'. The mass region between ~4 and ~4.2 GeV contains several bumps, with one very likely another radial excitation of the ψ. The $\psi(4414)$ fits fairly well as yet a third radial excitation. There is every reason to expect still higher mass radially excited states, but they become very difficult to distinguish from background because of the increasing total width and smaller coupling to e^+e^-.

With some recent additions to the list of known states, the evidence for radial excitations in the "old" meson spectrum is fairly convincing by itself. The only established[12] mesonic radial excitation for quite some time was the $\rho'(1600)$. In the last year or so it has been joined by a K' (1400), which was found[31] in an isobar analysis of the $K\pi\pi$ final state produced in $K^{\pm}$ p collisions at 13 GeV/c. It is a $J^P = 0^-$ state decaying to $K(\pi\pi)_{s\text{-wave}}$, so, as noted before, it must be a radial excitation of the ground state K(495). It has a possible partner in the $K^{*\prime}$ (1650), a vector meson found in some $K\pi$ phase shift solutions from the same experiment.[32] The situation in the later case is very similar to that for $\pi\pi$ phase shifts, where some solutions show the $\rho'(1600)$ rather distinctly.

The last few months have seen a population explosion among vector mesons composed of "old" quarks. The initial result from Orsay[33] was an indication of a resonance decaying to 5π near 1780 MeV. This has been followed by evidence for a relatively narrow bump at ~1820 MeV from Frascati.[34] Even more recent data indicates that the region from 1500 to 2000 MeV may be quite complicated with as many as half a dozen (or even more!) vector meson states found in that region.[35] Inasmuch as we do expect both L = 0 radial excitations and L = 2 vector mesons composed of $\bar{u}u$, $\bar{d}d$, and $\bar{s}s$ in that mass region, such a complicated situation is not totally unexpected. At still higher mass there are indications of a bump in inclusive K^* production in e^+e^- annihilation[36] near 2100 MeV (a ϕ''?) and also a bump in diffractive six pion photoproduction mass spectra[37] (a ρ''?) around 2200 MeV.

The situation for the mass spectrum of established ground state mesons, and their radial excitations with non-zero isospin is summarized in Fig. 1. Note the apparent regularity: $M_\rho^2 - M_\pi^2 \approx M_{K^*}^2 - M_K^2 \approx M_{D^*}^2 - M_D^2 \approx M_{K^{*\prime}}^2 - M_{K'}^2$. This is not true for the corresponding states composed of $(\bar{u}u + \bar{d}d)/\sqrt{2}$, $\bar{s}s$, or $\bar{c}c$ quarks (with isospin zero). Even so, Fig. 1 does suggest that there should be a π' in the 1300 to 1400 MeV mass range. Further, it is of considerable interest to see if L = 2 vector mesons lie nearby those radial excitations with L = 0, as seems to be the case with $\psi(3684)$ and $\psi(3772)$. Although much remains to be sorted out, nevertheless, both charmonium and the "old" meson spectroscopy emphatically indicate that a sequence of radial excitations does exist in the meson spectrum.

The other clear set of excitations in the meson spectrum is that corresponding to non-zero orbital angular momentum between the quarks. The only orbitally excited states explored experimentally with even moderate thoroughness are those with L = 1. We recall from Section I that the quark model rules say that for each quark flavor combination we have an L = 1, S = 0 state with $J^{PC} = 1^{+-}$ and L = 1, S = 1 states with $J^{PC} = 0^{++}$, 1^{++}, and 2^{++}.

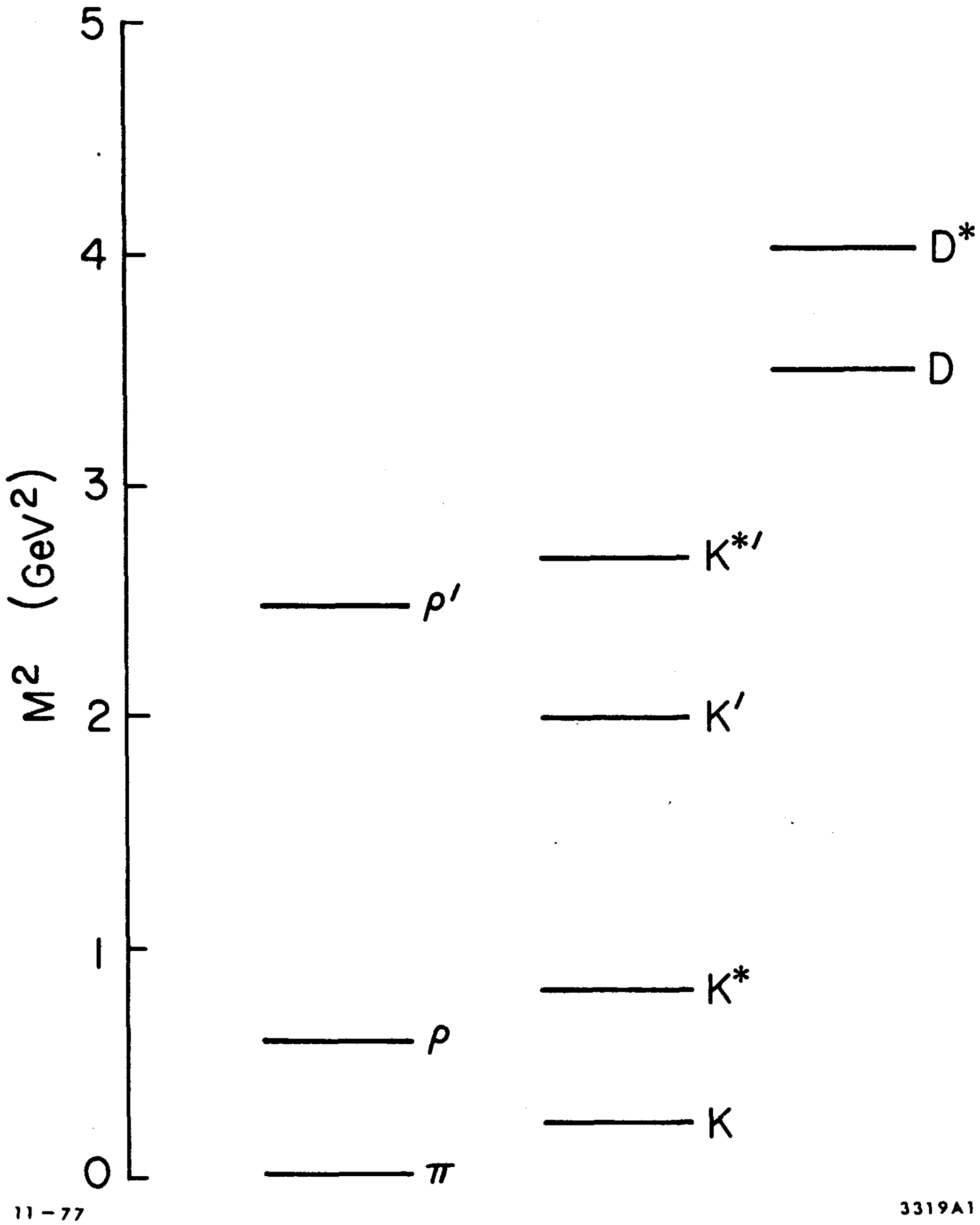

Fig. 1 Meson ground states and radical excitations with non-zero isospin.

The most spectacular examples of the L = 1, S = 1 states are the $\chi(3414)$, $\chi(3508)$, and $\chi(3552)$ levels of the charmonium ($\bar{c}c$) system.[2,38] The L = 1, S = 0 charmonium state has odd charge conjugation and will be difficult to find experimentally--so we shouldn't worry that it isn't an established state. In fact, we have every reason on the basis of charmonium to expect that the L = 1 meson states will be found in all possible quark flavor combinations.

For the $J^P = 2+$ states, this expectation is already fulfilled for the u, d, and s flavors. All the needed states are established, and we have the "ideal" or "magic" mixing situation shown in Table III.

TABLE III

$J^P = 2^+$ Mesons Composed of u, d, and s Quarks

Quark Flavor State	Observed Meson[12]
$\bar{d}u$, $(\bar{u}u - \bar{d}d)/\sqrt{2}$, $\bar{u}d$	$A_2(1310)$
$(\bar{u}u + \bar{d}d)/\sqrt{2}$	f(1270)
$\bar{s}u$ $\bar{s}d$	$K^*(1420)$
$\bar{d}s$ $\bar{u}s$	$\bar{K}^*(1420)$
$\bar{s}s$	f'(1515) [c]

The $J^P = 1^+$ states of u, d and s quarks are a traditional area of experimental confusion. However, in the last year or so the situation is beginning to clarify. The biggest single advance has been the evidence[39, 40, 41] for two Q mesons, $Q_1(\sim 1300)$ and $Q_2(\sim 1400)$, which are axial-vector states containing a strange quark and a u or d quark. The observed states are actually mixtures[40] of the S = 0 and S = 1 quark model states. The B(1235) meson is an established[12] candidate for the isospin one axial-vector state composed of u and d quarks with quark spin S = 0. The D(1285) (not to be confused with the charmed mesons) is the only established[12] isospin zero meson which likely has $J^P = 1^+$ (and from its positive charge conjugation would correspond to S = 1).

Along with the Q mesons, the traditional problem child of the axial-vector mesons is the A_1. Even here some real progress is being made.[41] Although earlier analyses of diffractive three pion production were never able to show evidence for a real resonance at the peak mass of ~1100 MeV, more recent theoretical work[42] with multichannel analyses do indicate resonance behavior, although perhaps at a higher mass (even possibly 1400 to 1500 MeV). At the same time, more direct experimental indications of a resonance decaying to $\pi\rho$ at ~1100 MeV come from several different experiments performed at CERN.[41] It seems unlikely that the uncertainty with regard to the A_1 will persist very much longer. With, in addition, the new evidence[43] for the heavy lepton decay $\tau \rightarrow A_1 \nu_\tau$, the establishment of a suitable isovector meson to match the L = 1, S = 1 axial-vector state of the quark model seems finally to be within sight.

While the situation for 1^+ states composed of u, d, and s quarks is considerably improved, that for the $J^P = 0^+$ states is more confusing than ever. The δ(970) seems healthy enough[12] as a candidate for the I = 1 state composed of u and d quarks. However, the s-wave $K\pi$ phase shift rises slowly and passes through 90° near 1250 MeV. If defined as a strange, $J^P = 0^+$ resonance,[12] it must be very broad (~450 MeV), and furthermore, the drop in elasticity and phase motion at higher mass (~1400 MeV) is then suggestive of another resonance in the same channel.[40] The isospin zero s-wave $\pi\pi$ phase shift does much the same thing: it passes slowly through 90° near 700 MeV [ϵ(700)? ?], exhibits a clear resonance which also couples to $K\bar{K}$ [S*(990)], and very likely shows another resonance, the ϵ'(1200), as a broad state. Discarding the ϵ(700), the latter state's width is ~600 MeV; otherwise it is narrow (~200 MeV).

The possibility that there are too many $I = 0$ and/or strange 0^+ states to fit the quark model is a big headache. Down through the ages, various explanations of this (e.g. dilatons,[44] glueballs,[45] cryptoexotics[7]) have been entertained which allow some or all of the observed 0^+ states to be other than $\bar{q}q$ $L = 1$ levels. On the other hand, if we throw out the $\epsilon(700)$ (and a higher mass K^* besides κ (1250) we have a "peculiar" SU(3) octet plus singlet[46] with respect to quark model mass formulas. It is difficult to be optimistic that this situation will be resolved soon. It is fortunate that we have the χ states, the full set of 2+ states composed of u, d, and s quarks, and the improving situation with 1^+ mesons to bolster our confidence that all the $L = 1$ levels will be found eventually in all quark flavor combinations.

At the next level of orbital excitation, $L = 2$, we expect $S = 0$ ($J^{PC} = 2^{-+}$) and $S = 1$ ($J^{PC} = 1^{--}$, 2^{--}, 3^{--}) states. Of these only the 3^- states are in good shape: the g(1690), $\omega^*(1675)$ and $K^*(1780)$ are all established[12] to have $J^P = 3^-$. The "ideal" or "magic" mixing pattern seems evident also, and we expect an $\bar{s}s$ (ϕ-like) state with $J^P = 3^-$ at 1850 to 1950 MeV. Other candidates for $L = 2$ levels, like the A_3 ($J^P = 2^-$) and L ($J^P = 2^-$) remain to be firmly established,[12] but enough has been found to give us assurance that all the $L = 2$ levels must exist for all quark flavor combinations.

When we get to $L = 3$, the only established state[12] is the h(2040), which fits as the $L = 3$, $S = 1$ isoscalar state composed of u and d quarks with $J^P = 4^+$. There are, however, some signs of a 4^+ K^* state[40] near 2100 MeV and also of the corresponding $I = 1$ non-strange state.[41]

At still higher mass, there are bumps[12] in the $\bar{p}p$ total cross section at ~2190 and ~2360 MeV. Recent results[47] from $\bar{p}p \to \pi^-\pi^+$ using both differential cross section and polarization measurements strongly suggest broad resonances at 2150, 2310, and 2480 MeV with $J^{PC} = 3^{--}$, 4^{++}, and 5^{--} respectively. This later result matches fairly well with an earlier fit[48] to angular distributions for $\pi^- p \rightarrow \bar{p}p\, p$ assuming one pion exchange, which suggested a sequence of resonances in the same mass range.

Thus it is suggestive, at the very least, that <u>many, broad, states occur at high masses</u>. There seem to be narrower states as well,[49] although these may have another origin. Certainly, looking at the Regge plot of spin versus mass squared in Fig. 2, there is no sign that even the conventional states on the leading trajectory do not continue without interruption well into the 2 GeV mass region. The major outstanding question is the existence and nature of other, non-$\bar{q}q$ states in the meson spectrum.

III. BARYONS

As already discussed in Section I the overall color singlet nature of the three quarks in a baryon results in a color part of the wave function which is completely antisymmetric. According to Fermi-Dirac statistics the remainder of the wave function must be symmetric.

For the ground state, with all quarks in relative s-waves and $L = 0$, we have a symmetric spatial wave function. If the total quark spin is $S = 3/2$, then the spin wave function is symmetric and the only remaining quantity, flavor, also must have a symmetric wave function. With four quark flavors from which to choose, there are 20 possible symmetric three quark flavor states. These are shown in Table IV, together with the corresponding observed baryon, if known.

In the case of total quark spin $S = 1/2$, it may be shown that the spin wave function is of "mixed symmetry." With a symmetric ground state spatial wave function, Fermi-Dirac statistics now demands a mixed symmetry flavor wave function. With four quarks, it turns out there are again 20 such quark flavor states. It is purely an accident that the number of flavor states is the same

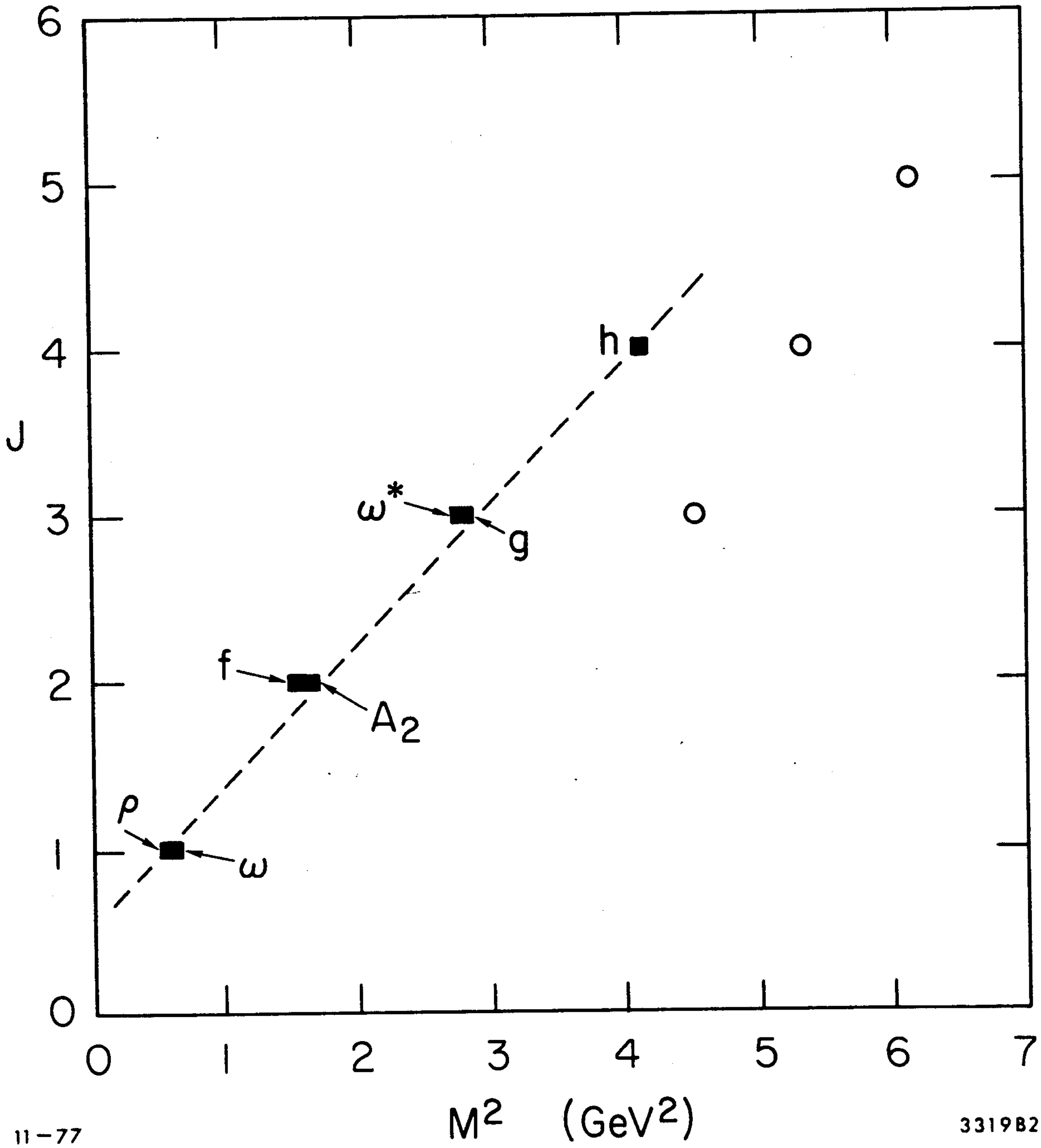

Fig. 2 Leading Regge trajectory for mesons (solid squares) and states established in Reference 47 (open circles).

TABLE IV

S = 3/2 Baryon Ground States

Quark Flavor States				Observed States[12,50]	
uuu	uud	udd	ddd	$\Delta^{++,+,o,-}(1232)$	
	uus	uds	dds	$\Sigma^{*+,o,-}(1385)$	
		uss	dss	$\Xi^{*o,-}(1530)$	
			sss	$\Omega^{-}(1670)$	
uuc	udc	ddc		Σ_c^* or $C_1^{*++,+,o}(2500?)$[51]	
	usc	dsc		$S^{*+,o}$	(?)
		ssc		T^{*o}	(?)
ucc	dcc			X_u^{*++}, X_d^{*++}	(?)
	scc			X_s^{*+}	(?)
ccc				Θ^{++}	(?)

as for a symmetric flavor wave function: as we will see below, this is not true when there are other than four quark flavors. The appropriate mixed symmetry states composed of u, d, s, and c quarks, together with their experimental counterparts, are shown in Table V.

It is instructive for some of the things to follow to exhibit the explicit quark model wave functions as they depend on spin and flavor. We denote by $u\uparrow$ an "up" quark with spin component $S_z = +1/2$, $u\downarrow$ an "up" quark with spin component

TABLE V

S = 1/2 Baryon Ground States

Quark Flavor States				Observed States[12,50]
	uud	udd		$N^{+,o}(940)$
	uus	{ud}s	dds	$\Sigma^{+,o,-}(1190)$
		[ud]s		$\Lambda(1115)$
		uss	dss	$\Xi^{o,-}(1320)$
uuc	{ud}c	ddc		Σ_c or $C_1^{++,+,o}(2426\,?)$[52]
	{us}c	{ds}c		$S^{+,o}$ (?)
		ssc		T^o (?)
	[us] c	[ds] c		$A^{+,o}(?)$
	[ud]c			Λ_c or C_o^+ (2260)[51]
ucc	dcc			X_u^{++} , X_d^+ (?)
	scc			X_s^+ (?)

{ } ≡ symmetrized, in flavor   [ ] ≡ antisymmetrized in flavor

$S_z = -1/2$, etc. Then the wave function for a Δ^{++} with $J_z = 3/2$ is simply, $u\uparrow u\uparrow u\uparrow$, while that for a Δ^+ with $J_z = 3/2$ is, $(1/\sqrt{3})\ (u\uparrow u\uparrow d\uparrow + u\uparrow d\uparrow u\uparrow + d\uparrow u\uparrow u\uparrow)$. In the case of a Δ^+ with $J_z = +\,1/2$, we must have complete symmetry in both spin and flavor, so the normalized wave function is:

$(1/\sqrt{9})$(u↑ u↑ d↓ + u↑ d↓ u↑ + d↓ u↑ u↑ + u↑ u↓ d↑ + u↑ d↑ u↓ + d↑ u↑ u↓ + u↓ u↑ d↑ + u↓ d↑ u↑ + d↑ u↓ u↑).

For the nucleon, say a proton with $J_z = 1/2$, one may construct the wave function in several ways. The simplest, perhaps, is to start with u and d quarks having $S = S_z = 0$. This is antisymmetric in spin, so we antisymmetrize in flavor also to obtain something symmetrical under overall interchange of the two quarks:

(u↑ d↓ - d↑ u↓ - u↓ d↑ + d↓ u↑)/2.

If we now add a third quark, u↑ and completely symmetrize it with the first two, we get on normalizing the result:

$(1/\sqrt{18})$ (2u↑ u↑ d↓ + 2u↑ d↓ u↑ + 2d↓ u↑ u↑ - u↑ d↑ u↓ - d↑ u↑ u↓ - d↑ u↓ u↑ - u↑ u↓ d↑ - u↓ u↑ d↑ - u↓ d↑ u↑).

To get the neutron wave function with $J_z = 1/2$ we need only make the interchange u ⟷ d. The other L = 0 baryon wave functions are constructed analogously, and can be obtained straightforwardly.

As a first use of these wave functions let us consider the static electromagnetic properties of baryons. We picture these as arising from those of the constituent quarks. We have been doing this all along for the charge:

$$Q\ (\text{hadron}) = \sum_i Q_i\ (\text{quark}). \qquad (1)$$

Now we do the same for the magnetic moments. In other words, we assume that

$$\vec{\mu}\ (\text{hadron}) = \sum_i \vec{\mu}_i\ (\text{quarks}). \qquad (2)$$

We define the magnetic moment for a particle of spin J as

$$\mu \equiv \langle J_z = J | \mu_z | J_z = J \rangle . \qquad (3)$$

This coincides with the usual definition for J = 1/2, 1, etc. With our previous assumption we have

$$\mu \;=\; \langle J_z = J \left| \sum_i \frac{eQi}{2m_i}\, \sigma_z^{(i)} \right| J_z = J \rangle \;. \qquad (4)$$

At this point the "quark masses," m_i, appearing in Eq. (4) are not defined and need not be directly related to the masses discussed in Section I. Setting $m_u = m_d$, we calculate the values of the baryon magnetic moments shown in Table VI, with the aid of the explicit wave functions developed above. There are also two transition moments that are experimentally accessible and calcuable in the same way. These are $\mu_{\Sigma\Lambda}$ and $\mu_{\Delta p}$, which are $-1/\sqrt{3}$ and $2\sqrt{2/3}$, respectively in the units of Table VI.

The comparison of these theoretical values with experiment is shown in Table VII. We fix $\mu_p = 2.79$ and calculate all other moments. In the column labeled $m_u/m_s = 1$, SU(3) symmetry is assumed. The value $m_u/m_s = 0.7$ corresponds to the constituent quark masses given in Table I and agrees somewhat better with experiment. The overall agreement with experiment is certainly very adequate, if not close to spectacular.

Radial excitations of the baryon ground state, as for meson radial excitations, differ only in having a different radial wave function and should have the same spin and flavor states available as the ground state. For S = 3/2 we then have a symmetric flavor wave function, while for S = 1/2 one of mixed symmetry. The number of possible baryon (three quark) flavor states as a function of the number of different quark flavors is given in Table VIII. Also shown is the number of flavor states times the number of S_z states available for the entire ground state or its radial excitation. We often refer to the set of these states by their total spin (S_z) and flavor multiplicity, e.g. for three quarks (u, d, s) it is the "56", made up of an SU(3) octet with S = 1/2 and a decuplet with S = 3/2.

TABLE VI

Baryon Magnetic Moments in the Quark Model

State	μ (units of $e/2m_u$)
Δ^{++}	2
Δ^{+}	1
Δ^{o}	0
Δ^{-}	-1
Σ^{*+}	$4/3 - 1/3\ (m_u/m_s)$
Σ^{*o}	$1/3 - 1/3\ (m_u/m_s)$
Σ^{*-}	$2/3 - 1/3\ (m_u/m_s)$
Ξ^{*o}	$2/3 - 2/3\ (m_u/m_s)$
Ξ^{*-}	$-1/3 - 2/3\ (m_u/m_s)$
Ω^{-}	$-(m_u/m_s)$
p	1
n	-2/3
Σ^{+}	$8/9 + (1/9)\ (m_u/m_s)$
Σ^{o}	$2/9 + (1/9)\ (m_u/m_s)$
Σ^{-}	$-4/9 + (1/9)\ (m_u/m_s)$
Λ^{o}	$-1/3\ (m_u/m_s)$
Ξ^{o}	$-2/9 - 4/9\ (m_u/m_s)$
Ξ^{-}	$1/9 - 4/9\ (m_u/m_s)$

TABLE VII

Comparison of Theory and Experiment for Baryon Magnetic Moments

Magnetic Moment	Theory $m_u/m_s = 1$	Theory $m_u/m_s = 0.7$	Experiment[12] (Nucleon Magnetons)
μ_p	2.79 (input)	2.79 (input)	2.79
μ_n	-1.86	-1.86	-1.91
μ_{Σ^+}	2.79	2.70	$2.62 \pm .41$
μ_{Σ^o}	.73	.84	
μ_{Σ^-}	-.93	-1.02	$-1.48 \pm .37$
μ_Λ	-.93	-.65	$-.67 \pm .06$
μ_{Ξ^o}	-1.86	-1.49	
μ_{Ξ^-}	-.93	-.56	$-1.85 \pm .75$
$\mu_{\Sigma\Lambda}$	-1.61	-1.61	$\pm\left(1.82^{+.25}_{-.18}\right)$[53]
$\mu_{\Delta p}$	2.63	2.63	2.6 to 3.4[54]

TABLE VIII

Multiplicity of the Baryon Ground State or its Radial Excitations

N = No. of Quark Flavors	No. of Baryon Flavor States S = 1/2	S = 3/2	No. of Spin Times Flavor States
1	0	1	4
2	2	4	20
3	8	10	56
4	20	20	120
5	40	35	220
6	70	56	364

Besides the ground state or its radial excitations, we will of course have the same accounting of baryon spin and flavor states whenever the quark spatial wave function is symmetric. For then the flavor times spin wave function is required to be symmetric, and we have exactly the same arguments on the available spin and flavor states that led us to Table VIII, for the ground state or its radial excitations.

For baryon orbital excitations one can in principle have quark spatial wave functions which are symmetrical, antisymmetrical, or of mixed symmetry. The lowest orbital excitation, that with L = 1, turns out to have a spatial wave function with mixed symmetry among the three quarks. For the case of quark spin S = 3/2 (a symmetric spin wave function), this forces a mixed symmetry flavor wave function. However, when S = 1/2 (mixed symmetry spin wave function) the overall Fermi-Dirac statistics can be satisfied with either a symmetrical, mixed symmetry, or antisymmetrical flavor wave function. The situation with regard to the multiplicity of baryon flavor states in this case is shown in Table IX.

TABLE IX

Multiplicity of the Baryon Orbital Excitations with Mixed Symmetry Spatial Wave Functions

	No. of Baryon Flavor States				No. of Spin
	S = 3/2	S = 1/2			Times
N = No. of Quark Flavors	Mixed	Antisym.	Mixed	Sym.	Flavor States
1	0	0	0	1	2
2	2	0	2	4	20
3	8	1	8	10	70
4	20	4	20	20	168
5	40	10	40	35	330
6	70	20	70	56	572

Again, such an array of spin and flavor states will arise any time the three quark spatial wave function is of mixed symmetry. The set of these spin and flavor states is then often referred to by their total spin times flavor multiplicity, e.g. for three quarks one has the "70", composed of an S = 3/2 SU(3) octet and an S = 1/2 SU(3) singlet, octet, and decuplet.

Aside from the observed charmed baryons, which are candidates for being members of the L = 0 ground state, only states composed of u, d and s quarks are known for baryons. Therefore, in discussing the observations of radially and orbitally excited baryonic levels,[55] we consider only states composed of three quarks. As indicated above, we refer to the multiplets of given L by their spin (S_z) times flavor multiplicity.

The first excited baryon level above the ground state is a 56, L = 0 multiplet, i.e. a radial excitation of the 56, L = 0 ground state. Its most familiar non-strange member is the Roper resonance, N*(1470). The radially excited counterpart of the 3-3 resonance is the Δ*(1690).

At slightly higher mass, on average, is a set of negative parity states which form a 70, L = 1 orbital excitation. All seven of the non-strange resonances needed to fill this multiplet are known to exist with the right spins and isospins—no more and no less than the expected states.

Above the 70, L = 1 there is another possible radial excitation of the ground state 56, L = 0. However, most of the evidence for this is based on the N*(1780) with $J^P = \frac{1}{2}^+$ and confirmation of the whole multiplet awaits evidence for some of the other states.

In the same mass range there is a further established multiplet, a 56, L = 2. Most, if not all of the six non-strange states sitting in this multiplet are found experimentally, including the long established N*(1688) with $J^P = \frac{5}{2}^+$ and the Δ*(1950) with $J^P = \frac{7}{2}^+$.

In the 2 GeV mass region there is fairly good evidence for a 70, L = 3 set of states. In particular the established N*(2190) and N*(2140) with $J^P = \frac{7}{2}^-$ and $\frac{9}{2}^-$ respectively, rather uniquely fit into just such a multiplet.

At still higher mass there are the established $J^P = \frac{9}{2}^+$ N*(2220) and the $\frac{11}{2}^+$ Δ*(2420). Even though essentially all the other states remain to be found, these two levels are very likely the first members of a 56, L = 4 multiplet.

Thus we see a fairly extensive sequence of radial and orbital excitations in the baryon spectrum, just as in the case of the meson spectrum. A few more multiplets are quite possible in the mass range discussed up to now (e.g. a 56, L = 2 radial excitation and a 70, L = 1 radial excitation).

The established multiplets so far all have the property that L even corresponds to a flavor times spin multiplicity of 56 while those with L odd have a multiplicity of 70. While this is trivial for the ground state, or first orbital excitation, it is entirely non-trivial that we do not see, say, 70, L = 0 and 70, L = 2 multiplets below 2 GeV. (These are expected in a harmonic oscillator potential to be degenerate with the 56, L = 2). The full significance of this for the quark-quark force remains to be seen. In fact, there are recent suggestions that the empirical connection of 56's and 70's with L even and odd, respectively, may break down: this is based on a $5/2^-$ Δ^* near 1960 MeV which would seem to fit best in a 56, L = 1 multiplet.[56]

At still higher mass spins and parities are unknown, but there are N* bumps at 2650 and 3030 MeV and Δ^*'s at 2850 and 3230 MeV. If one draws the leading Δ^* Regge trajectory (Fig. 3) it has a slope very much like that for the mesons (Fig. 2). Further, if we take the $\Delta^*(2850)$ and $\Delta^*(3230)$ as the next two states on the leading trajectory with $J^P = 15/2^+$ and $19/2^+$, respectively, then we have 5 states, all seemingly on a linear trajectory. As with the mesons, we have no reason to doubt that the baryon spectrum continues on to much higher masses, albeit with broader, low elasticity states, making it almost impossible to isolate individual levels and their quantum numbers.

IV. HADRON MASSES

As in our treatment of all other aspects of spectroscopy in these lectures, we discuss the subject of hadron masses within a picture of hadrons as composed of quarks. More particularly, we will work in a constituent or "atomic" model with quarks bound by an effective potential due to the action of colored gluons.[58,59]

In such a picture, hadron masses come from four sources:

(1) Quark masses;

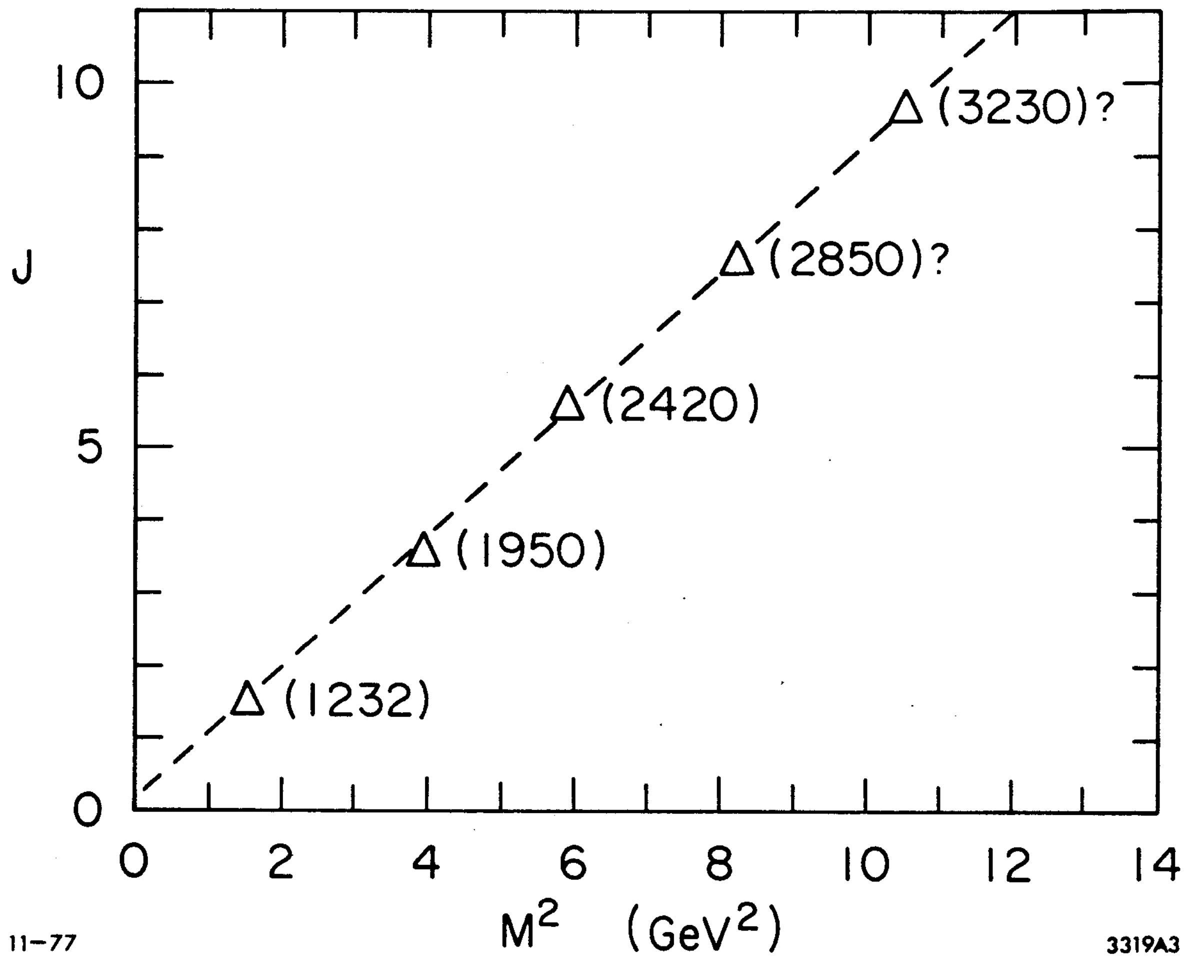

Fig. 3 Leading isospin 3/2 baryon Regge trajectory.

(2) The primary level of excitation of the potential, in other words the kinetic and potential energy of the quarks;

(3) The residual interactions between quarks of a spin-spin, spin-orbit or tensor force character, which split the principal levels of the potential;

(4) The gluons which carry energy (or mass) by themselves and also provide diagonal and off-diagonal elements to the meson mass matrix due to transitions of the form $\bar{q}_1 q_1 \longleftrightarrow \text{gluons} \longleftrightarrow \bar{q}_2 q_2$.

The division among categories (1) through (4) is somewhat arbitrary. For example, sources (2) and (3) may be thought of as coming from the same basic origin, the exchange of gluons between quarks. If we could solve QCD the quark-quark interaction should emerge in toto from theory and give us (2) and (3) in one swoop.

Further, different values of the quark mass could well result in making (2), (3) and/or (4) different for various quark flavors. Thus effects from (1) could, for example, actually manifest themselves as mass splittings through a difference of forces of type (3).

The effect of the gluon energy, source (4), is usually assumed to be the same for all baryons with given L. On the other hand, mesons, $\bar{q}_i q_j$ with $i \neq j$, are distinguished from those with $i = j$ for given L, S, and J by the gluon annihilation and creation terms noted in (4).

Let us then examine where various masses and mass differences arise from in terms of sources (1) through (4).

Masses of Orbital and Radial Excitations Relative to the Ground State

Almost by definition these come from source (2), the level of excitation of the overall binding potential. Examples are the mass splitting between the L = 0 and L = 1 mesons or L = 0 and L = 1 baryons. The $\omega - f$, $\rho - A_2$, $K^* - K^{**}(1420)$,

ϕ - f' and ψ-χ (3552) mass differences,[12] which are each ~450 MeV, are all of the type: 1^- (L = 0) - 2^+ (L = 1). From this pattern[57] we expect that the 2^+ D** is at M_{D*} + 450 MeV ≈ 2450 MeV and the 2^+ F** at M_{F*} ≈ 2600 MeV.

If we measure the L = 0 to L = 1 mass splitting for baryons by that between states with symmetric spin wave functions (or between those with mixed symmetry spin wave functions), we also find a value of ~450 MeV. As for the meson examples given above, this mass splitting seems to be independent of quark flavor.

Splitting of States with Given L and S, Different J

This arises from spin-orbit forces or tensor forces which fall into category (3). In atomic physics this is called fine structure. For mesons, such forces give different masses to the δ, A_1, and A_2, or the χ_0, χ_1, and χ_2. These mass differences are all in the range 100 to 200 MeV for mesons, but somehow turn out to be very much smaller for baryons, e.g. the near degeneracy of the $1/2^-$, $3/2^-$ and $5/2^-$ N*'s with L = 1 and S = 3/2.

If the spin-orbit force arose from an "effective vector" exchange between the quark and antiquark in a meson and the effective potential is attractive, then it is possible to show that the mass splitting is proportional to $\vec{L} \cdot \vec{S}$ with a positive coefficient. Since

$$\vec{L} \cdot \vec{S} = \frac{J(J+1) - L(L+1) - S(S+1)}{2} \quad ,$$

the L = 1, S = 1 meson states would be 0^+, 1^+, and 2^+ in order of increasing mass. This is just the case for χ_0, χ_1, and χ_2. In principle the tensor force could have ruined this ordering, but at least for charmonium it turns out to have a smaller, but non-zero, coefficient.[59]

There is no proof that the quark-quark force has to have an "effective vector" form. Of course, this would result automatically if one gluon exchange dominated. But in the case of charmonium this gives rise to mass splittings[58] which are too small by an order of magnitude. There is no reason to expect that one gluon exchange is the dominate source of the spin-orbit force for any of the other mesons either.

Splitting of States With the Same L, Different S

These again have a source (3) origin, but are of the spin-spin variety. Such terms result in the N - Δ, $\Sigma - \Sigma^*$, etc. mass difference for baryons and the $\pi - \rho$, K - K*, D - D*, etc. splittings among mesons. They also split the B relative to the A_2, A_1, and δ.

If the interaction between quarks has an effective vector character, then in a non-relativistic situation the spin-spin interaction contribution to the mass from quarks i and j has the form:

$$\Delta M_{s-s} \propto \frac{1}{m_i m_j} \vec{S}_i \cdot \vec{S}_j \, \vec{\nabla}^2 V(r_{ij}) \quad , \tag{5}$$

where V(r) is the effective potential in configuration space. With a single vector particle being exchanged, the proportionality constant in Eq. (5) has opposite signs for the case of two quarks (in a baryon) or a quark and antiquark (in a meson). However, if colored gluons are exchanged in color singlet hadrons, it turns out that the sign of the proportionality constant in Eq. (5) is the same for mesons and baryons.[58,59,60] Furthermore, the sign is such (positive) that with $\nabla^2 V(r)$ positive (as it is expected to be) the system with parallel quark spins has a higher energy than that with antiparallel spins. So if we accept the sign and general form of the spin-spin interaction that comes from colored gluon exchange, we predict that the ρ is heavier than the π, the K* heavier than the K, and Δ heavier than the nucleon. While just one gluon exchange is unlikely to dominate completely in all these cases where mass splittings are a few hundred MeV, the experimentally observed sign in mesons and baryons seems to indicate that gluon exchange has something to do with at least the qualitative nature of these splittings.

Because of the explicit quark masses in the non-relativistic form of ΔM_{s-s}, even states with the same quark flavors but different relative quark spin orientations have different masses. This may well be the origin of the mass difference between the Λ and Σ.[58,60]

To see this, assume that

$$\Delta M_{s-s} = c \sum_{i>j} \frac{\vec{s}_i \cdot \vec{s}_j}{m_i m_j} \tag{6}$$

where c is a positive constant for the L = 0 baryons. In the Σ, the u and d quarks are in a symmetrical (I = 1) flavor state and hence a symmetrical (S = 1) spin state. The total spin of all three quarks is 1/2. This leads to

$$\vec{s}_u \cdot \vec{s}_d = \frac{1}{4} \quad , \tag{7a}$$

$$\vec{s}_u \cdot \vec{s}_s = \vec{s}_d \cdot \vec{s}_s = -\frac{1}{2} \tag{7b}$$

Hence,

$$\Delta M_{s-s}(\Sigma^0) = c\left[\frac{1}{m_u m_d}\left(\frac{1}{4}\right) + \frac{1}{m_u m_s}\left(-\frac{1}{2}\right) + \frac{1}{m_d m_s}\left(-\frac{1}{2}\right)\right]$$

$$= \frac{c}{4m_u^2} - \frac{c}{m_u m_s} \quad , \tag{8}$$

on taking $m_u = m_d$. For the Λ, the u and d quarks are in antisymmetrical flavor (I = 0) and spin = 0) states. Then for the Λ^o,

$$\vec{s}_u \cdot \vec{s}_d = -3/4, \tag{9a}$$

and

$$\vec{s}_d \cdot \vec{s}_s = \vec{s}_u \cdot \vec{s}_s = 0, \tag{9b}$$

so that

$$\Delta M_{s-s}(\Lambda) = -\frac{3c}{4m_u^2} \quad . \tag{10}$$

Combining (8) and (10), we have

$$\begin{aligned} M(\Sigma^o) - M(\Lambda^o) &= \frac{c}{m_u^2} - \frac{c}{m_u m_s} \\ &= \frac{c}{m_u^2}\left(1 - \frac{m_u}{m_s}\right) . \end{aligned} \tag{11}$$

Since the strange quark is heavier than the up (or down) quark, we have $M_{\Sigma^o} > M_{\Lambda^o}$ in agreement with experiment.

It is interesting to note that the Λ_c and Σ_c would be split in mass by the same mechanism. With the charmed quark replacing the strange one,

$$M(\Sigma_c) - M(\Lambda_c) = \frac{c}{m_u^2}\left(1 - \frac{m_u}{m_c}\right) \quad . \tag{12}$$

Since $m_c > m_s$, this mass difference should be even larger than that between the Σ and Λ. If we identify the Σ_c with the BNL neutrino induced $\Lambda 4\pi$ system[52] at 2426 MeV and consider the Λ_c to be at 2260 MeV,[57] then this prediction is correct!

Splitting of States with Different Flavor

Such mass differences arise directly from source (1), but as we just saw they can arise indirectly from (3)[(or (2)]. The splitting of different flavor states with the same L, S, J and all other quantum numbers, allows us to estimate constituent quark mass differences. For example, we take

$$m_s - m_u \approx M(\Omega^-) - M(\Xi^*) \approx M(\Xi^*) - M(\Sigma^*)$$

$$\approx M(\Sigma^*) - M(\Delta) \approx 150 \text{ MeV} \tag{13}$$

Similarly,

$$m_c - m_s \approx M(\Sigma_c^*) - M(\Sigma^*) \approx 1150 \text{ MeV} \tag{14}$$

Essentially the same mass differences are obtainable by considering the ground state vector mesons, i.e. $2(m_c - m_s) \approx M(\psi) - M(\phi)$ and $2(m_s - m_u) = M(\phi) - M(\rho)$.

To get an absolute quark mass scale we must fix one of these masses. One way to do this is from charmonium, where calculations indicate $M_c \approx 1650$ MeV. Another way is to take $M_u \approx M_d$ to be $M_N/3$ or $M_\rho/2$, A third method is to take the expressions in terms of quark masses for the baryon magnetic moments in Section III very seriously. All these methods give the same answer:

$$m_u \approx m_d \approx 350 \text{ MeV}$$

$$m_s \approx 500 \text{ MeV}$$

$$m_c \approx 1650 \text{ MeV}. \tag{15}$$

These "constituent quark masses" were already given in Table I. We repeat the caveat given there: These are not real masses, but only parameters with the dimensions of mass that appear in certain equations discussed above. Other equations give other values, e.g. current quark masses.

In addition to quark masses, gluons (source (4)) can also give hadrons composed of different flavors different masses. In particular, consider the contributions to the meson mass illustrated in Fig. (4). For mesons with net flavor (i.e., $i \neq j$), the second diagram makes no contribution to the mass matrix, for the gluons do not carry flavor. But for mesons with no net flavor ($i = j$), the second diagram contributes. Suppose it has the same value for all i and k, i.e. is flavor independent. Then for mesons with no net flavor we have two extreme situations.

If the first diagram due to the quark masses dominates the mass matrix, then the $I = 0$ eigenstates are $(\bar{u}u + \bar{d}d)/\sqrt{2}$, $\bar{s}s$, and $\bar{c}c$. We have the situation of "magic mixing" at the SU(3) level. The vector mesons (ρ), ω, ϕ, ψ are a good example, as are the $J^P = 2^+$ and 3^- mesons.

On the other hand, if the second diagram due to annihilation into gluons dominates the mass matrix for N quark flavors, then its eigenstates are the $SU(N)_{flavor}$ singlet and non-singlet (s). For example, with u, d, and s quarks, the $I = 0$ eigenstates would be the SU(3) singlet and octet states, $(\bar{u}u + \bar{d}d + \bar{s}s)/\sqrt{3}$ and $(\bar{u}u + \bar{d}d - 2\bar{s}s)/\sqrt{6}$, respectively. The "old" pseudoscalar mesons are closer to, but not exactly in, this situation.

Furthermore, our assumption of the flavor independence of the second diagram is only approximate. Asymptotic freedom suggests that more gluons or higher mass of the meson makes the second diagram smaller.[16] Analysis of the situation with charmonium suggests a fairly big flavor dependence.[61] It will be interesting to test these ideas on the $\bar{s}s$ mesons in the $L = 1$ and $L = 2$ levels to see if the expected dependence on mass and gluon number is found experimentally.

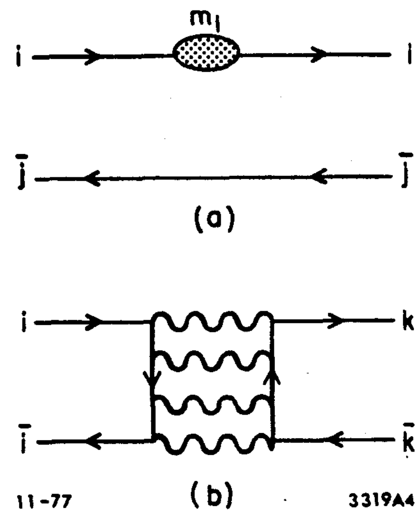

Fig. 4 (a) Meson mass matrix contribution due to quark mass.
(b) Meson mass matrix contribution due to gluon annihilation.

"Electromagnetic" Mass Differences

A particular example of some importance of the ideas we have been discussing is hadron electromagnetic mass differences. These have their origin in two physically distinct sources. First is the difference in mass of the u and d quarks. This is a source of type (1), which also has indirect effects of type (3). Second, there is explicit photon exchange between quarks, a manifest electromagnetic process.

By an accident (? ?) of nature the u - d mass difference and the effects due to photon exchange are of the same order, namely several MeV. Thus both are usually treated at the same time as "electromagnetic" mass differences.

These mass differences are characterized by several unique features. First, we can surely treat the splittings to lowest order in the perturbation. Further, we have a known interaction: one photon exchange in lowest order leads to a Coulomb interaction and to a magnetic dipole interaction. The first is proportional to the product of the charges, Q_iQ_j, of the quarks involved, while the second is proportional to the product of their charges and the dot product of their spins, $Q_iQ_j\vec{s}_i \cdot \vec{s}_j$. For the L = 0 baryons composed of u, d, and s, these quantities and the quark masses, summed over the appropriate flavors, are given in Table X.

We shall now assume that the mass of a state is determined by

$$M = M_o + \sum_{i=1}^{3} m_i + c_1 \sum_{i>j} Q_iQ_j + c_2 \sum_{i>j} Q_iQ_j\vec{s}_i \cdot \vec{s}_j \quad . \tag{16}$$

Here M_o depends on sources (2), (3) and (4) and consequently is different for each L, S, J, etc. All the "electromagnetic" effects are assumed to be in the last three terms: the difference in u - d quark masses in Σm_i, and the Coulomb and magnetic interactions in the last two terms, with c_1 and c_2 constants for a given excitation of the overall binding potential. We neglect a possible dependence of c_2 on the mass of the quarks involved (at least for u, d, s). Also neglected at this simplified level are indirect effects of type (3), e.g. differences in the spin-spin interaction (buried in M_o) arising from strong interaction gluon exchange because different mass quarks (u and d) are involved. These more indirect effects, which are not necessarily negligible, can be taken into account in a more sophisticated calculation.[62]

If we apply Eq. (16) to the L = 0 baryons,[58] then there are three parameters (m_u m_d, c_1, and c_2) that enter mass differences, while there are four independent octet baryon mass differences. There is, therefore, one relation:[63]

$$M(p) - M(n) + M(\Xi^o) - M(\Xi^-) = M(\Sigma^+) - M(\Sigma^-). \tag{17}$$

Experimentally, the left- and right- hand sides are -7.69 ± 0.6 MeV and -7.98 ± 0.08 MeV, respectively.

The three remaining independent mass differences may be solved for $m_u - m_d$, c_1, and c_2:

$$m_u - m_d = -1.9 \text{ MeV}, \tag{18a}$$

$$c_1 = 3.6 \text{ MeV}, \tag{18b}$$

$$c_2 = -7.2 \text{ MeV}. \tag{18c}$$

Looking back at Table X, we find that the proton-neutron mass difference of -1.3 MeV arises as -1.9 MeV, + 1.2 MeV, and -0.6 MeV from the $m_u - m_d$, Coulomb, and magnetic terms respectively. The correct experimental sign is due to $m_u - m_d$! In general, all three terms give comparable contributions to baryon mass differences. The magnetic term is not negligible.

TABLE X

"Electromagnetic" Terms Contributing to Ground State Baryon Masses

State	$\sum_i m_i$	$\sum_{i>j} Q_i Q_j$	$\sum_{i>j} Q_i Q_j \vec{s}_i \cdot \vec{s}_j$
p	$2m_u + m_d$	0	1/3
n	$m_u + 2m_d$	-1/3	1/4
Σ^+	$2m_u + m_s$	0	1/3
Σ^o	$m_u + m_d + m_s$	-1/3	0
Σ^-	$2m_d + m_s$	1/3	-1/12
Ξ^o	$m_u + 2m_s$	-1/3	1/4
Ξ^-	$m_d + 2m_s$	1/3	-1/12
Λ	$m_u + m_d + m_s$	-1/3	1/6
Δ^{++}	$3m_u$	4/3	1/3
Δ^+	$2m_u + m_d$	0	0
Δ^o	$m_u + 2m_d$	-1/3	-1/12
Δ^-	$3m_d$	1/3	1/12
Σ^{*+}	$2m_u + m_s$	0	0
Σ^{*o}	$m_u + m_d + m_s$	-1/3	-1/12
Σ^{*-}	$2m_d + m_s$	1/3	+1/12
Ξ^{*o}	$m_u + 2m_s$	-1/3	-1/12
Ξ^{*-}	$m_d + 2m_s$	1/3	+1/12
Ω^-	$3m_s$	1/3	+1/12

For the ground state decuplet one may now deduce the relations:[58]

$$M(\Delta^{++}) - M(\Delta^{+}) = M(p) - M(n) + M(\Sigma^{+}) + M(\Sigma^{-}) - 2M(\Sigma^{o}), \tag{19a}$$

$$M(\Delta^{+}) - M(\Delta^{o}) = M(\Sigma^{*+}) - M(\Sigma^{*o}) = M(p) - M(n), \tag{19b}$$

$$M(\Delta^{o}) - M(\Delta^{-}) = M(\Sigma^{*o}) - M(\Sigma^{*-}) = M(\Xi^{*o}) - M(\Xi^{*-}) \tag{19c}$$

$$= M(p) - M(n) - M(\Sigma^{+}) - M(\Sigma^{-}) + 2M(\Sigma^{o}).$$

Only the last of these is testable now with some sensitivity:
$M(\Xi^{*o}) - M(\Xi^{*-}) = -3.3 \pm 0.6$ MeV and $M(p) - M(n) - [M(\Sigma^{+}) + M(\Sigma^{-}) - 2M(\Sigma^{o})]$ $= -3.07 \pm 0.10$ MeV, in good agreement. Adding (19b) and (19c) we obtain

$$M(\Sigma^{*+}) - M(\Sigma^{*-}) = 2M(p) - 2M(n) - M(\Sigma^{+}) - M(\Sigma^{-}) + 2M(\Sigma^{o}). \tag{20}$$

The left- and right- hand sides of this later relation are -4.1 ± 1.5 MeV and -4.36 ± 0.10 MeV, so it is consistent with experiment within rather large errors.

This striking success for predicting ground state baryon electromagnetic mass differences on the basis of Eq. (16) is not repeated for mesons. If we stick to u, d, and s quarks there are only two independent mass differences: $M(\pi^{+}) - M(\pi^{o})$ and $M(K^{+}) - M(K^{o})$. Since Q_iQ_j and $Q_iQ_j\vec{S}_i \cdot \vec{S}_j$ are proportional for mesons of given total quark spin (like pseudoscalars), there are also only two independent parameters (say $m_u - m_d$ and c_1). Therefore we do not get mass formulas like Eqs. (17), (18), (19), and (20) in this case.

While there is no relation, we can still invert the equations relating the two mass differences to the two parameters. Since the pion must be a deeply bound, relativistic system there is no reason for Eq. (16), linear in $\sum_i m_i$ to be valid. Indeed if we push blindly ahead we find $m_u - m_d \approx -7.1$ MeV, which very much disagrees with the value derived from the baryons.

It appears that to describe the meson electromagnetic mass differences we need to go beyond our simplified formula, Eq. (16). Some success[62] has been reported, including values for the D and D* electromagnetic mass differences in agreement with experiment, by taking into account the difference in the strong spin-spin interaction due to $m_u - m_d$.

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FIGURE CAPTIONS

1. Meson ground states and radical excitations with non-zero isospin.

2. Leading Regge trajectory for mesons (solid squares) and states established in Reference 47 (open circles).

3. Leading isospin 3/2 baryon Regge trajectory.

4. (a) Meson mass matrix contribution due to quark mass.

   (b) Meson mass matrix contribution due to gluon annihilation.