## DECAYS OF A HEAVY LEPTON INVOLVING THE HADRÓNIC VECTOR CURRENT*

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#### Abstract

The decays of a heavy lepton involving the hadronic vector current are calculated from electron-positron annihilation data. The result,  and its implications are discussed.


## I. Introduction

Over the past few years evidence has been accumulating from electronpositron annihilation experiments for a class of events with low multiplicity and charged leptons among the final particles. The properties of these events, as measured in several independent experiments, are such that only a small fraction could originate from the production and weak semileptonic decay of charmed hadrons. The only surviving single explanation for these events is that they are due to the pair production and subsequent weak decay of a new charged heavy lepton. ${ }^{1-9}$

Such a lepton, called $\tau,{ }^{9}$ might be expected to couple to a neutrino, $\nu_{\tau}$, via the charged weak current. If this is the same charged weak current as that responsible for the leptonic and semileptonic decays of the "ordinary" particles, we must expect the decays $\tau^{-} \rightarrow \nu_{\tau} \mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}, \tau^{-} \rightarrow \nu_{\tau} \mu^{-} \bar{\nu}_{\mu}$ and $\tau^{-} \rightarrow \nu_{\tau}+$ (hadrons) ${ }^{-}$. This last decay, if pictured as occurring by production of a light quark pair which then dress themselves as hadrons, is naively expected (because of three colors) to occur at three times the rate of $\tau^{-} \rightarrow \nu_{\tau} \mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}$ or $\tau^{-} \rightarrow \nu_{\tau} \mu^{-} \bar{\nu}_{\mu}$.

These decays, $\tau^{-} \rightarrow \nu_{\tau}+$ (hadrons $^{-}$, are of considerable interest; for, not only does one want to know for theoretical reasons if the naively calculated rate agrees with the observed sum over the physical hadronic channels, but also experimentally these modes and their detailed properties serve to clarify the existence and nature of the $\tau$ and of its couplings.

A number of individual modes (like $\tau^{-} \rightarrow \nu_{\tau} \pi^{-}$) can be calculated from other known quantities (the pion decay constant). The Cabibbo allowed decays through the hadronic vector current may be related to the total cross section for $\mathrm{e}^{+} \mathrm{e}^{-}$ annihilation into hadrons through the isovector electromagnetic current. In the
past, several calculations of $\tau^{-} \rightarrow \nu_{\tau}+$ (hadrons) $^{-}$have been made combining known couplings to a few channels with estimates of others. ${ }^{10,11,12}$

In this paper we recalculate the decays through the hadronic vector current. We do this because previous partial calculations plus estimates can now be replaced by a direct integration of colliding beam data over the entire energy range relevant to $\tau$ decay. In the next section we recall the relevant formulas for $\Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+(\text { hadrons })^{-}\right)$through the hadronic vector current and show how the ratio of three charged prong to one charged prong decays can be calculated. Then in Section III we present the detailed input and output of the calculation assuming various masses for $\tau$ and $\nu_{\tau}$. Section IV is a discussion of our results and their comparison with present experimental information.

## II. Heavy Lepton Decay Rates Via the Hadronic Vector Current

The formula for the decay rate for $\tau^{-} \rightarrow \nu_{\tau} \mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}$ or $\nu_{\tau} \mu^{-} \bar{\nu}_{\mu}$, assuming the charged current has a $\mathrm{V} \neq \mathrm{A}$ form and is of universal strength at the $\tau-\nu \tau$ vertex, is

$$
\begin{equation*}
\Gamma\left(\tau^{-} \rightarrow \nu_{\tau} \mathrm{e}^{-\overline{\nu_{e}}}\right)=\frac{\mathrm{G}^{2} \mathrm{M}_{\tau}^{5}}{192 \pi^{3}} \tag{1}
\end{equation*}
$$

Here $G=1.02 \times 10^{-5} / \mathrm{M}_{\mathrm{N}}^{2}$ is the weak coupling constant, and $\mathrm{M}_{\tau}$, the mass of the $\tau$, is experimentally $1.9 \pm 0.1 \mathrm{GeV} .{ }^{3,8,9}$ We have assumed that all the final leptons may be taken as massless. With a massive neutrino the decay rate becomes

$$
\begin{equation*}
\Gamma\left(\tau^{-} \rightarrow \nu_{\tau} \mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}\right)=\frac{\mathrm{G}^{2} \mathrm{M}_{\tau}^{5}}{192 \pi^{3}} \mathrm{~F}(\Delta) \tag{2a}
\end{equation*}
$$

with

$$
\begin{equation*}
F(\Delta)=1-8 \Delta^{2}+8 \Delta^{6}-\Delta^{8}-12 \Delta^{4} \ln \Delta^{2} \tag{2b}
\end{equation*}
$$

and $\Delta=\mathrm{m}_{\nu_{\tau}} / \mathrm{M}_{\tau}$. The experimental upper bound on the neutrino mass, $\mathrm{m}_{\nu_{\tau}}$, is $0.6 \mathrm{GeV} .3,8,9$

The corresponding decay rate for $\tau^{-} \rightarrow \nu_{\tau}+$ (hadrons) $^{-}$, proceeding through the action of the strangeness non-changing hadronic vector current, is straightforward to calculate: ${ }^{11}$

$$
\begin{align*}
& \Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+(\text { hadrons })^{-}\right)= \\
& \quad=\frac{\mathrm{G}^{2} \cos ^{2} \theta}{96 \pi^{3} \mathrm{M}_{\tau}^{3}} \int_{0}^{\mathrm{M}_{\tau}^{2}} \mathrm{dQ}^{2}\left(\mathrm{M}_{\tau}^{2}-\mathrm{Q}^{2}\right)^{2}\left(\mathrm{M}_{\tau}^{2}+2 \mathrm{Q}^{2}\right) \frac{\sigma_{\left.\mathrm{e}^{+} \mathrm{e}^{-\left(Q^{2}\right.}\right)}^{\sigma_{\mathrm{pt}}\left(\mathrm{Q}^{2}\right)}}{} \tag{3}
\end{align*}
$$

where $\cos \theta_{c}$ is the cosine of the Cabibbo angle, $\left.\sigma_{\mathrm{e}^{+} \mathrm{e}^{-}}^{(1)} \mathrm{Q}^{2}\right)$ is the electronpositron cross section to annihilate into hadrons with total isospin one at $\mathrm{E}_{\mathrm{cm}}^{2}=\mathrm{Q}^{2}$, and $\sigma_{\mathrm{pt}}\left(\mathrm{Q}^{2}\right)=4 \pi \alpha^{2} / 3 Q^{2}$ is the cross section for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}$. The extension to the case of massive neutrinos is

$$
\begin{align*}
& \Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+(\text { hadrons })^{-}\right)= \\
& \begin{aligned}
=\frac{\mathrm{G}^{2} \cos ^{2} \theta_{\mathrm{c}}}{96 \pi^{3} \mathrm{M}_{\tau}^{3}} \int_{0}^{\mathrm{M}_{\tau}^{2}} \mathrm{dQ}^{2} & {\left[\mathrm{M}_{\tau}^{4}+\mathrm{m}_{\nu}^{4}+\mathrm{Q}^{4}-2 \mathrm{~m}_{\nu}{ }_{\tau}^{2} \mathrm{M}_{\tau}^{2}-2 \mathrm{~m}_{\nu}^{2} \mathrm{Q}^{2}-2 \mathrm{M}_{\tau}^{2} \mathrm{Q}^{2}\right]^{1 / 2} } \\
& \times\left[\mathrm{M}_{\tau}^{4}+\mathrm{m}_{\nu}^{4}-2 \mathrm{Q}^{4}-2 \mathrm{~m}_{\nu_{\tau}}^{4} \mathrm{M}_{\tau}^{2}+\mathrm{m}_{\nu}^{2}{ }_{\tau}^{2} \mathrm{Q}^{2}+\mathrm{M}_{\tau}^{2} \mathrm{Q}^{2}\right] \frac{\mathrm{e}^{++} \mathrm{e}^{-\left(\mathrm{Q}^{2}\right)}}{\sigma_{\mathrm{pt}}\left(\mathrm{Q}^{2}\right)}
\end{aligned} \tag{4}
\end{align*}
$$

which reduces to Eq. (3) when $\mathrm{m}_{\nu_{\tau}}=0$.
The term involving the strangeness changing vector current which we have neglected is expected to be of order $\tan ^{2} \theta_{c} \simeq 0.05$ relative to that which we are calculating. Furthermore, its main contribution, through $\tau^{-} \rightarrow \nu_{\tau}+\mathrm{K}^{*}(890)^{-}$,
may be calculated separately, as we will do in Section IV. For the range of integration in Eq. (3) or (4) of interest to us, purely multipion states very much dominate the final state hadron channels in electron-positron annihilation. The annihilation cross section into final states with total isospin one involves only those channels with even numbers of pions.

The $\pi \pi$ channel must be $\pi^{+} \pi^{-}$in electron-positron annihilation and $\nu_{\tau}+\pi^{0} \pi^{-}$in $\tau^{-}$decay, and so it results in a single charged prong for the final $\tau^{-}$decay products. The four pion channel must be either $2 \pi^{+} 2 \pi^{-}$or $\pi^{+} \pi^{-} 2 \pi^{\circ}$ in colliding beams ${ }^{13}$ and $\nu_{\tau}+\pi^{+} 2 \pi^{-} \pi^{\circ}$ or $\nu_{\tau}+\pi^{-} 3 \pi^{\circ}$ in $\tau^{-}$decay. The four pion states in colliding beams and $\tau^{-}$decay are total $I_{z}=0$ and -1 states, respectively, of the same total $\mathrm{I}=1$ state. This fact allows us to derive ${ }^{14} \mathrm{a}$ relation between the populations of the two charge states of four pions in colliding beams and the two charge states of four pions in $\tau^{-}$decay. For any invariant mass, Q , of the four pion system it is:

$$
\begin{equation*}
\frac{\mathrm{d} \Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\pi^{+} 2 \pi^{-} \pi^{\mathrm{o}}\right)}{\mathrm{d} \Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\pi^{-} 3 \pi^{\mathrm{o}}\right)}=1+2 \frac{\sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \pi^{+} \pi^{-} 2 \pi^{\mathrm{o}}\right)}{\sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow 2 \pi^{+} 2 \pi^{-}\right)} \tag{5}
\end{equation*}
$$

Thus the proportion of four charged pions out of all four pion final states in colliding beams tells us the proportion of three charged prong decays for $\tau^{-} \rightarrow \nu_{\tau}+(4 \pi)^{-}$. The relative number of three charged prong to one charged prong decays arising from $\tau \rightarrow \nu_{\tau}+2 \pi$ and $\tau \rightarrow \nu_{\tau}+4 \pi$, which is of some interest experimentally, then can be settled completely from electron-positron annihilation data.

## III. Experimental Input and Results

As input to Eq. (4) we need data on electron-positron annihilation into $\pi^{+} \pi^{-}, 2 \pi^{+} 2 \pi^{-}, \pi^{+} \pi^{-} 2 \pi^{\circ}, \ldots$ in the center-of-mass energy range from threshold to $\mathrm{M}_{\tau}$. For this purpose we have taken cross section data from experiments done at Orsay, ${ }^{15,18,19}$ Novosibirsk, ${ }^{16}$ and Frascati. ${ }^{17,20}$ Our method has been to use what we considered to be the best data on a particular process in a given energy range. We have not made a statistical average of all available data. On occasion we have interpolated experimental data points to get a cross section at a desired energy. Our specific choice of data is as follows:

## A. $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \pi^{+} \pi^{-}$

From $\mathrm{Q}=0.28$ to 0.90 GeV we use the Orsay ${ }^{15}$ fit (taking $\rho-\omega$ interference into account) to their data on $\left|F_{\pi}\left(q^{2}\right)\right|^{2}$ :

$$
\begin{equation*}
\left|F_{\pi}\left(Q^{2}\right)\right|^{2}=\frac{\mathrm{F}_{0}^{2} \mathrm{M}_{\rho}^{2} \Gamma_{\rho}^{2}}{\left(\mathrm{M}_{\rho}^{2}-Q^{2}\right)^{2}+M_{\rho}^{2} \Gamma_{\rho}^{2}\left(\frac{\mathrm{p}}{\mathrm{p}_{0}}\right)^{6}\left(\frac{\mathrm{M}_{\rho}}{\mathrm{Q}}\right)^{2}} \tag{6}
\end{equation*}
$$

where $Q$ is the total center-of-mass energy and $p$ the pion momentum. For this fit the rho mass $M_{\rho}=0.7754 \mathrm{GeV}, \Gamma_{\rho}=0.1496 \mathrm{GeV}, \mathrm{F}_{0}=5.83$, and $\mathrm{p}_{0}$, the pion momentum at the rho mass, is 0.3615 GeV . The cross section for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \pi^{+} \pi^{-}$is related to $\left|\mathrm{F}_{\pi}\left(Q^{2}\right)\right|^{2}$ by

$$
\begin{equation*}
\sigma\left(e^{+} \mathrm{e}^{-} \rightarrow \pi^{+} \pi^{-}\right)=\frac{\pi \alpha^{2}}{3 Q^{2}}\left(\frac{2 p}{Q}\right)^{3}\left|F_{\pi}\left(Q^{2}\right)\right|^{2} \tag{7}
\end{equation*}
$$

Between $Q=0.90$ and 1.34 GeV we use the recent Novosibirsk data ${ }^{16}$ on $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \pi^{+} \pi^{-}$, which are significantly above the $\rho$ meson tail calculated from

Eq. (6). Above 1.34 GeV the measurements ${ }^{17}$ of $\left|F_{\pi}\right|^{2}$ are consistent, within rather large crror bars, with Eq. (6) once again. We use this formula as input in this region, but in any case this domain makes a very small contribution to $\tau^{-} \rightarrow \nu_{\tau}+\pi^{-} \pi^{0}$.
B. $\quad \mathrm{e}^{+} \mathrm{e}^{-} \rightarrow 2 \pi^{+} 2 \pi^{-}$

Between $\mathrm{Q}=0.90$ and 1.34 GeV we use the recent data from Novosibirsk ${ }^{16}$ along with the Orsay data ${ }^{18}$ at $0.91,0.99$, and 1.076 GeV to guide us at the lower end. Above 1.34 GeV our input is based on the recent data ${ }^{19}$ from DCI at Orsay, as smoothed by a fit involving both interferring resonance and backgound contributions.
C. $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \pi^{+} \pi^{-} 2 \pi^{\mathrm{o}}$

Again we use the Novosibirsk data ${ }^{16}$ for this channel up to 1.34 GeV , with earlier Orsay results ${ }^{18}$ used to pin down the threshold behavior ( 0.9 to $\sim 1.0 \mathrm{GeV}$ ). Above 1.34 GeV , we turn to the DCI data ${ }^{19}$ on the sum of $\pi^{+} \pi^{-} 2 \pi^{\circ}, 2 \pi^{+} 2 \pi^{-} 2 \pi^{\circ}$, and $\pi^{+} \pi^{-} 4 \pi^{\circ}$. These join on well to the $\pi^{+} \pi^{-} 2 \pi^{\circ}$ Novosibirsk data ${ }^{16}$ at the lower end.
D. $\mathrm{e}^{+} \mathrm{e} \rightarrow 6 \pi$
 along with $\pi^{+} \pi^{-} 2 \pi^{\circ}$ from using the DCI data above 1.34 GeV . Direct measurements ${ }^{20}$ of $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow 3 \pi^{+} 3 \pi^{-}$, as well as diffractive photoproduction, ${ }^{21}$ show an effective threshold near 2 GeV .

The input cross sections are summarized in Tables I and II.

We cstimate the total error in our calculation due to statistical and systematic errors in the input data to be about $\pm 12 \%$. The largest part of this comes from the $\pi^{+} \pi^{-}$channel below 900 MeV , and is calculated from the statistical errors stated by the Orsay group on the parameters in Eq. (6) combined with their estimates of the systematic errors. ${ }^{15}$ That the errors due to the $\pi^{+} \pi^{-}$ data dominate the total error is not because the intrinsic statistical or systematic errors in that experiment are particularly large-just that the bulk of the answer comes from that source. Although we have assigned large systematic errors to the multipion data at higher energies, they do not make an important contribution to the overall errors because the magnitude of the multipion contributions is not large and we have added the errors from different channels and energy regions in quadrature.

It is convenient to state our results for $\Gamma\left(\tau \rightarrow \nu_{\tau}+(2 \mathrm{n}\right.$ pions $)$ ) in terms of its magnitude relative to that for $\Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\mathrm{e}^{-} \overline{\nu_{\mathrm{e}}}\right)$. For a $\tau$ mass of 1.9 GeV and massless $\nu_{\tau}$, we find a value for this ratio of 1.69. We expect a value of $1.5 \cos ^{2} \theta_{c}=1.43$ on the basis of the naive model where $\tau^{-} \rightarrow \nu_{\tau}+(2 \mathrm{n} \text { pions })^{-}$ is due to $\tau^{-} \rightarrow \nu_{\tau}+\bar{u} d$, with light $\bar{u}$ and d quarks coming in three colors. ${ }^{22}$ Our calculated value is within $20 \%$ of this naive result and is even closer to the result obtained with the logarithmic correction due to asymptotic freedom. ${ }^{23}$ The contributions to the total result of 1.69 come from individual channels as follows: 1.12 from $\pi^{+} \pi^{-}, 0.22$ from $2 \pi^{+} 2 \pi^{-}$, and 0.35 from $\pi^{+} \pi^{-} 2 \pi^{\circ}$ (plus the six pion channels involving $\pi^{{ }^{0}}{ }^{\mathbf{s}}$ ).

The variation in $\Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+(2 \mathrm{n} \text { pions })^{-}\right) / \Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}\right)$ with $\mathrm{M}_{\tau}$ is shown in Fig. $1\left(\mathrm{~m}_{\nu_{\tau}}=0\right)$. There is relatively little variation with $\mathrm{M}_{\tau}$ as long as it is in the 1.5 to 2 GeV range.

Similarly, the decay width for non-zero values of $\mathrm{m}_{\nu}$ (with $\mathrm{M}_{\tau}$ fixed at 1.9 GeV) is shown in Fig. 2. Here $\Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}\right)$ is computed from Eq. (2) with $\mathrm{m}_{\nu_{\tau}} \neq 0$. Only when the neutrino mass exceeds about 600 MeV does one see a fairly sizeable variation in the ratio $\Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+(2 \mathrm{n} \text { pions })^{-}\right) / \Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}\right)$.

Employing an integrated version of Eq. (5), we can calculate the ratio $\Gamma\left(\tau^{-}-\nu_{\tau}+\pi^{+} 2 \pi^{-} \pi^{0}\right) / \Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\pi^{-} 3 \pi^{\mathrm{o}}\right)$. For a nominal $\tau$ mass of 1.9 GeV and a massless $\tau$ neutrino this ratio is 4.18, if we assume that in our input data the six pion contribution is negligible compared to that from $\pi^{+} \pi^{-} 2 \pi^{\circ}$. In other words, under the same assumption $\sim 81 \%$ of $\tau$ decays involving four pions have three charged prongs. Since $\Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+(4 \pi)^{-}\right) / \Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}\right)$ is $\approx 0.57$, we conclude that $\Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\pi^{+} 2 \pi^{-} \pi^{0}\right) / \Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}\right) \approx 0.46$.

## IV. Discussion

Using data from electron-positron annihilation, we have calculated the decay rate for $\tau^{-} \rightarrow \nu_{\tau}+(2 \mathrm{n} \text { pions })^{-}$which proceeds through the hadronic vector weak current. There is in addition to what we have calculated a small contribution to $\tau$ decays coming from the strangeness changing vector current. This contribution is proportional to $\sin ^{2} \theta_{c}$ and is likely dominated by the $K^{*}(890)$ in the same way that the $\rho$ dominates the strangeness non-changing contribution. Assuming $\Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\mathrm{K}^{-}\right)=\tan ^{2} \theta_{\mathrm{c}} \Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\rho^{-}\right)$, we estimate $\Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\mathrm{K}^{*}{ }^{-}\right) / \Gamma\left(\tau \rightarrow \nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}\right)=0.05$.

So, the sum of the $\tau$ decay widths to $\nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}, \nu_{\tau}+\mu^{-} \bar{\nu}_{\mu}, \nu_{\tau}+(2 \mathrm{n} \text { pions })^{-}$, and $\nu_{\tau}+\mathrm{K}^{*}(890)^{-}$is $(1+0.98+1.69+0.05) \Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}\right)=$ $3.72 \Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\mathrm{e}^{-\bar{\nu}} \overline{\mathrm{e}}\right)$. This is a lower bound on the total width, and hence we have an upper bound on the branching ratio into $\nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}$ :

$$
\operatorname{BR}\left(\tau^{-} \rightarrow \nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}\right) \leq \frac{1}{3.72}=0.27
$$

While this bound applies for $\mathrm{M}_{\tau}=1.9 \mathrm{GeV}$ and $\mathrm{m}_{\nu_{\tau}}=0$, the results of the last section show that it is not sensitive to variations in these masses by several hundred MeV .

The experimental measurements ${ }^{9}$ of $\mathrm{BR}\left(\tau^{-} \rightarrow \nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}\right)$ are all smaller than our bound and typically less than about 0.2. Most measurements lie in the range 0.15 to 0.20 . Since the bound would be saturated if the only $\tau$ decays were into $\nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}, \nu_{\tau}+\mu^{-} \bar{\nu}_{\mu}$ and $\nu_{\tau}+$ hadrons through the hadronic vector weak current, we conclude that there must exist other decays. Of course, one does expect decays into $\nu_{\tau}+$ hadrons through the hadronic axial-vector weak current. Using our calculation for the vector current contribution we compute that the width for $\tau^{-} \rightarrow \nu_{\tau}+$ (hadrons $^{-}$arising from the axial-vector current is 2.95, 2.16 and 1.28 times $\Gamma\left(\tau^{-}-\nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}\right)$ for values of the $\tau^{-} \rightarrow \nu_{\tau}+\mathrm{e}^{-\bar{\nu}} \overline{\mathrm{y}}^{\text {b }}$ branching ratio of $0.15,0.17$, and 0.20 , respectively.

Part of these decays through the axial-vector current can be calculated from known quantities: $\Gamma\left(\tau^{-} \rightarrow \nu_{\tau} \pi^{-}\right)$and $\Gamma\left(\tau^{-} \rightarrow \nu_{\tau} \mathrm{K}^{-}\right)$just involve the additional knowledge of the pion and kaon decay constants. For $^{M_{\tau}}=1.9 \mathrm{GeV}$ and $\mathrm{m}_{\nu_{\tau}}=0$, one finds

$$
\frac{\Gamma\left(\tau^{-} \rightarrow \nu\right.}{\Gamma\left(\tau^{-}-\pi^{-}\right)} \tau^{\left.\mathrm{e}^{-} \overline{\nu_{\mathrm{e}}}\right)}=0.54
$$

and

$$
\frac{\Gamma\left(\tau^{-} \rightarrow \nu_{\tau} \mathrm{K}^{-}\right)}{\Gamma\left(\tau^{-} \rightarrow \nu_{\tau} \mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}\right)}=0.03
$$

However attempts to find the decay $\tau^{-} \rightarrow \nu_{\tau} \pi^{-}$have so far been unsuccessful, with some indication ${ }^{4}$ that the predicted rate is too large to be compatible with the experimental lack of observation.

In any case, even if the $\nu_{\tau} \pi^{-}$and $\nu_{\tau} \mathrm{K}^{-}$decays occur at the predicted rate, we have seen that the total width for $\tau^{-} \rightarrow \nu_{\tau}+$ (hadrons) $^{-}$proceeding through the axial-vector current is much larger. There must be decays through the axial-vector current other than $\nu_{\tau} \pi^{-}$and $\nu_{\tau} K^{-}$. Specifically, taking our calculation of the vector current decays and those through the axial-vector current involving only a $\pi^{-}$or $\mathrm{K}^{-}$, we still have $2.38,1.59$, and 0.71 times $\Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}\right)$ for the decay widths $\tau^{-} \rightarrow \nu_{\tau}+$ (hadrons $\neq \pi^{-}, \mathrm{K}^{-}$) through the axial-vector weak current when $\operatorname{BR}\left(\tau^{-} \rightarrow \nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}\right)$ is $0.15,0.17$, and 0.20 , respectively.

Recently there have been preliminary reports ${ }^{8,9,24}$ of the decay $\tau^{-} \rightarrow \nu_{\tau} \mathrm{A}_{1}^{-} \rightarrow \nu_{\tau}(3 \pi)^{-}$at roughly the level we are deducing here. ${ }^{25}$ Establishing this and the other semi-hadronic modes of the $\tau$ are important; for, if $\tau^{-}-\nu_{\tau}(3 \pi)^{-}, \tau^{-} \rightarrow \nu_{\tau} \pi^{-}$, and the decays through the vector current do not all occur at the rates discussed above, then the weak current involved in $\tau$ dccays is not the one responsible for all other weak decays observed until now.

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TABLE I

| $\mathrm{Q}(\mathrm{GeV})$ | $(\mathrm{A})$ | $(\mathrm{B})$ | $(\mathrm{C})$ |
| :--- | ---: | ---: | ---: |
|  |  |  |  |
| 0.91 | 133 | 0.0 | 0.0 |
| 0.93 | 115 | 0.0 | 0.0 |
| 0.95 | 60.3 | 0.0 | 2.0 |
| 0.97 | 58.9 | 0.0 | 4.0 |
| 0.99 | 62.3 | 1.0 | 6.0 |
| 1.01 | 49.3 | 1.5 | 8.0 |
| 1.03 | 40.6 | 2.0 | 10.0 |
| 1.05 | 40.9 | 3.9 | 7.6 |
| 1.07 | 41.9 | 3.0 | 25.5 |
| 1.09 | 20.0 | 7.0 | 25.7 |
| 1.11 | 32.5 | 5.0 | 18.7 |
| 1.13 | 37.5 | 5.2 | 35.1 |
| 1.15 | 19.1 | 9.6 | 20.9 |
| 1.17 | 21.9 | 10.2 | 29.8 |
| 1.19 | 17.2 | 10.4 | 22.5 |
| 1.21 | 14.2 | 11.7 | 35.1 |
| 1.23 | 10.1 | 12.7 | 30.1 |
| 1.25 | 5.7 | 15.8 | 37.1 |
| 1.27 | 7.6 | 13.6 | 41.6 |
| 1.29 | 4.8 | 17.1 | 19.4 |
| 1.31 |  | 20.0 | 22.6 |
| 1.33 |  |  | 36.0 |

(A) $\quad \sigma\left(\mathrm{e}^{+} \mathrm{e}^{-}-\pi^{+} \pi^{-}\right)(\mathrm{nb})($ Ref. 16).
(B) $\quad \sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow 2 \pi^{+} 2 \pi^{-}\right)(\mathrm{nb})($ Ref. 16,18$)$.
(C) $\quad \sigma\left(e^{+} e^{-}-\pi^{+} \pi^{-} 2 \pi^{\circ}\right)(n b)($ Ref. 16, 18).

## TABLE II

| $\mathrm{Q}(\mathrm{GeV})$ | $(\mathrm{A})$ | $(\mathrm{B})$ |
| :--- | :--- | :--- |
|  |  |  |
| 1.35 | 23.9 | 33.7 |
| 1.40 | 28.8 | 38.7 |
| 1.45 | 36.5 | 44.2 |
| 1.50 | 45.6 | 57.3 |
| 1.55 | 54.0 | 62.3 |
| 1.60 | 40.0 | 64.8 |
| 1.65 | 34.0 | 54.7 |
| 1.70 | 30.9 | 79.2 |
| 1.75 | 26.0 | 62.3 |
| 1.80 | 19.6 | 44.2 |
| 1.85 | 16.8 | 35.8 |
| 1.90 | 15.4 | 33.7 |
| 1.95 | 14.2 | 32.5 |

(A) $\quad \sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow 2 \pi^{+} 2 \pi^{-}\right)(\mathrm{nb})($ Ref. 19 $)$.
(B) $\quad \sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \pi^{+} \pi^{-} 2 \pi^{\mathrm{o}}, 2 \pi^{+} 2 \pi^{-} 2 \pi^{\mathrm{o}}, \pi^{+} \pi^{-} 4 \pi^{\mathrm{o}}\right)(\mathrm{nb})($ Ref. 19).

## References

1. M. L. Perl et al., Phys. Rev. Letters 35, 1489 (1975); M. L. Perl et al. , Phys. Letters 63B, 466 (1976); G. J. Feldman et al., Phys. Rev. Letters 38, 177 (1976).
2. M. Cavalli-Sforza et al. , Phys. Rev. Letters 36, 588 (1976).
3. H. Meyer in Deeper Pathways in High Energy Physics, A. Permutter and L. Scott, eds. (Plenum Press, New York, 1977), p. 351; J. Burmester et al., Phys. Letters 68B, 297 and 301 (1977); G. Knies, invited talk presented at the 1977 International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, Germany, August 25-31, 1977.
4. R. Brandelik et al. , DESY preprint 77/36, 1977 (unpublished); S. Yamada, invited talk presented at the 1977 International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, Germany, August 25-31, 1977.
5. A. Barbaro-Galtieri et al., Phys. Rev. Letters 39, 1058 (1977);
A. Barbaro-Galtieri, invited talk presented at the 1977 International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, Germany, August 25-31, 1977.
6. J. Kirkby, invited talk presented at the 1977 International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, Germany, August 25-31, 1977.
7. U. Camerini et al., University of Colorado preprint, 1977 (unpublished).
8. M. L. Perl et al. , SLAC preprint SLAC-PUB-1997, 1977 (unpublished).
9. A review is found in M. L. Perl, invited talk at the 1977 International

Symposium on Lepton and Photon Interactions at High Energies, Hamburg, Germany, August 25-31, 1977 and SLAC-PUB-2022, 1977 (unpublished).
10. H. B. Thacker and J. J. Sakurai, Phys. Letters 36B, 103 (1971).
11. Y. S. Tsai, Phys. Rev. D4, 2821 (1971).
12. K. Gaemers and R. Raitio, Phys. Rev. D14, 1262 (1976).
13. The $4 \pi^{\circ}$ channel is forbidden by charge conjugation invariance.
14. This relation may be derived in a particularly simple manner using the method of I. M. Shmushkevich, Dokl. Akad. Nauk SSSR 103, 235 (1955) by demanding equal $\pi^{+}, \pi^{-}$and $\pi^{\circ}$ populations when one averages over $\mathrm{I}_{\mathrm{Z}}=-1,0$, and +1 states with equal weight.
15. D. Benaksas et al., Phys. Letters 39B, 289 (1972).
16. V. Sidorov, in Proceedings of the XVII International Conference on High Energy Physics, Tbilisi, July, 1976 (Joint Institute for Nuclear Research, Dubna, 1977), Vol. II, p. B13.
17. V. Alles-Borelli et al., Phys. Letters 40B, 433 (1972); G. Barbiellini et al., Lett. Nuovo Cimento 6, 557 (1973); M. Bernardini et al., Phys. Letters 44B, 393 and 46B, 261 (1973); D. Bollini et al., Lett. Nuovo Cimento 14, 418 (1975).
18. G. Cosme et al. , Phys. Letters 40B, 685 (1972); G. Cosme et al. , Phys. Letters 63B, 349 (1976).
19. F. Laplanche, invited talk at the 1977 International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, Germany, August 25-31, 1977.
20. B. Bartoli et al. , Phys. Rev. D6, 2374 (1972).
21. See B. Knapp in Particles and Fields ' 76, H. Gordon and R. Peierls, eds. (Brookhaven National Laboratory, Upton, New York, 1977), p. A13.
22. The rate for $\tau^{-} \rightarrow \nu_{\tau}+\bar{u} d$ is three times that for $\tau^{-} \rightarrow \nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}$, but half occurs through the vector and half through the axial-vector weak current of the quarks.
23. C. S. Lam and T. M. Yan, Cornell University preprint CLNS-357, 1977 (unpublished).
24. J. Jaros, SLAC seminar, October, 1977, and private communication.
25. The experimental values reported, assuming the $\mathrm{A}_{1}$ decays to $\pi \rho$, are $\operatorname{BR}\left(\tau^{-} \rightarrow \nu_{\tau}+\mathrm{A}_{1}^{-}\right)=0.11 \pm 0.04 \pm 0.03$ from Ref. 3 and $0.20 \pm 0.08$ from Ref. 24. Our range for $\operatorname{BR}\left(\tau^{-} \rightarrow \nu_{\tau}+\right.$ (hadrons $\left.\neq \pi^{-}, \mathrm{K}^{-}\right)$) through the axialvector current is 0.14 to 0.36 .

## Figure Captions

Fig. 1 The ratio, $\mathrm{R}_{\mathrm{V}}$, of the width for $\tau^{-} \rightarrow \nu_{\tau}+(2 \mathrm{n} \text { pions })^{-}$, proceeding through the hadronic vector weak current, to that for $\tau \rightarrow \nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}$ as a function of $\mathrm{M}_{\tau} \cdot \mathrm{m}_{\nu_{\tau}}=0$.

Fig. 2 The dependence of the ratio $R_{V}$, as in Fig. 1 , on $m_{\nu}$ for $M_{\tau}=1.9 \mathrm{GeV}$. $\Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+\mathrm{e}^{-} \bar{\nu}_{\mathrm{e}}\right)$ is computed from Eq. (2) with the appropriate value of $m_{\nu}$.


Fig. 1


Fig. 2

