Longitudinal dynamics and chiral symmetry breaking in holographic light-front QCD

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The breaking of chiral symmetry in holographic light-front QCD is encoded in its longitudinal dynamics with its chiral limit protected by the superconformal algebraic structure which governs its transverse dynamics. The scale in the longitudinal light-front Hamiltonian determines the confinement strength in this direction: It is also responsible for most of the light meson ground state mass consistent with the GMOR constraint. In the limit of heavy quark masses we find a precise connection of longitudinal and transverse confinement scales consistent with HQET. Longitudinal confinement and the breaking of chiral symmetry are found to be different manifestations of the same underlying dynamics.

In spite of the important progress of Euclidean lattice gauge theory, a basic understanding of the mechanism of color confinement and its relation to chiral symmetry breaking in QCD has remained an unsolved problem. Recent developments based on superconformal quantum mechanics [1, 2] in light-front quantization [3] and its holographic embedding on a higher dimensional gravity theory [4] (gauge/gravity correspondence) have led to new analytic insights into the structure of hadrons and their dynamics [5–10]. This new approach to nonperturbative QCD dynamics, holographic light-front QCD, leads to effective semi-classical relativistic bound-state equations for arbitrary spin [11], and it incorporates fundamental properties which are not apparent from the QCD Lagrangian, such as the emergence of the hadronic mass scale, the prediction of a massless pion in the chiral limit, and the remarkable connections between meson, baryon and tetraquark spectroscopy across the full hadron spectrum [12–15]. Phenomenological extensions of the holographic QCD approach also predict the running of the QCD coupling $\alpha_s(Q^2)$ in the nonperturbative domain [16, 17] and provide nontrivial connections between the dynamics of form factors and polarized and unpolarized quark distributions with pre-QCD nonperturbative approaches such as Regge theory and the Veneziano model [18–20].

In this letter we examine the effect of longitudinal light-front dynamics for the computation of hadron masses, confinement, and chiral symmetry breaking. Although light-front holography, based on the Maldacena conjecture [4] and the superconformal algebraic structure in [2], determines the confinement potential in the light-front (LF) transverse coordinates in the zero quark mass chiral limit [9], an extension is required which incorporartes color-confining LF longitudinal dynamics for non-zero quark masses. This extension of holographic LF QCD should preserve its successful predictions as well as to insure 3-dimensional rotational invariance in the heavy-quark limit.

A simple ansatz to account for quark masses in holographic LF QCD was introduced in [21] based on the off-shell dependence of the LF wave function on the invariant mass which controls the bound state. For a two-parton state this amounts to the substitution

$$\frac{\mathbf{k}_\perp^2}{x(1-x)} \to \frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \tag{1}$$

in the ground state wave function since the right-hand side in (1) is the LF kinetic energy, in presence of quark masses, as well as the invariant mass squared $s = (p_q + p_{\bar q})^2$ of the $q\bar q$ pair. The variable x in (1) is the LF longitudinal momentum fraction $x = k^+/P^+$ and $\mathbf{k}_\perp$ is the relative transverse momentum. The substitution (1) leads to the longitudinal ground-state wave function [10, 21]

$$\chi(x) = \mathcal{N} e^{-\frac{1}{2\lambda}\left(\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)}, \tag{2}$$

with normalization $\mathcal{N}$. The first-order shift in the hadron masses computed in [10, 21] has a limited range of application since it is assumes that quark masses are small and that the contribution from longitudinal dynamics is also small. We show here how these limitations can be overcome by a nonperturbative computation based on a convenient eigenfunction expansion. In doing so, we will also find new important qualitative properties not apparent in our previous treatment [10, 21].

We start from the semiclassical LF transverse [5] and longitudinal [22, 23] Hamiltonian wave equations for mesons

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_T(\zeta)\right)\phi(\zeta) = M_T^2 \phi(\zeta), \tag{3}$$

$$\left(\frac{m_q^2}{x} + \frac{m_{\bar q}^2}{1-x} + U_L(x)\right)\chi(x) = M_L^2\,\chi(x), \tag{4}$$

where the variable ζ in (3) is the invariant transverse variable, $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$, with $\mathbf{b}_\perp$ the transverse impact distance conjugate to the relative transverse momentum $\mathbf{k}_\perp$, and L is the relative LF orbital angular momentum $L \equiv |L^z|_{max}$. The longitudinal dynamical model in [22] is based in 't Hooft model for large-N two-dimensional QCD [24] which, as in Ref. [23], can be combined with the holographic LF transverse equation (3) to incorporate massive quarks.

We write the meson LF wave function ψ as

$$\psi(x,\zeta,\varphi) = \sqrt{\frac{x(1-x)}{2\pi\zeta}}\, e^{iL\varphi}\chi(x)\phi(\zeta), \tag{5}$$

with normalization $\int_0^1 dx\, \chi^2(x) = 1$ and $\int_0^\infty d\zeta\, \phi^2(\zeta) = 1$, where we have factored out the longitudinal, transverse and orbital dependence. This factorization of the wave function follows if the effective LF Hamiltonian can be written as the sum of longitudinal and transverse components: $H^{LF} = H^{LF}_L + H^{LF}_T$. The longitudinal mass M_L^2 appears as a separation constant of the full invariant LF Hamiltonian [22], $H^{LF}|\psi\rangle = M^2|\psi\rangle$; therefore, the mass-squared eigenvalues become the sum $M^2 = M_L^2 + M_T^2$. We have included in (5) the normalization factor $\sqrt{x(1-x)}$ which arises from the precise mapping of AdS form factors to light-front physics in the limit of zero quark masses [25].

The transverse LF equation (3) has a similar structure as the wave equations derived in five-dimensional AdS provided that one identifies $\zeta = z$ [5], the holographic fifth-dimensional coordinate of AdS. This precise mapping allows us to relate the LF confinement potential U_T to the dilaton profile which modifies AdS space [10]. The assumption of superconformal algebra then uniquely determines the form of the transverse confining potential for both mesons and nucleons [7, 8]: For mesons it is given by [8, 26]

$$U_T(\zeta) = \lambda^2\zeta^2 + 2\lambda(J-1). \tag{6}$$

In the factorized approximation, the radial and orbital excitations are determined by the transverse potential (6) with eigenvalues [10]

$$M_T^2(n,J,L) = 4\lambda\left(n + \frac{J+L}{2}\right), \tag{7}$$

and eigenfunctions

$$\phi_{n,L}(\zeta) = \lambda^{(1+L)/2}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\lambda\zeta^2/2}L_n^L(\lambda\zeta^2). \tag{8}$$

For the longitudinal component we will adopt the effective potential introduced by Li, Maris, Zhao and Vary in [23] to generate a convenient orthonormal basis functions in the LF longitudinal momentum fraction x. It is given by

$$U_L(x) = -\sigma^2\partial_x\left(x(1-x)\,\partial_x\right), \tag{9}$$

and contains the term $\sigma^2 x(1-x)\tilde{z}^2$ required to form an oscillator potential in the LF longitudinal as well as in the transverse directions. The longitudinal spatial variable $\tilde{z}$ conjugate to the longitudinal momentum-x, $\tilde{z} \sim i\partial_x$, is the frame-independent Ioffe coordinate of Miller and Brodsky [27]. The potential (9) was introduced in the context of basis light-front quantization [28, 29] and was further used in [30–33].

The scale σ in (9) is the longitudinal confinement scale and has units of mass. In contrast, the transverse confinement scale λ in (6) has dimensions of mass squared, but both scales are connected in the heavy quark mass limit. To show this, consider the limit $m_q, m_{\bar{q}} \to m_Q, m_{\bar{Q}} \gg k_\perp, k_z,\ \lambda \to \lambda_Q$. In the non-relativistic limit we find

$$x = \frac{m_Q + k_z}{m_Q + m_{\overline{Q}}}, \quad \overline{x} = \frac{m_{\overline{Q}} - k_z}{m_Q + m_{\overline{Q}}}, \tag{10}$$

which leads to the non-relativistic rotationally-invariant potential

$$U(r) \to V(r) = \frac{U(r)}{m_Q + m_{\overline{Q}}} = \tfrac{1}{2}\mu\,\omega^2 r^2, \tag{11}$$

and the constraint

$$\omega = \sigma = \frac{\lambda_Q}{m_Q + m_{\overline{Q}}}, \tag{12}$$

where $\mu = \frac{m_Q m_{\bar{Q}}}{m_Q + m_{\overline{Q}}}$ and $\mathbf{r}^2 = \mathbf{b}_\perp^2 + b_z^2$, where b_z is the canonical conjugate to k_z, $b_z = i\partial_{k_z}$.

In order to compute the longitudinal meson mass contribution for an arbitrary LF wave function $\chi(x)$, it is convenient to perform an expansion in terms of the complete basis of orthonormal functions generated by the longitudinal LF Hamiltonian equation (4) for the specific potential (9)

$$\chi_\ell^{\alpha,\beta}(x) = N x^{\alpha/2}(1-x)^{\beta/2}P_\ell^{(\alpha,\beta)}(1-2x). \tag{13}$$

Thus,

$$\begin{aligned} M_L^2 &= \\ &\int_0^1 dx\,\chi(x)\left[-\sigma^2\partial_x\left(x(1-x)\partial_x\right) + \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}\right]\chi(x) \\ &= \sum_\ell C_\ell^2 M_L^2(\alpha,\beta,\ell), \end{aligned} \tag{14}$$

where

$$M_L^2(\ell,\alpha,\beta) = \tfrac{1}{4}\sigma^2(\alpha+\beta+2\ell)(2+\alpha+\beta+2\ell), \tag{15}$$

with $\alpha = 2m_q/\sigma$ and $\beta = 2m_{\bar{q}}/\sigma$ as shown in the Appendix. For the invariant mass ansatz Eq. (2) we use the above expressions to compute M_L^2 using the complete eigenfunction basis (13)

$$\mathcal{N}\exp\left\{-\frac{\sigma^2}{8\lambda}\left(\frac{\alpha^2}{x} + \frac{\beta^2}{1-x}\right)\right\} = \sum_\ell C_\ell\,\chi_\ell(x). \tag{16}$$

In practice, we need to know the value of the scale σ and the quark masses to compute M_L^2. In the heavy quark limit Eq. (12) coincides with the heavy-quark effective theory (HQET) result [34], which requires that the confining scale is proportional to the mass of the heavy meson: $\sqrt{\lambda_M} = C\sqrt{M}$ [13]. The value is $C = 0.49 \pm 0.02$ GeV$^{1/2}$ for $M \geq 1.8$ GeV [15], namely $\sigma \simeq C^2 = 0.24$ GeV. Assuming that the value of the longitudinal confinement scale is approximately constant at all scales we can determine the effective light quark masses m_u and m_d from the measured pion mass and determine the strange quark mass, m_s, from the kaon mass using (14): The value of the $\phi(1020)$ mass is then a prediction. Notice that the $\phi(1020)$ vector meson also has the transverse mass component $M_T = \sqrt{2\lambda}$ from the spin-spin interaction in supersymmetric LF holographic QCD [10, 26] with $\sqrt{\lambda} = 0.523$ GeV.

TABLE I. Lowest expansion coefficients C_ℓ in (16).

	$\ell=0$	$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$	$\ell=5$	$\ell=6$
$C(ud)$	0.998	0	0.055	0	0.010	0	-0.003
$C(u\bar{s})$	0.967	-0.231	0.100	-0.006	-0.009	0.013	-0.016
$C(s\bar{s})$	0.998	0	0.038	0	-0.045	0	-0.024
$C(u\bar{c})$	0.958	-0.267	0.097	-0.012	-0.003	0	-0.007
$C(c\bar{c})$	0.999	0	0.016	0	-0.020	0	-0.003

We show in Table I the values of the lowest expansion coefficients. The results for the light meson masses in Fig. 1 correspond to the values $m_u = m_d = 28$ MeV and $m_s = 326$ MeV. Meson masses are determined from the stability plateau in Fig. 1. For light quark masses values above $\ell_{max} \simeq 20$ introduce large uncertainties in the numerical evaluations from highly oscillatory integrands.

The distribution amplitude (DA) [35], $X(x) \equiv \sqrt{x(1-x}\,\chi(x)$, for the pion, kaon and J/Ψ mesons are shown in Fig. (2). Very few modes in the basis expansion (16) are required to reproduce the invariant mass wave function result. The DAs predicted by holographic LF QCD at the initial nonperturbative scale should then be evolved to the relevant scale using the ERBL equation [35–37]. The Dyson-Schwinger results for the pion DA [38] are very similar to the chiral result $X(x) = \sqrt{x(1-x)}$ from LF holographic mapping [25].

Since the lowest mode $\ell = 0$ in the expansion (16) accounts for 99 % of the pion probability, we can write to a very good approximation

$$M_\pi^2 = \sigma(m_u + m_d) + (m_u + m_d)^2, \tag{17}$$

which has the same linear dependence in the quark mass as the Gell-Mann-Oakes-Renner (GMOR) relation for the light quark mass expansion [39]

$$M_\pi^2 f_\pi^2 = -\tfrac{1}{2}(m_u + m_d)\langle \bar{u}u + \bar{d}d\rangle + \mathcal{O}\big((m_u + m_d)^2\big), \tag{18}$$

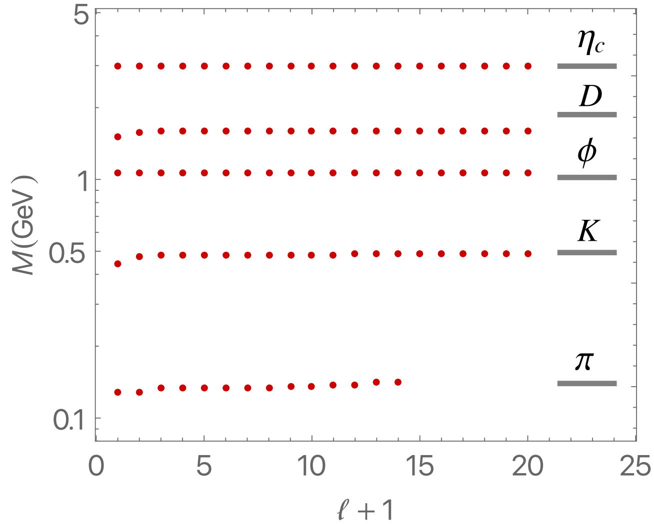

FIG. 1. Stability analysis and numerical evaluation of meson masses from (14).

where the vacuum condensate $\langle\bar{\psi}\psi\rangle \equiv \frac{1}{2}\langle\bar{u}u + \bar{d}d\rangle$ plays the role of a chiral order parameter.

We can extend our analysis to the heavy quark sector provided that longitudinal and transverse dynamics can be factored out to a first approximation. In contrast with the light quark mass sector, $m_q, m_{\bar{q}} \ll \sigma$, most of the hadron mass in the heavy sector, $m_Q, m_{\bar{Q}} \gg \sigma$, comes from quark masses. The expansion coefficients of the invariant mass wave function (16) for the the uc and cc meson are shown in Table I. We determine the effective charm quark mass from the η_c using (14) and compute, for example, the mass of the D meson as a prediction. We find for M_D a value within 14% of its measured value for $m_c \simeq 1.4$ GeV. Our simple approximation does not include one-gluon exchange, which becomes relevant for heavy quark masses [23].

We have shown how the introduction of an effective LF longitudinal potential leads to a GMOR type relation where an effective longitudinal scale accounts for a significant part of the ground state light meson mass and is also responsible for confinement in the longitudinal direction. Following [23], we choose a potential which generates a complete basis function and reduces to a rotational-invariant oscillator in the limit of heavy quark masses, therefore establishing a connection with the holographic LF transverse scale. The origin and physical interpretation of the longitudinal scale, which has the role of a condensate, remains to be explored. In QCD lattice field theory, for example, the condensate $\langle\bar{\psi}\psi\rangle$ originates from the condensation of quark-antiquark pairs in the vacuum [40]. The structure of the instant-form vacuum is sampled in the Euclidean region where non-trivial field configurations provide a mechanism for symmetry breaking through the Banks-

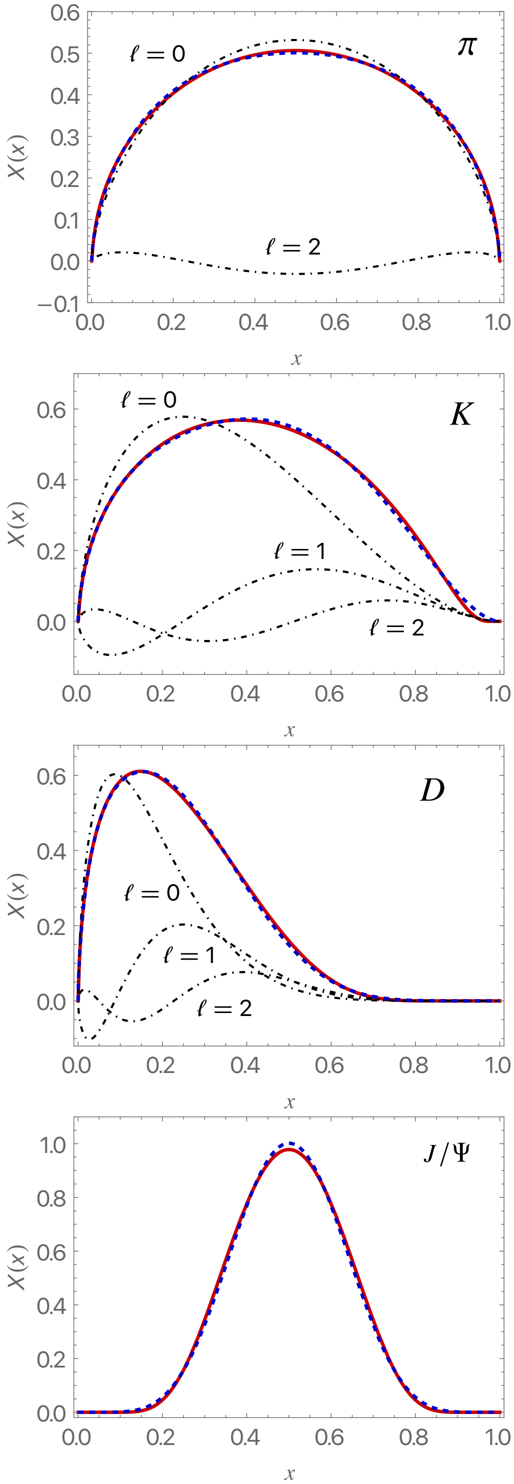

FIG. 2. Light-front distribution amplitudes $X(x)$ for the π, K, D and J/Ψ mesons: the red curve is the invariant mass result, dot dashed black curves are individual modes in the expansion (16), dashed blue curve represent the sum of modes in the figure. Notice that the J/Ψ result is well described by the zero mode alone.

Casher relation, $\langle\bar{\psi}\psi\rangle = -\pi\rho(0)$, with $\rho(0)$ the density or Dirac-zero modes [41]. However, the relation between chiral symmetry breaking and confinement remains elusive. In this context, it has been argued that chiral symmetry breaking condensates, usually viewed as a constant mass scale which fill all spacetime, are instead contained within hadrons (in-hadron condensate), therefore a property of hadron dynamics [42]. The light-front semiclassical approximation described here would favor a dynamical, rather than spontaneous, underlying mechanism of chiral symmetry breaking since the longitudinal scale σ determines the confinement strength in the longitudinal direction as well as the effective scale of chiral symmetry breaking.

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Appendix: Jacobi polynomials and solution to the longitudinal Hamiltonian equation. The Jacobi polynomials $P_n^{(\alpha,\beta)}(z)$ are solution of the differential equation

$$(1-z)^{-\alpha}(1+z)^{-\beta}\partial_z\left((1-z)^{\alpha+1}(1+z)^{\beta+1}\partial_z\, u(z)\right) + n(n+a+b+1)u(z) = 0, \tag{19}$$

which is orthogonal in the interval $[-1,1]$ with weight $(1-z)^{\alpha}(1-z)^{\beta}$. Performing the change of variable $z = 1-2x$ we find

$$x^{-\alpha}(1-x)^{-\beta}\partial_x\left(x^{\alpha+1}(1-x)^{\beta+1}\partial_x\, u(x)\right) + n(n+a+b+1)u(x) = 0, \tag{20}$$

with the solution $P_n^{(\alpha,\beta)}(1-2x)$ orthogonal in the interval $[0,1]$ with weight $x^{\alpha}(1-x)^{\beta}$.

Consider now the eigenvalue equation

$$\left(-\partial_x\left(x(1-x)\partial_x\right) + \frac{1}{4}\left[\frac{\alpha^2}{x} + \frac{\beta^2}{1-x}\right]\right) v(x) = \nu^2 v(x). \tag{21}$$

Writing $v(x) = x^{\alpha/2}(1-x)x^{\beta/2}w(x)$ and substituting in (21) we find that $w(x) = P_n^{(\alpha,\beta)}(1-2x)$. Therefore the normalized solution to (21)

$$\chi_n^{\alpha,\beta}(x) = N x^{\alpha/2}(1-x)^{\beta/2}P_n^{(\alpha,\beta)}(1-2x), \tag{22}$$

with eigenvalues

$$\nu^2 = \frac{1}{4}(\alpha+\beta+2n)(2+\alpha+\beta+2n), \tag{23}$$

and normalization

$$N = \sqrt{1+\alpha+\beta+2n}\,\sqrt{\frac{\Gamma(1+n)\Gamma(1+\alpha+\beta+n)}{\Gamma(1+\alpha+n)\Gamma(1+\beta+n)}}. \tag{24}$$

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