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Exotic states in a holographic theory *

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Abstract

Supersymmetric Light Front Holographic QCD is a holographic theory which predicts the existence of tetraquarks. The masses of those states, together with a discussion of its limitations are given.

Keywords: tetraquarks, exotic states, holographic models

1. Introduction

Light front holographic QCD[1–3] (LFHQCD) is a bottom-up holographic theory. Supersymmetric LFHQCD (supersymmetric LFHQCD) is a semiclassical model for hadron physics based on the equivalence [4] of a classical 5 dimensional Theory (AdS_5) and a superconformal 4-dimensional quantum field theory (AdS/CFT) and on a quantization in the light front (LF) frame. The breaking of the conformal symmetry, necessary for the generation of a discrete mass spectrum, is achieved by admitting that the resulting LF Hamiltonian is a linear combination of generators of the superconformal graded algebra [5, 6]. In this way the form of the interaction is completely fixed and in the limit of massless quarks it is determined by one constant λ, representing the QCD interaction. The spin dependence is taken from the tensor structure of the fields in the in the 5-dimensional classical theory [7], and the quark mass dependence from the LF Hamiltonian. In this way the number of parameters is minimal, like in lattice gauge theory. supersymmetric LFHQCD theory is not a supersymmetric field theory with squarks and gluinos, but a semi classical theory with supermultiplets of hadrons.

Supersymmetric LFHQCD describes not only hadron spectra but gives also insight in dynamical and structural properties of strong interactions [2]

2. Supersymmetric Light Front Holographic QCD

As mentioned above supersymmetric LFHQCD is based on broken superconformal quantum mechanics [6, 9, 10] The Hamiltonian (translation operator in time) H of superconformal quantum mechanics has the same form as that of the light front Hamiltonian derived from AdS_5. It is the anti-commutator of a fermionic operator Q, $H = \{Q, Q^\dagger\}$. Another fermionic operator S of the graded superconformal algebra is the "square root of the special conformal transformation" K : $K = \{S, S^\dagger\}$. In a two-component Hilbert-space $\mathcal{L}_2(\mathcal{R}_1) \oplus \mathcal{L}_2(\mathcal{R}_1)$, where the upper component is defined as the bosonic component and the lower one as

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the fermionic one, the two fermionic operators can be represented as: $Q = \begin{pmatrix} 0 & -\partial_x + \frac{f}{x} \\ 0 & 0 \end{pmatrix}, S = \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix}$. In supersymmetric LFHQCD a scale is introduced by changing the Hamiltonian H from $H = \{Q, Q^\dagger\}$ to

$$G = \{R_\lambda, R_\lambda^\dagger\} \text{ with } R = Q + \lambda S \tag{1}$$

The operators Q and S have different dimensions and therefore the linear combination of the two therefore must contain a dimensionful constant. This constant λ in 1 with the dimension of a squared mass is the scale of supersymmetric LFHQCD.

The resulting hadron mass spectra of the Hamiltonian $G = \{R_\lambda, R_\lambda^\dagger\}$ for Mesons and Baryons are:

$$\begin{aligned} M_M^2 &= 4\,\lambda\,(n + L + S/2) + \Delta(m_1^2, m_2^2) \\ M_B^2 &= 4\,\lambda\,(n + L + S/2 + 1) + \Delta(m_1^2, m_2^2, m_3^2) \end{aligned} \tag{2}$$

The parameters from a fit of all light hadron spectra come out to be:
$\sqrt{\lambda} = 0.523$ GeV and the masses for the up and down quark: $m_q = 0.046$ GeV, and for the strange quark $m_s = 0.357$ GeV

The analytical expressions for the term in eqs. (2) which describes the influence of the quark masses is [3, 10, 11, 13, 15]

$$\Delta M_n^2(m_1, \cdots m_n) = (-2\lambda^2)\frac{\partial}{\partial\lambda} \log K(\lambda) \text{ with}$$

$$K(\lambda) = \int_0^1 dx_1 \cdots dx_n \delta(x_1 + \cdots x_n - 1)\, e^{\frac{-1}{\lambda}\left(\frac{m_1^2}{x_1} + \cdots \frac{m_n^2}{x_n}\right)}$$

The holographic equivalence is based on a large number of colours [4] and therefore the accuracy in this $1/N_c \to \infty$ approximation is limited to approximately ± 120 MeV.

In Fig. 1 the value of the dynamical scale $\sqrt{\lambda}$ is shown, as determined from the different hadronic and mesonic channels; within the expected accuracy it clearly indicates the meson-baryon supersymmetry.

In Fig. 2 some leading meson and baryon trajectories are shown. The meson-baryon symmetry is evident. The lowest mesonic state, which in a chiral theory has mass 0, has no super-partner. In the supersymmetric theory it plays the role of a vacuum state [5]

For hadrons containing heavy quarks the conformal symmetry strongly broken, but due to the constraints imposed by supersymmetry [5] and heavy quark effective theory [14] the form of the interaction remains unchanged, ony the numerical value of the scale λ changes from the values for light quarks from $\sqrt{\lambda} = 0.523$ GeV for heavy quarks to a value

$$\sqrt{\lambda_Q} = C\,\sqrt{M_M} \tag{3}$$

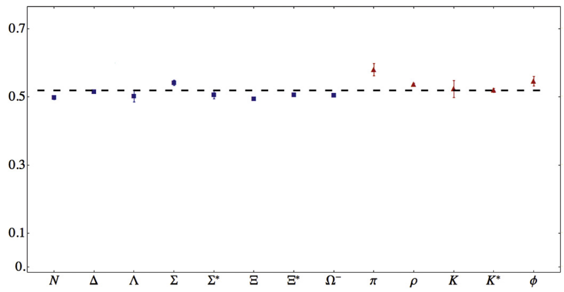

Figure 1: The dynamical scale $\sqrt{\lambda}$ as determined from different hadronic and mesonic channels

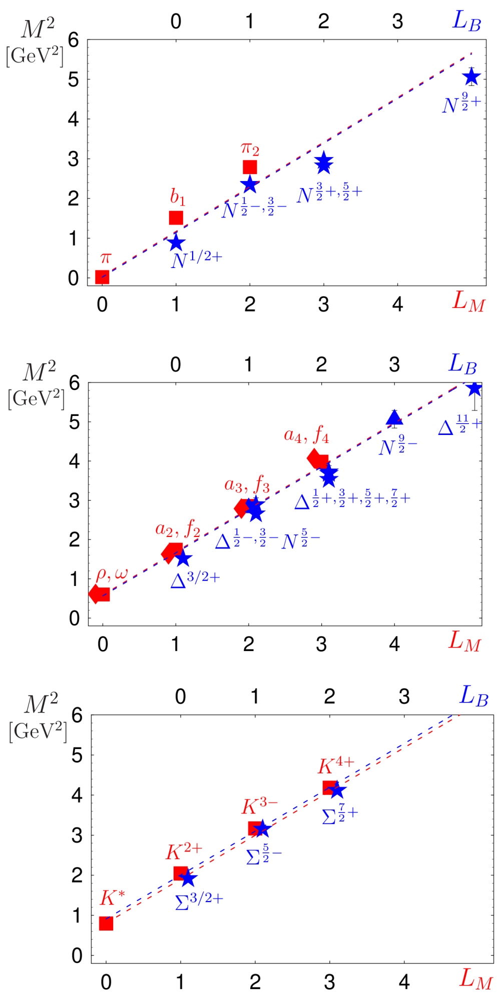

Figure 2: Leading light mesonic and baryonic trajectories; the dashed curves are the predictions of supersymmetric LFHQCD, eqs. 2

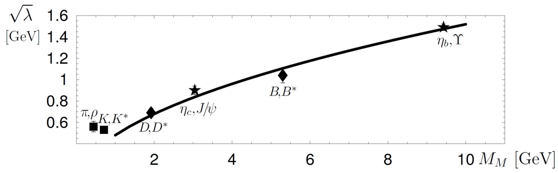

Figure 3: The value of the dynamical scale $\sqrt{\lambda}$ as determined from the lowest lying mesonic sates containing one or two heavy quarks. The solid line is the curve $\sqrt{\lambda_Q} = 0.49\,\sqrt{M_M}$ [GeV]

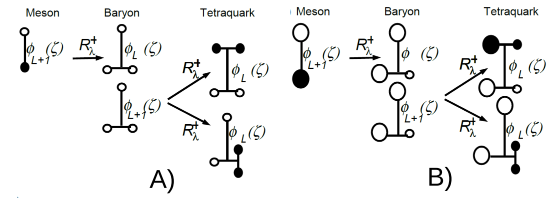

Figure 4: The interpretation of superconformal quantum mechanics in terms of LFHQCD. The operator $R_\lambda^\dagger$ transforms a quark (open circle) or an antiquark (filled circle) of the meson into pair of antiparticles of the same colour, that is a quark into an anti-diquark cluster with color 3 or an antiquark into a diquark cluster of color $\bar{3}$. In the same way a constituent of the baryon is transformed into a two-particle cluster of the same color. Therefore the resulting hadron is a tetraquark, consisting of a diquark - anti-diquark cluster

where $C \approx 0.49\,\sqrt{\text{GeV}}$ [15] and M_M is the mass of the lightest meson containing the heavy quarks, see Fig. 3.There are less data, but there is no experimental contradiction to the meson-baryon supersymmetry; Errors somewhat larger, due to uncertainty of λ_Q, a rough (optimistic) estimate is $\Delta M \approx 150$ MeV.

2.1. *Completion of the Supermultiplet*

Up to now we have supermultiplets of one meson and two baryons, one with positive and one with negative chirality: ϕ_{L_M} ; $\psi^+_{L_M-1}, \psi^-_{L_M}$ (note that in LF quantization the LF angular momentum of the negative chirality state is by one unit higher than that of the positive chirality state. The parity is determined by the LF angular momentum of the leading twist, that is positive chirality state [3]).

In a complete supermultiplet the number of bosons equals the number of fermions, we thus miss an additional boson. This is indeed included in the formalism [10, 13] and the mass of this additional states is given by supersymmetric LFHQCD. The fermionic operator R_λ, see (1), transforms a bosonic meson wave function with LF angular momentum L_M into a a fermionic baryon wave function with $L_B = L_M - 1$ and chirality + $R_\lambda : \phi_{L_M} = \psi^+_{L_M-1}$. It also transforms the negative chirality baryon wave function $\psi^+_{L_M-1}$ of a baryon which has LF angular momentum L_M into a a bosonic wave function with LF angular momentum $L_M - 1$:

Table 1: Predicted masses of light tetraquarks in supersymmetric LFHQCD. [...] indicated a diquark cluster in a total quark spin $S = 0$ state, (...) a cluster in a $S = 1$ state. All diquarks are assumed to be in the lowest possible state.

q-cont.	s	I	J^{PC}	$n+L$	Mass [GeV]	$n+L$	Mass [GeV]
$[\overline{ud}][ud]$	0	0	$L^{(-1)^L\,(-1)^L}$	0	1.10	1	1.52
$[\overline{ud}](qq)$	0	1	$(L+1)^{(-1)^L\,\pm}$	0	1.33	1	1.70
$[\overline{qs}][qs]$	0	0,1	$L^{(-1)^L\,(-1)^L}$	0	1.42	1	1.76
$[\overline{sq}](sq)$	0	0,1	$(L+1)^{(-1)^L\,\pm}$	0	1.60	1	1.91
$[\overline{ud}][sq]$	1	$\frac{1}{2}$	$L^{(-1)^L}$	0	1.23	1	1.61
$[\overline{ud}](sq)$	1	$\frac{1}{2}$	$(L+1)^{(-1)^L}$	0	1.43	1	1.77
$[\overline{sq}](qq)$	1	$\frac{1}{2}, \frac{3}{2}$	$(L+1)^{(-1)^L}$	0	1.43	1	1.77
$[\overline{sq}](ss)$	1	$\frac{1}{2}$	$(L+1)^{(-1)^L}$	0	1.60	1	1.91
$[\overline{ud}](ss)$	2	0	$(L+1)^{(-1)^L}$	0	1.60	1	1.91

$$R_\lambda \psi^+_{L_M} = \Phi_{L_M} \tag{4}$$

This additional bosonic state Φ_L completes the supersymmetric quadruplet of two mesons and two fermions which can be arranged into a matrix

$$\Psi = \begin{pmatrix} \phi_L & \psi^-_L \\ \psi^+_{L-1} & \Phi_{L-1} \end{pmatrix} \quad \text{with} \quad G \cdot \Psi = M^2\,\Psi \tag{5}$$

This additional bosonic state can in terms of LFHQCD be interpreted as a tetraquark, the argument is given in the capt of Fig. 4; it consists of a diquark with color 3 and an anti-diquark with color $\bar{3}$, the theorem of spin and statistics limits the possible states in LFHQCD, a diquark of identical quarks in the lowest lying state must be symmetric under the combined spin and isospin transformation, since it is antisymmetric in color, namely in a 3 od $\bar{3}$ state.

As mass of the tetraquark one obtains

$$M_T^2 = 4\,\lambda\,(n + L + S/2 + 1) + \Delta(m_1^2, m_2^2, m_3^2, m_4^2) \tag{6}$$

In Tab. 1 the lowest lying light tetraquarks are displayed, for hadrons with total isospin 0 additional possible non-perturbative effects have to be taken into account, see [16]. In accordance with the the spectra of mesons and baryons we expect an accuracy of $\approx$ 120 MeV. Possible candidates for a quadruplet are $a_2(1320)$ $\Delta(1232)$ $a_1(1260)$.

If the meson in the quadruplet consists of two heavy quarks, e.g. c and $\bar{c}$ we have the possible tetraquarks in the quadruplet, one with hidden [17, 18] or one with open [19] charm, see Fig. 4, B, where the large circles represent a charm or beautiful quark. In the first case, the heavy quark of the baryon is transformed in a heavy

Table 2: Some candidates for multiplets with hidden charm or hidden beauty

Hadron	q-cont.	n, L	J^{PC}	M_{theo} [GeV]	Cand.	Mass [GeV]
			$c\,\bar{c}$			
Tetraq.	$[\overline{cq}](cq)$	0, 0	1^{++}	3.87	$\chi_{c1}(3872)$	3.872
			1^{+-}	3.87	$Z_c(3900)$	3.886
Baryon	$(cq)c$	0, 0	$\frac{3}{2}^+$	3.76	Ξ_{cc} $\frac{1}{2}$ or $\frac{3}{2}$	3.621
Meson	$(\bar{c}\,c)$	0,1	2^{++}	3.67	$\chi_{c2}(1P)$	3.556
			$b\,\bar{b}$			
Tetraq.	$[\overline{bq}](bq)$	1, 0	1^{+}	10.65	$Z_b(10610)$	10.610
Baryon	$(bq)b$	1, 0	$\frac{3}{2}^+$	10.46	$\Xi_{bb}(?)$	?
Meson	$(\bar{b}\,b)$	1,1	2^{++}	10.34	$\chi_{b2}(2P)$	10.268

Table 3: Masses of hadrons containing two heavy quarks. The two last columns show the lightest strong decay channel and its threshold.

Mesons, $L_M = 1$			Baryons, $L_B = 0$			Tetraquark, $L_T = 0$				
q-cont	J^PC	name [MeV]	q-cont	J^P	name [MeV]	q-cont	J^P	$\approx M$ [MeV]	decay	thresh. [MeV]
$c\bar{s}$	1^+	$D_{s1}(2460)$	csq	$\frac{1}{2}^+$	$\Xi_c(2467)$	$cq\overline{sq}$	0^+	2550	$D_s\pi$	2048
$c\bar{s}$	2^+	$D^*_{s2}(2569)$	csq	$\frac{3}{2}^+$	$\Xi_c(2645)$	$cq\overline{sq}$	1^+	2730	$D^*_s\pi$..	2250
$c\bar{c}$	1^{+-}	$h_c(3525)$	ccq	$\frac{1}{2}^+$	$\Xi_{cc}(3614)$	$cq\overline{cq}$	0^+	3660	$\eta_c\pi\pi$..	3270
$c\bar{c}$	2^{++}	$\chi_{c2}(3565)$	ccq	$\frac{3}{2}^+$	$\Xi_{cc}(3770)$	$cc\overline{qq}$	1^+	3870	D^*D..	$3880^{(!)}$
$b\bar{b}$	1^{+-}	$h_b(9899)$	bbq	$\frac{1}{2}^+$	$\Xi_{bb}(9830)$	$bq\overline{bq}$	0^+	10020	$\eta_b\pi\pi$..	9680
$b\bar{b}$	2^{++}	$\chi_{b2}(9912)$	bbq	$\frac{3}{2}^+$	$\Xi_{bb}(10040)$	$bb\overline{qq}$	1^+	10230	B^*B..	$10800^{(!)}$
$b\bar{c}$	1^+	6550	bcq	$\frac{1}{2}^+$	6660	$bc\overline{qq}$	0^+	6810	BD..	$7150^{(!)}$

anti-diquark, in the latter case the light quark into a light diquark. Some of the new heavy bosons can in this way be explained as tetraquarks. In Tab. 2 some candidates for hidden charm or beauty are displayed.

For states with open beauty the mass of the lowest lying tetraquark could be below the threshold for a strong decay, as predicted by potential models [20]. As can be seen from Tab. 3, the existence of such states is also predicted by supersymmetric LFHQCD, namely a tetraquark with two b-quarks and one with a b and a c quark. The predicted masses are 570 and 340 MeV below the strong decay threshold, respectively. The tetraquark with open charm is predicted to be very near the threshold, so with the estimated uncertainty of ±180 MeV no conclusion can be drawn.

For tetraquarks containing four heavy quarks, not only conformal symmetry but also supersymmetry in the multiplets is strongly broken; the application of the formalism is therefore very questionable. Nevertheless, the mass formula (6) can be applied to those states without any additional free parameter if we insert in (3) for M_M the sum of the quark masses. The results are

Table 4: Tentative assignment of tetraquark masses according for four heavy quarks

	$cc\bar{c}\bar{c}$	$bb\bar{b}\bar{b}$	$cc\bar{b}\bar{b}$
L =0, S = 0	6.470	19.110	12.830
L =0, S = 1	6.680	19 340	13.060
L =1, S = 0 or L =0, S =2	6.880	19.570	13.280
L = 1, S =1	7.080	19.790	13.490

displayed in Tab. 4. It should be noted that spin and statistics demands that the lightest diquark cluster, consisting of quarks of the same flavor, must have total quark spin 1. Curiously the mass of the newly discovered $X(6900)$ [21] fits very well with the $L = 0, S = 2$ tetraquark state.

3. Final remarks and conclusions

There has been speculation on the existence of exotic quark states [22] since the establishment of the quark model. Supersymmetric light front holographic QCD predicts the existence of these states and its masses in dependence on the quantum numbers without introducing new parameters. The accuracy of the mass predictions is limited to ≈ 120 MeV for states composed of light quarks, and ≈ 180 for those containing heavy quarks. Although the mass values are given by the theory there remains, however, the problem of particle mixing, since the exotic states can mix with the conventional states without violating the Zweig-Iizuka rule. But also in more elaborate treatments, the results of supersymmetric LFHQCD could be a valuable input.

Whereas tetraquarks are a consequence of supersymmetric LFHQCD, pentaquarks are not; they are, however, not excluded. In supersymmetric LFHQCD the pentaquark could be the member of a new supermultiplet consisting of a tetraquark, a pentaquark with two chiralities, and a hexaquark.

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