# DAMA/LIBRA data: A Second Harmonic Analysis<sup>1</sup>

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The DAMA/LIBRA collaboration has recently published the final results of their phase 2 run. Combining this run with their prior two runs (DAMA/NaI and phase 1 runs, in total spanning  $\sim$  16 years), they find an annual modulation amplitude  $A_1 = 0.0102 \pm 0.0008$  cpd/kg/keV (a 12.9 $\sigma$  confidence level). There is significant tension between this result and that of other dark matter (DM) experiments, in particular those detectors based upon W and, more significantly, Xe. Employing the inelastic dark matter (iDM) scattering model of Smith and Weiner, a recent paper, based upon a proposed DM candidate <sub>m</sub>H (magnetic hydrogen), argues that this tension can be resolved in a certain region of the iDM parameter space. (The volume of this parameter space, afforded by uncertainties in cosmological parameters and the unknown masses of the constituents of mH, is actually quite large.) However, this solution (which would kinematically preclude all DM events in the W and Xe experiments) entails that the DAMA/LIBRA signal would be highly clipped (essentially no DM events in the winter), resulting in a rather large second harmonic amplitude. In this paper we further analyze the DAMA/LIBRA data (We derive our input data from their published figures.) in an effort to determine if the fit to the data is improved by including a second harmonic and what the amplitude of that harmonic might be. Combining all three of their data runs, we obtain an  $A_1$  in good agreement with their above noted published result (offering validation for our analytical approach) and find that there is some evidence for a non-zero second harmonic:  $A_2 = 0.0011 \pm 0.0009 \text{ cpd/kg/keV}$  (a confidence level of ~1.2 $\sigma$ ). Implications of this result for  $A_2$  are discussed.

Key Words: Dark matter, DAMA/LIBRA, magnetic hydrogen, magnetic monopoles

#### I. INTRODUCTION

The tension between the results of the DAMA/LIBRA (D/L) collaboration and the other direct dark matter (DM) searches has been growing for the last two decades. The DAMA/NaI experiment [1] reported (in 2000) an annual modulation signal amplitude at the  $4\sigma$  level. [This annual modulation is attributed to the earth's annual revolution about the sun, which, by adding to and then subtracting from the motion of the sun through the galaxy, imparts an annual velocity modulation to the (presumed) incoming DM particles (often labelled by the symbol  $\chi$ ) impinging on the earth and hence on the DM detector.] This D/L result was in conflict with a contemporaneous CDMS experiment, based upon Ge [2], which saw no DM signal. To offer a resolution to this tension, Smith and Weiner proposed an inelastic dark matter (iDM) scattering model [3], in which the

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incoming DM particle  $\chi$  would inelastically scatter into a nearby  $\chi^*$  final state, which would carry the same quantum numbers. This iDM model disfavors detectors employing lighter scattering nuclei. The DM mass  $m_{\chi}$  and the energy splitting  $\delta_{\rm E}$  (between  $\chi$  and  $\chi^*$ ) are adjustable input parameters in such an iDM analysis. Depending upon assumptions, Smith and Weiner found various regions of plausible solutions, specifically mentioning one with  $m_{\chi} = 70 \text{ GeV/c}^2$  and  $\delta_{\rm E} =$ 105 keV. Assumed halo properties, as well as other cosmological factors, also comprise inputs to this multi-parameter iDM space, which can be varied (within appropriate limits) in search of plausible solutions.

As time passed, however, DM experiments using liquid Xe have accumulated a considerable amount of data, significantly increasing the tension with the D/L result; because Xe is heavier than I, the iDM model as originally conceived<sup>2</sup> does not apply (Ge is lighter than I). In order to offer a possible solution to this increased tension, Chang, Lang, and Weiner [5] proposed that the active scattering nucleus in the D/L experiment, which uses NaI(Tl), would be the Tl nucleus: Tl is heavier than Xe. But they acknowledged that the Tl molar doping fraction (~10<sup>-3</sup>) could be a problem (because it would depress the expected D/L rate well below what might reasonably be expected).

Recently, the D/L collaboration published their final results for their phase 2 experimental data taking run [6]. Combining this result with their earlier two runs (DAMA/NaI [7] and DAMA/LIBRA phase 1 [8]), they obtain an annual modulation amplitude of  $A_1 = 0.0102 \pm 0.0008$  cpd/kg/keV (a 12.9 $\sigma$  confidence level). This result was for a detected signal in the range of 2-6 keV. Thus, the tension between the D/L and Xe results has become extreme. A recent Xe result [9], analyzing 1t × yr of data, reports no significant DM events above background, setting an upper limit of the elastic spin-independent WIMP–nucleon cross section  $S_{SI} = 4.1 \times 10^{-47}$  cm<sup>2</sup> for a  $\chi$  mass 30 GeV/ $c^2$ . This is about 5 orders of magnitude below that implied by the D/L modulation amplitude<sup>3</sup> (not even counting the Tl doping fraction of 10<sup>-3</sup>). But to date, no plausible non-DM explanation has been identified for the D/L result.

To search for a resolution to this tension, a recent paper [10] has proposed a new candidate for DM: Magnetic Hydrogen  $(_mH)$ .<sup>4</sup> In particular, Ref. [10] searched for a region of plausibility in the above mentioned multi-parameter space in which an inelastic collision of  $_mH$  with the Tl dopant of the NaI(Tl) in the D/L detector would excite  $_mH$  from an initial 1S state to a final 2S state. Assuming that  $_mH$  is a proper magnetic analogue to H, this would be a natural examplar of the iDM

<sup>&</sup>lt;sup>2</sup> It is relevant to note that since the I nucleus has a large magnetic moment relative to that of Xe, Chang, Weiner, and Yavin proposed a magnetic iDM (or MiDM) [4] in which the incoming  $\chi$  would inelastically scatter off of the nuclear magnetic moment in the DM detector. However, as a practical matter, the rate advantage that such a scattering would afford in support of the MiDM model is still insufficient to resolve the I–Xe tension.

<sup>&</sup>lt;sup>3</sup> The D/L collaboration actually views their result as "model independent." The conversion of their result to a WIMP–nucleon cross section is performed by others, e. g., authors of DM review papers.

 $<sup>{}^{4}</sup>$  mH is comprised of a magnetic electron me of unit magnetic charge 1*e* bound to a magnetic proton mp also of unit magnetic charge 1*e* (but of opposite polarity), analogous to the usual electrically bound H. The name "magneticon" has been proposed for these spin ½, magnetically charged fermions [10].

model as proposed in Ref. [5]. It is also convenient that the masses of the constituents of <sub>m</sub>H would nicely map (using the known physics of H) onto the  $m_{\chi}$  and  $\delta_{\rm E}$  of the iDM model. (It is relevant to remark that a large range of magneticon masses are within easy reach of the experiments at the LHC [12]. But a dedicated effort would be required to observe them.) These constituent masses and the cosmological parameters (in particular, the parameters of the putative DM halo) constitute the axes of the relevant multi-parameter iDM space.

The analysis in Ref. [10] used a semiclassical formulation for the inelastic <sub>m</sub>H–nuclear scattering cross section. Two criteria were set to define success in the search for a region of plausibility residing in this multi-parameter space: 1) that there would be sufficient events to explain the D/L results, and 2) that the null results of the other direct detection DM experiments could also be explained. Without going into the details of the analysis of Ref. [10], it was found that there was ample room in the parameter space to accommodate the observed D/L modulation amplitude. And, at the same time, there is a large region in which the kinematic requirements of inelastic scattering quite easily preclude the possibility that mH would scatter off of the Xe nucleus. (Tuning of the result can be accomplished by using the leeway afforded by the constituent magneticon masses and by the experimental uncertainties of the cosmological parameters; <sup>5</sup> Ref. [10] used  $m_{\rm me} = 12 \, {\rm GeV}/c^2$ and  $m_{\rm mp} = 106 \, {\rm GeV}/c^2$  as representative.) But to kinematically eliminate events of mH scattering off of the W nucleus (in the CRESST II experiment)<sup>6</sup> it was necessary to move to a location near the kinematic boundary of the parameter space, such that the D/L data would be significantly clipped (no events in the winter). The result of this clipping would be that the time waveform of the D/L signal would be expected to contain a large 2<sup>nd</sup> harmonic. While the D/L collaboration did perform a periodogram analysis<sup>7</sup> on their signal, such an analysis (being in the form of a power spectrum) is not as sensitive (it looks for all possible periodicities) as a direct Fourier analysis for a second harmonic amplitude would be. (A Fourier analysis for the 2<sup>nd</sup> harmonic asks only one question.) Hence, we were motivated to perform this (Fourier) analysis ourselves, using the published figures of the D/L results.<sup>8</sup> (For technical details, see Appendices A and B.)

#### **II. RESULTS**

In Table 1 we give our best fit values for the fundamental amplitude  $A_1 \pm S_1$  (and the value of  $\chi^2$ ) for our analysis in comparison to those of the D/L collaboration. (The second harmonic

<sup>&</sup>lt;sup>5</sup> All of the parameters need not be called into play, as there are significant statistical correlations between certain pairs, e. g., the DM escape velocity  $v_{esc}$  and the  $v_{min}$  for the iDM interaction [10].

<sup>&</sup>lt;sup>6</sup> The CRESST experiments, which report no DM events, measure two parameters for each event candidate: light output and heat output, defining their region of interest in terms of these two signal levels. There is a summary description of their data runs in Ref. [10]. In a more recent series of experiments, CRESST III has changed the focus of their research to lighter DM particles, for which the putative iDM D/L scattering events would be well outside their Region of Interest.

<sup>&</sup>lt;sup>7</sup> Periodogram analysis is discussed in Ref. [13].

<sup>&</sup>lt;sup>8</sup> While it would be preferable to use the actual D/L data, presumably due to the COVID-19 pandemic, the D/L collaboration has not responded to our queries about a possible second harmonic.

amplitude  $A_2$  was, of course, set equal to 0 for these fits.) All data sets are the D/L 2-6 keV data, except for the phase 2 (1-6) keV data set which is specifically labelled by superscript. We note here that based upon what we consider known physics, for all of these fits we used a fixed annual period of one year (365.25 days) with the crest occurring at day 152.5 [6].<sup>9</sup> Then, in addition to the fits to the 2-6 keV data sets, to afford the most statistically powerful fit, we also fit to the three D/L data runs using the 1-6 keV data set: NaI+p1+p2<sup>(1-6)</sup>.

Data set	$A_1 \pm S_1$	$\chi^2/d.o.f.$	$A_1 \pm S_1$	$\chi^2$ /d.o.f.
NaI	$0.0198 \pm 0.0032$	32.3/36	$0.0192 \pm 0.0031$	$[7]^{10}$
Phase 1	$0.0098 \pm 0.0013$	28.1/49	$0.0096 \pm 0.0013$	29.3/49 [8]
Phase 2	$0.0097 \pm 0.0012$	52.64/51	$0.0095 \pm 0.0011$	[6] <sup>11</sup>
Phase $2^{(1-6)}$	$0.0105 \pm 0.0011$	52.1/51	$0.0105 \pm 0.0011$	50.2/51 [6]
NaI+p1+p2	$0.0099 \pm 0.0008$	134/138	$0.0102 \pm 0.0008$	113.8/138 [6]
NaI+p1+p2 <sup>(1-6)</sup>	$0.0107 \pm 0.0008$	123.67/138	This fit not done.	

**Table 1.** Our fits (on left) for A<sub>1</sub> compared to those of the D/L collaboration (on right).

It is gratifying to see that our results are in good agreement with those of the D/L collaboration,<sup>12</sup> offering validation for our data development as described in Appendix A.

Data set	$A_2 \pm S_2$	$\chi^2$ /d.o.f.	$A_1 \pm S_1$	$\chi^2$ /d.o.f.
NaI	$0.0022 \pm 0.0041$	32.0/35	$0.0195 \pm 0.0032$	32.0/35
Phase 1	$0.0016 \pm 0.0031$	26.9/48	$0.0098 \pm 0.0013$	26.9/48
Phase 2	$0.00024 \pm 0.0012$	52.60/50	$0.0097 \pm 0.0012$	52.60/50
Phase $2^{(1-6)}$	$0.0008 \pm 0.0011$	51.6/50	$0.0105 \pm 0.0011$	51.6/50
NaI+p1+p2	$0.00096 \pm 0.0009$	132.9/137	$0.0099 \pm 0.0009$	132.9/137
NaI+p1+p2 <sup>(1-6)</sup>	$0.0011 \pm 0.0009$	122.04/137	$0.0107 \pm 0.0008$	122.04/137

**Table 2.** Our fits (on left) for  $A_2$  and then our fits for  $A_1$  (on right) using those  $A_2$  fits.

The values for  $A_1$  in Table 2 are to be compared to those given in Table 1, which used  $A_2 = 0$  as input. One can see that there is essentially no movement in the  $A_1$  fit when the deduced value of  $A_2$  is employed as input. (The statistical correlation between different harmonics, as revealed by the data fits, is expected to be small.)

<sup>&</sup>lt;sup>9</sup> One could also fit for the period and phase (The D/L collaboration does both cases – with and without fixing the period and phase.), but we believe that the physics argument to fix the period and phase is sound. In any case, the D/L collaboration fits show very little movement whether the period and phase are fixed or not fixed.

<sup>&</sup>lt;sup>10</sup> The tables given by the D/L collaboration in Ref. [7] don't include the minimum  $\chi^2$  value for their fits.

<sup>&</sup>lt;sup>11</sup> The tables given by the D/L collaboration in Ref. [6] don't include the minimum  $\chi^2$  values for this fit. <sup>12</sup> It is to be expected that some "noise" will be introduced by our using "hand derived" data from the D/L published figures.



Fig. 1. Data points with error bars and the best fit waveform for  $A_1$  and  $A_2$  as given in final row of Table 2.

### **III. SUMMARY AND CONCLUSIONS**

The work in this paper was undertaken to investigate the result found in Ref. [10], that postulated that <sub>m</sub>H, in the framework of the iDM model [5], might afford a plausible solution to the extreme tension between the D/L results and those of other direct detection DM experiments, in particular those using Xe, e. g., Ref. [9], or W (CRESST<sup>13</sup>). In Ref. [10], it was assumed that the D/L signal was the result of iDM scattering of <sub>m</sub>H off of a Tl nucleus in the NaI(Tl) of the D/L detector. In particular, this scattering would excite the <sub>m</sub>H from the atomic 1S state to the 2S state. (Given the assumptions of this specific calculation, there is insufficient energy in the center of momentum frame (with Tl scattering) to reach even the 3S state.)

With this iDM assumption for the D/L signal, there is a range of possible masses for the constituents of  $_{m}$ H (as well as a significant range of assumed halo parameters, given the experimental uncertainties) that kinematically precludes any iDM events associated

<sup>&</sup>lt;sup>13</sup> There are numerous CRESST publications that bear on this question. For a convenient list and some relevant discussion, see Appendix J in Ref. [10].

with mH-Xe scattering (exciting the mH into the 2S atomic state), satisfying both criteria above. ( $m_{\rm me} = 12 \,{\rm GeV/c^2}$  and  $m_{\rm mp} = 108 \,{\rm GeV/c^2}$  were chosen as representative.) Ref. [10] further explored this region in order to also preclude by the kinematics of the iDM model any such iDM scattering events associated with mH-W (W is also lighter than Tl). To achieve this kinematic exclusion for W, a satisfactory solution was found close to the kinematic boundary afforded by the iDM model. While the desired kinematic exclusion was achieved, it was found that the waveform of the D/L data would be severely clipped (on the order of 50%). That is, the CRESST detector would see no events at all, while the D/L detector would get their events for about one half of the year (essentially in the spring and summer). As a consequence, this particular solution entailed the prediction that there would be a large 2<sup>nd</sup> harmonic (~50% of the fundamental) in the D/L data.

The results of our analysis given in Table 1 are seen to be in good agreement with those of D/L, indicating that our analytical approach is sound. However, we see in Table 2 that our best fit for  $A_2$  is positive, but within ~1 $\sigma$  of a null result. (Our best fit value of  $A_2$  is ~10% of  $A_1$ , and, as would be expected, in phase.) At the same time, this result is ~4 $\sigma$  away from that associated with the solution found in Ref. [10] (~50% of  $A_1$ ). Hence, the conclusion here is that the solution proposed in Ref. [10] is highly unlikely (though not totally ruled out). However, we observe that there still is a considerable region in the multiparameter space that might, by exploiting a different D/L signal (but still in the framework of the iDM model) still satisfy, but with a higher probability, the two criteria cited above (an adequate explanation of the D/L rate and at the same time an explanation for the null results in the other direct detection experiments, in particular those employing Xe or W). It is our plan to explore these possibilities in a later paper.

#### **APPENDIX A. Data development**

The data were collected from downloaded pdf copies of the D/L papers. In particular, we used the bottom plot of Fig. 2 from "Final model independent result of DAMA/LIBRA–phase1," (Ref. [8]) for the Phase1 (2-6 keV) data; the plots in Fig. 2 and Fig. 3 from "First model independent results from DAMA/LIBRA–phase2," (Ref. [6]) for the Phase2 (respectively, the 1-6 and 2-6 keV) data, and finally, the bottom plot in Fig. 2 from "First results from DAMA/LIBRA and the combined results with DAMA/NaI," (Ref. [14]) for the NaI (2-6 keV) data.

The plot images were displayed on an LG 27UL600-W 4K UHD monitor for maximum screen resolution. We then used a screen capture program [HyperSnap 8 version 8.16.17 (64-bit)] to produce images of sections of the plots. HyperSnap 8 has the ability to exactly repeat a previous screen capture. This allowed us to first capture the vertical left-hand scale with maximum possible zoom and then, while maintaining this zoom level, sliding the plot image over the same capture region of the screen and capturing sequential sections of the plot ensuring the exact same vertical scale for each

plot capture. In addition, we independently screen captured the bottom scale of the plot and used the bottom day scale to identify the days of the maximum (dashed)<sup>14</sup> and minimum (dotted) lines in the waveform plot. We were then able to measure an offset to each data point with respect to either a maximum line or a minimum line thereby obtaining a day value for each data point. We also obtained the day bin width for each data point by measuring a scale factor from the day scale. The plot images were carefully printed out making sure the printing scale was a constant for all of the images. Using a 20 cm ruler (VANCO 282-20), we were then able to measure the vertical sigma values and horizontal bin widths as well as the vertical and horizontal location of each data point from the paper printouts. These results were then converted into a data spread sheet for use as input files for the MATLAB fitting program, described in Appendix B.

### **APPENDIX B. Fitting procedures**

Our fitting procedures to extract the first harmonic amplitude  $(A_1)$  and the second harmonic amplitude  $(A_2)$  from the D/L data follow the well-known method of least squares, or minimum  $\chi^2$ analysis.<sup>15</sup> As described in Appendix A, we have extracted from the several D/L data runs (as published in their figures) a set of N independent (residual) measurements  $y_i$  at points  $x_i$ , where  $x_i$ represent the center of the  $i^{th}$  data bin (in days from a first day). Associated with these  $y_i$  are the (assumed) standard deviations  $\sigma_i$  of those residuals, which are represented by the error bars on the  $y_i$  in the figures. The bin widths for each i varies, and are indicated by a notation (analogous to the error bars but) extending horizontally in both directions. We note that while the center of the bin should correspond to the sinusoidal amplitude at the central day, the fact that the data are collected in bins of finite width means that, due to the curvature of a sine wave (or cosine wave) a normalized mean number of counts will be somewhat diminished below the actual central value. This diminishment can be taken into account by use of a (sin x)/x diminishment function ( $D_i$ ), where xis  $\frac{1}{2}$  the total bin width.<sup>16</sup> It is evident that when the bins are narrow,  $D_i \Longrightarrow 1$ , while a bin halfwidth of  $\pi$  (covering a full sinusoidal cycle) yields  $D_i = 0$ , as required.

The above assertion re  $D_i$  can be easily demonstrated by integrating a cosine wave (or a sine wave) over an angular segment of full width  $\Delta \theta$ :

$$\frac{1}{\mathsf{D}q} \int_{q-\mathsf{D}q/2}^{q+\mathsf{D}q/2} \cos q dq = \frac{1}{\mathsf{D}q} \left[ \sin q \right]_{q-\mathsf{D}q/2}^{q+\mathsf{D}q/2} = \frac{1}{\mathsf{D}q} \left[ \sin(q+\mathsf{D}q/2) - \sin(q-\mathsf{D}q/2) \right] = \cos q \frac{\sin(\mathsf{D}q/2)}{(\mathsf{D}q/2)},$$
(B1)

where we have included the appropriate normalization factor  $(1/\Delta\theta)$  in front of the integral. Looking forward to our minimum  $\chi^2$ , in which  $x_i$  is in days, we write our diminishment factors as:

<sup>&</sup>lt;sup>14</sup> The D/L collaboration used  $t_0$  as day 152.5 (June 2<sup>nd</sup>) as this reference point.

<sup>&</sup>lt;sup>15</sup> A good exposition on the minimum  $\chi^2$  method (as well as other approaches to data analysis, and numerous references) can be found in Section 36, Statistics (by G. Cowan, p. 390) of Ref. [15].

<sup>&</sup>lt;sup>16</sup> The *x* of  $D_i$  is not to be confused with the  $x_i$  of the data points.

$$D_i^{(j)} = \frac{\sin(2\rho j \ hw_i / 365.25)}{2\rho j \ hw_i / 365.25} \quad \text{with } j = 1, 2 \text{ for } A_1 \text{ or } A_2.$$
(B2)

The bin half width  $hw_i$  is also in days, as obtained from the D/L figures for the  $i^{th}$  data point, as described in Appendix A.



Fig. 2. The  $D_i$  for the three (final) D/L data sets: DAMA/NaI, 37 points; D/L phase1, 50 points; and D/L Phase2, 52 points. ( $\overline{D}_i^{(1)} = 0.968$  and ( $\overline{D}_i^{(2)} = 0.877$ )

Including this refinement,<sup>17</sup> we write:

$$C^{2} = \bigotimes_{i=1}^{N} \frac{(y_{i} - F(x_{i}; A_{j})^{2})}{S_{i}^{2}},$$
(B3)

where

$$F = A_1 D_i^{(1)} \cos \frac{2\rho(x_i - d_0)}{365.25} + A_2 D_i^{(2)} \cos \frac{4\rho(x_i - d_0)}{365.25}$$
(B4)

<sup>&</sup>lt;sup>17</sup> We do not know if the D/L collaboration used this refinement, but it is evident that its effect is more important for the second harmonic than it is for the fundamental.

and  $d_0$  is the day of the cosine crest. It can be another parameter to be fit for, or, based upon the details of the earth's orbit around the sun, set to 152.5; we do the latter, but D/L does both.

As usual, the best fit for  $A_j$  is that value that minimizes the  $\chi^2$  function as given by Eq. (B3). And as prescribed in Ref. [15], the  $\sigma_j$  is found by finding that value of A at the point for which  $C_{1S}^2 = C_{\min}^2 + 1$ . For the purposes of illustration, we include Fig. 3 and Fig. 4 showing the  $\chi^2$  parabolas for  $A_1$  and  $A_2$ , also including the  $C_{1S}^2 = C_{\min}^2 + 1$  line for the determination of the standard deviations. We note that, as expected, the magnitude of these  $\sigma$  s are comparable.



Fig. 3.  $\chi^2$  parabola for  $A_1$  (with  $A_2 = 0$ ), and including the  $C_{1S}^2 = C_{\min}^2 + 1$  line (in red) for the determination of the standard deviation.



Fig. 4.  $\chi^2$  parabola for  $A_2$  (with  $A_1 = 0.0107$ ), and including the  $C_{1S}^2 = C_{\min}^2 + 1$  line (in red) for the determination of the standard deviation.

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