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PION-NUCLEON AMPLITUDES AROUND THE ZERO-MOMENTUM POINT*

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ABSTRACT

From an optimally stable extrapolation of π p elastic amplitudes, correctly containing all observed SU_2 non-invariances in the hadron spectrum, we determine $G^2/4\pi = 13.16$ and $\Sigma_{\pi N} = 40$ MeV. The first value confirms a previous, independent determination by Samaranayake and Woolcock, equally taking into account SU_2-symmetry violations; the much larger, widely accepted values for the second, around 60 MeV, are shown to be due both to the incorrect hadron spectrum and to the amplitude used in their derivation.

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A year ago we presented preliminary results of an extrapolation of πp elastic amplitudes to very low momenta [1]. Those results, from the Carleman weight function technique developed by Ciulli and co-workers [2,3], had to be optimally stable, i.e., their error bounds had to saturate the Nevanlinna lower bound [3].

We wish to discuss here an analysis of more accurate and exhaustive extrapolations of the same data [4,5], whose complete details will be presented elsewhere [6]. The method consists [2,3,6] of evaluating the integrals

$$\hat{I}_{j,n} = \left(2\pi i \mathscr{G}_j(0)\right)^{-1} \int_{\Gamma_1} z^{n-1} \hat{F}_j(z)(1+z)^{\beta_j} \mathscr{G}_j(z)\, dz \tag{1}$$

(n = 0 or 1) where $\Gamma = \Gamma_1 \cup \Gamma_2$ is the unit circle in the variable

$$z(\omega^2, \nu^2) = \left(\sqrt{\nu_t^2 - \omega^2} - \sqrt{\nu_t^2 - \nu^2}\right) \Big/ \left(\sqrt{\nu_t^2 - \omega^2} + \sqrt{\nu_t^2 - \nu^2}\right) \tag{2}$$

for real ω^2 (with the ν^2-plane cut from ν_t^2 to ∞), and $\hat{F}_j$ are the experimental histograms for the functions F_j, defined in Table I, on the portion Γ_1 of the cut.

The weight functions $\mathscr{G}_j$ are constructed, from the errors ϵ_j and the bounds M_j for the products $F_j(z)(1+z)^{\beta_j}$, so that $|\mathscr{G}_j|$ will be proportional to ϵ_j^{-1} on Γ_1 and to M_j^{-1} on Γ_2; using the Schwartz-Villat formula, we have [2,3,6]

$$\mathscr{G}_j(z) = \exp(2\pi i)^{-1}\left(\int_{\Gamma_1} \ell n(\lambda_j/\epsilon_j) \frac{\zeta+z}{\zeta-z} \frac{d\zeta}{\zeta} + \int_{\Gamma_2} \ell n(\lambda_j/M_j) \frac{\zeta+z}{\zeta-z} \frac{d\zeta}{\zeta}\right). \tag{3}$$

The Cauchy theorem in the z-plane unit circle Γ gives then the equalities, within the Nevanlinna bounds $\delta_j = \lambda_j / \mathscr{G}_j(0)$ for all n,

$$\rightarrow \hat{I}_{j,n} = \tilde{F}_j(\omega^2, t)\,\delta_{0,n} + \frac{G^2}{4\pi}\Bigg[d_j\,\delta_{0,n} + \\ + \frac{r_j}{\omega_B^2 - \omega^2}\left(\delta_{0,n} - z_B^{n'}(1+z_B)^{\beta_j}\left(\frac{\nu_t^2 - \omega^2}{\nu_t^2 - \omega_B^2}\right)^{1/2} \frac{\mathcal{G}_j(z_B)}{\mathcal{G}_j(0)}\right)\Bigg] , \tag{4}$$

where we have used the decomposition $F_j = F_j^{\text{Born}} + \tilde{F}_j$, and written

$$F_j^{\text{Born}}(\omega^2, t) = \frac{G^2}{4\pi}\left(d_j + \frac{r_j}{\omega_B^2 - \omega^2}\right) , \tag{5}$$

with ω_B^2 and z_B denoting the neutron pole position respectively in the ν^2 and in the z-planes.

The "Born-term stripped" amplitudes $\tilde{F}_j$ have been computed in the intervals $\mu^2/2 \geq \omega^2 \geq 0$ and $2\mu^2 \geq |t|$, retaining all known SU_2 non-invariant effects, but such evident second-order e.m. effects as the Coulomb corrections and the radiative capture channel, which are explicitly subtracted; we have then kept non-zero mass splittings both in the $\Delta(3/2^+)$ complex [4] and in the nucleon doublet (affecting deeply the expressions for F_j^{Born}) as well as the $\pi^- p$ unphysical range from the charge-exchange channel [7].

The n=1 integrals for the $B^{(+)}/\omega$ amplitude yield the value for the πN coupling constant

$$G^2/4\pi = 13.161 \pm 0.137 \tag{6}$$

much smaller than usually quoted values [8], confirming the only previous estimate including SU_2 violations [7]; since these latter are now clearly evident in the physical region [4], we feel that the "traditional" value for G has to be reconsidered.

The $n=0$ integrals, corrected according to (4), give the "Born-term stripped" amplitudes [9] whose values can be found tabulated in ref. [6]; here we shall parametrize these results with simple one-particle, on-mass-shell exchanges [8], normalized to the zero-momentum theorems [10]; these allow a reduction of the number of free parameters, with respect to two-variable polynomial expansions, once we use our information on πp resonance parameters and such phenomenological ideas as vector-meson dominance (VMD).

To fix the normalizations, we can use current algebra and PCAC at zero pion momenta [10]

$$\lim_{q_1^2,\, q_2^2,\, \omega^2 \to 0} \tilde{A}^{(+)}\left(\omega^2, t=0; q_1^2, q_2^2\right) = -2\Sigma_{\pi N}/f_\pi^2 , \tag{7a}$$

$$\lim_{q_1^2,\, q_2^2,\, \omega^2 \to 0} \tilde{A}^{(-)}\left(\omega, t=0; q_1^2, q_2^2\right)/\omega = 1/f_\pi^2 , \tag{7b}$$

$$\lim_{q_1^2,\, q_2^2,\, \omega^2 \to 0} \tilde{B}^{(-)}\left(\omega^2, t=0; q_1^2, q_2^2\right) = 0 \ ; \tag{7c}$$

moving away from the point $q_1^2 = q_2^2 = \omega^2 = t = 0$ along the line $\omega^2 = 0$, $t = q_1^2 + q_2^2$, $q_1^2 = q_2^2$ we can avoid divergencies connected with s and u-channel discontinuities and write a simple phase representation for $F_j(\omega^2=0, t) = F_j(t)$

$$F_j(t) = F_j(0) \prod_z \left(\frac{t_z - t}{t_z}\right) \exp\left(\frac{t}{\pi} \int_{4\mu^2}^{\infty} \frac{\phi_j(s)\, ds}{s(s-t)}\right) =$$

$$= \pm F_j(0) \left(1 + \epsilon_j(t)\right) \tag{8}$$

where a minus sign enters only for $A^{(+)}$ and $C^{(+)}$, which both have an Adler zero, and ~~we~~ expect, at least for $t \lesssim 4\mu^2$, $\epsilon_j \simeq 0\,(t/1\ \mathrm{GeV}^2)$. Then the zero-momentum theorems become, on the mass-shell,

$$\tilde{A}^{(+)}(0,\, 2\mu^2) = 2\,\Sigma_{\pi N}(1+\epsilon_+)/f_\pi^2 + G^2/m_p \tag{9a}$$

$$\tilde{C}^{(+)}(0,\, 2\mu^2) = 2\,\Sigma_{\pi N}(1+\epsilon_+)/f_\pi^2 \tag{9b}$$

$$\lim_{\omega\to 0} \tilde{C}^{(-)}(\omega,\, 2\mu^2)/\omega = (1+\epsilon_-)/f_\pi^2 - 2\,G^2/(m_n+m_p)^2; \tag{9c}$$

all other amplitudes are proportional to the ratio $\epsilon_j/(m_n - m_p)^{n_j}$, with $n_j \geq 1$, and therefore hard to predict, due to our poor knowledge of the numerator and the smallness of the denominator.

Assuming the Adler zero to lie at $t = q_1^2 + q_2^2 \simeq \mu^2$, regardless of the momenta, and neglecting far-away zeros, we can estimate the corrections $\epsilon_\pm$ at the lowest order in μ^2 from Watson's theorem and $\pi\pi$ S and P waves [11] as $\epsilon_+ \simeq 0.101$ and $\epsilon_- \simeq 0.069$, with a $\sim 10\%$ "theoretical" uncertainty.

We have then only three free parameters to normalize our amplitudes at $\omega^2 = 0$ and $t = 2\mu^2$, the so-called Cheng-Dashen-Weinstein (CDW) point [12].

We then assume their ω^2 and t dependence to be described by low-mass exchanges in the s, u, and t-channels. Limiting ourselves to masses $M^2 \lesssim 2\ \mathrm{GeV}^2$, we can expect only the exchanges listed in table 2 to contribute appreciably to these dependences.

The s and u-channel exchanges $\Delta(3/2^+)$ and $N^*(1/2^+)$ can be evaluated in a narrow-width approximation [8] from resonance parameters, including the observed SU_2 non-invariances [4] as $M(\Delta^{++}) \neq M(\Delta^0)$ and $G(\Delta^{++}p\pi^+) \neq G(\Delta^0 p\pi^-)$, determined directly from our input [4,5]. Once their contributions (we shall always conveniently normalize them to zero at the CDW point) are subtracted, we expect that no appreciable ω^2 dependence should remain in the C-odd amplitudes and in $\hat{B}^{(+)}/\omega$ (indicating with a "hat" the subtraction of the normalized Δ and N* contributions), and this is confirmed, within errors, by our extrapolations.

The only appreciable t-channel exchange in C-odd amplitudes remains the $\rho(1^{--})$ meson, and we use VMD to fix its couplings as [8]

$$G_{\rho\pi\pi} = G^V_{\rho N\bar{N}} = f_\rho \ , \quad G^T_{\rho N\bar{N}} = (\mu_p - \mu_n) f_\rho \tag{10}$$

and all C-odd amplitudes reduce to a single free parameter, $\tilde{B}^{(-)}(0, 2\mu^2)$, which we have determined with a fit at $t = 2\mu^2$ only, allowing then checks of both the Adler-Weisberger relation (9c) and of VMD (10).

C-even amplitudes $\hat{A}^{(+)}$, $\hat{B}^{(+)}/\omega$, and $\hat{C}^{(+)}$ should then be described by $\epsilon(0^{++})$ and $f(2^{++})$ exchanges [13] in the t channel; the second should account for both the residual ω^2 dependence in $\hat{A}^{(+)}$ and $\hat{C}^{(+)}$ and the t dependence in $\hat{B}^{(+)}/\omega$, leaving the ϵ exchange to give the t dependence in $\hat{A}^{(+)}(0,t)$ and $\hat{C}^{(+)}(0,t)$.

Such a six-parameter parametrization agrees rather well with our extrapolations but for $\tilde{C}^{(-)}/\omega$, where a strange hump at $t \lesssim 0$ had us worried for quite a while. However, the appearance of a similar, smaller feature in $\hat{C}^{(+)}$ and the position of the two have allowed us to track them back to a probable under-estimate of the errors on the S_{11} wave in ref. [4]. Indeed such effects are washed out by an increase in these errors, but due to the arbitrarity of such a correction, we choose to present our results without further tampering.

A very interesting result can be derived from the zero-momentum theorem (9b) with our estimate of ϵ_+, i.e., the value for the sigma term

$$\Sigma_{\pi N} = 40 \pm 9 \text{ MeV} \quad . \tag{11}$$

This value being about 20 MeV below the usually quoted values [8], the reader may wonder at the source of such a huge discrepancy. This source becomes, however, clear once one notes that those values were obtained in the limit $m_n = m_p$ from the non-spin-flip amplitude $T^{(+)}$, for which, in this limit, we have

$$\tilde{C}^{(+)}\left(0, 2\mu^2, m_n = m_p\right) = \tilde{T}^{(+)}\left(0, 2\mu^2, m_n = m_p\right) + \frac{G^2}{m_p} \frac{\mu^2}{2m_p^2 - \mu^2}, \quad (12)$$

instead of the exact answer $\tilde{C}^{(+)}(0, 2\mu^2) = \tilde{T}^{(+)}(0, 2\mu^2)$, obtained keeping track of the n-p mass difference. This introduces a spurious correction, i.e., to take a specific case,

$$(1 + \epsilon_+) \cdot \left[\Sigma_{\pi N}(m_n = m_p) - \Sigma_{\pi N}\right] = 19 \text{ MeV} \quad (13)$$

for Langbein's value [14] $G^2/4\pi = 14.3$, bringing our value (11) to an "on-mass-shell, SU_2-symmetric" value $(1 + \epsilon_+) \cdot \Sigma_{\pi N}(m_n = m_p) = 63 \pm 10$ MeV, to be compared with his estimate [14] of 61 ± 16 MeV.

We hope that what we have here called $\Sigma_{\pi N}(m_n = m_p)$ will no longer be confused with the true sigma term $\Sigma_{\pi N}$ appearing in the theorem (7a), ending for good the speculations about its "abnormally large" value, while the new value for G should end those on an equally "abnormally large" correction ϵ_{GT} to the Goldberger-Treiman relation [16]. Indeed, our value leads to

$$\epsilon_{GT} = 1 + g_A(0)\,(m_n + m_p)/(\sqrt{2}\, f_\pi\, G) = (1.3 \pm 1.1) \times 10^{-2}, \quad (14)$$

consistent with estimates of the 3π discontinuity for the axial divergence form factor, $\epsilon_{GT} \sim 0(\mu^2/1 \text{ GeV}^2)$, and a world almost symmetric under chiral $SU_2 \times SU_2$.

The same cannot be said for chiral $SU_3 \times SU_3$; indeed, in terms of such a symmetry broken by a $(3, \bar{3})$ piece in the Hamiltonian [17], our value (11) corresponds to a contribution by its unitary-singlet piece of as much as 1/3 to the average baryon-octet mass. We shall then expect no particular success from perturbations around a chiral $SU_3 \times SU_3$ symmetry limit; however, our value is not so far from expectations in such a model [18] to upset our belief in generalized, octet PCAC. Indeed, kaon PCAC results in a $(3,\bar{3})$ model are not as far from physical reality [19] as the big naive scale $\epsilon_8/\epsilon_0 \simeq -1.25$ would lead us to believe. Whether this success is connected to the existence of a much larger "basic scale," set by some underlying, even worse broken, chiral $SU_n \times SU_n$ $(n \geq 4)$ symmetry, or to an as yet unforeseen dynamical accident, it is beyond our purpose and our means to speculate.

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Table 1

j	F_j	j	r_j	d_j
1	$A^{(+)}$	3/2	$-\frac{4\pi\omega_B(m_n - m_p)}{m_p}$	$-\frac{4\pi(m_n - m_p}{m_n + m_p}$
2	$B^{(+)}/\omega$	-1/2	$4\pi/m_p$	0
3	$A^{(-)}/\omega$	0	$-\frac{4\pi(m_n - m_p)}{m_p}$	0
4	$B^{(-)}$	0	$4\pi\omega_B/m_p$	0
5	$C^{(+)}$	3/2	$-\frac{4\pi\omega_B(m_n - m_p - \omega_B)}{m_p}$	$-\frac{4\pi(m_n - m_p)}{m_n + m_p}$
6	$C^{(-)}/\omega$	0	$-\frac{4\pi(m_n - m_p - \omega_B)}{m_p}$	0

Table 2

Channel(s)	Exchange(J^{PC})	Mass (MeV)	Coupling Constant(s)
s, u	$\Delta^{+}(3/2^{+})$	$\underline{1231.1}$	$G(\Delta^{++}p\pi^{+})^{2}/4\pi = \underline{14.9}$
s, u	$\Delta^{0}(3/2^{+})$	$\underline{1232.5}$	$G(\Delta^{0}p\pi^{-})^{2}/4\pi = \underline{15.3}$
s, u	$N^{*0}(1/2^{+})$	$\underline{1466}$	$G(N^{*}N\pi)^{2}/4\pi = \underline{2.06}$
t	$\rho(1^{--})$	$\underline{772.3}$	$G_{\rho\pi\pi}Q^{V}_{\rho N\bar{N}}/4\pi = \underline{2.26}$
			$G_{\rho\pi\pi}G^{T}_{\rho N\bar{N}}/4\pi = \underline{10.6}$
t	$\epsilon(0^{++})$	$\underline{993}$	$G_{\epsilon\pi\pi}G_{\epsilon N\bar{N}}/4\pi = 28.7 \pm 10.5$
t	$f(2^{++})$	$\underline{1271}$	$G_{f\pi\pi}G^{(1)}_{fN\bar{N}}/4\pi = .24 \pm .64$
			$G_{f\pi\pi}G^{(2)}_{fN\bar{N}}/4\pi = -.39 \pm .64$

Amplitudes	Free Normalizations (from $t = 2\mu^2$ only)
$\lim_{\omega\to 0} \tilde{B}^{(+)}(\omega, 2\mu^{2})/\omega$	-3.107 ± 0.206
$\tilde{C}^{(+)}(0, 2\mu^{2})$	0.719 ± 0.297
$\tilde{B}^{(-)}(0, 2\mu^{2})$	8.643 ± 0.229

All underlined parameters have been fixed during our fits.