

CP VIOLATION, RADIATIVE SYMMETRY BREAKING EFFECTS,
AND HIGGS AND FERMION MASSES*

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ABSTRACT

Existence of CP violation is shown to require a bound on a Higgs particle mass in several examples. A lower bound which depends on the fermion mass is derived from one-loop approximation in an Abelian theory. Two $SU(2) \times U(1)$ examples are studied. The first produces radiative CP violation by electron and muon loops when a neutral Higgs mass is bounded by limits determined by electron and muon masses. The CP violating vacuum phase θ is bounded by $\tan \theta \leq 2 \frac{M_e}{M_\mu} / \left(1 - \left(\frac{M_e}{M_\mu}\right)^2\right)$. The usual four quarks couple to the two Higgs doublets as in the scheme proposed by Weinberg, and CP is violated by charged Higgs exchange. No third Higgs doublet is needed, but the bound on one neutral Higgs mass is so low that unacceptably large effects arise in thermal neutron-electron scattering. A second scheme with spontaneous P and CP violation by fermion loops, where heavy leptons and super heavy quarks are included, produces a light, neutral Higgs mass which is estimated to be 300 MeV. Higgs effects are within experimental limits. The vacuum phase angle θ is bounded by light to heavy fermion mass ratios, $M_f/M_{f'}$, according to $\tan \theta \leq 2 \frac{M_f}{M_{f'}} / \left(1 - \left(\frac{M_f}{M_{f'}}\right)^2\right)$.

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I. INTRODUCTION

The problem of incorporating CP violation in gauge theories is to show that the smallness of the effect is plausible. The two basic approaches loosely divide into explicit CP violation and spontaneous CP violation. Believability of any gauge scheme depends both on how well it accommodates existing data and on how tightly it relates CP violation parameters to other small parameters in such a way that the CP effects are unavoidably small.

The arguments which work so well in understanding the suppression of $\Delta S=1$ neutral current effects³ have recently been extended by B. W. Lee to the suppression of CP violation⁴ and muon and electron number nonconservation.⁵ These latter arguments apply to cases where the Lagrangian is not required to be CP conserving (or electron number and muon number conserving). The trick is to introduce conditions on the multiplet choices for a given gauge group in such a way that dangerous gauge field couplings are forbidden in lowest order and cancellations occur in higher orders in a natural³ way. As usual, suppression depends on having a small quark to gauge boson mass ratio, $M_q^2/M_W^2 \ll 1$. The CP violation experimentally observed in the kaon system fixes the arbitrary CP-violating phase or phases in such an approach.

In the spontaneously broken CP schemes,² the suppression is plausible to the extent that Higgs boson masses, M_H , are believed to be much larger than quark and lepton masses, M_f , since CP violation occurs predominantly by Higgs exchange and the violation amplitudes are suppressed by factors of M_f^2/M_H^2 . The division between explicit and spontaneous CP violation is a loose one because examples such as the one discussed by Weinberg⁶ do not require CP invariance of the Lagrangian, but the quark sector multiplets and coupling to scalar bosons are nonetheless so restricted that the only appreciable CP violation effects occur

via charged Higgs exchange, which looks more like the consequence of spontaneously broken CP theories. In the framework of spontaneously broken P and CP symmetry, another form of suppression on CP violating effects has been shown to occur in one vector-like model because of the relationship between the spontaneously generated CP violation angle and the ratio of light to heavy fermion masses.^{7,8} In this latter case, the angle is required to be small if the mass ratios are small. This CP angle limitation is an extra suppression in addition to the one which occurs because of M_f^2/M_H^2 mass ratios. In this approach the CP violating phase is not arbitrary, but is related to ratios of light to heavy quark and/or lepton masses.

In the work referred to in the above remarks, the nature of the vacuum and the Higgs mass matrix is presumed to be decided in the tree approximation. The attractive idea that CP violation might be decided only after radiative effects are included⁹ and be therefore small, has been considered with rather discouraging results by Georgi and Pais,¹⁰ who studied radiative perturbations to a CP conserving or CP indeterminate tree-approximation vacuum. They showed that if the CP character of the vacuum is undetermined at the tree approximation level, and if there is a zero-mass scalar field at that level which gains mass due to radiative effects (the accidental symmetry situation), then it is possible that CP violation too will appear when radiative (loop) effects are included. The contrived examples given in Ref. 10 demonstrate how difficult it is to construct a realistic gauge theory with accidental symmetry which produces CP violation only after radiative perturbations are included.

A hint of an intermediate point of view is found in an $SU(2) \times U(1)$ model with two complex Higgs doublets when the one quartic coupling term in the classical potential which could contain a CP phase is set equal to zero so that fermion

loop terms are necessary to decide the CP question.⁸ This is reminiscent of the Coleman-Weinberg^{11, 12} demonstration that gauge theories in which scalar mass terms are set equal to zero exhibit spontaneous symmetry breakdown that can only be discovered when radiative gauge-boson loops are included. In Appendix B of Coleman and Weinberg's paper it is shown that spontaneous breakdown occurs even when a small, positive mass term is present in the Higgs model. Recently Linde^{13, 14} and Weinberg¹⁵ have shown by several examples that the question of stable, broken-symmetry solutions to gauge theories can depend upon the ratio of Higgs to gauge boson masses in the theory or, equivalently, the ratio of quartic scalar self coupling to gauge coupling. The relevance to the Georgi-Pais¹⁰ observations about radiative CP violation is that for a range of parameters of the Lagrangian, some of the radiative effects will be comparable in magnitude to some of the terms in the classical potential. The arguments based on perturbative shifts to the vacuum potential must be modified. One expands about an effective potential which includes loop effects of the same size as tree approximation terms. Linde¹⁴ argues that the question of whether or not such effects are dynamical in origin is terminological, since one can change the renormalization conditions so that Coleman and Weinberg's result in the Higgs model is obtained if $\lambda = 3e^4/16\pi^2$ even though the Lagrangian has a negative scalar mass term. The result is nontrivial nonetheless, since the nature of the vacuum symmetry is not decided merely by rigging the form of the classic potential to look as if the symmetry is broken "by definition".

In this paper I select several models for which the above considerations apply to CP violation. In Section II a simple Abelian model studied by T. D. Lee,² but with an extra interchange symmetry on the complex Higgs scalars, is shown to break the gauge symmetry and CP symmetry only when one Higgs scalar mass

is larger than a value determined by the lepton mass, which enters through lepton-loop corrections to the effective potential. This Abelian model exhibits CP (and gauge) symmetry violation in the tree approximation alone when the Higgs mass is large. This is not true in the more realistic $SU(2) \times U(1)$ example studied in Section III, where it is only the electron and muon loop effects which lead to CP-violation when one of the quartic scalar couplings is of order g^4 , where g is a lepton-Higgs Yukawa coupling. It is shown in Section III that in the model presented by Weinberg,⁶ this mechanism can supply the CP violation for quark interactions via the charged Higgs propagator without introducing extra Higgs doublets decoupled from quarks. Not surprisingly, one of the neutral Higgs masses for this range of parameters of the theory is very small, $O(M_e)$. Such a small mass for a neutral scalar can almost certainly be excluded by experiment.¹⁶ In Section IV I consider a spontaneous P and CP violating model with heavy quarks and leptons⁷ which set a larger bound on the neutral scalar mass, and it is plausible that this Higgs scalar has escaped detection. The models of Sections III and IV reveal interesting relationships between the CP violating phase, fermion mass ratios, and the mass of a light neutral Higgs boson. Results are summarized and conclusions drawn in Section V. Two appendices contain details of mass bounds and Higgs couplings.

II. AN ABELIAN EXAMPLE WITH TWO COMPLEX FIELDS

In this section a simple Abelian gauge theory example is used to illustrate the role that fermion loops can play in spontaneous CP violation. The idea, as described in the introduction, is to study the effective potential for that range of parameters where loop contributions are necessary to determine the symmetry of the vacuum.¹¹ The system is made of two complex spin-zero field ϕ_1 and ϕ_2 , a massless gauge field B_μ , a massless left-handed charged fermion ℓ_L and a

massless, neutral right-handed fermion ℓ_R .² For simplicity, an exchange symmetry between ϕ_1 and ϕ_2 is assumed. Time reversal, but not parity, is taken to be a good symmetry of the Lagrangian.

The Lagrangian density of the system is expressed as

$$\mathcal{L} = -g(\phi_1 + \phi_2)\bar{\ell}\ell_R + \text{H. c.} - V(\phi) + \mathcal{L}(\phi, B, f) + \mathcal{L}(\ell, B, f) \quad (2.1)$$

where

$$\begin{aligned} -V(\phi) = & \mu^2(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) - \lambda\left[(\phi_1^\dagger\phi_1)^2 + (\phi_2^\dagger\phi_2)^2\right] - 2\sigma(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \\ & - \frac{\delta}{2}\left[(\phi_1^\dagger\phi_2)^2 + (\phi_2^\dagger\phi_1)^2\right] - \frac{\epsilon}{2}(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1)(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) \quad . \end{aligned}$$

$\mathcal{L}(\phi, B, f)$ and $\mathcal{L}(\ell, B, f)$ represent standard gauge-field terms with gauge coupling f whose details are unnecessary here.

The classical potential $V(\phi)$ supports a CP violating vacuum

$$\langle\phi_1\rangle = \frac{\rho_1 e^{i\theta}}{\sqrt{2}} \quad , \quad \langle\phi_2\rangle = \frac{\rho_2 e^{i\theta}}{\sqrt{2}} \quad (2.2)$$

which will have $\rho_1 = \rho_2 = \rho$ and $\theta \neq 0$ when $\mu^2 > 0$, $|\frac{\epsilon}{2}| < 1$, $\lambda - \frac{\delta^2}{8\epsilon} > \frac{1}{2}(\sigma - \delta - \frac{\epsilon}{4\delta}) > 0$.

Let us now consider the effects which arise when δ and ϵ are of order g^4 , so that fermion loop corrections must be included in order to decide the question of CP violation, which originates in the vacuum phase angle θ .² The fermion 1-loop corrections, with renormalization defined at nonzero but otherwise arbitrary scalar field values,¹¹ are sufficient for the purposes of this paper. By suitably defining a mass parameter M (not a renormalization mass), the $\frac{\delta}{2}(\phi_1^\dagger\phi_2)^2 + \text{H. c.}$ term can be absorbed in the g^4 fermion term of the effective

potential V_{eff} , which I write as

$$\begin{aligned} -V(\phi_1, \phi_2) = & \mu^2(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) - \lambda[(\phi_1^\dagger\phi_1)^2 + (\phi_2^\dagger\phi_2)^2] - 2\sigma(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \\ & - \frac{\epsilon}{2}(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1)(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) + \frac{1}{16\pi^2} M_\ell^4 \ln(M_\ell^2/M^2) \\ & + \dots \end{aligned} \quad (2.3)$$

where $M_\ell^2 = g^2(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 + \phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1)$ and the dots indicate θ -independent 1-loop terms and higher loop corrections.

It is sufficient to study the dependence of V on ρ_1 , ρ_2 and θ , defined in (2.2), and $V(\rho_1, \rho_2, \theta)$ is expressed as

$$\begin{aligned} -V(\rho_1, \rho_2, \theta) = & \frac{\mu^2}{2}(\rho_1^2 + \rho_2^2) - \frac{\lambda}{4}(\rho_1^4 + \rho_2^4) - \frac{\sigma}{2}\rho_1^2\rho_2^2 - \frac{\epsilon}{4}(\rho_1^2 + \rho_2^2)\rho_1\rho_2 \cos \theta \\ & + \frac{1}{16\pi^2} \frac{g^4}{4}(\rho_1^2 + \rho_2^2 + 2\rho_1\rho_2 \cos \theta) \ln \left[\frac{g^2(\rho_1^2 + \rho_2^2 + 2\rho_1\rho_2 \cos \theta)}{M^2} \right]. \end{aligned} \quad (2.4)$$

The conditions that $V(\rho_1, \rho_2, \theta)$ be at an extremum are satisfied when

$$\begin{aligned} \rho_1 = \rho_2 = \rho \quad , \\ \epsilon = \frac{g^2}{4\pi^2} \frac{M_\ell^2}{\rho^2} \left(\ln \left[\frac{M_\ell^2(\rho, \theta)}{M^2} \right] + \frac{1}{2} \right) \quad , \end{aligned} \quad (2.5)$$

and

$$\frac{\mu^2}{\rho} - \lambda - \sigma = -\frac{1}{2}(1 - \cos \theta)\epsilon \quad .$$

Here the fermion and gauge-boson masses, acquired in the spontaneously broken solution, are

$$M_\ell^2 = g^2 \rho^2 (1 + \cos \theta) \quad (2.6)$$

and

$$M_B^2 = f^2 \rho^2 \quad .$$

In order that the extremum defined by Eq. (2.4) be a stable minimum, it is necessary that

$$\frac{\partial^2 V}{\partial \theta^2} = - \frac{(1-\cos \theta)}{8\pi^2} g^2 \rho^2 M_\ell^2 \left(\ln \frac{M_\ell^2}{M^2} + \frac{3}{2} \right) > 0 \quad (2.7)$$

and that

$$V(\rho, \rho, \theta) = - \frac{\mu^2}{2} \rho^2 + \frac{g^2 \rho^2}{8\pi^2} M_\ell^2 < 0 \quad (2.8)$$

to the 1-loop level of approximation to which we are working. The masses of the three Higgs scalars at this level are given by

$$\begin{aligned} M_1^2 &= 2\mu^2 + \mathcal{O}(g^4) \\ M_2^2 &= 4\lambda \rho^2 - 2\mu^2 + \mathcal{O}(g^4) \end{aligned} \quad (2.9)$$

and

$$M_3^2 = (1-\cos \theta) g^2 f^2 \frac{M_\ell^2}{M_B^2} \left(\ln \frac{M_\ell^2}{M_\ell^2} - \frac{3}{2} \right)$$

Equations (2.7)-(2.9) illustrate the basic point that the presence or not of CP violation depends upon relationships between the fermion mass and other mass parameters of the theory. Equation (2.7) shows that a $\theta \neq 0$, CP violating minimum of the effective potential occurs only if

$$M_\ell^2 < M^2 e^{-3/2} \quad (2.10)$$

Likewise Eq. (2.8) shows that this minimum is stable, lower than the symmetric minimum,¹⁷ if

$$M_\ell^2 < 2\pi^2 \frac{M_1^2}{g^2} + \mathcal{O}\left(\frac{f^4}{g^2}, \frac{\lambda^2}{g^2}, \frac{\sigma^2}{g^2}\right) \quad (2.11)$$

These inequalities are analogous to the ones found between the Higgs mass and gauge bosons mass in the Higgs model by Linde and in the Weinberg-Salam model by Linde^{13,14} and Weinberg¹⁵ who required stability of the spontaneously broken solution at the one-loop level.

The specific nature of the CP violating amplitudes for a model like the one outlined above has been discussed by Lee,² who worked in the tree approximation. My question was whether or not the presence of CP violation imposes restrictions on the mass parameters of the model. The answer is yes, the presence of CP violation does impose lepton mass constraints. I will not elaborate on the nature of the CP violating Higgs exchange mechanism, which is discussed in detail by Lee.² Let us next consider more realistic $SU(2) \times U(1)$ models which have several leptons, gauge bosons, and charged as well as neutral Higgs particles to see if relationships among fermion masses, Higgs boson masses, and the CP violation parameters can be found.

III. THE ELECTRON-MUON MASS RATIO AND CP VIOLATION IN AN ALMOST STANDARD $SU(2) \times U(1)$ MODEL

A. The Higgs Potential

Those versions of the $SU(2) \times U(1)$ gauge theory which include extra spin-zero doublets^{2,6,7,8,18} can have fermions whose masses depend upon the relative phase angles between the vacuum expectation values of the doublets. Fermion loop terms in the effective potential can therefore influence the resolution of the CP violation issue.

In this section, I will consider an $SU(2) \times U(1)$ model which has only the standard³ fermion fields $e, \mu, \nu_e, \nu_\mu, u, d, s, c$ with left-handed coupling to the gauge bosons, but has one extra Higgs doublet. Unlike the Abelian example discussed in the last section, there will be no spontaneous CP violation in the theory,

and no CP violation at all in the quark sector, if the tree approximation alone is considered. The key question is whether CP violating solutions exist when fermion loop corrections are competitive with a certain quartic term in the scalar self-couplings. This question intimately involves the Higgs mass which depends on the small quartic coupling.

The general form of the potential for the spin-zero doublets ϕ_1 and ϕ_2 is taken to be

$$\begin{aligned} V(\phi_1, \phi_2) = & \mu_1^2 \phi_1^\dagger \phi_1 + \mu_2^2 \phi_2^\dagger \phi_2 + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 \\ & + \sigma_2 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \sigma_2 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) + \epsilon (\phi_1^\dagger \phi_2)^2 + \epsilon^* (\phi_2^\dagger \phi_1)^2, \end{aligned} \quad (3.1)$$

where symmetry under $\phi_1 \rightarrow -\phi_1$ is assumed, ϕ_1 and ϕ_2 have CP transformations $\phi_i \rightarrow e^{i\eta_i} \phi_i^\dagger$ $i=1, 2$, and for the present $\epsilon = \epsilon^*$ will be assumed. The discrete symmetry $\phi_1 \rightarrow -\phi_1$ could be spatial parity,¹⁹ or it could be an ad hoc symmetry designed to eliminate unwanted Yukawa couplings,⁶ such as flavor-changing neutral Higgs couplings. The latter view will be taken in this section. Parity conservation at the Lagrangian level will be assumed in the example studied in the next section.

Analyzing the extremal condition

$$\frac{\partial V(\rho_1 e^{i\theta}, \rho_2)}{\partial \theta} = 0, \quad ,$$

where

$$\langle \phi_1 \rangle \equiv \begin{pmatrix} 0 \\ \frac{\rho_1 e^{i\theta}}{\sqrt{2}} \end{pmatrix}, \quad \langle \phi_2 \rangle \equiv \begin{pmatrix} 0 \\ \frac{\rho_2}{\sqrt{2}} \end{pmatrix} \quad (3.2)$$

one finds that $\text{Im} \left[\epsilon (\phi_1^\dagger \phi_2)^2 + \text{H. c.} \right] = 0$ and no CP violation is possible. With no loss of generality, $\theta=0$ can be chosen and $\epsilon < 0$ is required to ensure that

$$M_{H_0}^2 = -8(\rho_1^2 + \rho_2^2)\epsilon > 0 \quad (3.3a)$$

where

$$H_0 = \frac{1}{\sqrt{2}} \frac{(\rho_2 (\text{Im } \phi_1^0 \cos \theta - \text{Re } \phi_1^0 \sin \theta) - \rho_1 \text{Im } \phi_2^0)}{\sqrt{\rho_1^2 + \rho_2^2}} \quad (3.3b)$$

is a physical, neutral Higgs field. However if ϵ is of the order of fourth power of the Yukawa couplings of H_0 to fermions, then the fermion loop corrections to the effective potential must be included to answer the CP question perturbatively. The potential (3.1) was chosen because only the last term breaks an independent ϕ_1 and ϕ_2 global phase symmetry, which distinguishes the quartic coupling constant ϵ as the only source of mass for one of the Higgs bosons, H_0 , and the only support for the phase angle θ .²⁰

B. Leptons

To pursue this point further, let us consider the Yukawa Lagrangian of massless leptons

$$\begin{aligned} \mathcal{L}_Y(e, \mu) = & \sqrt{2} \bar{\psi}_L^e (-g_1 \phi_1 + g_2 \phi_2) e_R \\ & + \sqrt{2} \bar{\psi}_L^\mu (g_1 \phi_1 + g_2 \phi_2) \mu_R + \text{H. c.} \end{aligned} \quad (3.4)$$

where ψ_L^e, ψ_L^μ are the usual left-handed lepton doublets of $SU(2) \times U(1)$ and e_R, μ_R are the right-handed singlets. $\mathcal{L}_Y(e, \mu)$ is invariant under the transformation $\phi_1 \rightarrow -\phi_1$ and $e \leftrightarrow \mu$,²¹ consistent with the symmetry of $V(\phi_1, \phi_2)$, Eq. (3.1). For our purposes, Eq. (2.4a) has the desirable consequence that the leptonic loop contributions to $V_{\text{eff}}(\langle \phi_1 \rangle, \langle \phi_2 \rangle)$ depend upon the relative phase, θ , between

$\langle \phi_1 \rangle$ and $\langle \phi_2 \rangle$. Therein lies the origin of CP violation. For the present, I take g_1 and g_2 to be relatively real, so CP is a good symmetry of $\mathcal{L}_Y(e, \mu)$.

When the 1-loop corrections to the effective potential are included, it is possible to choose a mass parameter M^2 such that the quartic coupling constants in Eq. (3.1) are redefined and leptonic 1-loop corrections absorb the term $\epsilon(\phi_1 \phi_2)^2 + \text{H.c.}$ This potential reads

$$\begin{aligned} V_{\text{eff}}(\phi_1, \phi_2) = & \mu_1^2 \phi_1^\dagger \phi_1 + \mu_2^2 \phi_2^\dagger \phi_2 + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 \\ & + \sigma_1 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \sigma_2 (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \\ & - \frac{M_\mu^2(\phi_1, \phi_2)}{16\pi^2} \ln \left[\frac{M_\mu^2(\phi_1, \phi_2)}{M^2} \right] \\ & - \frac{M_e^4(\phi_1, \phi_2)}{16\pi^2} \ln \left[\frac{M_e^2(\phi_1, \phi_2)}{M^2} \right] + \dots, \end{aligned} \quad (3.5)$$

where

$$M_{\mu, e}^2(\phi_1, \phi_2) = g_1^2 \phi_1^\dagger \phi_1 + g_2^2 \phi_2^\dagger \phi_2 \pm g_1 g_2 (\phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1)$$

and the dots indicate θ -independent one-loop terms and higher loop corrections.

The dimensional parameter M^2 has replaced the dimensionless ϵ .^{11, 12} Let us again seek the minimum of the effective potential with respect to θ in order to study CP effects. We have

$$\frac{\partial \mathcal{L}}{\partial \theta} = 0 = \frac{2g_1 g_2 \rho_1 \rho_2 \sin \theta}{16\pi^2} \left[2M_\mu^2 \ln \frac{M_\mu^2}{M^2} + M_\mu^2 - 2M_\ell^2 \ln \frac{M_e^2}{M^2} - M_e^2 \right]. \quad (3.6)$$

The masses M_μ^2 and M_e^2 are expressible as

$$M_{\mu, e}^2 = g_1^2 \rho_1^2 + g_2^2 \rho_2^2 \pm 2g_1 g_2 \rho_1 \rho_2 \cos \theta. \quad (3.7a)$$

The extremal condition (3.6) has a solution for $\theta \neq 0$ if the mass parameters obey the relation

$$2M_\mu^2 \ln \frac{M_\mu^2}{M^2} + M_\mu^2 - 2M_e^2 \ln \frac{M_e^2}{M^2} - M_e^2 = 0 \quad (3.8)$$

Equation (3.8) has solutions when

$$0 < \frac{M_e^2}{M^2} < e^{-3/2} \text{ and } e^{-3/2} < \frac{M_\mu^2}{M^2} < e^{-1/2} \quad (3.9)$$

The inequalities (3.9) are seen quite clearly on a sketch of $2x \ln x + x$, whose double values correspond to solutions of Eq. (3.8). This is shown in Fig. 1.

To leading order in g_1^4 , $g_1^2 g_2^2$ and g_2^4 , the field H_0 , defined by Eq. (2.3b), is an eigenmode which has mass

$$M_{H_0}^2 = \frac{(G_F/\sqrt{2})^{-1}}{64\pi^2} \frac{\tan^2 \theta}{4\rho_1^2 \rho_2^2} (M_\mu^2 - M_e^2) \left(\frac{M_\mu^2 + M_e^2}{M_\mu^2 - M_e^2} \ln \frac{M_\mu^2}{M_e^2} - 2 \right) \quad (3.10)$$

and g is the SU(2) gauge group coupling constant. Equation (3.10) shows that the small electron to muon mass ratio ensures that $M_{H_0}^2 > 0$ and that the $\theta \neq 0$, CP violating solution is indeed a minimum. Specifically, positivity of $M_{H_0}^2$ demands that M_e and M_μ satisfy the condition

$$\frac{M_\mu^2 + M_e^2}{M_\mu^2 - M_e^2} \ln \frac{M_\mu^2}{M_e^2} - 2 > 0 \quad (3.11a)$$

which when combined with (3.9) means that

$$\frac{M_\mu^2}{M_e^2} > 1.5 \quad (3.11b)$$

If $V(\rho_1 e^{i\theta}, \rho_2) < V(\rho_1, \rho_2) < 0$, then the CP violating vacuum is stable. A simple calculation yields the result

$$V(\rho_1 e^{i\theta}, \rho_2) = \frac{\mu_1^2 \rho_1^2 + \mu_2^2 \rho_2^2}{4} + \frac{1}{16\pi^2} \frac{(M_e^4 + M_\mu^4)}{2} \quad (3.12)$$

The $\rho_1 \neq 0$, $\rho_2 \neq 0$, $\theta \neq 0$ minimum is stable when

$$\frac{\mu_1^2 \rho_1^2 + \mu_2^2 \rho_2^2}{4} < -\frac{1}{16\pi^2} \frac{(M_e^4 + M_\mu^4)}{2} < 0 \quad (3.13)$$

Since

$$\frac{M_e^4 + M_\mu^4}{2} = (g_1^2 \rho_1^2 + g_2^2 \rho_2^2)^2 + 4g_1^2 g_2^2 \rho_1^2 \rho_2^2 \cos \theta$$

has a maximum at $\theta=0$, it is clear that the $\theta \neq 0$ minimum is lower than $\theta=0$. The CP violating solution is stable.

Referring to Eqs. (3.7a), (3.9), (3.10), (3.11), and (3.13), we see that the electron and muon masses, the CP violating phase θ , and the mass of one neutral Higgs boson are all tied together in a remarkably tight way. From Eq. (3.7a) alone it follows that $M_e/M_\mu \ll 1 \Rightarrow |g_1 \rho_1| \approx |g_2 \rho_2|$ and $\theta \approx 0$. Specifically, $\sin \theta \leq \frac{2M_e M_\mu}{M_e^2 + M_\mu^2}$ and

$$\frac{M_\mu - M_e}{2} \leq \left| \frac{g_1 \rho_1}{g_2 \rho_2} \right| \leq \frac{M_e + M_\mu}{2} \quad (3.7b)$$

are the limits on θ and $g_1 \rho_1 / g_2 \rho_2$ which are imposed by the lepton mass identification. The spontaneous generation of a CP violating phase occurs if the inequalities (3.9) are satisfied. Rephrasing Eq. (3.9) by substitution of Eq. (3.8) and

(3.10), we find that the inequality

$$\ln \frac{M_\mu}{M_e} \geq 8\pi^2 \frac{G_F}{\sqrt{2}} \frac{M_{H_0}^2 4\rho_1^2 \rho_2^2}{(M_\mu^2 - M_e^2) \tan^2 \theta} \geq \ln \frac{M_\mu}{M_e} - 1 \quad (3.14)$$

must be satisfied if CP violation is to occur.²² Again my point is illustrated that CP violation is sometimes possible only under very restricted conditions on the mass parameters of a theory.

According to Eq. (3.14) the largest value of $\tan \theta$ corresponds to the largest possible value of M_{H_0} consistent with the bounds for fixed $\rho_1 \times \rho_2$. However, small values of $\rho_1 \times \rho_2$ can yield large values of M_{H_0} . What can one say about $\rho_1 \times \rho_2$? There is no constraint due to gauge boson masses, since $M_W^2 = g^2(\rho_1^2 + \rho_2^2)/8$ puts no condition on $\rho_1 \times \rho_2$. By inspection of the lepton mass formulas (3.7a) and (3.7b), we notice that since $|g_1 \rho_1 / g_2 \rho_2| \approx 1$ and

$$\frac{g_1^2 \rho_1^2 + g_2^2 \rho_2^2}{2} = M_\mu^2 + M_e^2 \quad \text{then} \quad \rho_1 / \rho_2 \ll 1 \Rightarrow \frac{g_1}{g_2} \approx \frac{\rho_2}{\rho_1} \gg 1.$$

Care must be taken to avoid $g_1 \approx \mathcal{O}(1)$ if g_1 enters as an expansion parameter in perturbation theory, so $\rho_1 \times \rho_2$ cannot assume arbitrarily small values. It is possible to turn directly to experiment to bound the quantity $\rho_1 \times \rho_2$. For example, parameters measured in μ decay bound the allowed contribution from the charged Higgs-particle H^+ , and the analysis of Appendix A shows that

$$4\rho_1^2 \rho_2^2 > \frac{(M_e^2 + M_\mu^2)}{M_{H^+}^2} \times \frac{1}{4NG_F^2} \quad (3.15)$$

where M_{H^+} is the mass of the charged Higgs and N is, roughly speaking, the fraction of the strength of G_F with which the charged Higgs boson may contribute to a four point interaction without violating the experimental limits set by

universality and electron helicity in $\mu \rightarrow e \nu \bar{\nu}$ measurements. The consequent bound on M_{H_0} is then

$$M_{H_0}^2 \leq N M_{H^+}^2 \frac{G_F}{\pi^2 \sqrt{2}} \frac{(M_\mu^2 - M_e^2)^2}{M_\mu^2 + M_e^2} \ln \frac{M_\mu}{M_e} . \quad (3.16)$$

For illustration one can chose $M_{H^+}/M_P = 10^2$, $N=10^{-1}$ and obtain $M_{H_0} \lesssim \frac{1}{2} M_e$, so that it is clear that the neutral Higgs boson H^0 , a "would be Goldstone boson" in the approximate symmetry sense,²⁰ is extremely light in this picture. The question of whether or not it is too light to be in accord with data on neutron electron scattering,^{16, 23} for example, depends upon the Yukawa couplings of Higgs particles to quarks. I next turn to the question of wedding the above 4-lepton scheme to a 4-quark scheme.

C. Enter the Quarks

A simple and attractive way to introduce four quarks is the one chosen by Weinberg,⁶ who writes the Yukawa interaction as

$$\begin{aligned} \mathcal{L}_Y(\text{quarks}) = & \sum_{i,j=1}^2 \Gamma_{ij}^{(1)} \bar{N}_{iR} \left(\phi_{1PjL}^* + \phi_{1NjL}^* \right) \\ & + \sum_{i,j=1}^2 \Gamma_{ij}^{(2)} \bar{P}_{iR} \left(\phi_{2PjL}^0 - \phi_{2NjL}^+ \right) + \text{H. c.} \end{aligned} \quad (3.17)$$

which again satisfies the discrete symmetry $\phi_1 \rightarrow -\phi_1$, with the charge -1/3 quarks N_R simultaneously changing sign $N_R \rightarrow -N_R$, just as the leptonic and scalar potentials do. The neutral Higgs couplings to physical quarks thereby conserve strangeness and charm automatically.^{6, 4} By suitable choice of the arbitrary CP transformation phases η_1 and η_2 on ϕ_1 and ϕ_2 respectively, the Yukawa interaction between ϕ_1 , ϕ_2 and the physical quarks conserves CP even if CP invariance is not imposed on the Lagrangian (3.17). However, CP violation in the quark sector can occur by charged Higgs exchange if the parameter A,

defined by⁶

$$A = \langle T \{ \phi_1^{+*} \phi_2^+ \} \rangle_{q=0} / \langle \phi_1 \rangle^* \langle \phi_2 \rangle \quad (3.18a)$$

and which through one loop is equal to

$$A = \left(\frac{\partial^2 V_{\text{eff}}(\phi_1, \phi_2)}{\partial \phi_1^{+*} \partial \phi_2^+} \right)^{-1} \bigg|_{\substack{\phi_1 = \langle \phi_1 \rangle \\ \phi_2 = \langle \phi_2 \rangle}} / \langle \phi_1 \rangle^* \langle \phi_2 \rangle \quad (3.18b)$$

is complex. One can readily verify that $V_{\text{eff}}(\phi_1, \phi_2)$ as given by Eq. (3.5), when combined with condition (3.8), produces a real value of A . Therefore, if CP is conserved at the Lagrangian level, there is no CP violation in the quark sector due to Higgs exchange even if CP is spontaneously broken and CP violation occurs in the lepton-Higgs interactions.²⁴ The situation changes, however, if Eqs. (3.1) and (3.4) are not CP conserving.²⁵

In order to account for CP violation in the 1-loop potential which stems from CP violating phases in the Lagrangian, there are two changes which must be made in the effective potential, Eq. (3.5). First, the parameter M^2 which was introduced to absorb the effects of the term $\epsilon(\phi_1^\dagger \phi_2)^2 + \text{H.c.}$, in the classical potential must be complex. In addition, the lepton masses now depend on the relative phase between the Yukawa couplings g_1 and g_2 , defined in Eq. (3.4).²⁵ Including the new effects, we must replace the last two terms in the potential (3.5) by the expression

$$V'_{\text{eff}}(\text{lepton loop}) = -\frac{1}{16\pi^2} \left\{ M_\mu^4 \ln \frac{M_\mu^2}{|M|^2} + M_e^4 \ln \frac{M_e^2}{|M|^2} + 4\Omega |g_1|^2 |g_2|^2 \rho_1^2 \rho_2^2 \sin 2(\theta_{12} + \theta) \right\}, \quad (3.19)$$

where

$$M_{\mu,e} = |g_1|^2 \rho_1^2 + |g_2|^2 \rho_2^2 \pm 2|g_1||g_2|\rho_1\rho_2 \cos(\theta_{12}+\theta) ,$$

$$M^2 = |M^2| e^{i\Omega}$$

and

$$e^{i\theta_{12}} = g_1 g_2^* / |g_1| |g_2| .$$

By extremizing Eq. (3.19) with respect to θ , one finds that

$$\Omega = \sin(\theta_{12}+\theta) \left[M_\mu^2 \left(\ln \left(\frac{M_\mu^2}{|M^2|} \right) + \frac{1}{2} \right) - M_e^2 \left(\ln \left(\frac{M_e^2}{|M^2|} \right) + \frac{1}{2} \right) \right] . \quad (3.20)$$

If Ω is set equal to zero, the previous mass relationships (3.7), (3.9), (3.10), and (3.13) all hold with the replacement $\theta \rightarrow \theta_{12}+\theta$ everywhere. This special case is analogous to the mass relationships which follow from requiring that coefficients of bilinear terms in the potential be zero at each order of the loop expansion.^{11, 14, 26}

The coefficient A , defined in Eq. (3.18), determines whether or not CP violation occurs in the quark sector. The value of $\text{Im } A$ is expressible as

$$\text{Im } A \cong \frac{4|g_1|^2 |g_2|^2 \cos \theta' X (\sin 2(\theta_{12}+\theta) - \sin(\theta_{12}-\theta))}{(M_\mu^2 - M_e^2) \left(2M_H^2 + \frac{G_F}{\sqrt{2}} \rho_1 \rho_2 + \frac{X|g_1||g_2| \cos(\theta_{12}-\theta)}{\cos 2(\theta_{12}+\theta)} \right)^2} , \quad (3.21)$$

where

$$X \equiv M_\mu^2 \left(2 \ln \frac{M_\mu^2}{|M|^2} + 1 \right) - M_e^2 \left(2 \ln \frac{M_e^2}{|M|^2} + 1 \right) \quad \text{and} \quad \theta \ll 1, \quad \theta_{12} \ll 1$$

are assumed, a result whose only beauty is that it is not zero and therefore produces CP violation in charged Higgs exchange between quarks. The positivity of $\partial^2 V / \partial \theta^2$ determines whether or not the extremum defined by Eq. (3.20) is a minimum. The details of this evaluation are presented in Appendix B, and the relevant point is that

$$M_e^2 < |M^2| e^{-3/2} < M_\mu^2 < |M^2| e^{-1/2} ,$$

where $(\theta_{12} + \theta) \ll 1$ because $M_e / M_\mu \ll 1$ as discussed above Eq. (3.7b), suffices to produce a minimum at the Ω value given by Eq. (3.20).

I conclude that Weinberg's picture of "natural" CP violation⁶ in a 4-quark and 4-lepton $SU(2) \times U(1)$ scheme can be implemented with only two Higgs doublets if the parameter ϵ in Eq. (2.1) is of order g^4 , where g is a Yukawa coupling of leptons to scalars.

As shown in Appendix A, the e and μ couplings to the charged Higgs scalar H^+ are equal when $\rho_1 \rho_2 / (\rho_1^2 + \rho_2^2) \ll 1$, which is also the condition that M_{H_0} be as large as possible. Under this condition, the CP violating effects in e and μ final states of K_L decay will be the same, in agreement with experiment.³⁰ This contrasts with Weinberg's remark⁶ that electron couplings to H^+ should be negligible.

A discussion of the mass and quark couplings of neutral Higgs particles associated with the (approximate) eigenmode H_0 , Eq. (3.3b), is given in Appendix B. Although there are several adjustable parameters and free quark model calculations of hadronic properties are, at best, order of magnitude indicators, it is shown that the estimates of the effects of the neutral Higgs particles are orders of magnitude too large to have escaped detection in thermal neutron-electron scattering. The tiny mass estimate of the Higgs particle which

is responsible for this large effect is a consequence of the smallness of electron and muon masses compared to the weak interaction mass scale. In the following section I turn to a final example of CP violating, fermion-loop effects in a model which has heavy leptons and quarks (the C-quark is light in this context), and where parity and CP are spontaneously broken symmetries.^{7,8}

IV. A SPONTANEOUS P AND CP BREAKING MODEL:

HEAVY LEPTONS AND QUARKS

An example of an $SU(2) \times U(1)$ model which links the CP violation angle θ , the mass ratios of light to heavy fermions, and the mass of a neutral Higgs boson has been discussed several times in the literature.^{7,8,19} The quartic potential is the same as in Eq. (3.1), but the pivotal position of the Higgs boson H_0 in determining the CP character of the model was overlooked in Ref. 7 and only partially appreciated in Ref. 8. For the purposes of this paper, the model provides an illustration of a bound on the mass of H_0 , M_{H_0} , which depends upon heavy fermion masses. The range of values of M_{H_0} is thereby lifted, and it becomes plausible that the H_0 could have thus far escaped detection.^{16,23}

The essential point of the model under discussion is that parity and CP are symmetries of the Lagrangian. The field ϕ_1 is a pseudoscalar, $P\phi_1 P^{-1} = -\phi_1$, while ϕ_2 is a scalar. Parity therefore plays the role of the ad hoc $\phi_1 \rightarrow -\phi_1$ symmetry of Eqs. (3.1) and (3.4). Four leptons are left-hand coupled to charged weak currents, while four are right-hand coupled. All neutral leptons are massless, and lepton number is separately conserved for e , μ , and their right-handed counterparts. The quarks similarly break up into left- and right-hand sectors, but the lack of mixing between right and left must be enforced order by order (does not follow naturally from symmetries of the Lagrangian).^{4,6}

The left-handed leptons and quarks are identified as e, μ, ν_e, ν_μ and d, s, u, c as in the standard model, while the right-handed leptons and quarks are called E, M, ν_E, ν_M and d', s', u', c' . Since CP is a symmetry of the Lagrangian the only CP phase that enters is the relative $\langle \phi_1 \rangle \leftrightarrow \langle \phi_2 \rangle$ phase, θ . Fermion masses break up into light (left) and heavy (right) partners f_i, f'_i according to

$$M_{f_i, f'_i}(\langle \phi_1 \rangle, \langle \phi_2 \rangle) = (g_1^i)^2 \langle \phi_1^\dagger \phi_1 \rangle + (g_2^i)^2 \langle \phi_2^\dagger \phi_2 \rangle \mp g_1^i g_2^i \langle \phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1 \rangle \quad (4.1)$$

or

$$M_{f_i, f'_i}^2 = \rho_1^2 (g_1^i)^2 + \rho_2^2 (g_2^i)^2 \mp 2\rho_1 \rho_2 g_1^i g_2^i \cos \theta$$

where g_1^i and g_2^i are the Yukawa couplings of the i^{th} left/right fermion pair and $f_i = e, \mu, d, s, u, c$ and $f'_i = E, M, d', s', u', c'$. Phases are chosen so that the minus sign in Eq. (4.1) goes with the light fermions f_i while the plus sign goes with heavy fermions. It is possible to identify the E or M with the anomalous (e, μ) signal observed at SPEAR.²⁷ However, this model would be eliminated as a realistic description of low energy weak interactions if the V+A coupling is ruled out when muon momentum cutoff effects are thoroughly understood in the sequential decay interpretation of the (e, μ) signal.²⁷

I will skip details of the model, which have been discussed in Ref. 7, and turn directly to analysis of the θ -dependent part of the effective potential. By redefinition of the quartic couplings, Eq. (3.1), when $\epsilon \sim \mathcal{O}(g_1^4)$, the effective potential including 1-fermion-loop contributions can be written in the same

form as Eq. (3.5),

$$\begin{aligned}
 V_{\text{eff}}(\phi_1, \phi_2) = & \mu_1^2 \phi_1^\dagger \phi_1 + \mu_2^2 \phi_2^\dagger \phi_2 + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 \\
 & + \sigma_1 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \sigma_2 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) \\
 & - \frac{1}{16\pi^2} \sum_i \left[M_{f_i}^4 \ln \frac{M_{f_i}^2}{M^2} + M_{f'_i}^4 \ln \frac{M_{f'_i}^2}{M^2} \right] + \dots
 \end{aligned} \tag{4.2}$$

Again a mass parameter M^2 has replaced ϵ .

The value of the CP violating vacuum phase, θ , is determined by the extremum condition

$$\begin{aligned}
 \frac{\partial V}{\partial \theta}(\rho_1 e^{i\theta}, \rho_2) = & - \frac{4\rho_1 \rho_2 \sin \theta}{16\pi^2} \sum_i g_1^i g_2^i \left[M_{f'_i}^2 \left(2 \ln \frac{M_{f'_i}^2}{M^2} + 1 \right) \right. \\
 & \left. - M_{f_i}^2 \left(2 \ln \frac{M_{f_i}^2}{M^2} + 1 \right) \right] = 0 \quad . \quad .
 \end{aligned} \tag{4.3}$$

A CP violating $\theta \neq 0$ solution to Eq. (4.3) is

$$M_{f_i}^2 \left(2 \ln \frac{M_{f_i}^2}{M^2} + 1 \right) = M_{f'_i}^2 \left(2 \ln \frac{M_{f'_i}^2}{M^2} + 1 \right) , \quad \text{each } i , \tag{4.4}$$

which is simple but implies an unattractive mass spectrum. As illustrated in Fig. 2, the smallest unprimed mass, the electron, is paired with the highest primed mass. The second-to-smallest unprimed mass is paired with the second-to-largest primed mass, etc. To identify the lepton E or M with the τ ²⁷ would force all heavy quark masses to be below 2 GeV, and would force the charmed meson mass to a value less than $2 \text{ GeV}/\sqrt{2.72}$.

An unorthodox interpretation of the τ given by Ma, Pakvasa and Tuan²⁸ could be implemented in the present model and at the same time preserve the

solution (4.4). The τ would be identified with the charged Higgs particle H^\pm , and an extra $\phi_1 \leftrightarrow \phi_2$ symmetry would be imposed to ensure a lepton conservation pattern on the Higgs decay. The light, neutral Higgs boson which is needed in that picture would be a result of the small value of ϵ in Eq. (3.1) and would admit spontaneous CP violation in the model via charged Higgs exchange, as worked out in Ref. 7. The heavy leptons in this solution, Eq. (4.4), would be heavier than the heaviest quarks, an unusual mass pattern, and the scheme survives only by very careful adjustment of charged Higgs couplings in order to make the 3-body decays $H^\pm \rightarrow H^0 e^\pm \nu_e$ ($H^0 \mu^\pm \nu_\mu$) big enough to explain the SPEAR (e, μ) events.²⁷ This interpretation seems therefore rather unconvincing. Let us look at a different solution to Eq. (4.3).

Equation (4.3) can also be satisfied for $\theta \neq 0$ by cancellations among terms of different i . Let us suppose that one mass is much larger than all the rest (the right-handed counterpart, c' , of the charmed quark is a plausible candidate. The very heavy quark will be called c' from now on.)²⁹ It is then necessary that this heaviest mass $M_{c'}$ satisfies

$$2 \ln \frac{M_{c'}^2}{M^2} + 1 \approx \frac{M_i^2}{M_{c'}^2} \ll 1 \quad (4.5)$$

where i refers to any other fermion. Otherwise it would be impossible to cancel the term $M_{c'}^2 \left(2 \ln \left(M_{c'}^2 / M^2 \right) + 1 \right)$ against the smaller mass terms. If all of the primed masses were taken to be equal, as in Ref. 7, then Eq. (4.5) would be true for all of them.

The Higgs mode H_0 , defined by Eq. (3.3b), has a mass approximately given by

$$M_{H_0}^2 = - \frac{\tan^2 \theta}{16\pi^2} \frac{(G_F/\sqrt{2})^{-1}}{4\rho_1^2 \rho_2^2} \sum_i \left(M_{f_i}^2 - M_{f_i}^2 \right) \left(\ln \frac{M_{f_i}^2}{M^2} + \ln \frac{M_{f_i}^2}{M^2} + 3 \right). \quad (4.6)$$

Now if $M_{c'}^2$ is much larger than any of the other masses, then Eq. (4.5) is necessary and

$$M_{H_0}^2 \simeq - \frac{1}{16\pi^2} \frac{\tan^2 \theta}{4\rho_1^2 \rho_2^2} (G_F/\sqrt{2})^{-1} M_{c'}^4 \left(\ln \frac{M_{c'}^2}{M_{c'}^2} + 2 \right). \quad (4.7)$$

This corresponds to a stable solution lower in energy than the CP conserving $\theta=0$ vacuum when $M_{c'}^2 > 2.72 M_c^2$, which is certainly required by experiment under the assumption we have made that $M_{c'} \gg M_u, M_d, M_s \gg M_c, M_u, M_d, M_s$. Taking $\tan \theta \approx 2M_e/M_E$ (assume that M_e/M_E is less than any other low/high fermion mass ratio), $\rho_1=\rho_2$ and $M_{c'} \approx 10^2 M_\rho$ we gain an estimate

$$M_{H_0}^2 \approx \frac{5}{\pi^2} \frac{M_e^2}{M_E^2} \left(\frac{G_F}{\sqrt{2}} \right) M_{c'}^4 \approx (10 M_e)^2 \quad (4.8)$$

This estimate rises to $(300 \text{ MeV})^2$ when the limit on $\rho_1 \rho_2$, Eq. (A.3b), is used. As discussed in Appendix B, the mass and couplings of this neutral Higgs H_0 , to leptons and quarks are such as to make its effects in n-e scattering unobservable.

The message of the present section is that low-mass Higgs particles can be linked with the presence of a small CP vacuum breaking phase and small ratios of standard fermions to heavy counterparts. In the model chosen here for illustration, the heavy fermions are right-handed but are forced to have no mixing with normal quarks, and therefore do not produce a high γ anomaly in antineutrino reactions on hadron targets. The low mass, neutral Higgs particle

H_0 which is necessarily attendant to the CP violation has properties that make it very difficult to detect.¹⁶

V. A BRIEF SUMMARY, AND CONCLUSIONS

The question of CP violation in a gauge theory is only settled by a close look at the values that coupling parameters assume in that theory. In an Abelian example, it was established that spontaneous CP violation can only occur for values of one Higgs mass M_{H_0} which are greater than a lower bound which can depend critically on the lepton mass. This result is patterned after the one obtained by Linde^{13, 14} and Weinberg¹⁵ that gauge symmetry breaking itself can only occur when the Higgs mass is larger than a lower bound fixed by gauge boson and fermion masses. My discussion, like theirs, is carried out in a 1-loop approximation but should not be substantially affected by higher loop considerations in weak coupling theories.

Taking up several examples of $SU(2) \times U(1)$ weak gauge theories, I showed that there is a range of parameters in these cases where there is an approximate global symmetry of the Higgs potential,²⁰ one Higgs boson is very light, and CP violation can occur. If the tree approximation alone dominates the effective potential, CP violation is not possible in these models. Only if lepton loop corrections are important, as they are in the cases where one Higgs is light, do the CP violating solutions emerge. When CP violation does occur, small fermion mass ratios such as M_e/M_μ , or $M_c/M_{c'}$, $\ll 1$, where $m_{c'}$ is the mass of a very heavy right-handed counterpart of the charmed quark, imply that the spontaneous CP violating phase θ between the different Higgs vacuum expectation values is likewise small. For example, in a four lepton model, $\sin \theta = 2M_e M_\mu / (M_e^2 + M_\mu^2)$,

$$M_{H_0}^2 \leq \frac{(G_F/\sqrt{2})^{-1}}{8\pi^2} \frac{\tan^2 \theta (M_\mu^2 - M_e^2)}{4\rho_1^2 \rho_2^2} \ln \frac{M_\mu}{M_e} .$$

Experimental constraints on the mass and couplings of a neutral scalar boson indicate that more than the standard leptons and quarks are needed. Heavy leptons and quarks are necessary to make the mechanism considered in this paper a realistic one. The model presented in Section IV illustrated how this would work.

The conclusion which I draw from the study of these gauge models is that small discrete symmetry violation such as CP violation may be understood from the radiatively induced symmetry breakdown point of view with comparatively few free parameters if there exists at least one very low mass, neutral Higgs boson. This point of view can be complementary to the attractive "natural" suppression of CP violation,⁴ as exemplified by the model studied in Section III. If strictly spontaneous P and CP violation are enforced, the dynamical effects as presented here provide an alternative to natural suppression. This case was illustrated by the model of Section IV. The distinguishing feature of spontaneous CP breaking, clearly demonstrated by the models studied in this work, is that the CP violating phase can be directly related to fermion mass ratios and not just appear as a Cabibbo-type parameter to be fixed by direct comparison to CP violating amplitudes.

APPENDIX A

In this appendix, the couplings of the charged Higgs to the electron and muon for the model of Section II are given, and the bound on $\rho_1\rho_2$ which is implied by the bound on charged Higgs exchange in $\mu \rightarrow e\nu\bar{\nu}$ is worked out.

The physical, charged Higgs particle H^+ is related to the charged components of ϕ_1 and ϕ_2 by

$$\frac{\rho_1}{\sqrt{\rho_1^2 + \rho_2^2}} H^+ = \phi_2^{(+)} , \quad \frac{\rho_2}{\sqrt{\rho_1^2 + \rho_2^2}} e^{i\theta} H^+ = -\phi_1^{(+)} .$$

The lepton mass terms can be rendered real, γ_5 -free, if redefinitions of e_R and μ_R are chosen to be

$$e_R \rightarrow e_R e^{-i\alpha_e} \quad \text{and} \quad \mu_R \rightarrow \mu_R e^{-i\alpha_\mu}$$

with

$$\tan \alpha_e = \frac{g_1 \rho_1 \sin \theta}{g_1 \rho_1 \cos \theta - g_2 \rho_2} \quad \text{and} \quad \tan \alpha_\mu = \frac{g_1 \rho_1 \sin \theta}{g_1 \rho_1 \cos \theta + g_2 \rho_2} .$$

The effective Yukawa interaction between H^+ and the leptons takes the form

$$\begin{aligned} \mathcal{L}_{Y(H, \ell \nu_\ell)} = & \sqrt{2} \left(\frac{M_\mu^2 + M_e^2}{\sqrt{2} G} \frac{1}{4\rho_1^2 \rho_2^2} - \sqrt{2} G M_e^2 \right)^{1/2} e^{i\eta_e} \bar{\nu}_e e_R H^+ \\ & + \sqrt{2} \left(\frac{M_\mu^2 + M_e^2}{\sqrt{2} G} \frac{1}{4\rho_1^2 \rho_2^2} - \sqrt{2} G M_\mu^2 \right)^{1/2} e^{i\eta_\mu} \bar{\nu}_\mu \mu_R H^+ + \text{H.c.} \end{aligned} \quad (\text{A.1})$$

The phase angles η_e and η_μ are defined by

$$\eta_e = -\alpha_e + \tan^{-1} \left(\frac{g_1 \rho_2 \sin \theta}{\sqrt{g_1^2 \rho_2^2 + g_2^2 \rho_1^2 + 2g_1 g_2 \rho_1 \rho_2 \cos \theta}} \right)$$

and

$$\eta_\mu = -\alpha_\mu + \tan^{-1} \left(\frac{g_1 \rho_2 \sin \theta}{\sqrt{g_1^2 \rho_1^2 + g_2^2 \rho_1^2 - 2g_1 g_2 \rho_1 \rho_2 \cos \theta}} \right)$$

The additional 4-point interaction for $\mu \rightarrow e \nu \bar{\nu}$ gains a term from H^+ exchange equal to

$$\mathcal{L}_{H^+}(\mu \rightarrow e \nu \bar{\nu}) = \frac{G}{\sqrt{2}} w e^{i\Delta} \left[-\bar{e} (1-\gamma_5) \nu_e \bar{\nu}_\mu \mu + \bar{e} (1+\gamma_5) \nu_e \bar{\nu}_\mu \gamma_5 \mu \right] + \text{H. c.}$$

The parameter w measures the effect of the charged scalar exchange. The $e^{i\Delta}$ factor is irrelevant for our purposes. The muon decay parameters K and h ³⁰ take the values

$$K = 16 \left(1 + \frac{1}{4} w^2 \right)$$

and

$$h = \frac{-1 + \frac{1}{4} w^2}{1 + \frac{1}{4} w^2}.$$

The universality of the weak current is modified by the presence of w in K , and the helicity of the electron deviates from -1 to the extent $w \neq 0$ in h .

The relationship between w^2 and $X = 4\rho_1^2 \rho_2^2$ is given by

$$w^2 = \frac{1}{4} \left[\frac{M_\mu^2 + M_e^2}{2 \left(\frac{G}{\sqrt{2}} \right)^2 M_{H^+}^2} \right]^2 \left(\frac{1}{X} - \left(\frac{2G}{\sqrt{2}} \right)^2 \frac{M_e^2}{M_e^2 + M_\mu^2} \right) \left(\frac{1}{X} - \left(\frac{2G}{\sqrt{2}} \right)^2 \frac{M_\mu^2}{M_e^2 + M_\mu^2} \right) \quad (\text{A. 2})$$

Designate by N^2 the experimental bound on w^2 , where $N^2 \approx 10^{-2}$ from universality and $N^2 \approx .25$ from the helicity. The upper bound on $\rho_1^2 \rho_2^2$ occurs when $\rho_1^2 = \rho_2^2$ because $2(\rho_1^2 + \rho_2^2) = (G_F/\sqrt{2})^{-1}$ is a constraint. It is easy to check that the upper bound is below either of the zeros of (A.2). Therefore, a lower bound

on $X = 4\rho_1^2 \rho_2^2$ consistent with experimental bounds on w^2 may be obtained by ignoring the constant terms in the parentheses in (A.2) compared to $\frac{1}{X}$. We find

$$4\rho_1^2 \rho_2^2 > \frac{\left(M_e^2 + M_\mu^2\right)}{M_1^2} \cdot \frac{1}{4NG^2} \quad (\text{A. 3a})$$

This is the result used in Section III. A similar bound is easily found for the model of Section IV, namely

$$4\rho_1^2 \rho_2^2 > \frac{M_E M_M}{M_{H^+}^2} \cdot \frac{1}{4NG^2} \quad (\text{A. 3b})$$

An interesting point related to Weinberg's⁶ remarks about CP effects in semileptonic K decays is that the coupling pattern (A.1) is entirely different from the one he assumes for leptons. One finds that when $\rho_1 = \rho_2$ or $g_1 = g_2$,

$$g_{H^+e\nu} \sim M_\mu G_F^{1/2} \quad \text{and} \quad g_{H^+\mu\nu} \sim M_e G_F^{1/2},$$

the opposite from usual expectations. However, when $\rho_1 \rho_2 / \rho_2^2 + \rho_1^2 \ll 1$,

$$g_{H^+e\nu} \simeq g_{H^+\mu\nu} \sim M_\mu G_F^{1/2}$$

which restores a kind of e- μ universality in the Higgs couplings and therefore in CP violation effects. Experiments on $K_L \rightarrow \ell \mu \nu$ CP violation charge asymmetries indicate no difference between e and μ final states.³¹ In the context of the model of Section III, this indicates $\rho_1 \rho_2 / \rho_1^2 + \rho_2^2 \ll 1$, which also helps raise the neutral Higgs mass and is an improvement from the phenomenological standpoint.¹⁶

APPENDIX B

Several computations and approximations referred to in Sections III and IV are listed in this appendix for completeness. Specifically, the mass and Yukawa couplings of the special Higgs particle H_0 are listed.

The expression for M_{H_0} for the case when $g_{1,2} \neq g_{1,2}^*$ in the model of Section III reads

$$M_{H_0}^2 = \frac{1}{8\pi^2 \left(\frac{G}{\sqrt{2}}\right) \rho_1 \rho_2 \cos \theta'} \left\{ + \frac{(\cos^2 \theta' + \cos^2 2\theta')}{\cos 2\theta'} \left(M_\mu^2 \ln \frac{M_\mu^2}{M^2} + M_\mu^2 - M_e^2 \ln \frac{M_e^2}{M^2} - M_e^2 \right) \right. \\ \left. + 2(M_\mu^2 - M_e^2) \left(\frac{\cos^4 \theta' + \sin^4 \theta'}{\cos 2\theta'} \right) - \sin^2 \theta' \left(M_\mu^2 \ln \frac{M_\mu^2}{M^2} - M_e^2 \ln \frac{M_e^2}{M^2} \right) \right\}, \quad (B.1)$$

where $\theta' \equiv \theta_{12} + \theta$. To insure that $M_{H_0}^2 > 0$, it is sufficient to have $\theta' \ll 1$, as required by the small M_e/M_μ mass ratio, and M^2 chosen so that

$$M_e^2 < M^2 e^{-3/2} < M_\mu^2 < M^2 e^{-1/2}.$$

It is then possible to make positive each term in the expression in braces in Eq. (B.1) and guarantee that the CP violating vacuum is stable.

Yukawa Couplings of H_0

The Higgs couplings for the models of Sections III and IV are given below, as well as the mass estimates and contributions to the effective neutron form factor slope relevant to electron-neutron scattering.

The mass of H_0 can be written as

$$M_{H_0} \cong \begin{cases} \frac{3}{4\pi} \left(\frac{G_F}{\sqrt{2}}\right)^{-1/2} \frac{M_\mu^2 \tan \theta}{4\rho_1 \rho_2} & , \quad \text{Section III} \\ \frac{3}{4\pi} \left(\frac{G_F}{\sqrt{2}}\right)^{-1/2} \frac{M_{c'}^2 \tan \theta}{4\rho_1 \rho_2} & , \quad \text{Section IV} \end{cases} \quad (B.2a)$$

$$(B.2b)$$

and M_e is neglected compared to M_μ in Section III and any fermion masses are neglected compared to the mass of c' , $M_{c'}$, in Section IV. We have the bounds

$$\tan \theta \leq \frac{2M_e}{M_\mu} \sim 10^{-2} \quad \text{Section III}$$

$$\tan \theta \leq \frac{2M_e}{M_E} \sim 5 \times 10^{-4} \quad \text{Section IV .}$$

Using the super-heavy mass assumption that $M_{c'} = 10^2 M_p$,²⁹ the H_0 Higgs mass upper bounds for the cases $\rho_1 = \rho_2$ and $\rho_1 \rho_2 > F(8M, NG)^{-1}$, where $F = M_\mu$ for the model of Section III and $F = \sqrt{M_E M_M}$ for that of Section IV as discussed in Appendix A, are shown in Table I. For the cases that $M_{H_0} \lesssim 1$ MeV, there is a very strong effect by neutral Higgs exchange on the neutron form factor at $q^2 \approx 0$ in thermal neutron scattering from electrons. Assuming scalar coupling only between H_0 and electrons, which can be guaranteed if $\rho_1 = \rho_2$ and $g_1 = g_2$ or if $\cos \theta = -(g_2^2 - g_1^2 / \rho_2^2 - \rho_1^2) \rho_1 \rho_2 / g_1 g_2$, the relevant scalar coupling constants are

$$g_{H_0 \bar{e}e} \cong \begin{cases} \frac{-4\pi}{3} \frac{M_{H_0}}{M_e} & \text{Section III} & (B.3a) \\ \frac{-4\pi}{6} \frac{M_E^2}{M_{c'}^2} \frac{M_{H_0}}{M_e} & \text{Section IV} & (B.3b) \end{cases}$$

for the electron, and

$$g_{H_0 \bar{u}u} \cong \begin{cases} \frac{\rho_1}{\rho_2} \sqrt{2G_F} M_u & \text{pseudoscalar, Section III} & (B.4a) \\ \frac{4\pi}{6} \frac{M_{u'}^2}{M_{c'}^2} \frac{M_{H_0}}{M_u} & \text{(scalar) Section IV} & (B.4b) \end{cases}$$

for the up quark (for down quark, $\rho_1 \leftrightarrow \rho_2$ in (B.4a)). The modification to the neutron form factor slope which results if $M_{H_0} \lesssim 1$ MeV is approximately

$$\Delta a_N = \frac{1}{\alpha} \frac{g_{H_0 e \bar{e}} g_{H_0 u \bar{u}}}{M_{H_0}^2} \quad (\text{B.5})$$

ignoring quark mass difference and binding effects. This Δa_N contribution (B.5) should be less than ~ 0.05 (GeV/c) $^{-2}$ in order that electron deuteron and thermal neutron electron experiments be compatible.²³ The various cases are shown in Table II, which indicates that only in the model of Section IV in the case that $\rho_1 \rho_2 \ll \rho_1^2 \rho_2^2$, so that $M_{H_0} \gg M_e$, is gross disagreement with data avoided.

Acknowledgments

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REFERENCES AND FOOTNOTES

1. R. Mohapatra, Phys. Rev. D 6, 2023 (1972); A. Pais, *ibid.* 8, 625 (1973).
2. T. D. Lee, Phys. Rev. D 8, 1226 (1973), and Phys. Reports 9C, 143 (1974).
3. S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D 2, 1285 (1970);
S. L. Glashow and S. Weinberg, Phys. Rev. D 15, 1958 (1977).
4. B. W. Lee, "Gauge Theories of Microweak CP Violation," Fermilab Pub
76/101-THY (December 1976).
5. B. W. Lee and R. E. Schrock, "Natural Suppression of Symmetry Violation
in Gauge Theories—Muon and Electron—Lepton Number Nonconservation,"
Fermilab Pub 77/21-THY (February 1977).
6. S. Weinberg, Phys. Rev. Lett. 37, 657 (1976).
7. D. W. McKay, Phys. Rev. D 13, 645 (1976).
8. D. W. McKay, Phys. Rev. D 15, 905 (1977).
9. A. Zee, Phys. Rev. D 9, 1772 (1974); R. N. Mohapatra, Phys. Rev. D 9,
3461 (1974).
10. H. Georgi and A. Pais, Phys. Rev. D 10, 1246 (1974).
11. S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).
12. A recent thorough analysis of the gauge-independence of the Coleman-
Weinberg results of $\lambda\phi^4$ and the Higgs models to all orders of perturbation
theory is given by Iliopoulos and Papanicolaou, Nucl. Phys. B105, 77 (1976).
Earlier references on this question can be found there.
13. A. D. Linde, Phys. Lett. 62B, 435 (1976).
14. A. D. Linde, "Dynamical Symmetry Restoration and Constraints on Masses
and Coupling Constants in Gauge Theories," ICTP preprint IC/76/26. (To
be published.)
15. S. Weinberg, Phys. Rev. Lett. 36, 294 (1976).

16. Search for evidence of very light Higgs bosons is reviewed by Ellis, Gaillard and Nanopoulos, Nucl. Phys. B106, 292 (1976). Detailed discussions are given only for the single Higgs scalar of the type in the original $SU(2) \times U(1)$ gauge scheme (S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam in Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8), edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367). However, even for more complicated neutral Higgs systems it appears likely that a neutral scalar with mass $\sim m_e$ also contradicts n-e scattering data.
17. The question of decay of a quasi-stable CP breaking vacuum is not addressed here. The possibility of tunneling between equivalent, CP violating vacua was pointed out by Lee (Ref. 2), who did not pursue the question. Methods to compute tunneling have recently been discussed by S. Coleman ("The Fate of the False Vacuum: Semi-Classical Theory," Phys. Rev. (to be published)).
18. Many recent papers have incorporated extra Higgs multiplets in order to obtain lepton universality in Higgs coupling, P violation, CP violation, or electron and muon lepton number nonconservation or some combination of the above. Very little serious consideration has been given to phenomenological consequences of low mass Higgs particles in these schemes. An exception is the work of Deshpande and Ma ("P and CP Nonconservation through Higgs Exchange in Gauge Models with Right-Handed Charged Currents," Oregon preprint OITS-70 (November 1976), and "Possible P and T Nonconservation in ψ' Decays," IOTS-72 (December 1976)) who consider the effects due to low mass Higgs bosons in an $SU(2) \times U(1)$ model with two Higgs doublets and one triplet. The CP violation is patterned after that of Weinberg, Ref. 6, but

the addition of a right-handed quark doublet destroys the natural CP and flavor conservation in the quark sector.

19. P. Fayet, Nucl. Phys. B74, 14 (1974); D. W. McKay and H. Munczek, Phys. Rev. Lett. 34, 432 (1975); D. W. McKay (Refs. 7 and 8).
20. This extra global symmetry can be viewed as an approximate symmetry, and the H_0 a would-be Goldstone associated with the global symmetry. This observation merely motivates the distinction between H_0 and the other, higher mass Higgs particles. Peccei and Quinn ("Constraints Imposed by CP Conservation in the Presence of Instantons," Stanford ITP-572) have recently shown that just such a global symmetry of $V(\phi)$ is needed to guarantee P and CP conservation to arbitrary orders of strong coupling when strong interactions are introduced through a non-Abelian gauge theory of quarks and massless gauge fields. If the weak interaction model under discussion here is joined directly to a strong SU(3) color group, then P and CP conservation in the strong interactions are not naturally conserved. A mysterious adjustment of parameters must be achieved at every order of perturbation theory. The H_0 Higgs field would not be a Goldstone boson in the absence of the ϵ term in Eq. (3.1), since it gains extra mass from strong interaction instanton contributions to the effective potential.

Peccei and Quinn point out that the model for CP violation presented by Weinberg⁶ must have at least four Higgs doublets, not three, in order to satisfy their criterion that a chiral U(1) be an invariance of the starting Lagrangian. The lepton-loop mechanism proposed in the present work reduces to three the minimum number of Higgs doublets. One Higgs doublet must be allowed an arbitrary phase variation, and it would couple to quarks but not to leptons.

21. Taken by itself, $\mathcal{L}_Y(e, \mu)$ is indistinguishable from a universal lepton-Higgs coupling scheme $g[\bar{\psi}_{e_L} \phi_1' e_R + \bar{\psi}_{\mu_L} \phi_2' \mu_R + \text{H. c.}]$ as in S. Barr and A. Zee, "A New Approach to the Electron-Muon Mass Ratio," Princeton preprint (1976) or in D. Gromes, Nuovo Cimento Lett. 10, 209 (1974).

The Higgs Lagrangian distinguishes ϕ_1 and ϕ_2 , however.

22.

$$\frac{G_F}{\sqrt{2}} = \frac{1}{2(\rho_1^2 + \rho_2^2)} ; \quad M_W^2 = \frac{g^2}{8} (\rho_1^2 + \rho_2^2) .$$

23. S. L. Adler, R. F. Dashen, and S. B. Treiman, Phys. Rev. D 10, 3728 (1974).
24. The original two-Higgs doublet potential (3.1) yields real A, regardless of the phase of ϵ as pointed out by Weinberg.⁶
25. The potential (3.1), taken by itself, cannot violate CP since the arbitrary ϕ_1 and ϕ_2 CP phases can be chosen so that $\epsilon(\phi_1^\dagger \phi_2)^2 \leftrightarrow \epsilon^*(\phi_2^\dagger \phi_1)^2$ under a CP transformation. In the presence of other interactions, however, a matching of phase with respect to ϵ will leave CP violation in other parts of the Lagrangian, or vice-versa. In particular, I will use Weinberg's choice that the quark Yukawa couplings be CP conserving.⁶ This leaves a phase for ϵ which must be dealt with. There will be no phase relevant to CP coming from quark-loops. Lepton loops renormalize the phase of ϵ , however.
26. E. Gildener and S. Weinberg, Phys. Rev. D 13, 3333 (1976).
27. M. L. Perl et al., Phys. Rev. Lett. 35, 1489 (1975); M. L. Perl et al., Phys. Lett. 63B, 117 (1976); discussion of the most recent data is given in M. L. Perl, Stanford Linear Accelerator Center preprint SLAC-PUB-1923 (April 1977), to be published in Proceedings of the XII Rencontre de Moriond, edited by Tran Thanh Van, RMIEM, Orsay.

28. E. Ma, S. Pakvasa, and S. F. Tuan, "Has a Higgs Particle Been Seen?," University of Hawaii preprint UH-511-231-77.
29. Hadronic gauge field corrections to the quark loops are not accounted for here. The threat that very heavy quarks of the type discussed in this section poses for natural suppression of higher order effects in flavor changing neutral currents is studied by E. Poggio and H. Schnitzer (Phys. Rev. D 15, 1975 (1977)). Basically, small mixing between light and heavy quarks must be enforced, as done here. The question of heavy quark effects on broken symmetry stability is not pursued in this paper.
30. The notation is that of E. D. Commins, Weak Interactions (McGraw-Hill, New York, 1973).
31. S. Gjesdal et al., Phys. Lett. 52B, 113 (1974). I am grateful to V. Luth for bringing this reference to my attention. Complete references are listed in the discussion of K_L^0 in Particle Data Group, Rev. Mod. Phys. 48, 2, Part II (April 1976).

TABLE CAPTIONS

- I. Estimates of the mass of the light Higgs scalar H_0 for two different assumptions about the modulus of the vacuum expectation values. The mass of the charged Higgs scalar, M_{H^+} , is taken to be $M_{H^+} = 10^2 M_p$, $N=10^{-1}$ as described in Appendix A, and $F=M_\mu$ in the case of Section III model and $F = \sqrt{M_E M_M}$ for Section IV model.
- II. Estimates of the contribution to an effective neutron form factor slope, Δa_N , which would result from H_0 exchange for the various cases corresponding to those of Table I. The upquark coupling is used as a crude estimate of the neutron coupling to H_0 with $5M_u=M_\mu$ assumed. $\Delta a_N < .05 \text{ (GeV/c)}^2$ is necessary for agreement between eD and ne scattering data when $M_{H_0} \lesssim 1 \text{ MeV}$. The lower right corner, $\rho_1 \rho_2 = \sqrt{M_E M_M} (8G_F N M_1)$ model IV case, has an M_{H_0} value $\approx 300 \text{ MeV}$ which is far outside the $M_{H_0} \lesssim 1 \text{ MeV}$ range where this Δa_N argument is applicable. The value of $1/\alpha (g_{H_0 \tilde{e}e} g_{H_0 \bar{\mu}\mu} / M_{H_0}^2)$ is listed nonetheless for comparison between high and low mass H_0 exchange at $q^2=0$.

TABLE I

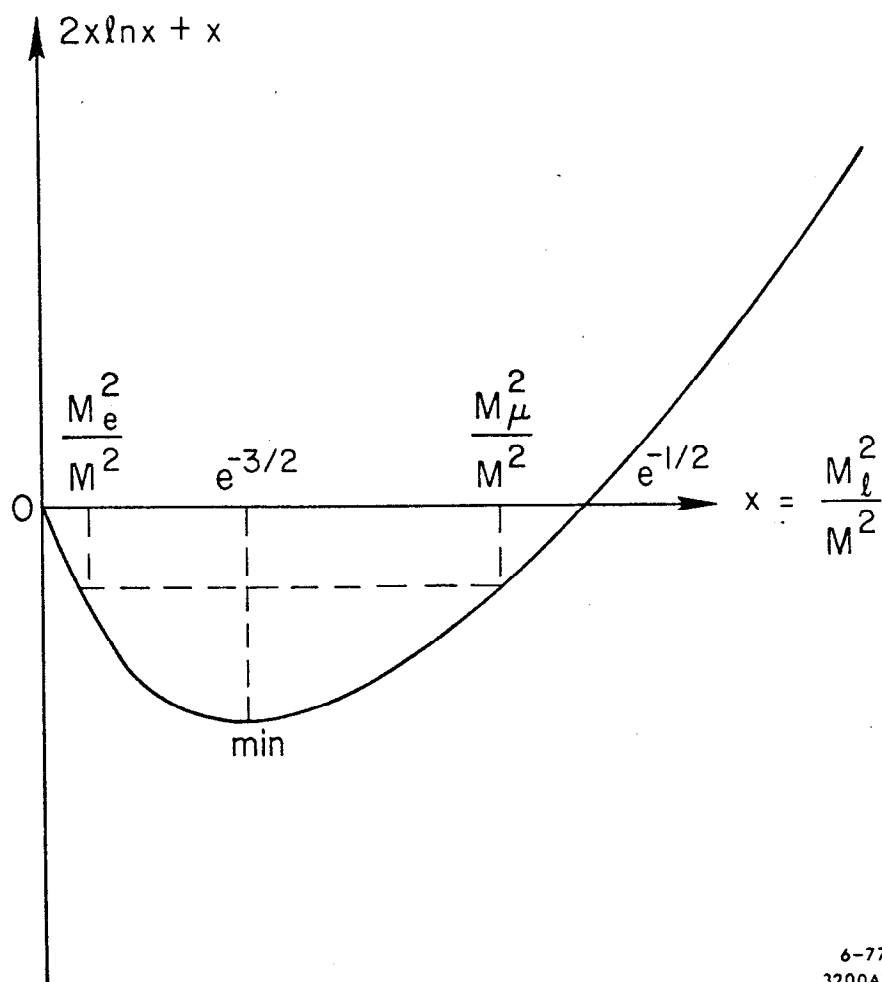
	Model Section III	Model Section IV
$\rho_1 = \rho_2$	$M_{H_0} = 6 \times 10^{-4} \text{ MeV}$	$M_{H_0} = 5 \text{ MeV}$
$\rho_1 \rho_2 = F(8G_F NM_1)^{-1}$	$M_{H_0} = 0.5 \text{ MeV}$	$M_{H_0} = 300 \text{ MeV}$

TABLE II

	Model Section III	Model Section IV
$\rho_1 = \rho_2$	$\Delta a_N = 4 \times 10^8 \text{ (GeV/c)}^2$ $5M_u = M_\mu$	$\Delta a_N = 14 \text{ (GeV/c)}^2$
$\rho_1 \rho_2 = F \left(8G_F N M_1 \right)^{-1}$	$\Delta a_N = 10^5 \text{ (GeV/c)}^2$	$\Delta a_N < .01 \text{ (GeV/c)}^2$

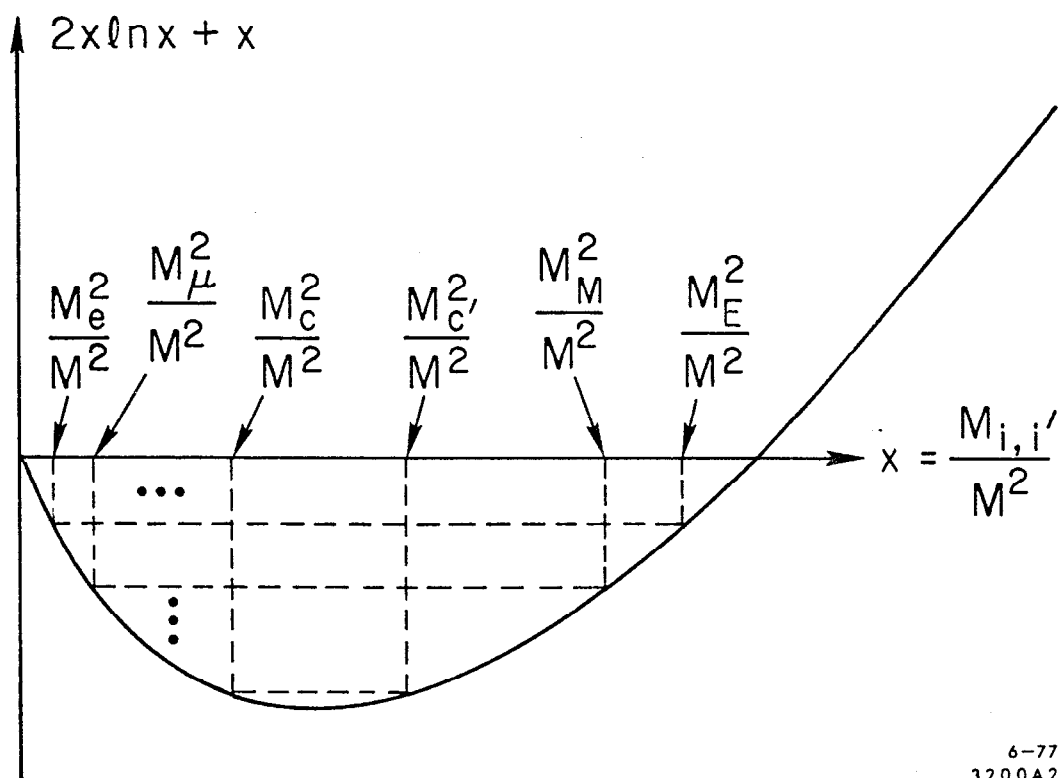
FIGURE CAPTIONS

1. Graph of the function $2x \ln x + x$ vs. $x = M_\ell^2/M^2$, where M_ℓ is lepton mass and M^2 is the mass parameter which replaces ϵ in Eq. (3.1). Double values of this function correspond to solutions of the minimization of the one-loop effective potential in the model of Section III.
2. Same plot as Fig. 1, with the mass pattern indicated which would emerge if the minimization of the effective potential were satisfied term by term in the model of Section IV, Eq. (4.3).



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Fig. 1



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Fig. 2