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$\mu \to e+\gamma$: WHY SO INTERESTING ?*

B. Humpert
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

The attempts at an implementation of muon number nonconservation in the gauge theory framework are presented. After discussing the experimental and theoretical constraints in constructing models, we focus on the proposed explanations for a possible violation of muon number and investigate the experimental implications.

The background of $\mu \to e+\gamma$ due to radiative muon decay is analyzed with the conclusion that it cannot explain the six events seen at SIN.

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I. INTRODUCTION

Recent months have witnessed considerable excitement in weak interaction theory which was initiated by reports of unpublished experimental results. Theorists constructed explanations, sometimes behind closed doors, and papers appeared at a rate of two per day.[1] Why was there so much interest in the particular reaction $\mu \to e + \gamma$ - even without useful experimental results?

The successful unification of weak and electromagnetic interactions within the framework of non-Abelian gauge theories led to the triumphant discoveries of neutral currents and most recently to the detection of charmed particles.[2] One would be tempted to conclude that the first $SU_2 \times U_1$ model by Weinberg and Salam[3] correctly describes all known data and phenomena in weak interaction physics. However, in recent years more and more experimental information has accumulated which clearly indicates inconsistencies and hints at needed extensions. Many attempts at generalization of the "minimal" model have therefore been proposed recently which extend the number of quarks, leptons, gauge bosons, and needed Higgs particles beyond the simplest construction. At present, there exists a great deal of latitude in generalizing the minimal model, a freedom which can only be reduced by more experimental and theoretical limitations. The reaction $\mu \to e + \gamma$, currently being sought at various "meson factories", can provide new constraints in this undertaking. In the minimal $SU_2 \times U_1$ model it could not occur; attempts to accommodate it in a more general framework immediately led to the more fundamental question: How can muon number nonconservation be incorporated into gauge theories?

Study of these questions revealed that violation of muon number can easily be obtained and it is rather the experimentally suppressed rate which needs to be understood. The explanations put forward can be grouped as follows:

(i) more Higgs doublets

(ii) left-handed mixing schemes

(iii) right-handed mixing schemes

(iv) other schemes.

The focus on these theoretical aspects of gauge theories also naturally leads to inspection of a number of other muon number nonconserving processes[4] such as

- $\mu \to e+\gamma$ and $\mu \to ee\bar{e}$
- $\mu + (A, Z) \to e + (A, Z)$
- $K_L \to e\bar{\mu}$ and $K \to \pi e\bar{\mu}$

all of which provide constraints on weak model building and the mechanisms of lepton number violation.

One of the most interesting aspects of muon number nonconservation is that its explanation requires Feynman diagrams with new virtual leptons and/or bosons. Study of higher order weak interaction effects leads us to hypothesize upon the existence of new, as yet undiscovered, leptons in the theory whose existence may only be verified after years of effort.

This paper gives an introduction to the present problems in weak interaction theory and seeks to describe the theoretical and phenomenological questions arising in the experimental search for muon number violating transitions. Data on the well-known muon decay modes have become available[5] and will be analyzed. In particular, we will concentrate on the radiative decay $\mu \to e\bar{\nu}_e \nu_\mu \gamma$ and discuss its electron and photon energy distribution.

This paper is organized as follows: In Section II we briefly present the minimal (or standard) $SU_2 \times U_1$ model with its basic theoretical characteristics and the attempts at its generalization;[16] we sketch its successes and shortcomings in describing the available data. Section III reviews the theoretical explanations for muon number violation in weak interactions and discusses the predictions and implications of these schemes for the above reactions. Section IV is devoted to an analysis of the recently available data on μ-decay from SIN[5] and gives predictions for the background reaction $\mu \rightarrow e\bar{\nu}_e \nu_\mu \gamma$. Our conclusions are presented in Section V.

II. $SU_2 \times U_1$ MODELS AND BEYOND

The successful unification of weak and electromagnetic interactions within one framework provided the instrument required for a satisfactory description of weak processes at very high energies. The construction of this theory is not unique but the elegance of the first, most simple, $SU_2 \times U_1$ model is considered as the most convincing guide for a generalization of this framework, including more than the four familiar leptons.

1. The Minimal $SU_2 \times U_1$ Model

Leptons are grouped in left-handed doublets

$$L \equiv \left\{ \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \right\} \qquad (\ )_L \equiv \left(\frac{1+\gamma_5}{2}\right)(\ )$$

and right-handed singlets

$$R \equiv \left\{ e_R, \mu_R \right\} \qquad (\ )_R \equiv \left(\frac{1-\gamma_5}{2}\right)(\ )$$

where e, μ, ... stand for the electron, ... fields. The Lagrangian then reads

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_I$$

with

$$\mathscr{L}_0 = \mathscr{L}(\vec{A}^\mu) + \mathscr{L}(B^\mu) + \mathscr{L}(R) + \mathscr{L}(L)$$

and

$$\mathscr{L}_I = \sum \left[\frac{g}{2} \bar{L}(\vec{\not{A}} \cdot \vec{\tau}) L - \frac{g'}{2}(\bar{L}\not{B}L + 2\bar{R}\not{B}R) \right]$$

$\vec{A}^{\mu}$ and B^{μ} are a triplet and a singlet of four-component vector fields. Linear combinations of these vector fields describe the charged and neutral W-bosons and the photon.

Notice that the first term in $\mathcal{L}_I$ describes the SU_2 part and the second one, which commutes with the generators of the SU_2 group must be of U_1-type. Thus the minimal group of this model is $SU_2 \times U_1$. Therefore, once the doublets and singlets of the assumed leptons are formed and the group structure is specified, the interaction Lagrangian is determined. In the above form all particles are supposed to be massless. The masses are generated through spontaneous symmetry breaking by the coupling of (at least) one doublet of scalar fields $\begin{pmatrix}\phi^+\\ \phi^o\end{pmatrix}$ to the vector fields $\vec{A}^{\mu}$ and B^{μ} and the fermions in the theory. One can now go through the same chain of arguments in constructing the weak interaction Lagrangian for quarks. We therefore can limit ourselves from now on to the grouping of the fermions, the vector bosons, and the Higgs mesons of the minimal model, which is well known.[6]

The charged and neutral currents are easily evaluated

$$j^{\mu}_{W^{\pm}} = \Sigma \bar{\ell}_1 \gamma^{\mu}(1-\gamma_5)\ell_2$$

$$j^{\mu}_{Z} = \frac{1}{2} \Sigma[\bar{\ell}_1\gamma^{\mu}(1-\gamma_5)\ell_1 - \bar{\ell}_2\gamma^{\mu}(1-\gamma_5)\ell_2] - 2\sin^2\theta_W \cdot j^{\mu}_{em}$$

where ℓ_1 (ℓ_2) refers to the upper (lower) component of the doublets and j^{μ}_{em} stands for the electromagnetic current. θ_W is the Weinberg angle.

This framework can easily be generalized by admitting more leptons in the $SU_2 \times U_1$ scheme or by using a different classification group for the leptons with possibly more leptons. The constraints in such an undertaking are: close analogy between the hadronic and leptonic sector (which of course can be given up), simplicity and theoretical attractiveness, and correct description of the experimental results.

2. Properties of the Minimal Model[6]

The model presented above implies a number of fundamental theoretical and phenomenological consequences which are expected to be satisfied by any more general scheme.

(i) The theory is free of anomalies and can be renormalized.

(ii) The quark and lepton masses, as well as the Cabibbo angle, are parameters of the theory. No mass relation exists, nor does there exist any understanding of the mass pattern. The masses of the vector bosons are related by $m_Z = m_W/\cos\theta_W$ due to their generation by spontaneous symmetry breaking; the magnitude of the masses is determined by the coupling parameters g and g' and the vacuum expectation value $<\phi^o>$, which also determines the Higgs-meson mass.

(iii) Universality of weak couplings still holds; the muon doublet couples to the intermediate vector bosons with the same strength as does the electron doublet.

(iv) There is no CP-violation in the minimal model with one Higgs doublet only. It can however be included in three different ways: (1) We remain within the framework of the standard model but introduce a larger set of Higgs doublets; this permits introduction of an arbitrary relative phase parameter between the interactions of different Higgs particles, which will result in CP-violation. (2) The right-handed quarks are grouped in doublets too, which results in new currents. (3) In the framework of the standard model with V-A structure, more quark-doublets are introduced; at least six quarks then are needed.

(v) Charm is the only new degree of freedom which can be produced in deep-inelastic lepton-scattering and in e^+e^- annihilation processes. A number of new weak interaction models with additional quarks and leptons have been suggested, which, however, in most cases are lacking compelling theoretical elegance and/or have to be abandoned if their predictions are confronted with the data.

(vi) The neutral current Z^{o} is a superposition of vector and axial vector currents; consequently Z^{o} exchange leads to parity violation. However, models with six (or more) basic fermions and left- as well as right-handed doublets lead to parity conserving neutral currents.

(vii) The flavor-changing neutral currents and the $\Delta S = 2$ effects are naturally suppressed to order $O(G_F)$ and $O(\alpha G_F)$. This suppression takes place in any $SU_2 \times U_1$ model under the following conditions: (1) Quarks of a given charge and chirality have the same weak $\vec{T}^2$ and T_3. (2) Quarks of a given charge receive their mass <u>either</u> from a gauge-invariant bare mass term <u>or</u> from their couplings with a single neutral Higgs field. These results have been extended to suppress naturally $\Delta S = 2$ transitions which may (or may not) violate CP-invariance. The needed condition, additional to the ones given above (without the bare mass option), is: (3) quarks of charge q and quarks of charge $q \pm 1$ do not belong to the same multiplet for at least one chirality.[7]

(viii) Baryon and lepton numbers are conserved and in addition electron number and muon number are <u>separately</u> conserved. This is a limitation of the minimal model which can easily be modified in many ways such that muon number is no longer conserved. In analogy to the suppression mechanisms in the hadronic sector, Lee and Shrock[26] have determined the general conditions, in the framework of $SU_2 \times U_1$, under which muon number violating transitions are possible, but naturally suppressed, i.e., suppressed like

$G_F \cdot \alpha \cdot \frac{\Delta m^2}{M_W^2}$. These are: (1) leptons of a given charge receive their mass from couplings with a single neutral Higgs field, (2) leptons of a given charge and chirality have the same weak $\vec{T}$ and T_3, and (3) leptons of charge q and leptons of charge q ± 1 do not belong to the same isomultiplet for at least one chirality. Through specific model studies they found that, for a measurable effect of muon number violation, a model must include at least one massive neutral or doubly charged heavy lepton which is coupled to both electron and muon.

3. Phenomenological Constraints

The most impressive successes of non-Abelian gauge theories in general and the minimal $SU_2 \times U_1$ model in particular are, without doubt, the discoveries of neutral currents and most recently of charmed particles. We therefore briefly enumerate in the following the data characteristics which stand in favor of the gauge theory framework together with the recently accumulated experimental evidence that extension of the minimal $SU_2 \times U_1$ model might be considered. These are:

(i) The charged heavy leptons $\tau^{\pm}$, which most likely couple to their own neutrinos ν_τ, have to be included.[8]

(ii) The ratio of deep-inelastic charged current cross sections $R = \sigma(\bar{\nu} \to \mu^+)/\sigma(\nu \to \mu^-)$ is expected to remain constant ($\sim 1/3$) as we go to higher energies - instead it rises substantially around $E_\nu \sim 80$ GeV. The average value of the $\frac{d\sigma}{dy}(\bar{\nu} \to \mu^+)$ -distribution also rises in this region, whereas for neutrinos it does not change significantly with increasing energy.[9] Qualitatively such an effect was expected in strong interaction dynamics due to scaling violations implied by an underlying field theory (QCD). The experimental effect is however larger; it is well described by assuming that another b-quark exists which is grouped together with a u-quark in a right-handed doublet.[10]

(iii) The ratios of neutral to charged currents

$$R_\nu = \frac{\sigma(\nu \to \nu)}{\sigma(\nu \to \mu^-)} \quad , \quad R_{\bar{\nu}} = \frac{\sigma(\bar{\nu} \to \bar{\nu})}{\sigma(\bar{\nu} \to \mu^+)}$$

are both nonzero. The measurement of these ratios proved to be the key stone in the establishment of neutral currents. The "ratio of ratios"

$$R_\nu \equiv \frac{R_{\bar{\nu}}}{R_\nu} = \frac{\sigma(\bar{\nu} \to \bar{\nu})}{\sigma(\nu \to \nu)} \sim 0.5$$

is significantly different from unity.[11]

(iv) Similar results exist for elastic neutrino-proton scattering where one finds

$$\sigma(\bar{\nu} p \to \bar{\nu} p) < \sigma(\nu p \to \nu p) \quad .$$

In ν_e-e and ν_μ-e scattering the situation is less clear.[12]

(v) In the minimal model, Z^o-exchange leads to parity violation which is presently tested in atomic physics experiments on bismuth. Two groups have recently published a result that seems to be significantly below the predictions of the minimal model which would indicate no parity violating terms in neutral currents. The theoretical and experimental analysis of this experiment is still in progress.[13]

(vi) The value of $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ provides us with a measure of the sum of the squared charges of all fundamental pointlike fermions which are produced at a given energy. In the range $4.5 \leq E_{c.m.} \leq 8$ GeV the value of R is approximately constant and is given by $R \sim 5 - 5.5$. The minimal model predicts $3\frac{1}{3}$. Approximately 2 ± 1 units of R remain unexplained.[14] They may be due to additional quarks and/or additional leptons.

The absolute minimum would be one charged lepton or one quark with $Q = +\frac{2}{3}$.

(vii) Most recently the production of six trimuon events has been reported which are considered as possible evidence for the sequential decay of new heavy leptons,[15] and the existence of neutral heavy leptons is anticipated.[16]

Besides the above-presented experimental information which imposes strong constraints on "model-building" in weak interaction theory, more restrictions are expected from data in the near future:

(i) If muon number violating transitions indeed are found the minimal model needs modification to include and predict the correct rate of such effects. Possible explanations are given in Section III.

(ii) Experimental tests of CP-violation are not simple, but might lead to positive results in charmed particles decays.[17]

4. New Models[6]

Despite the constraints imposed by phenomenology and by the expected theoretical properties to be satisfied by a "good" theory, there is still considerable freedom in such constructions which is reflected by the wide variety of new models being proposed. In the framework of $SU_2 \times U_1$ we mention four types of models:

- Extended Minimal Model: The minimal model is enlarged by adding the quark doublet $\binom{t}{b}_L$ in the hadronic sector and the lepton doublet $\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$ in the leptonic sector; their right-handed partners as well as all possible other new fermions (such as neutral heavy leptons) are assumed to be in the right-handed singlet part.
- The Vector Model: A substantially different way of extending the minimal $SU_2 \times U_1$ model is to accept left- and right-handed doublets only and to

exclude any singlet parts in the hadronic as well as leptonic sector. The smallest vectorlike scheme of this kind is based on six quark flavors and six lepton flavors which include new neutral and charged heavy leptons. This model is distinct in that it reduces to a pure vector theory if the masses of leptons and quarks are set to zero; neutral currents are therefore vectors and have no intrinsic axial vector parts.

Parity violation is not intrinsic (as for instance in the 'extended standard model') but is due to the presence of parity-violating terms in the fermion mass matrix.

- The Ambidextrous Model:[18] Since the "high-y anomaly" is not satisfactorily explained by the extended standard model and the vector model removes the parity-violating characteristics believed to be intrinsic in weak interactions, a unifying model has been proposed which goes beyond the established group structure of the simplest model and uses $(SU_2)_L \times (SU_2)_R \times U_1$. Six quark and six lepton flavors are assumed which include heavy neutral and charged leptons. All flavors of $(SU_2)_L$ are singlets under $(SU_2)_R$ and vice versa and only $(E^o, M^o, U^o)_L$ are overall singlets under both groups. This model goes beyond the conventional three vector bosons $(W^{\pm}, W^o)$ and introduces a pair of triplets $\vec{W}_{L,R} \in (SU_2)_{L,R}$ and a singlet $X \in U_1$ - thus seven gauge bosons in all. In addition at least three multiplets of Higgs mesons are assumed.
- Triplet Models:[19] The models presented above all assume two-dimensional extensions of the minimal model. One can assume three-dimensional extensions instead - still within the framework of $SU_2 \times U_1$. Such suggestions admit the appearance of doubly charged leptons (E^{++}, M^{++}) and new quarks (a, v) with the unusual charges 5/3 and -4/3. At least two Higgs triplets are

then required and many scalar particles, some doubly charged, are expected. In contrast to the minimal model, the neutral current coupling to the electron and muon is purely vector. A variation of this scheme, putting the doubly charged leptons together with electron and muon in right-handed doublets, leads to parity violation twice as strong as in the minimal model. A spectacular rise of R in e^+e^- annihilation should be seen as we go to higher energies.

Obviously one can completely abandon $SU_2 \times U_1$ models and go to <u>more complex group structures</u>. However the problem then arises of how to select the right group from the variety of possibilities.[2] One way is to try higher dimensional groups like $SU_3 \times U_1$ ... seeking to incorporate all theoretical properties and data characteristics assembled so far. Weinberg and Lee[20] have recently proposed such a model which accounts for the trimuon events.[15] The eight quarks of this model are grouped in two left- and two right-handed triplets whereas the twelve leptons, six "conventional" and six new, are in three left-handed and three right-handed triplets. This theory needs an octet and one singlet of vector fields V_a^μ which act between the different groupings of the fermions. The interaction Lagrangian which contains the full symmetry structure of the model reads:

$$\mathscr{L}_I = \sum_{L,R} \left[\frac{g}{2} \left\{ \bar{L}(\not{V}_a \lambda_a) L + \bar{R}(\not{V}_a \lambda_a) R \right\} - \frac{g'}{2} \left\{ \bar{L} \not{V}_o L + \bar{R} \not{V}_o R \right\} \right]$$

In general, in such models strangeness-changing neutral currents are not suppressed and the universality of lepton couplings is destroyed. However, these characteristics can be maintained by imposing a discrete symmetry R under which gauge bosons and right-handed fermions are invariant whereas left-handed fermions change sign.

The similar space-time structure of quarks and leptons has led to the hypothesis that they might belong to the same multiplet of fundamental fermions. If weak and electromagnetic interactions are described by a (Weinberg-Salam) vector gauge theory and the strong interactions are described by a vector gauge theory (Quantum Chromo Dynamics) as well, then perhaps there is one large representation including all gauge bosons. Such a point of view has initiated the search for a larger group G which can incorporate $(SU_3)_{color}$ and $(SU_2 \times U_1)_{weak}$ in one "grand unification". As a minimal example, SU_5 has been suggested,[21] while the group E_7 is considered as an interesting candidate for a maximal model.[22] The E_7-scheme is maximal in the sense that it incorporates all quarks and all leptons in one multiplet. Its main predictive power is a consequence of the explicit set of fermions which cannot be extended. The main difficulties with these unification schemes are:

(i) New gauge bosons (leptoquarks) are predicted which convert quarks into leptons and vice versa.

(ii) Baryon number is violated and/or lepton number is conserved.

What do we conclude from the above? Determining the correct extension of the simplest unifying scheme is difficult, mainly because the number of existing fermions in the theory is unknown and since possible extensions of its group structure are almost unconstrained by the experiment. More theoretical and experimental constraints are needed. One such new constraint, the violation of muon number, will be considered in the following section.

III. $\mu \to e+\gamma$: NONCONSERVATION OF MUON NUMBER

The study of muon number violation within the framework of gauge theories leads to a wealth of possible explanations.[1] These theoretical investigations stimulated the exploration of further experimental tests for such a property in nature. Aside from the decay $\mu \to e+\gamma$, which obviously could give direct proof, other reactions have been suggested which will be presented before concentrating on the proposed mechanisms for muon number nonconservation.

1. Experimental Tests

The proposed experimental tests can be grouped into three classes: muon decays, K-decays and μ-e conversion on nuclei.

μ-decays: The decay $\mu \to e+\gamma$ is described by the matrix element

$$M(\mu \to e+\gamma) = \bar{u}_e(f_{M1}+f_{E1}\gamma_5)\, i \cdot \epsilon^{\alpha}\sigma_{\alpha\beta}\cdot q^{\beta}\, u_{\mu}\left(\frac{e}{m_{\mu}}\right) \tag{3.1}$$

leading to the decay width

$$\Gamma(\mu \to e+\gamma) = \left(\frac{\alpha m_{\mu}}{2}\right)\left(|f_{M1}|^2 + |f_{E1}|^2\right) \tag{3.2}$$

The angular distribution of the outgoing electron (positron) with respect to the initial polarization vector $\hat{\mu}$ is

$$I_{\pm}(\theta) = 1 \mp \alpha \cos\theta \tag{3.3}$$

where $\cos\theta \equiv \hat{p}_e \cdot \hat{\mu}$; the asymmetry parameter α is

$$\alpha = \frac{2\mathrm{Re}(f^*_{M1}\, f_{E1})}{|f_{M1}|^2 + |f_{E1}|^2} \tag{3.4}$$

Therefore measurement of the electron decay's angular distribution determines the relative sizes of f_{M1} and f_{E1} which are specified differently by different models of this decay. The dominant μ-decay mode is $\mu \to e\bar{\nu}_e \nu_\mu$ with a width:

$$\Gamma_\mu = \frac{G_F^2 \, m_\mu^5}{192 \, \pi^3} \qquad (3.5)$$

In gauge theories $\mu \to e+\gamma$ only occurs through virtual transitions (Fig. 1) which can involve intermediate Higgs-, W- and Z^o-bosons and a photon as well as the allowed charged and neutral leptons of the theory.

The decay $\mu \to ee\bar{e}$, as will be seen later, can be decisive in distinguishing between different muon number violating mechanisms. Such a process certainly can occur through photon exchange. Substantial contributions can also come from Z^o-exchange or a loop involving charged W-bosons (Fig. 2).

<u>K-decays</u>: The decay $K_L \to e\bar{\mu}$ as compared to $K_L \to \mu\bar{\mu}$ can give further information; it forms the hadronic analogue to $\mu \to ee\bar{e}$ apart from the fact that Z^o-exchange is excluded due to the GIM-mechanism which suppresses all strangeness changing neutral currents. The (μ, e) pair on one side of the diagrams in Fig. 2 is replaced by the (s, d) quark-pair which forms the K-meson. Thus the loop-diagram of Fig. 3 is appropriate for the description of $K_L \to e\bar{\mu}$; $K_L \to \mu\bar{\mu}$ however is not adequately described and requires consideration of the $\gamma\gamma$ exchange which turns out to be substantial. Similarly search for $K \to \pi e\bar{\mu}$ and its comparison with the muon number conserving process $K \to \pi e\bar{e}$ will be interesting since this process is described by $s \to de\bar{\mu}$ which is just Fig. 3 with the d-quark line crossed.

<u>μ-e conversion</u>[23]: A muon trapped in the field of a nucleus with atomic number A and charge Z is usually converted to a neutrino by the process

$$\mu + (A, Z) \to \nu_\mu + (A, Z-1)$$

However if muon number is not conserved, the conversion

$$\mu + (A,Z) \rightarrow e + (A,Z)$$

can not be excluded and in fact is predicted to be quite substantial in specific models. A number of authors investigated the theory of this process about 25 years ago—experimental searches are expected to provide results in the near future.[24] The theoretical description of μe-conversion requires consideration of Z^o-exchange between the lepton pair and the quark-pair as well as investigation of the resulting W-loops (see Fig. 4). The $\mu \rightarrow e$ vertex is described by the most general Lorentz-invariant ansatz for $\langle \mu | J^{\mu} | e \rangle$ which introduces the four form factors f_{E0}, f_{E1}, f_{M0}, f_{M1}; their size is specified by the chosen diagrams. The relative rate of the above two processes then is

$$\begin{aligned} R(\mu \rightarrow e) &\equiv \frac{\omega(\mu \rightarrow e)}{\omega(\mu \rightarrow \nu)} = 32\pi^2 \alpha^2 Z \ |F_{NN'}|^2 \cdot \xi(f) \\ \xi(f) &\equiv \frac{|f_{E0}+f_{M1}|^2 + |f_{M0}+f_{E1}|^2}{8G_F^2 \cdot m_\mu^4} \end{aligned} \tag{3.6}$$

$F_{NN'}$ is the inelastic nucleon form factor and Z represents the nucleon charge: $Z \cdot |F_{NN'}|^2 \sim 6\text{-}7$.

In the preceding subsection we have presented experiments where muon-number violation could be detected and introduced the Feynman diagrams for the description of such phenomenon. The essential questions now are:

(i) what are the leptonic fermions in the theory and how are they related?

(ii) what is their coupling strength?

(iii) what is their dynamical behavior, V+A or V-A?

The answer to these questions is closely related to the choice of a gauge theory model; once it is fixed the counting rates for the above processes can formally be determined and the only undetermined quantities remaining are masses, mixing parameters, Higgs couplings, etc.

2. Higgs Mesons[25]

The introduction of a second doublet of Higgs particles in the minimal $SU_2 \times U_1$ model is a first possibility leading to muon number nonconservation. Doublets, consisting of charged and neutral Higgs fields, couple to the leptons in $SU_2 \times U_1$ invariant manner through

$$\mathscr{L}_{\text{Higgs}} = \sum \lambda_i \, \bar{L} \Phi_i R \, , \qquad \Phi_i \equiv \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix} \tag{3.7}$$

The leading contributions to $\mu \to e\gamma$ decay are not the one-loop diagrams of the type shown in Fig. 1b since the Higgs-lepton couplings are weak; such diagrams make a small contribution as compared to the two-loop graphs (Fig. 5) in which the Higgs bosons couple only once to leptons. The ratio of these two contributions is

$$\frac{1\text{ loop}}{2\text{ loop}} \approx \frac{2\pi}{\alpha}\left(\frac{m_\mu}{m_H}\right)^2 \quad . \tag{3.8}$$

For $m_H > 3$ GeV the two-loop diagram clearly dominates and the branching ratio to the standard weak process becomes

$$BR(\mu \to e+\gamma) \simeq 3\left(\frac{\alpha}{\pi}\right)^3 \sim 4 \times 10^{-8} \quad . \tag{3.9}$$

It is interesting to note that this value is independent of any masses in the theory. Since $f_{E1} \simeq f_{M1}$, the angular distribution for $\mu^+ \to e^+ + \gamma$ will be $(1-\cos\theta)$. What is the size of the other muon number violating processes? $\mu \to e\gamma\gamma$ could in principle occur through single Higgs-exchange however the counting rate is substantially below $\mu \to e+\gamma$ and therefore this process is expected to be dominated by inner bremsstrahlung. The reaction $\mu \to 3e$ also can occur via tree

Higgs-exchange with

$$\mathrm{BR}(\mu \to 3e) \simeq \frac{m_\mu^2 m_e^2}{m_H^4} \times 0(1) \lesssim 10^{-11} \tag{3.10}$$

which indicates that this mechanism is strongly suppressed for larger values of the Higgs boson mass. Muon number violating K-decays such as $K_L \to e\bar{\mu}$ and $K \to \pi\bar{\mu}e$ could in principle be substantial; however the K_L-K_S mass difference and the $K_L \to \mu\bar{\mu}$ rate then would become too large; if only one Higgs doublet couples to the quarks all such decays are forbidden in lowest order and no problem arises.

$\mu N \to eN$ conversion by single Higgs exchange is suppressed with respect to the analogous neutrino transition $\mu N \to \nu N$ by a factor

$$R(\mu \to E) \approx \frac{A^2}{Z} |F|^2 \cdot \frac{m_\mu^2 m_{N^*}^2}{m_H^4} \sim 4 \cdot 10^{-9} \tag{3.11}$$

which is just below the experimental limit if $m_H = 30$ GeV and if the nucleon mass arising from the "bare" quark masses $m_N^* = 100$ MeV; for smaller Higgs boson masses a much larger rate could be obtained. A and Z are the atomic number and the charge of the nucleus and F is the elastic form factor at $-q^2 = m_\mu^2$.

3. Left-Handed Mixing[26]

In the hadronic sector of the minimal $SU_2 \times U_1$ model the d and s quarks appear as mixed states. The assumption of small neutrino masses ($m_{\nu_e} < 30$ eV, $m_{\nu_\mu} < 500$ eV) then admits muon number violation through a leptonic version of the Cabbibo rotation:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{3.12}$$

Such a theory allows $\mu \to e+\gamma$ transitions since ν_e and ν_μ may appear as mixed intermediate states in the loops of Fig. 1 giving:

$$BR(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \left(\sin\phi \cos\phi \frac{m_{\nu_1}^2 - m_{\nu_2}^2}{M_W^2} \right)^2 \tag{3.13}$$

Even if a large violation of muon number is introduced by a large mixing angle, the induced rate is far too small to be measurable: $BR(\mu \to e\gamma) \lesssim 10^{-26}$; this can not be the correct explanation of such effect. The above method however indicates how sizable effects of muon number nonconservation in the framework of $SU_2 \times U_1$ still can be achieved if new particles, grouped in doublets or triplets, are admitted.

The most obvious extension of the minimal $SU_2 \times U_1$ model is obtained by adding a third (or more) left-handed doublet(s) $\begin{pmatrix} L \\ U^- \end{pmatrix}_L$ . . . where L here stands for massive (or massless) neutral leptons and U^- for the charged ones. Their right-handed partners are all singlets. This generalized model, as it stands, still can not accommodate muon number violation unless mixing of the neutral lepton states L_j through the matrix ξ is allowed: $(L_j') = (\xi_{ij}) \cdot (L_i)$. The amount of muon number nonconservation then is hidden in the matrix elements ξ_{ij}.

The evaluation of the rates for $\mu \to e\gamma$, $\mu \to 3e$ and $\mu \to e$ conversion needs consideration of the Feynman diagrams in Figs. 1, 2, 4 with L being the only intermediate lepton. The result is

$$\begin{aligned} BR(\mu \to e\gamma) &= \frac{3\alpha}{32\pi} \delta^2 \\ BR(\mu \to 3e) &= \frac{3}{2^8} \left(\frac{\alpha}{\pi}\right)^2 (\delta\Lambda)^2 \\ R(\mu \to e) &= \frac{1}{2^8} \left(\frac{\alpha}{\pi}\right)^2 \cdot \left(\delta(1+\Lambda)\right)^2 \cdot Z \cdot |F_{NN'}|^2 \end{aligned} \tag{3.14}$$

where $\delta \equiv |\xi_{23} \cdot \xi^*_{13}| \cdot \epsilon$ and $\epsilon \equiv m_L^2/M_W^2$. Λ is a complicated function dependent on $\ln \epsilon$, the charge Z and atomic number A of the nucleus, the weak hypercharge Y_S of S^+ and the Weinberg angle θ_W; its explicit form is given in Ref. 27 (Eq. (14b)). Note that the $\mu \to e\gamma$ transition depends only on the form factors f_{M1} and f_{E1}, whereas $\mu \to 3e$ is a function of f_{E0} and f_{M0}; both types of form factors influence μ–e conversion. All three rates depend in a simple way on the mixing parameters of the theory; the amount of muon number violation is the same in all three processes. The ξ_{ij} are only constrained by the requirement of μ-e universality, giving $|\xi^*_{13}\xi_{23}| \lesssim 10^{-1} - 10^{-2}$. However there are differences in the contributing and dominating diagrams which substantially influence the "branching ratios" of the various processes and consequently the L-mass dependences are different. $\mu \to e\gamma$ is dominated by Fig. 1a with a charged W-boson loop where the outgoing photon is attached. $\mu \to 3e$ and $\mu \to e$ conversion are dominated by Z-exchange and the W-loops (Fig. 2) which bring the additional $\ln \epsilon$ factor. Choosing $m_L \sim 10$ GeV, $M_W \sim 60$ GeV and $|\xi^*_{13}\xi_{23}| \sim 0.05 - 0.005$, the numerical values given in Table I result. Note that $\mu \to 3e$ is on the %-level of $\mu \to e\gamma$. The ratio of these two processes is independent of the mixing parameters and proportional to $\ln^2 \epsilon$. The origin of this term can be traced back to the Z-exchange diagram. We thus obtain information on the mass of the neutral heavy lepton. Comparing $R(\mu \to e)$ from μe-conversion with the branching ratio for $\mu \to e\gamma$ one finds the surprisingly large number of $\sim$30. Thus if a left-handed neutral heavy lepton is the reason for muon number violation such effect should be detectable in μe-conversion experiment. The angular distribution of the positron in $\mu^+ \to e^+\gamma$ is (1–cos θ) due to the left-handedness of the new L-lepton. If $L \equiv \nu_L$ the rate falls below the measurable values and other mechanisms have to be sought; the massiveness of the neutral heavy lepton is essential. Estimates for $K_L \to e\bar{\mu}$ and $K \to \pi e\bar{\mu}$ lead to unmeasurably small values.

An alternative method of extending the minimal $SU_2 \times U_1$ model, while retaining a left-handed scheme, is obtained if doubly charged heavy leptons (≡ 'heptons') are admitted which are added to the "old" doublets forming now triplets.[19] Heptons are supposed to be mixed states of two mass eigenstates with masses m_{L_1} and m_{L_2} with a mixing angle ϕ; mixing with other lepton states does not occur. Lepton number violation here proceeds in close analogy to the above scheme; in all loop diagrams the propagating leptonic fermions now are the heptons. The rates for $\mu \to e\gamma$, $\mu \to 3e$ and for $\mu \to e$ conversion are:

$$
\begin{aligned}
BR(\mu \to e\gamma) &= \frac{75\alpha}{32\pi}\,\delta^2 \\
BR(\mu \to 3e) &= \frac{1}{6}\left(\frac{\alpha}{\pi}\right)^2 \cdot \delta'^2 \\
R(\mu \to e) &= \left(\frac{\alpha}{3\pi}\right)^2 \cdot \delta'^2 \cdot Z\,|F_{NN'}|^2
\end{aligned}
\tag{3.15}
$$

where

$$\delta \equiv \sum_j c_{1j}\,c_{2j}\,\epsilon_j\ , \qquad \delta' \equiv \sum_j c_{1j}\,c_{2j}\,\ln\epsilon_j$$

with

$$\epsilon_j \equiv \frac{m_{L_j}^2}{M_W^2}\ .$$

The expansion coefficients $c_{1j}(c_{2j})$ parametrize the amount of $e(\mu) - L_j$ coupling and are therefore proportional to the mixing angle ϕ, thus $\delta \equiv \sin\phi\,\cos\phi\,(\epsilon_1 - \epsilon_2)$ and similarly for δ'. The most interesting predictions here are the large rates for $\mu \to 3e$ and for μe-conversion in comparison to $\mu \to e\gamma$; one finds $\frac{\mu \to 3e}{\mu \to e\gamma} \approx 10$ and $\frac{\text{conversion}}{\mu \to e\gamma} \sim 200$. The first ratio is $(m_L)^{-4}$ mass-dependent and was obtained for typical values $m_L \sim 4$ GeV whereas in the second one no mass seems to appear. No predictions exist for the K-decays. The numerical values of this model are summarized in Table I.

4. Right-Handed Mixing[28]

In seeking an extension of the minimal model in a way substantially different from a left-handed scheme, one is naturally led to consider the possibility of introducing neutral heavy leptons in right-handed doublets. The characteristic properties of such vector-like schemes have been presented earlier. The new neutral leptons which group together with a right-handed electron and a right-handed muon are assumed to be mixed like in the neutrino case (see Eq. (3.12)) with mass eigenstates N_1 and N_2 whereas the charged leptons are not. Of course more involved mixing schemes with more doublets are quite possible. The scheme introduced sofar admits only right-handed couplings of the two massive neutral leptons with their leptonic partners. There is however a small probability of left-handed couplings to e_L, μ_L which comes through diagonalization of the lepton mass matrix. Such a step is needed in order to have cancellation of the leading contributions in the one-loop diagrams for $\mu \to e+\gamma$ transitions such that the muon number changing effects remain on the $0\left(G_F^2\right)$-level.[28] What are the predictions of this model?

The rates for the processes under consideration are determined by the formulas

$$\mathrm{BR}(\mu \to e\gamma) = \frac{75}{32}\left(\frac{\alpha}{\pi}\right)\left(\sum_j \delta_j\right)^2$$

$$\mathrm{BR}(\mu \to 3e) = \frac{3}{2^8}\left(\frac{\alpha}{\pi}\right)^2\left(\sum_j \delta_j\Lambda_j\right)^2$$

$$\mathrm{R}(\mu \to e) = \frac{1}{2^8}\left(\frac{\alpha}{\pi}\right)^2\left(\sum_j \delta_j(1+\Lambda_j)\right)^2 \cdot Z\ |F_{NN'}|^2 \tag{3.16}$$

where δ_j, ϵ_j and Λ_j are defined as in Eq. (3.14) for each neutral heavy lepton L_j; the mixing parameters ξ_{ij} are replaced by cos ϕ and sin ϕ, thus $\delta_1 \equiv \sin\phi\ \cos\phi\ \epsilon_1$, $\delta_2 \equiv -\sin\phi\ \cos\phi\ \epsilon_2$, etc. The calculations leading to these results show that the RR transitions in Fig. 1 give the "right" order of magnitude, however LR transitions which are possible due to mixing of the left-handed neutrinos with the neutral heavy leptons, give contributions six times bigger and with opposite sign; therefore the difference of a factor 25 in the final result! It is again interesting to compare the relative rates which reveal $\frac{\mu \to 3e}{\mu \to e\gamma} \sim 3\%$ and $\frac{\text{conversion}}{\mu \to e\gamma} \sim 40\%$ and indicate that $\mu \to e$ conversion is substantial in this model whereas $\mu \to 3e$ is on the few percent level. More precise predictions depend on the parameters of the theory such as the masses of the neutral heavy leptons, the amount of mixing, etc. $K_L \to e\bar{\mu}$ decay is expected on the percent level of $K_L \to \mu\bar{\mu}$ whose branching ratio is $\sim 10^{-8}$, but $K \to \pi e\bar{\mu}$ is strongly suppressed relative to $K \to e\bar{\mu}$ and offers little hope as a test. Because of the ideas described above, neutral heavy leptons have recently come into prominence in theoretical and experimental research. They are expected to be detectable in e^+e^--beams at PEP/PETRA energies through the $e\pi$ and $\mu\bar{\mu}\nu$-decay modes.[16] Searches in neutrino induced reactions might lead to their discovery and the semi-leptonic decay products of charmed particles could contain N's.[29]

5. Other Schemes

The above presented explanations of muon number nonconservation rely, from a phenomenological point of view, on the most obvious possibilities for extension of the minimal model. There exist a few attempts which employ less familiar assumptions but can not be ruled out at the present time as a possible explanation.

Bigger Groups? Once the framework of $SU_2 \times U_1$ is abandoned the existence of more W-bosons is inevitable. One might try $SU_3 \times U_1$ as has been suggested and investigated in a different context by a number of authors.[20] A representative example was presented in Section II.4.

In analogy to the hadron sector where SU_4 flavor now "dominates the scene" one might suggest $SU_4 \times U_1$ for the lepton sector.[30] Twelve new W-bosons result, but no new leptons have to be introduced. The "right" transition rates for muon number violating processes can be accommodated by appropriate mixing of the charged W-bosons.

Based on an attempt to accommodate strong, electromagnetic, weak,... interactions in one unifying scheme in which parity and time reversal (discrete symmetries which stand on an equal footing with the discrete symmetries of SU_n(strong) $\times$ SU_n(weak)) are spontaneously broken, one might be tempted to consider the consequences of a theory with the group structure $(SU_2)_L \times (SU_2)_R \times (U_2^V)$. The elements of $(SU_2)_L$ are the doublets of the minimal model whereas the elements of $(SU_2)_R$ are doublets which contain new neutral heavy leptons together with electron and muon. Four W_L^μ and four W_R^μ are assumed to exist. $\mu \to e\gamma$ transition with a rate just below the present experimental limit can be explained; $\mu \to 3e$ is dominated by Dalitz pairs.[31]

Muon Quantum Number? Without introducing any new leptonic particles muon number violation can be achieved by the introduction of an extra quantum number which all fields other than muon and its neutrino must have. In addition to the fermions of the minimal model, right-handed electron and muon neutrinos, grouped as singlets, are assumed.[32] A minimum of 5 complex doublets of Higgs fields is required which are characterized by the new muon quantum number. The resulting rates for the processes, discussed earlier, are given in Table I. The most spectacular characteristic of this model is that there is no angular asymmetry in polarized muon decay!

H-Boson? In the framework of the extended minimal model with six leptons and six quarks, the existence of a scalar boson H has been postulated which is assumed to be a singlet under the gauge groups SU_2 and U_1;[33] therefore its coupling to left- and right-handed singlets and doublets becomes possible. The counting rates are summarized in Table I. Depending on the relative coupling strengths of the H-boson to the SU_2-doublets and the U_1-singlets, the asymmetry parameter in polarized muon decay takes a value between -1 and +1.

IV. ANALYSIS OF MUON DECAY

The experimental search for muon number violating processes is at present in progress. Muon decay, in particular the reaction $\mu \to e+\gamma$, is considered as one of the best possibilities to uncover such an effect. However the signal from the competing radiative decay channel gives rise to a substantial background. We therefore present here a theoretical analysis, based on standard V-A weak interaction theory, or the reactions $\mu \to e\bar{\nu}_e\nu_\mu$ and $\mu \to e\bar{\nu}_e\nu_\mu+\gamma$ and compare our results with the preliminary data recently available from SIN.[5]

1. $\underline{\mu \to e\bar{\nu}_e\nu_\mu}$-

This muon decay mode dominates by orders of magnitude. It is theoretically described by lowest order weak interaction theory, with the resultant electron energy spectrum

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2\, m_\mu^2}{12\,\pi^3}\, E_e^2 \left(3 - \frac{4E_e}{m_\mu}\right) \tag{4.1}$$

and the total width as given in Eq. (3.5).[34] In Fig. 6 we compare the experimental results (curve 1) with this prediction (curve 3) and show, by a Monte Carlo simulation, the influence of a 10% apparatus resolution. Agreement between theory and experiment can not be fully achieved since radiative corrections have not yet been accounted for. If included, they have the effect of shifting the maximum of curve 2 towards lower energy values.

2. $\underline{\mu \to e\bar{\nu}_e\nu_\mu+\gamma}$

This process is theoretically described by the two Feynman diagrams in Fig. 7 whose matrix elements can easily be determined. The theoretical evaluation of the differential distributions has been given in Ref. 34 in full generality.

For the simplified case of V-A theory we find the structure:

$$d\Gamma = \text{const.}\ dx \frac{dy}{y} d\Omega_e\, d\Omega_\gamma\, \mathscr{F}(x, y, \Delta) \tag{4.2}$$

where

$$\mathscr{F}(\ldots) = \frac{F_{-1}}{\sigma(\Delta)} + \sum_{n=0}^{2} \Delta^n \cdot F_n$$

$$\sigma \equiv \Delta + \frac{2m_e^2}{m_\mu^2 x^2}, \qquad \Delta \equiv 1 - \cos\theta_{e\gamma} \tag{4.3}$$

and

$$\text{const} \equiv \frac{G_F^2\, m_\mu^5\, e^2}{3 \cdot 2^{16}\, \pi^6} \tag{4.4}$$

Instead of the electron and photon energies, the normalized variables $x = \frac{2E_e}{m_\mu}$ and $y = \frac{2E_\gamma}{m_\mu}$ have been used. The functions F_j depend only on the variables x and y; their explicit forms are given in the appendix of Ref. 24. The form of the differential distribution reveals that the photon energy distribution is peaked at its lower end, since $d\Gamma \propto \frac{dE_\gamma}{E_\gamma}$, and thus has the typical shape of a bremsstrahlung spectrum with an infrared divergence. Its angular dependence comes only through the relative angle between the photon and the electron $\theta_{e\gamma}$; it is strongly peaked for $\theta_{e\gamma} \sim 0^o$ due to the electron-photon correlation which reflects itself in the denominator function of the first term in Eq. (4.3).

In order to compare the theoretical predictions with the data we have carried out a Monte Carlo simulation of this process assuming a 10% energy resolution. In Fig. 8 the experimental (curve 1) and the theoretical (curve 2) photon energy distributions are compared. The analogous simulated energy distribution for the electrons coming from $\mu \to e\bar{\nu}_e \nu_\mu + \gamma$ is given in Fig. 9 (curve 3, dN/dE_e). For later purposes, where the background rate of $\mu \to e+\gamma$

will be estimated, we also determined the same energy spectra of the electrons and photons restricting their relative angle to cos $\theta_{e\gamma} < -0.98$. In Fig. 9 the experimentally found electron energy spectrum (curve 1) is presented and the result of our Monte Carlo simulation is shown in curve 2 of the same figure. The analogous distributions for the photon are given in Fig. 10; curve 1 shows the data and curve 2 the simulation. There is good agreement between theory and experiment.[36] However this distribution depends quite sensitively on the cos $\theta_{e\gamma}$ cut; a choice of cos $\theta_e < -0.8$ results in substantial forward peaking. Such behavior is reflected in the curves of Fig. 11 where we have assumed the $\theta_{e\gamma}$-angle restrictions: cos $\theta_{e\gamma} < -0.98$ (curve 1) and cos $\theta_{e\gamma} < -0.96$ (curve 2). In addition the electron energy is constrained to the small strip at the upper end of the spectrum: $E_e > 47$ MeV. One notices a strong peaking of the photon spectrum near $E_\gamma \sim 0$ for cos $\theta_{e\gamma} < -0.96$, which however is less pronounced if cos $\theta_{e\gamma} < -0.98$ is imposed. Note that the two curves have a substantially different number of events: curves 1 has ~100 events (scale on left) and curve 2 has ~300 events (scale on right). We finally stress that all curves are intended as illustrations of the shapes of the various distributions; their sizes, in particular the experimental relative to the theoretical ones, may not be quantitatively compared!

3. Background Rate for $\mu \to e + \gamma$

The reaction $\mu \to e + \gamma$, if existent, is expected to lead to events characterized by $\theta_e \sim 180^\circ$, $E_e \approx E_\gamma \approx \frac{1}{2} m_\mu$. The only alternative source of such events could be the reaction $\mu \to e\bar{\nu}_e\nu_\mu\gamma$ commonly termed as "background" (apart from other effects). In order to determine how many events one should expect from this process under the above kinematical restrictions, we have carried out a Monte Carlo simulation and have determined the suppression factors. We write

$$N = N_0 \times BR_\gamma \times \left(\frac{\Delta\Gamma}{\Gamma_\gamma}\right)_{\theta_{e\gamma}} \times \left(\frac{\Delta\Gamma}{\Gamma_\gamma}\right)_{E_e} \times \left(\frac{\Delta\Gamma}{\Gamma_\gamma}\right)_{E_\gamma} \Delta\Omega \qquad (4.5)$$

N is the number of $e\gamma$ events expected in the limited kinematical region specified above and N_0 is the total number of considered muon decays. The branching ratio for the radiative decay mode $\mu \to e\bar{\nu}_e\nu_\mu\gamma$ has been calculated in Ref. 34: $BR_\gamma \sim 10^{-4}$. As we restrict the kinematical region, the number of events reduces; this is represented by the factors $(\Delta\Gamma/\Gamma_\gamma)_x$. Numerical values for various $\theta_{e\gamma}$- , E_e- and E_γ-cuts are given in Table II; they have been obtained by assuming a 10% energy resolution in the electron and the photon energy. There is substantial dependence on the $\theta_{e\gamma}$-cut and on the E_γ-cut but relatively little dependence on the cut in the electron energy. The factor $\Delta\Omega$ stands for all further reductions due to the geometry of the apparatus and detection equipment which depends on the individual experiment. If we assume an angle cut of $\cos\theta < -0.96$ and E_e, $E_\gamma > 51$ MeV and $N_0 = 10^{11}$ events, leaving all further reductions aside, we find

$$N \lesssim 0.14 \cdot \Delta\Omega \quad .$$

We expect $\Delta\Omega \sim 0.1$ and conclude that even with more generous energy and angle cuts no events of the type $\mu \to e\bar{\nu}_e\nu_\mu\gamma$ can have been detected. The six events found in the SIN-experiment[35] have to be explained by a different source, one of it being the reaction $\mu \to e+\gamma$.

V. CONCLUSION

The implementation of muon number nonconservation in a gauge theory framework which can account for the known data does not lead to any contradictions. Violation of muon number is expected to appear once the existence of new leptonic particles is admitted, as have been anticipated by a variety of other theoretical and phenomenological arguments. It can occur through: additional Higgs doublets, massive right or left-handed doublets, doubly charged leptons. If new leptons are admitted, mixing schemes make such effect possible. It is one of the most interesting aspects of muon number nonconservation that the existence of new particles is anticipated which appear as intermediate leptons (or bosons) in the loop contributions. This is a consequence of the fact that, due to the smallness of such effect, only higher order terms of the perturbation expansion are relevant. All schemes predict $BR(\mu \to e+\gamma) \sim 10^{-9}$ (although these predictions strongly depend on the parameters of the particular theory) but they can be distinguished by the amount of muon number violation in other reactions (see Table I). The experimental verification of the $\mu \to e+\gamma$ transition poses problems, one of it being the radiative decay channel, whose dynamical characteristics we have analyzed. In particular we found that the recently discussed six $\mu \to e+\gamma$ events[35] can hardly be understood as due to the radiative decay channel.

From the above we conclude that muon number violation, viewed from gauge theories, has to be considered as an almost natural consequence of such framework and, if found, is expected to give substantial information about its structure. The question whether nature chooses this freedom remains for the time being to be settled by the experiment. We look ahead to new exciting discoveries.

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TABLE I

Predicted branching ratios and further characteristics of the proposed explanations for muon number violating transitions

	$BR(\mu \to e\gamma)$	$\frac{d\Gamma}{d\cos\theta}(\mu^+ \to e^+\gamma)$	$BR(\mu \to 3e)$	$\frac{\mu \to 3e}{\mu \to e\gamma}$	$R(\mu \to e)$	$BR(K_L \to e\bar{\mu})$	$BR(K \to \pi e\bar{\mu})$	$\frac{K_L \to e\bar{\mu}}{K_L \to \mu\bar{\mu}}$	$\frac{K \to \pi e\bar{\mu}}{K \to \pi e\bar{e}}$	Uncertainties
Exp. Values	$< 2.2 \times 10^{-8}$	?	$< 1.9 \times 10^{-9}$	< ?	$< 1.6 \times 10^{-8}$	$< 1.6 \times 10^{-9}$	$< 1.4 \times 10^{-8}$	?	?	
Higgs Bosons	$< 4 \times 10^{-8}$	$1 - \cos\theta$	$\lesssim 10^{-11}$	$\lesssim 2 \times 10^{-4}$	$\sim 4 \times 10^{-9}$	$\lesssim 10^{-14}$	$< 10^{-14}$	-		m_H, Higgs couplings
LH - Mixing (doublets)	$\sim 10^{-9}$	$1 - \cos\theta$	$\sim 10^{-11}$	$\sim 6 \times 10^{-2}$	3×10^{-8}	$< 10^{-12}$	$\sim 10^{-11}$	$< 10^{-7}$	$< 10^{-14}$	m_L, Mixing Parameters
LH - Mixing (triplets)	$\sim 10^{-9}$	$1 - \cos\theta$	$\lesssim 10^{-8}$	$\lesssim 10^{1}$	$\lesssim 10^{-7}$	?	?	$\sim 10^{-2}$	$\sim 10^{-6}$	$m_{E^{++}}, m_{M^{++}}$ Mixing Angle ϕ
RH - Mixing (doublets)	$\lesssim 4 \times 10^{-10}$	$1 + \cos\theta$	$\sim 10^{-11}$	$\sim 10^{-2}$	$\lesssim 10^{-10}$	$\sim 10^{-10}$	$\lesssim 10^{-12}$			m_{N_i}, Mixing Angle ϕ
$SU_2 \times U_1 \times U_1$	$\lesssim 5 \times 10^{-9}$	1	$\lesssim 5 \times 10^{-9}$	~ 1	?	$\sim 10^{-10}$	$\sim 5 \times 10^{-7}$	2×10^{-2}	~ 1	-
H-Boson	$\sim 2 \times 10^{-9}$	$(1 \pm \cos\theta)$	$\sim 10^{-10}$	$\sim 10^{-1}$	?	$< 10^{-12}$	?	?	?	m_H, m_{N_i} H-couplings

TABLE II

Reduction factors due to kinematical constraints imposed on the reaction $\mu \rightarrow e\bar{\nu}_e \nu_\mu + \gamma$

$\cos\theta_{e\gamma}$	$(\Delta\Gamma/\Gamma_\gamma)$
< -0.98	2.5×10^{-4}
< -0.96	5.0×10^{-4}
< -0.82	2.7×10^{-3}

E_e(MeV)	$(\Delta\Gamma/\Gamma_\gamma)$ 10% resolution
> 51.3	1.2×10^{-2}
> 49.8	1.3×10^{-2}
> 47.0	2.0×10^{-2}

E_γ(MeV)	$(\Delta\Gamma/\Gamma_\gamma)$ 10% resolution	$(\Delta\Gamma/\Gamma_\gamma)$ no resolution
> 51.3	2.4×10^{-3}	5.0×10^{-4}
> 49.8	4.2×10^{-3}	2.5×10^{-3}
> 47.0	8.6×10^{-3}	1.3×10^{-2}

FIGURE CAPTIONS

1. Diagrams contributing to the transition $\mu \rightarrow e+\gamma$. Wavy lines represent photon and W, Z-boson, and dashed lines stand for Higgs particles.
2. Diagrams contributing to the decay mode $\mu \rightarrow ee\bar{e}$.
3. Diagrams contributing to the $K_L \rightarrow e\bar{\mu}$ decay mode. The Z-exchange vanishes due to the GIM mechanism.
4. Diagrams contributing to μ-e conversion.
5. Two-loop graphs contributing to $\mu \rightarrow e+\gamma$ transitions in the Higgs particle scheme.
6. Electron energy distribution of the decay $\mu \rightarrow e\bar{\nu}_e\nu_\mu$.

   Curve 1: Data from Ref. 5.

   Curve 2: Monte Carlo simulation assuming standard V-A weak interaction theory and an energy resolution of 10%.

   Curve 3: Theoretical prediction using Eq. (4.1).
7. Feynman diagrams describing the radiative decay mode $\mu \rightarrow e\bar{\nu}_e\nu_\mu+\gamma$ in lowest order V-A weak interaction theory.
8. Photon energy distribution of the decay mode $\mu \rightarrow e\bar{\nu}_e\nu_\mu+\gamma$.

   Curve 1: Data from Ref. 5.

   Curve 2: Monte Carlo simulation using standard V-A weak interaction theory and 10% energy resolution.
9. Electron energy distribution of the decay mode $\mu \rightarrow e\bar{\nu}_e\nu_\mu+\gamma$.

   Curve 1: Data from Ref. 5 with angular constraint $\theta_{e\gamma} \sim 180^\circ$.

   Curve 2: Monte Carlo simulation with angular constraint $\cos\theta_{e\gamma} < -0.98$ and 10% energy resolution.

   Curve 3: Monte Carlo simulation with no angular constraint and 10% energy resolution.

10. Photon energy distribution of the decay mode $\mu \rightarrow e\bar{\nu}_e \nu_\mu + \gamma$ with the imposed constraint $\theta_{e\gamma} \sim 180^o$.

Curve 1: Data from Ref. 5.

Curve 2: Monte Carlo simulation using $\cos \theta_e < -0.98$ and a 10% energy resolution.

11. Photon energy distribution of the decay mode $\mu \rightarrow e\bar{\nu}_e \nu_\mu + \gamma$ with the constraint $\theta_{e\gamma} \sim 180^o$ and $E_e > 47$ MeV.

Curve 1: Monte Carlo simulation using $\cos \theta_{e\gamma} < -0.98$.

Curve 2: Monte Carlo simulation using $\cos \theta_{e\gamma} < -0.96$.

Both curves with an energy resolution of 10%.

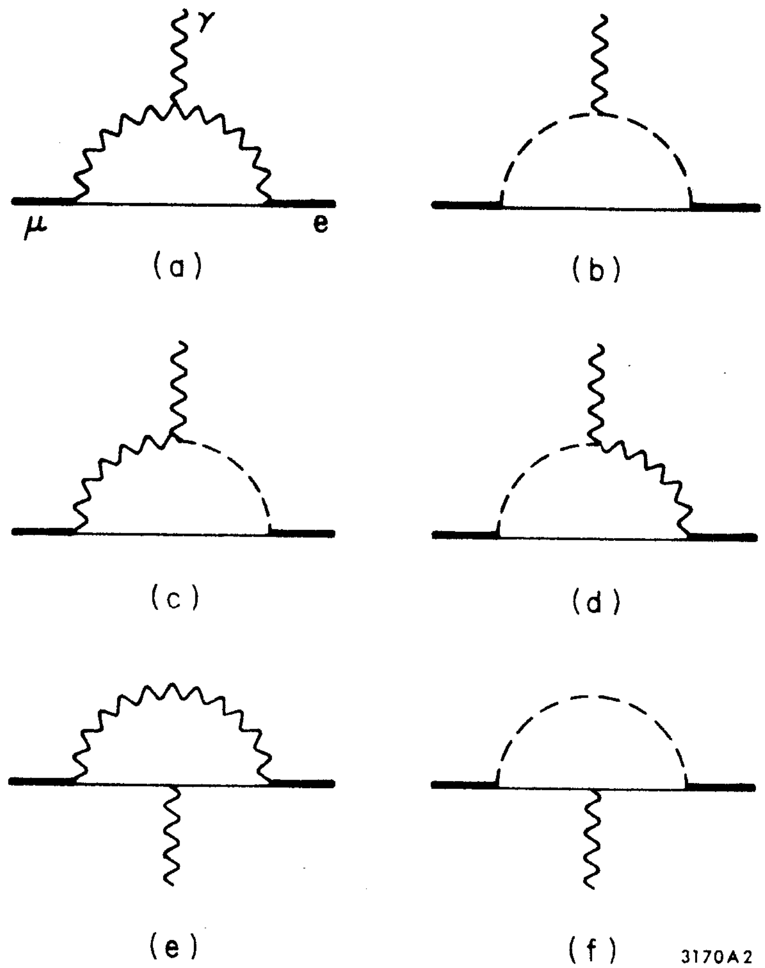
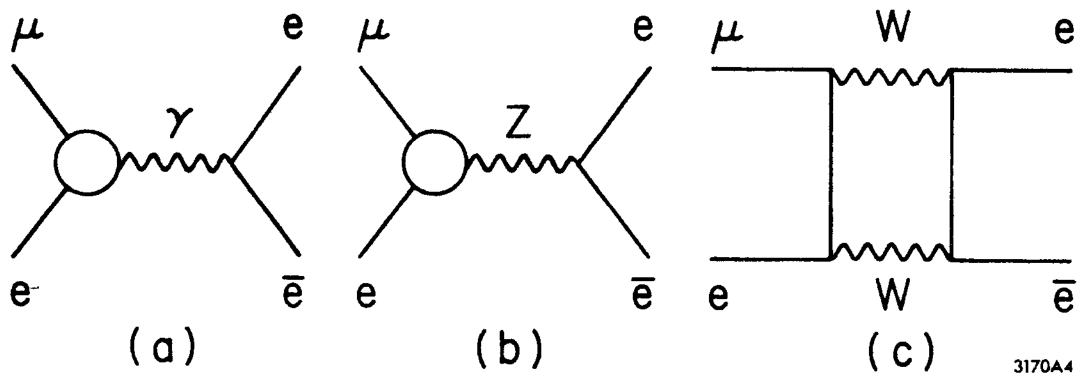
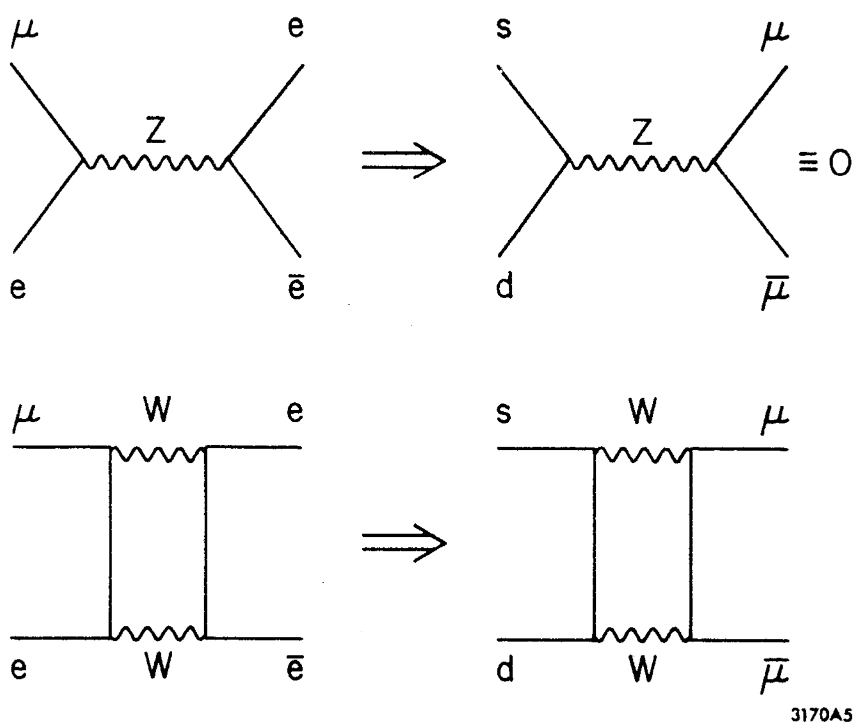


Fig. 1



μ
e
γ
e⁻
ē
(a)
μ
e
Z
e
ē
(b)
μ
W
e
e
W
ē
(c)
3170A4

Fig. 2



μ
e
Z
e
ē
s
μ
Z
d
μ̄
≡ 0
μ
W
e
e
W
ē
s
W
μ
d
W
μ̄
3170A5

Fig. 3

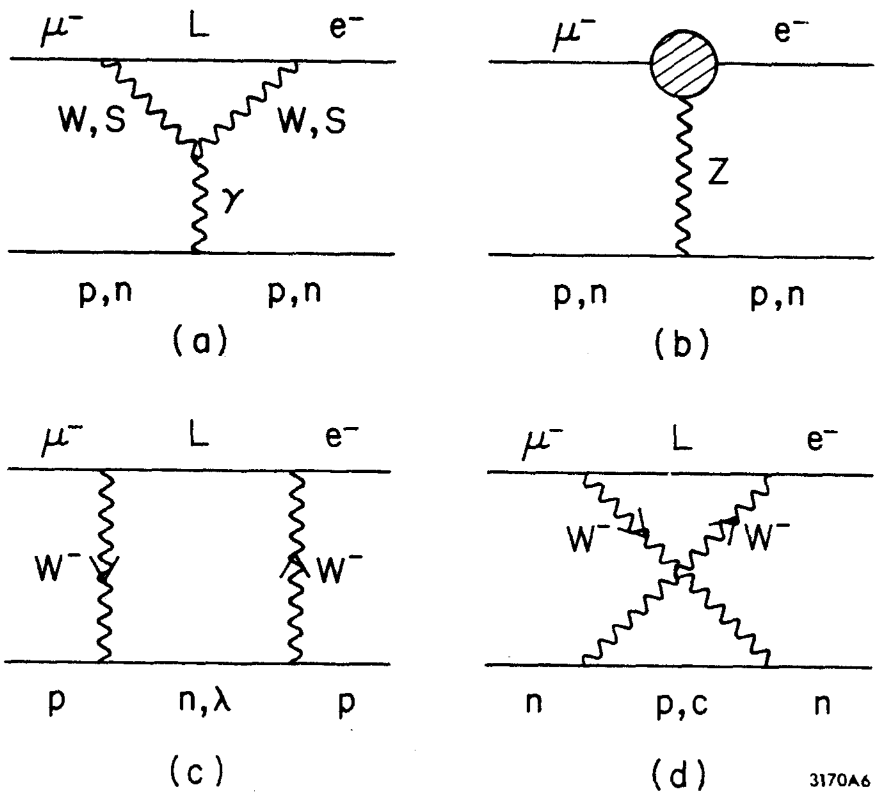
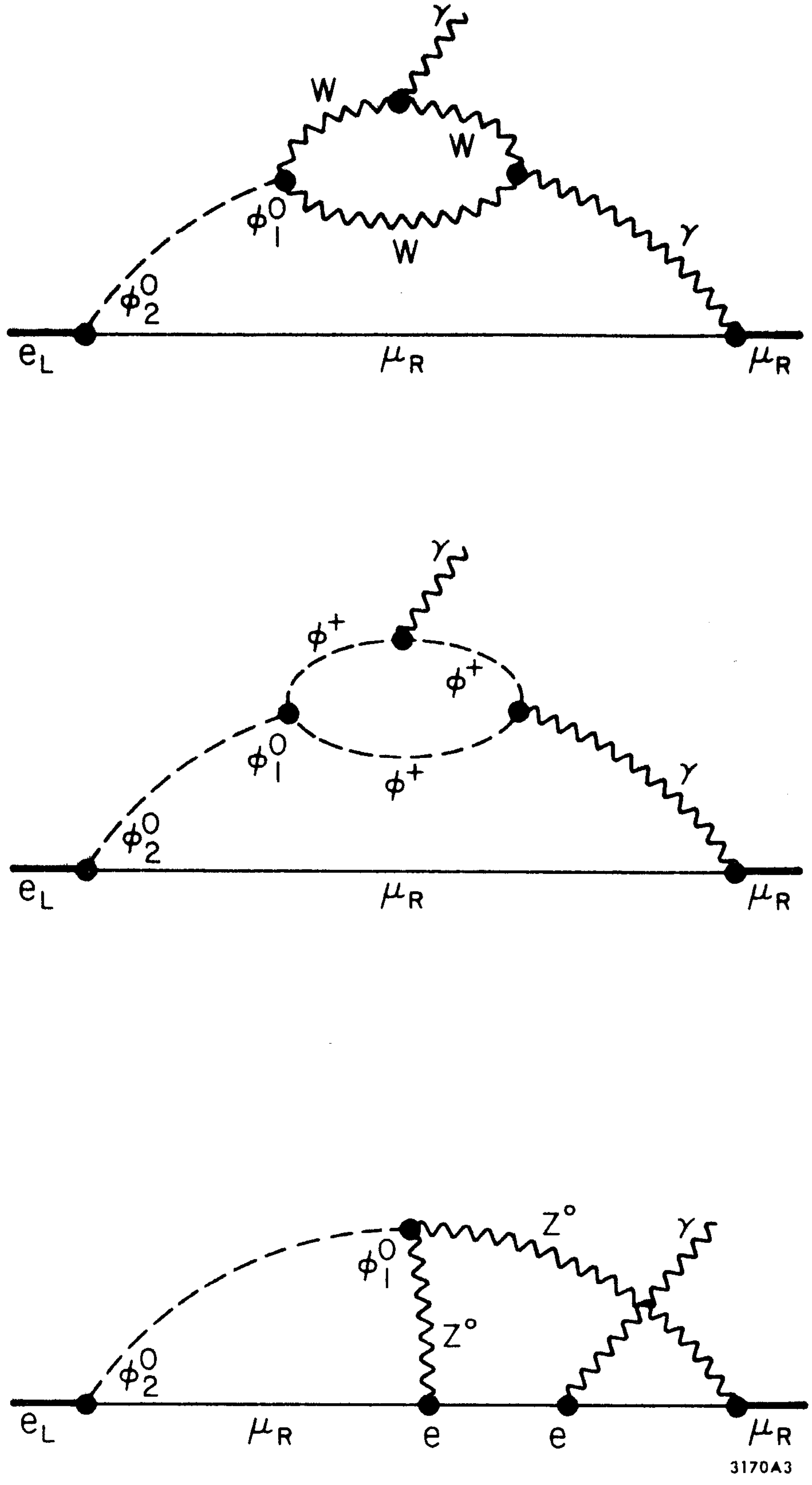


Fig. 4



γ
W
W
W
ϕ_1^0
γ
ϕ_2^0
e_L
μ_R
μ_R
γ
ϕ^+
ϕ^+
ϕ_1^0
ϕ^+
γ
ϕ_2^0
e_L
μ_R
μ_R
Z°
γ
ϕ_1^0
Z°
ϕ_2^0
e_L
μ_R
e
e
μ_R
3170A3

Fig. 5

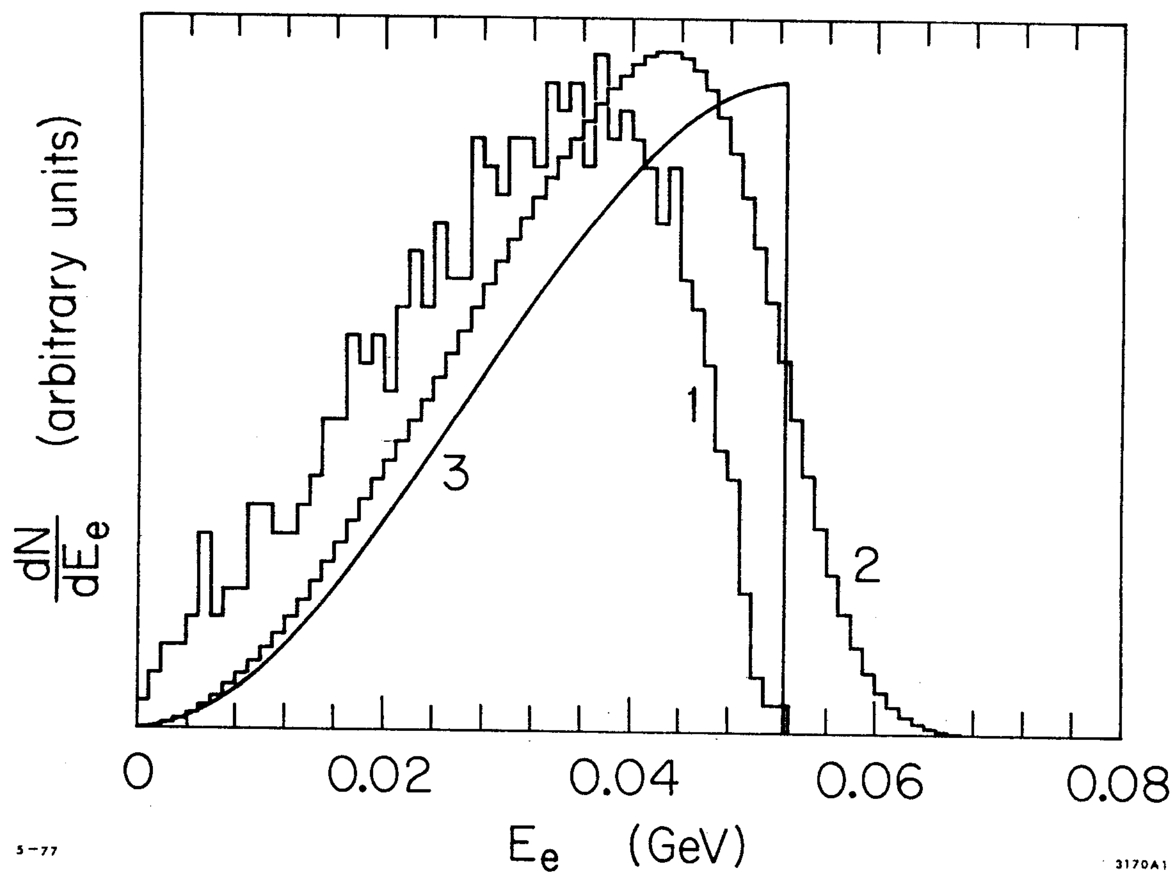


Fig. 6

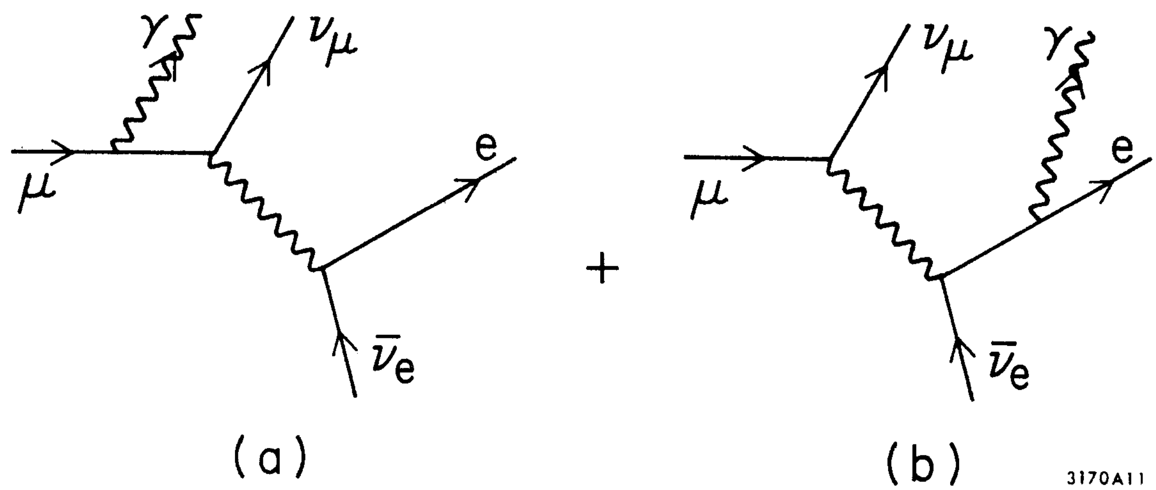
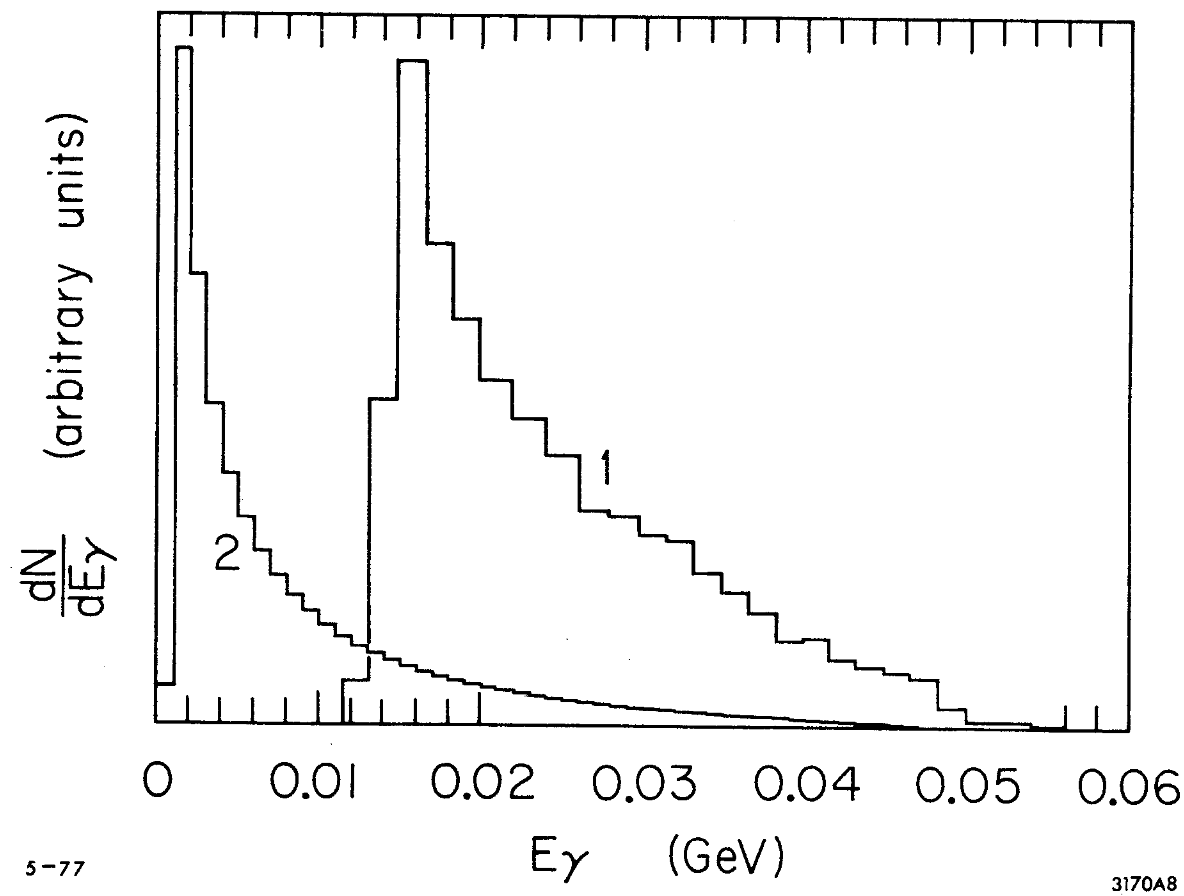
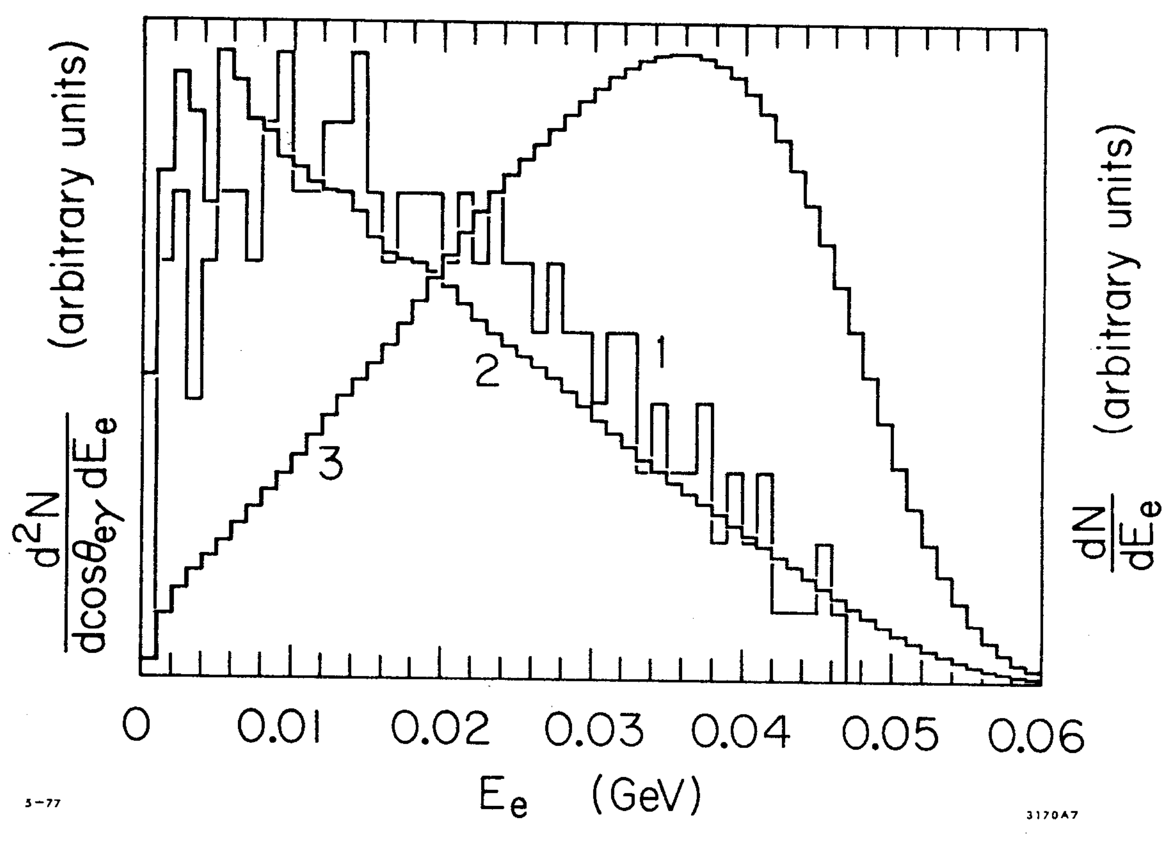


Fig. 7



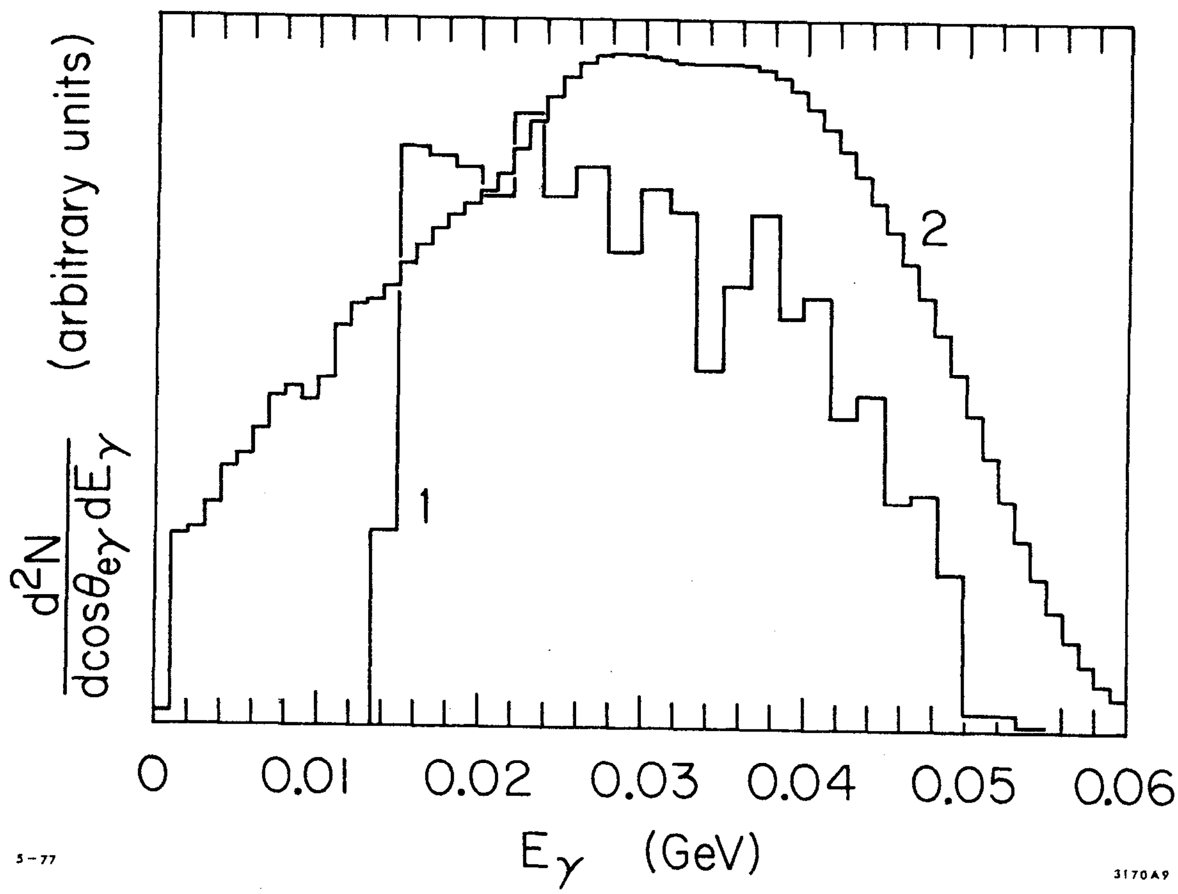
dN/dEγ (arbitrary units)
1
2
0
0.01
0.02
0.03
0.04
0.05
0.06
Eγ (GeV)
5-77
3170A8

Fig. 8



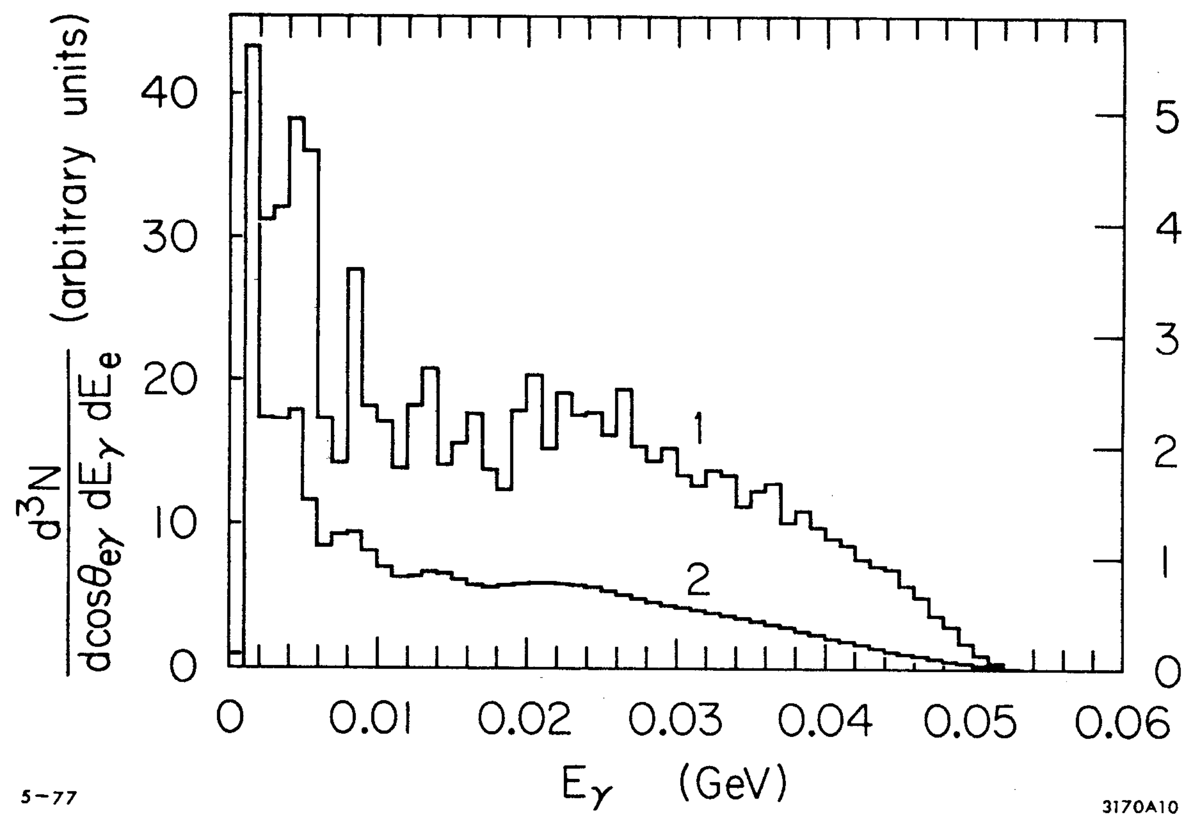
$\frac{d^2N}{d\cos\theta_{e\gamma}\,dE_e}$ (arbitrary units)
$\frac{dN}{dE_e}$ (arbitrary units)
1
2
3
0
0.01
0.02
0.03
0.04
0.05
0.06
E_e (GeV)
5-77
3170A7

Fig. 9



$\frac{d^2N}{d\cos\theta_{e\gamma}\, dE_\gamma}$ (arbitrary units)
1
2
0
0.01
0.02
0.03
0.04
0.05
0.06
E_γ (GeV)
5-77
3170A9

Fig. 10



$\frac{d^3N}{d\cos\theta_{e\gamma}\, dE_\gamma\, dE_e}$ (arbitrary units)
40
30
20
10
0
5
4
3
2
1
0
1
2
0
0.01
0.02
0.03
0.04
0.05
0.06
E_γ (GeV)
5-77
3170A10

Fig. 11