SLAC-PUB-1927
April 1977
(T/E)

HIGH MOMENTUM TRANSFER ELASTIC e-d SCATTERING

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ABSTRACT

We have calculated the deuteron electromagnetic form factor to all orders of q^2/M^2 in the impulse approximation. Our results are compared to the data for selected deuteron wave functions. We also extract the ultra high q^2 limit of our results, and obtain most naturally the same q^{-10} falloff predicted by the quark model.

(Submitted to Phys. Rev. Lett.)

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*Research supported by the National Science Foundation under Grant No. PHY 75-15986.

†Work supported by the Energy Research and Development Administration.

‡Supported in part by the National Science Foundation.

§A. P. Sloan Foundation Fellow.

Recent measurements[1] have made it necessary to calculate electron deuteron elastic scattering without making nonrelativistic approximations or q^2/M^2 expansions. Here we report on a relativistic calculation of the deuteron electromagnetic form factors in the impulse approximation (RIA), retaining terms to all orders in q^2/M^2. Two effects are included in this relativistic treatment. First, relativistic kinematics is used throughout. Second, the two nucleons in the deuteron cannot both be on shell. We have included the most important consequence of the latter by allowing the interacting nucleon to be off shell, which requires that all four invariants (or, equivalently, four wave functions) be retained in the deuteron-nucleon-nucleon vertex.[2] We obtain the three deuteron form factors as functionals of the four deuteron wave functions.

We shall present two aspects of our results in this letter. We first examine the ultra high q^2 limit of our results, discussing its implications, and then compare numerical results for selected deuteron wave functions with the recent data at high q^2.

The key to understanding the high q^2 behavior of the form factor lies in examining the q^2 dependence of the generic overlap integral

$$I = \int d^3p \, u(k_1^2) \, u'(k_2^2) \tag{1}$$

where u and u' are any two of the deuteron wave functions. The arguments are the magnitudes of the relative momenta of the incoming and outgoing deuterons evaluated in their respective rest frames. This is related to the three-momentum of the on-mass-shell spectator, $\vec{p}$, and the momentum transfer $\vec{q}$ in the Breit or brick wall frame by

$$k_{1,2}^2 = p_\perp^2 + \left(\frac{D_0 p_\parallel \pm \frac{1}{2} q E_p}{M_d}\right)^2 \tag{2}$$

where $p_\perp$ and $p_\parallel$ are components perpendicular and parallel to $\vec{q}$, $D_0 = (M_d^2 + q^2/4)^{\frac{1}{2}}$, and $E_p = (M^2 + p^2)^{\frac{1}{2}}$. If we expand the wave functions in a series of Hulthén-like functions

$$u(p) = \sum_i c_i (p^2 + \beta_i^2)^{-1} , \tag{3}$$

we find that for very large momentum transfer

$$I(q) \sim \begin{cases} \dfrac{K_1}{q^3} & \text{if } \sum_i c_i \neq 0 \qquad (4a) \\[2ex] \dfrac{K_2}{q^7} & \text{if } \sum_i c_i = 0 \ , \ \text{but } \sum_i c_i \beta_i \neq 0 \ , \qquad (4b) \end{cases}$$

where

$$K_1 = 8\pi^2 M_d^3 \left(\sum_i c_i\right) \left(\sum_j c_j \frac{1}{\gamma_j} \arctan \frac{\gamma_j}{\beta_j}\right)$$

$$K_2 = 32\pi^2 \frac{M_d^7}{M^4} \left(\sum_i c_i \beta_i^2\right) \left(\sum_j c_j \beta_j\right) \tag{5}$$

and

$$\gamma_i = \sqrt{M^2 - \beta_i^2} \ .$$

It may be seen that the second result is the one which is natural for the deuteron. The high momentum behavior of the vertex functions or wave functions may be determined by studying a covariant wave equation obtained by restricting one particle to the mass shell.[3] If the binding is due to one boson exchanges and if each BNN vertex has a form factor which goes like[4] $(t+p_1^2+p_2^2)^{-1}$, where t is the momentum transfer through the boson, and p_1^2 and p_2^2 are the nucleon four-momenta squared, then the momentum space wave functions used in

Eq. (1) must fall like $1/k_i^4$ and may be arbitrarily well approximated by a sum of Hulthén functions with $\sum_i c_i = 0$. If the nucleons and bosons were themselves elementary particles, there should be no BNN form factor, the wave functions would fall like $1/k_i^2$, and we would get $\sum_i c_i \neq 0$.

The e-d differential cross section is

$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{Mott}} \left(A(q^2) + B(q^2)\tan^2\frac{\theta}{2}\right) = \left.\frac{d\sigma}{d\Omega}\right|_{\text{Mott}} F_d^2(q^2,\theta) \tag{6}$$

and an examination of the detailed formulas relating A and B to integrals like Eq. (1) yields

$$F_d(q^2,\theta) \underset{\theta \text{ fixed}}{\sim} \begin{cases} q^{-2}\, F_N(q^2) & \sum c_i \neq 0 \quad \text{(7a)} \\ q^{-6}\, F_N(q^2) & \sum c_i = 0\,, \quad \text{(7b)} \end{cases}$$

where F_N is the nucleon isoscalar form factor. The first result is consistent with work[5] which showed that for a system composed of <u>n</u> elementary constituents the electromagnetic form factor should behave as $(q^2)^{1-n}$. The second result shows that the RIA for the deuteron electromagnetic form factor falls like q^{-10}, provided F_N falls like q^{-4}. This is the same result predicted[5,6] on the basis of the quark model.[7]

We note an interesting consequence of the above result concerning the relative size of the RIA and meson exchange contributions to $F_d(q^2,\theta)$. Since the quark model and the RIA both fall like q^{-10}, their difference - presumably the meson exchange effects - must fall like q^{-10} (or faster). While the constants multiplying the falloffs of the RIA and meson exchange effects remain to be determined, there is no <u>a priori</u> reason to expect one to dominate the other.

Since the q^{-10} prediction follows from both the quark model and the (relativistically calculated) conventional n-p bound state model of the deuteron, it might be regarded as a fairly secure prediction. Nevertheless, one should ask how well it is verified by the present data. The cynic will point out that if one fits the data for $q^2 \geq 0.8$ GeV^2 to a monomial q^{-n}, the best fit has $n = 5.5$. However, in order to fairly compare the q^{-10} prediction to the data, one needs to estimate the nonleading terms, which are important in the q^2 region where the form factor has been measured. For the n-p bound state model, the numerical evaluations of the complete formulas will be presented shortly, and one will see that while the asymptotic falloff is q^{-10}, the falloff at finite q^2 is slower, and indeed follows the trend of the data. For the quark model, a detailed examination has indicated[6] that the form $(1+q^2/m_0^2)^{-1}F_N^2(q^2/4)$ is appropriate for the deuteron form factor. The data[1,8] divided by this factor, with a scaling mass $m_0^2 = 0.28$ GeV^2, are shown in Fig. 1a. Note the flatness of the implied curve for $q^2 \gtrsim 1$ GeV^2. The data, thus properly analyzed, certainly do not disagree with the q^{-10} prediction. On the other hand, let us also compare the data to the odd possibility that the nucleons and the bosons that bind them together are elementary, i.e., the BNN form factors are unity, but the nucleons still have their measured electromagnetic form factors. This leads to a leading q^{-6} falloff for the deuteron electromagnetic form factor, as given by Eq. (7a). The data divided by $(1+q^2/m_0^2)^{-1}F_N(q^2)$ [the form of the first factor gives the correct normalization at $q^2=0$, but differs from a pure q^{-2} falloff only at low q^2] are plotted in Fig. 1b. The flatness of the curve above $q^2 \gtrsim$ is again striking. We must conclude that, because of the importance of the nonleading terms, the data do not yet distinguish the models, and that we must go to higher q^2 to clearly see the leading falloff.

This leads us naturally to the question of the predictions for ^{3}He and ^{4}He. While the quark model predicts a falloff of q^{-16} and q^{-22} for these two cases, the actual falloff in the q^2 region of a few $(GeV)^2$ would be much slower because of the large masses which enter the specific scaling prediction of Brodsky and Chertok.[6] If, for example, the quark predictions are compared with the forms $q^{-4}F_N(q^2)$ (for ^3He) and $q^{-6}F_N(q^2)$ (for ^4He), which follow from the same assumptions leading to (7a), one finds less than a factor of two difference in the range $2(GeV)^2 < q^2 < 8(GeV)^2$.

We now turn to the second part of this letter; we discuss our results for $A(q^2)$ for $q^2 < 6\ (GeV)^2$ as numerically evaluated for several selected wave functions and plotted along with the data in Fig. 2.

There are three theoretical curves in this figure. In each calculation, the nucleon isoscalar form factor was given a dipole form with a $(mass)^2$ of 0.71 $(GeV)^2$. The curve labeled Reid Soft Core is the nonrelativistic impulse approximation with Reid soft core wave functions.[9] The curve labeled Reid Relativistic is a calculation with the relativistic formulas using the Reid soft core wave functions for the usual S and D states, and setting the two additional wave functions to zero. The difference between these two curves is due entirely to treating the kinematics to all orders in q^2/M^2.

The Reid wave functions, however, were obtained from a nonrelativistic Schrödinger equation, and it should be clear that a consistent evaluation of the form factors requires wave functions which were themselves calculated relativistically. There is no consensus on what the best such wave functions are and the curve labeled $\lambda_\omega = 1$ is representative of recent calculations fully described elsewhere.[10]

From the figure we see that the relativistic effects tend to decrease the form factor at $q^2 \approx 2\ (GeV)^2$ and that the calculated form factors can vary by an order of magnitude near $q^2 = 6\ (GeV)^2$. Note also that our calculations fall systematically below the data, so that there is room for other processes, such as meson exchange corrections[11] or contributions to the impulse approximation with the spectator off shell, to make up the difference. Also, further work on the high momentum components of the wave functions is needed, and this, along with relativistic evaluations of the meson exchange effects, will clarify the results.

A detailed report on this work will be presented elsewhere.

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7. It should be noted that both the result (7b) and the q^{-10} prediction of the quark model depend on the assumption that the relativistic wave function is not zero at the origin. Interestingly enough, if one makes the same assumption for the nonrelativistic wave function, and further assumes that there are BNN form factors which go like t^{-1}, then one also obtains the result (7b) from the nonrelativistic impulse approximation. However, while we believe that it is reasonable to assume that the relativistic wave functions

are finite at the origin, the nonrelativistic wave functions currently in use are zero there because of the repulsive nature of the nonrelativistic potential at short distances, and in this case one would obtain a form factor going like two powers of q^2 faster than the result (7b).

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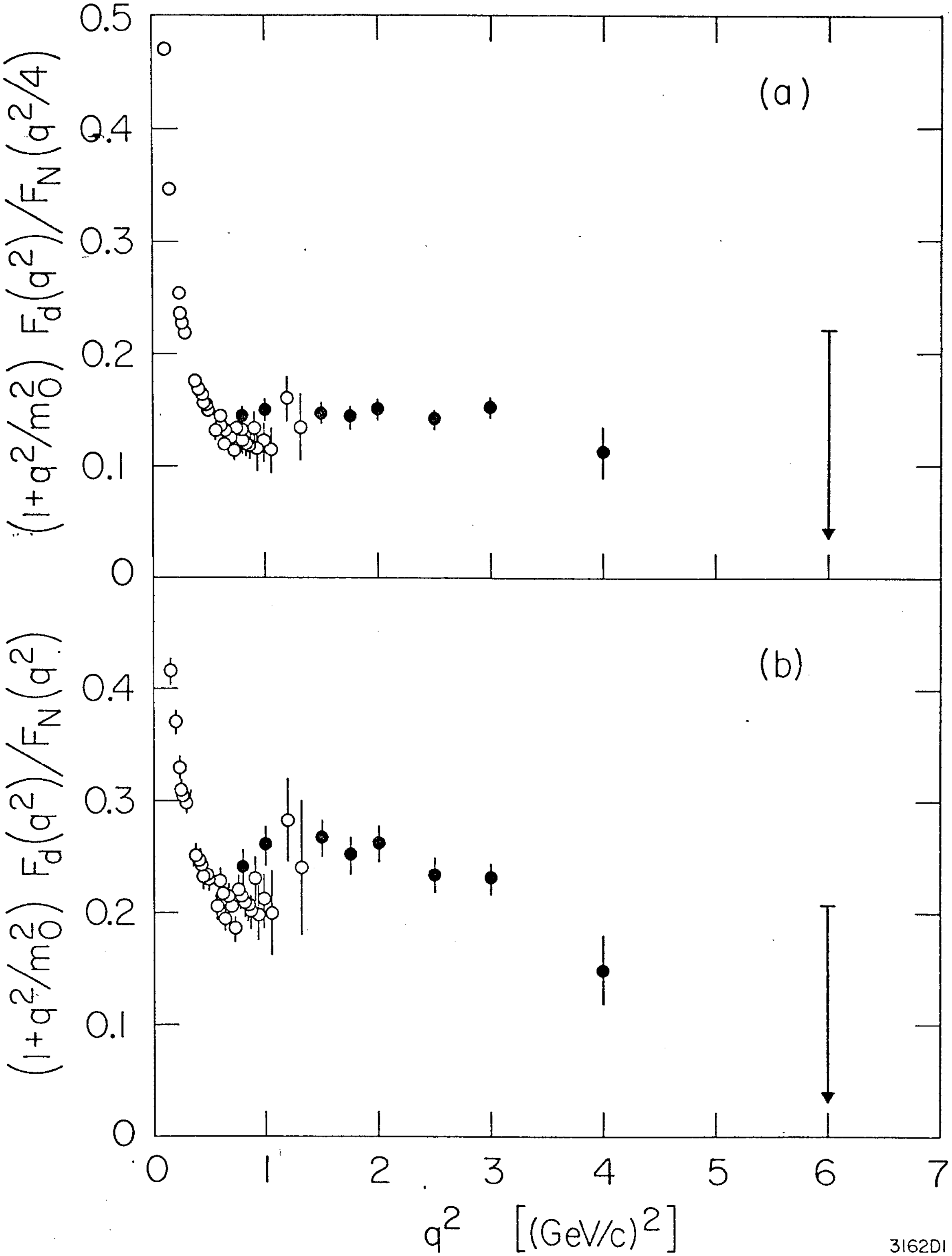
Figure Captions

1. (a) The deuteron form factor data compared to $\left(1+\frac{q^2}{m_0^2}\right)^{-1} F_N^2(q^2/4)$; this is the quark model prediction of Ref. 6.

   (b) The data compared to $\left(1+\frac{q^2}{m_0^2}\right)^{-1} F_N(q^2)$. The points ● are from Ref. 1, points ○ from Ref. 8. In both figures $m_0^2 = .28\ (\mathrm{GeV})^2$.

2. The data for $A(q^2)$ from Ref. 1 compared to the RIA for selected wave functions described in the text.



(a)
(b)
$(1+q^2/m_0^2)\ F_d(q^2)/F_N(q^2/4)$
$(1+q^2/m_0^2)\ F_d(q^2)/F_N(q^2)$
0.5
0.4
0.3
0.2
0.1
0
0 1 2 3 4 5 6 7
q^2 [(GeV/c)^2]
3162D1

Fig. 1

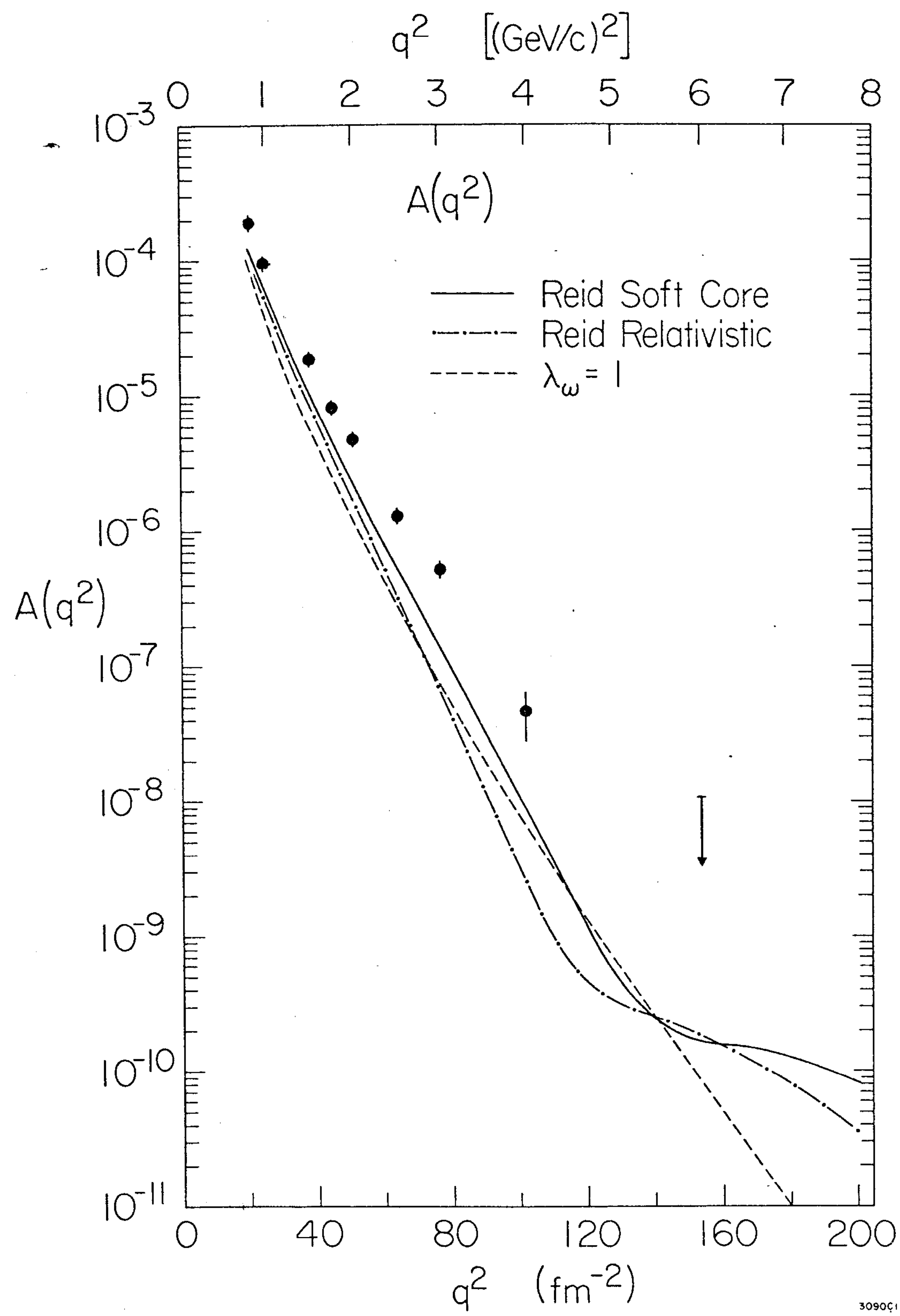

q2 [(GeV/c)2]
0 1 2 3 4 5 6 7 8
10-3
10-4
10-5
10-6
10-7
10-8
10-9
10-10
10-11
A(q2)
Reid Soft Core
Reid Relativistic
λω = 1
A(q2)
0 40 80 120 160 200
q2 (fm-2)
3090C1

Fig. 2