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THE DECAY $\mu \to e\gamma$ IN MODELS WITH NEUTRAL HEAVY LEPTONS*

James D. Bjorken and Kenneth Lane
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Steven Weinberg†
Department of Physics
Stanford University, Stanford, California 94305

ABSTRACT

We explore possible muon number nonconserving processes in gauge theories in which the right-handed muon and electron appear along with neutral heavy leptons in $SU(2) \otimes U(1)$ doublets. The same mechanism which gives a mass to the electron and muon is expected to mix the left-handed neutral leptons with the neutrinos. In the simplest case, this leads to a rate for $\mu \to e\gamma$ 25 times larger than previously calculated in such models. Other phenomenological consequences of these theories are discussed. Formulas are given for the magnetic dipole transition matrix element in general gauge theories, and these are used to derive general conditions for suppression of $\mu \to e\gamma$ to acceptable levels.

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†During 1976-77, on leave from Harvard University, Cambridge, Massachusetts.

I. INTRODUCTION

A wide range of renormalizable gauge theories of the weak and electromagnetic interactions predict that muon-number-nonconserving processes like $\mu \to e\gamma$ should be very slow, even if the Lagrangian does not conserve muon number. Thus, the stringent experimental limits on muon nonconservation[1] do not yet reveal whether muon number conservation is really a fundamental symmetry of nature.

One hint is provided by the well-known analogy between strangeness and muon number: they both are "flavors" which distinguish the bottom members of weak SU(2) doublets. Nonconservation of strangeness is an experimental fact, so we suspect that muon number is also not conserved. The avalanche of theoretical papers[2] which have emerged in the short span of three months, just from the stimulus of a tenuous rumor of an observation of the process $\mu \to e\gamma$, is ample evidence that muon nonconservation is indeed a natural theoretical possibility.

Cheng and Li[3] have proposed an attractive mechanism for muon-nonconservation, which leads to an estimated branching ratio of magnitude interesting from the point of view of both theory and experiment. Working in the context of the standard $SU(2) \otimes U(1)$ weak and electromagnetic gauge theory, Cheng and Li couple the electron and muon via right-handed currents to a corresponding pair of heavy neutral leptons N_e^0, N_μ^0, which are allowed to mix. The radiative transition $\mu \to e\gamma$ then proceeds at the one-loop level of radiative corrections via the diagrams shown in Fig. 1.

In their original work, Cheng and Li did not deal in detail with the origin of the muon and electron masses, and considered only right-handed couplings at the two lepton-W vertices shown in Fig. 1. However, whatever mechanism produces the muon and electron masses will generally also mix the heavy neutral leptons with the neutrinos. This gives rise to additional terms in the amplitude for $\mu \to e\gamma$, in which one of the right-handed couplings in Fig. 1 is replaced by a left-handed coupling.

At first glance, despite a suppression factor of order m_μ/m_W at the left-handed vertex, this additional "left-right" term seems to be much too big. A simple estimate of this term would give a result larger than the "right-right" term calculated by Cheng and Li by a factor of order m_W^2/m_N^2. However, a detailed calculation reveals a remarkable cancellation of leading terms. When the dust settles, the "left-right" term turns out to be of the same form as the "right-right" term, differing only in that its sign is opposite and that it is six times larger. Hence the $\mu \to e\gamma$ rate in this model is expected to be just 25 times larger than originally calculated by Cheng and Li.

The purpose of this paper is to document this phenomenon. In Section II we consider the nature of the mixing of neutrinos with neutral heavy leptons expected in the Cheng-Li model. In Section III we set up the calculation of the amplitude for $\mu \to e\gamma$. It turns out to be convenient to do this in a general gauge theory, and in a general renormalizable gauge, if for no other reason than that it is easiest to see how the many individually gauge-dependent terms combine and cancel in the context of a general formalism. We also discuss the general condition for the sort of cancellation of leading terms which we have found in the Cheng-Li model, and which seems to be required by the present stringent experimental limit on the $\mu \to e\gamma$ branching ratio. In Section IV we apply the general results of Section II to the Cheng-Li model, and compute the branching ratio for $\mu \to e\gamma$. Section V deals with some other consequences of this model.

II. DESCRIPTION OF THE MODEL

The Cheng-Li model is based on the familiar $SU(2) \otimes U(1)$ gauge group of the weak and electromagnetic interactions.[4] However, instead of the usual pairs of left-handed doublets $(\nu_e, e^-)_L$; $(\nu_\mu, \mu^-)_L$ and right-handed singlets e^-_R, μ^-_R, the leptons in this model form two left-handed doublets, two right-handed doublets, and two left-handed neutral singlets:

$$\begin{pmatrix} a_i^o \\ \ell_i^- \end{pmatrix}_L \quad \begin{pmatrix} b_i^o \\ \ell_i^- \end{pmatrix}_R \quad c_{iL}^o \tag{2.1}$$

with i a two-valued index. We do not yet assume that these fields correspond to lepton states of definite mass.

With two pairs of neutral left-handed fields but only one pair of neutral right-handed fields, this model must evidently contain two left-handed neutral leptons of zero mass, provided overall fermion-number is conserved. These we of course identify with the two neutrinos, ν_e and ν_μ. Mass terms are to be introduced so that the remaining left- and right-handed pairs of neutral lepton fields correspond to two heavy neutral lepton states N_1^o and N_2^o, with masses M_1 and M_2.

Cheng and Li introduce neutral mass terms which couple only b_{iR}^o with c_{jL}^o. Thus, in their work, the fields a_{iL}^o are associated purely with the neutrinos, while b_{iR}^o and c_{iL}^o have no neutrino components. This picture would be correct for instance if the only Higgs bosons which could generate lepton masses were all doublets, and if there were some sort of global symmetry which prohibited <u>bare</u> lepton mass terms.

However, it is also necessary to include masses for the charged leptons. Indeed, if the charged leptons were massless, we could <u>define</u> the μ^- and e^- in the Cheng-Li model so that the matrix element for $\mu \to e\gamma$ vanished. Cheng and Li simply insert mass terms for e^- and μ^-, without introducing any mass terms which couple a^o_{iL} to b^o_{jR}. This is certainly a possible picture. For instance, it can be arranged by introducing a mixture of Higgs singlets and triplets, whose vacuum expectation values are adjusted so that they couple ℓ^-_{iL} with ℓ^-_{jR}, but not a^o_{iL} with b^o_{jR}.

We consider it to be much more reasonable and natural to suppose that the charged leptons receive their masses either from an SU(2)-invariant bare mass term, or from the vacuum expectation values of singlet Higgs boson fields. The effective mass terms in the Lagrangian (after spontaneous symmetry breaking) would then take the form

$$\mathscr{L}_{\text{mass}} = -m_{ij}\left(\bar{a}^o_{iL} b^o_{jR} + \bar{\ell}^-_{iL}\ell^-_{jR}\right) - M_{ij}\ \bar{c}^o_{iL} b_{jR} + \text{h.c.} \tag{2.2}$$

The 2×2 matrices m_{ij} and M_{ij} are in general neither diagonal nor even Hermitian. However, we can subject the doublet fields to transformations

$$a^o_{iL} \to U_{ij} a^o_{jL} \qquad \ell^-_{iL} \to U_{ij}\ell^-_{jL}$$

$$b^o_{iR} \to V_{ij} b^o_{jR} \qquad \ell^-_{iR} \to V_{ij}\ell^-_{jR}$$

with the unitary matrices U and V chosen so that $U^{-1}mV$ is real and diagonal. We shall henceforth assume that this has already been done, so that m takes the form:

$$m = \begin{pmatrix} m_e & 0 \\ 0 & m_\mu \end{pmatrix} \tag{2.3}$$

The fields ℓ_{iL}^- and ℓ_{iR}^- are thus identified for i=1 and 2 with the left- and right-handed parts of the electron and muon fields.

With b_{iR}^o now fixed by the condition that m_{ij} be real and diagonal, it is not generally possible to choose the lepton fields so that M_{ij} is also diagonal. Therefore the neutral lepton fields a_i^o, b_i^o, c_i^o must be expressed as linear combinations of the fields ν_e, ν_μ, N_1^o, N_2^o of definite mass. To accomplish this we introduce the matrix-notation

$$a_{iL}^o = A_{ij}\nu_j + B_{ij}N_{jL} \tag{2.4}$$

$$c_{iL}^o = C_{ij}\nu_j + D_{ij}N_{jL} \tag{2.5}$$

$$b_{iR}^o = E_{ij}N_{jR} \tag{2.6}$$

where the ν_j are the ν_e, ν_μ eigenstates and the N_j are mass-eigenstates. Inserting this into Eq. (2.2), and demanding a diagonal mass-matrix in the transformed basis, leads to the conditions

$$\left(A^\dagger m+C^\dagger M\right)E = 0 \tag{2.7}$$

$$\left(B^\dagger m+D^\dagger M\right)E = m_N \equiv \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \tag{2.8}$$

In addition, we have the unitarity constraints

$$EE^\dagger = 1 \tag{2.9}$$

$$AA^\dagger + BB^\dagger = 1 \tag{2.10}$$

$$AC^\dagger + BD^\dagger = 0 \tag{2.11}$$

$$CC^\dagger + DD^\dagger = 1 \tag{2.12}$$

We can use (2.11) to eliminate the unknown matrix M from Eqs. (2.7) and (2.8), and obtain a relation between B and E:

$$Bm_N = mE \tag{2.13}$$

By choices of phases of the lepton fields one may reduce E to a real orthogonal matrix.

$$E = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \tag{2.14}$$

so that (2.13) gives

$$B = mE\,m_N^{-1} = \begin{pmatrix} \dfrac{m_e\cos\phi}{M_1} & \dfrac{m_e\sin\phi}{M_2} \\ -\dfrac{m_\mu\sin\phi}{M_1} & \dfrac{m_\mu\cos\phi}{M_2} \end{pmatrix} \tag{2.15}$$

We can also define the neutrino states ν_e, ν_μ so that A is Hermitian; the unitarity relation (2.10) then allows us to express A in terms of B:

$$A = [1 - BB^\dagger]^{1/2}$$

or, to first order in m_e^2/m_N^2 and m_μ^2/m_N^2:

$$A \simeq 1 - \frac{1}{2}\begin{pmatrix} m_e^2\left(\dfrac{\cos^2\phi}{M_1^2} + \dfrac{\sin^2\phi}{M_2^2}\right) & m_e m_\mu \cos\phi\sin\phi\left(\dfrac{1}{M_2^2} - \dfrac{1}{M_1^2}\right) \\ m_e m_\mu \cos\phi\sin\phi\left(\dfrac{1}{M_2^2} - \dfrac{1}{M_1^2}\right) & m_\mu^2\left(\dfrac{\sin^2\phi}{M_1^2} + \dfrac{\cos^2\phi}{M_2^2}\right) \end{pmatrix} \tag{2.16}$$

To summarize, we have obtained a description of the $SU(2)\otimes U(1)$ doublets in terms of mass eigenstates, as follows:

$$\begin{pmatrix} (1-\epsilon_1)\nu_e + \epsilon_3\nu_\mu + \epsilon_4 N_1 + \epsilon_5 N_2 \\ e^- \end{pmatrix}_L$$

$$\begin{pmatrix} (1-\epsilon_2)\nu_\mu + \epsilon_3\nu_e + \epsilon_6 N_1 + \epsilon_7 N_2 \\ \mu^- \end{pmatrix}_L$$

$$\begin{pmatrix} N_1 \cos\phi + N_2 \sin\phi \\ e^- \end{pmatrix}_R$$

$$\begin{pmatrix} -N_1 \sin\phi + N_2 \cos\phi \\ \mu^- \end{pmatrix}_R \qquad (2.17)$$

The ϵ_i are given by the formulae

$$\epsilon_1 \approx \frac{1}{2} m_e^2 \left(\frac{\cos^2\phi}{M_1^2} + \frac{\sin^2\phi}{M_2^2} \right) \qquad \epsilon_4 = \frac{m_e \cos\phi}{M_1}$$

$$\epsilon_2 \approx \frac{1}{2} m_\mu^2 \left(\frac{\sin^2\phi}{M_1^2} + \frac{\cos^2\phi}{M_2^2} \right) \qquad \epsilon_5 = \frac{m_e \sin\phi}{M_2}$$

$$\epsilon_3 \approx \frac{1}{2} m_e m_\mu \cos\phi \sin\phi \left(\frac{1}{M_1^2} - \frac{1}{M_2^2} \right) \qquad \epsilon_6 = -\frac{m_\mu \sin\phi}{M_1}$$

$$\epsilon_7 = \frac{m_\mu \cos\phi}{M_2} \qquad (2.18)$$

III. CALCULATION OF THE DECAY AMPLITUDE IN GENERALIZED FORMALISM

We describe here our calculation, to one-loop order, of the transition magnetic form factor F_1 defined by the electromagnetic current matrix element

$$
\begin{aligned}
&<\ell_2(p-q)|j^\lambda(0)|\ell_1(p)> \equiv \bar{u}_2(p-q)T^\lambda u_1(p) \\
&T^\lambda = F_1(q^2)\,\tfrac{i}{2}\sigma^{\lambda\mu} q_\mu + F_2(q^2)\left(q^\lambda q^\mu - q^2 g^{\lambda\mu}\right)\gamma_\mu
\end{aligned}
\tag{3.1}
$$

where $\sigma^{\lambda\mu} = \left[\gamma^\lambda, \gamma^\mu\right]/2i$. The calculation will be carried out for an arbitrary unified gauge model of weak and electromagnetic interactions. We assume throughout that fermions ℓ_1 and ℓ_2 are distinct in that they correspond to different eigenstates of the zeroth-order fermion mass matrix. It follows that T^λ vanishes at $q^2=0$ in zeroth order. The form (3.1) for T^λ, a consequence of current conservation, guarantees that F_1 and the monopole form factor F_2 will be finite in renormalizable gauges. Note that both F_1 and F_2 may contain terms proportional to γ_5.

Our calculation will apply to processes of the type $\mu \to e\gamma$, to which only the transition moment $F_1(0)$ contributes. Both form factors contribute to such processes as $\mu \to 3e$ and $\mu + \mathcal{N}(A,Z) \to e + \mathcal{N}(A,Z)$. If they occur at all, these transitions are very rare, and whether F_1 and F_2 can be small enough in a given model is a basic concern. While F_1 is gauge-invariant and completely specifies the $\mu \to e\gamma$ process, F_2 is not, and T^λ does not describe the complete amplitude for processes such as $\mu \to 3e$. There are additional contributions from box graphs, massive neutral gauge boson exchange, and Higgs meson exchange. We have not computed these contributions and so do not report our results for the gauge-dependent F_2. We mention, however, that in many models the gauge-invariant part of T^λ controls the rates for $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$. For example, models in which $\mu \rightarrow e\gamma$ proceeds via photon emission from a virtual heavy charged lepton can give an unacceptably large $\mu \rightarrow 3e$ rate via the monopole form factor F_2.

The general notation used by one of us[5] in discussing perturbative symmetry breaking is very convenient for computing F_1 in an arbitrary unified gauge model. In an R_ξ gauge,[6] the effective interaction Lagrangian for gauge bosons A^α_μ, fermions ψ_n, Higgs scalars $\phi_i = \phi_i^\dagger$, and spinless fermion ghosts ω_α is

$$\begin{aligned}
\mathscr{L}_I = &-\frac{1}{2}\left(\partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu\right) C_{\alpha\beta\gamma} A^{\beta\mu} A^{\gamma\nu} \\
&- \frac{1}{4} C_{\alpha\beta\epsilon} C_{\gamma\delta\epsilon} A^\alpha_\mu A^\beta_\nu A^{\gamma\mu} A^{\delta\nu} \\
&- i\, \partial_\mu \phi_i \theta^{ij}_\alpha \phi_j A^{\alpha\mu} + (\theta_\alpha \theta_\beta \lambda)_i \phi_i A^\alpha_\mu A^{\beta\mu} \\
&+ \frac{1}{2} (\theta_\alpha \theta_\beta)_{ij} \phi_i \phi_j A^\alpha_\mu A^{\beta\mu} + \bar{\psi} \gamma^\mu t_\alpha A^\alpha_\mu \psi \\
&- \bar{\psi} \Gamma_i \phi_i \psi - \frac{1}{6} f_{ijk} \phi_i \phi_j \phi_k - \frac{1}{24} f_{ijk\ell} \phi_i \phi_j \phi_k \phi_\ell \\
&- \partial_\mu \omega^*_\alpha C_{\alpha\beta\gamma} \omega_\beta A^{\gamma\mu} - \xi^{-1} \omega^*_\alpha \omega_\beta (\theta_\alpha \theta_\beta \lambda)_i \phi_i
\end{aligned} \tag{3.2}$$

with ξ a free parameter which defines the gauge. The fermion and Higgs generators t_α and θ_α satisfy

$$\begin{aligned}
&\left[t_\alpha, t_\beta\right] = iC_{\alpha\beta\gamma} t_\gamma \qquad \left[\theta_\alpha, \theta_\beta\right] = iC_{\alpha\beta\gamma}\theta_\gamma \\
&\theta_\alpha = \theta^\dagger_\alpha = -\theta^*_\alpha \\
&\gamma_0 \left[t_\alpha, \gamma_0 \Gamma_i\right] = -\theta^{ij}_\alpha \Gamma_j
\end{aligned} \tag{3.3}$$

The Higgs fields have been shifted by their vacuum expectation values λ_i, so that $\langle\Phi_i\rangle_0 = 0$ in lowest order. The gauge-boson, fermion, Higgs, and ghost propagators are

$$\Delta_{\alpha\mu,\beta\nu}(k) = -g_{\mu\nu}(k^2-\mu^2)^{-1}_{\alpha\beta} + (\xi-1)\,k_\mu k_\nu\left[(k^2-\mu^2)(\xi k^2-\mu^2)\right]^{-1}_{\alpha\beta} \tag{3.4}$$

$$S(k)_{nm} = (\not{k}-m)^{-1}_{nm} \tag{3.5}$$

$$\begin{aligned}\Delta_{ij}(k) &= (k^2-M^2)^{-1}_{ij} - (\theta_\alpha\lambda)_i(\theta_\beta\lambda)_j\left[k^2(\xi k^2-\mu^2)\right]^{-1}_{\alpha\beta} \\ &= \left(\frac{\mathscr{P}}{k^2-M^2}\right)_{ij} - \xi(\theta_\alpha\lambda)_i(\theta_\beta\lambda)_j\left[\mu^2(\xi k^2-\mu^2)\right]^{-1}_{\alpha\beta}\end{aligned} \tag{3.6}$$

$$\Delta^{\omega}_{\alpha\beta}(k) = \xi(\xi k^2-\mu^2)^{-1}_{\alpha\beta} \tag{3.7}$$

where the gauge boson mass matrix is $\mu^2_{\alpha\beta} = (\lambda\theta_\alpha)_i(\theta_\beta\lambda)_i$, the fermion fields are defined so that $m = m_0 + \Gamma_i\lambda_i$ is γ_5-free, and $\mathscr{P}_{ij}$ projects onto the physical Higgs scalar subspace,

$$\begin{aligned}\mathscr{P}_{ij} &= \delta_{ij} + (\theta_\alpha\lambda)_i\,\mu^{-2}_{\alpha\beta}(\theta_\beta\lambda)_j \\ \mathscr{P}_{ij}(\theta_\alpha\lambda)_j &= M^2_{ij}(\theta_\alpha\lambda)_j = 0 \qquad \text{for all } \alpha\end{aligned} \tag{3.8}$$

From now on, the gauge index γ is reserved for the photon (in general, a massless linear combination of gauge bosons), so that

$$\theta_\gamma\lambda = \mu^2_{\alpha\gamma} = \left[t_\gamma, m\right] = \left[t_\gamma, \Gamma_i\lambda_i\right] = 0 \tag{3.9}$$

Naturally, we assume t_γ does not connect fermions ℓ_1 and ℓ_2. We further assume that electric charge is the only unbroken gauge symmetry.

To compute F_1, we need consider only the proper graphs in Fig. 1. Improper graphs involving the fermion self-energy and graphs involving the gauge boson polarization tensor contribute only to F_2 provided the fermions ℓ_1 and ℓ_2 are on the mass shell. The calculation is straightforward, following closely the method used in Ref. 5. Identities such as

$$S(p-k)\,\not{k}t_\beta = S(p-k)\,m_\beta + S(p-k)\,\gamma_0 t_\beta \gamma_0 S^{-1}(p) - t_\beta \tag{3.10}$$

were particularly useful; here

$$m_\alpha = \gamma_0 \left[t_\alpha, \gamma_0 m\right] = \Gamma_i(\theta_\alpha \lambda)_i \tag{3.11}$$

and we set $S^{-1}(p)$ (or $S^{-1}(p-q)$) on the extreme right (or left) equal to zero. Also useful is the relation

$$C_{\gamma\delta\epsilon}\left(\frac{\mu^2}{K^2-\mu^2}\right)_{\alpha\epsilon}\left(\frac{1}{k^2-\mu^2}\right)_{\beta\delta} = C_{\gamma\delta\epsilon}\left(\frac{1}{K^2-\mu^2}\right)_{\alpha\epsilon}\left(\frac{\mu^2}{k^2-\mu^2}\right)_{\beta\delta} \tag{3.12}$$

which follows from Eq. (3.9).

Then, by writing out all diagrams, using only the previous identities and neglecting everything not contributing to F_1, one finds massive and intricate cancellations of gauge dependent terms. Denoting by T_1^λ that part of T^λ which contributes to F_1, we find for the gauge-independent remainder:

$$\begin{aligned} T_1^\lambda = \int \frac{d^4k}{(2\pi)^4} \Bigg\{ & \frac{2i\,C_{\gamma\delta\epsilon}\ \gamma^\lambda t_\alpha\, S(p-k)\,\not{q} t_\beta - \not{q} t_\alpha\, S(p-k)\,\gamma^\lambda t_\beta}{(K^2-\mu^2)_{\alpha\epsilon}\,(k^2-\mu^2)_{\beta\delta}} \\ & + iC_{\gamma\delta\epsilon}(2k-q)^\lambda \left[\frac{\gamma^\mu t_\alpha\, S(p-k)\,\gamma_\mu t_\beta}{(K^2-\mu^2)_{\alpha\epsilon}\,(k^2-\mu^2)_{\beta\delta}} + \frac{m_\alpha S(p-k) m_\beta}{(K^2-\mu^2)_{\alpha\epsilon}\left[\mu^2(k^2-\mu^2)\right]_{\beta\delta}}\right] \\ & + \frac{\gamma^\mu t_\alpha\, S(p-q-k)\,\gamma^\lambda t_\gamma\, S(p-k)\,\gamma_\mu t_\beta}{(k^2-\mu^2)_{\alpha\beta}} + \frac{m_\alpha\, S(p-q-k)\,\gamma^\lambda t_\gamma\, S(p-k) m_\beta}{\left[\mu^2(k^2-\mu^2)\right]_{\alpha\beta}} \\ & - (2k-q)^\lambda\, \Gamma_i S(p-k)\, \Gamma_j \left(\frac{\mathscr{P}}{K^2-M^2}\,\theta_\gamma\,\frac{\mathscr{P}}{k^2-M^2}\right)_{ij} \\ & - \Gamma_i S(p-q-k)\,\gamma^\lambda t_\gamma\, S(p-k)\,\Gamma_j \left(\frac{\mathscr{P}}{k^2-M^2}\right)_{ij} \Bigg\} \end{aligned} \tag{3.13}$$

where $K=k-q$ and $1/A_{\alpha\beta} \equiv (A^{-1})_{\alpha\beta}$. Since we have assumed that the only unbroken gauge symmetry is that of electric charge, only massive gauge boson exchange and physical Higgs scalar exchange enters this expression if ℓ_1 and ℓ_2 are distinct. If $\ell_1=\ell_2$, photon exchange contributes in the fifth term (only) of Eq. (3.13).

We may extract F_1 from Eq. (3.13) by the following procedure:

(i) For simplicity, imagine that we have diagonalized the gauge boson and physical Higgs meson mass matrices so that $\mu^2_{\alpha\beta}=\mu^2_\alpha\delta_{\alpha\beta}$ and $\mathscr{P}_{ij}M^2_{jk}=M^2_i\delta_{ik}$. Then t_α and Γ_i are understood to be the gauge and Higgs couplings to fermions in this diagonal-mass basis. Note, for example, that the first term in Eq. (3.13) is now written

$$2\int\frac{d^4k}{(2\pi)^4}\,\frac{\gamma^\lambda t_\alpha\, S(p-k)\, \not{q}\left[t_\alpha, t_\gamma\right] - \not{q}t_\alpha\, S(p-k)\,\gamma^\lambda\left[t_\alpha, t_\gamma\right]}{\left(K^2-\mu^2_\alpha\right)\left(k^2-\mu^2_\alpha\right)} \tag{3.14}$$

where a sum over α $(\neq\gamma)$ is understood, and we have used Eq. (3.3).

(ii) After doing the Feynman integrals in the usual way, convert all $\not{p}$ and $(\not{p}-\not{q})$ factors to their mass shell values by moving them to the extreme right or left and using (with definitions $\bar{t}_\alpha=\gamma_0 t_\alpha\gamma_0$, $\overline{m}_\alpha=\gamma_0 m_\alpha\gamma_0$, $\Gamma^\dagger_i=\gamma_0\Gamma_i\gamma_0$)

$$\begin{aligned} \ldots\not{p}t_\alpha &= \ldots\bar{t}_\alpha m & \qquad t_\alpha(\not{p}-\not{q})\ldots &= m\bar{t}_\alpha\ldots \\ \ldots\not{p}\Gamma_i &= \ldots\Gamma^\dagger_i m & \qquad \Gamma_i(\not{p}-\not{q})\ldots &= m\Gamma^\dagger_i\ldots \end{aligned} \tag{3.15}$$

Remember that m is a matrix which need not commute with t_α or Γ_i.

(iii) Noting that, even though F_1 is a matrix and may contain terms proportional to γ_5,

$$\begin{aligned} &\bar{u}_2(p-q)\,\tfrac{i}{2}\,\sigma^{\lambda\mu}q_\mu F_1(q^2,m)\,u_1(p) \\ &\qquad = \tfrac{1}{2}\,\bar{u}_2(p-q)\left[m\gamma^\lambda F_1 + F_1\gamma^\lambda m - (2p-q)^\lambda F_1\right]u_1(p) \end{aligned} \tag{3.16}$$

and that, by Eq. (3.1), this is the only source of terms proportional to p^λ. We therefore may drop all terms proportional to γ^λ or to q^λ and then make the replacement

$$p^\lambda \longrightarrow -\frac{i}{2}\sigma^{\lambda\mu} q_\mu = -\frac{1}{4}\left[\gamma^\lambda, \not{q}\right] \tag{3.17}$$

For $q^2=0$ and in the approximation that external fermion masses p^2 and $(p-q)^2$ are much less than gauge boson and Higgs masses, μ_α^2 and M_i^2, we find

$$F_1 = F_1^{\text{Gauge}} + F_1^{\text{Higgs}} \tag{3.18}$$

where

$$\begin{aligned} F_1^{\text{Gauge}} = \frac{i}{8\pi^2}\int_0^1 dx(1-x)\Big\{ & -(1-x)(2-x)\left[m t_\alpha D_\alpha t_\alpha t_\gamma + \bar{t}_\alpha D_\alpha \bar{t}_\alpha t_\gamma m\right] \\ & + (2-x)\left[m t_\alpha D_\alpha t_\gamma t_\alpha + \bar{t}_\alpha D_\alpha t_\gamma \bar{t}_\alpha m\right] \\ & + 4(1-x)\bar{t}_\alpha m D_\alpha t_\alpha t_\gamma - 4\bar{t}_\alpha m D_\alpha t_\gamma t_\alpha \\ & - \frac{1}{2}x(1-x)\left[m\bar{m}_\alpha \mu_\alpha^{-2} D_\alpha m_\alpha t_\gamma + m_\alpha \mu_\alpha^{-2} D_\alpha \bar{m}_\alpha t_\gamma m\right] \\ & + \frac{1}{2}x\left[m\bar{m}_\alpha \mu_\alpha^{-2} D_\alpha t_\gamma m_\alpha + m_\alpha \mu_\alpha^{-2} D_\alpha t_\gamma \bar{m}_\alpha m\right] \\ & - x m_\alpha m \mu_\alpha^{-2} D_\alpha m_\alpha t_\gamma + \left(\frac{x}{1-x}\right) m_\alpha m \mu_\alpha^{-2} D_\alpha t_\gamma m_\alpha \Big\} \end{aligned} \tag{3.19}$$

and

$$\begin{aligned} F_1^{\text{Higgs}} = \frac{i}{8\pi^2}\int_0^1 dx(1-x)\Big\{ & \frac{x(1-x)}{2}\left[m\Gamma_i^\dagger D_i \Gamma_i t_\gamma + \Gamma_i D_i \Gamma_i^\dagger t_\gamma m\right] \\ & - \frac{x}{2}\left[m\Gamma_i^\dagger D_i t_\gamma \Gamma_i + \Gamma_i D_i t_\gamma \Gamma_i^\dagger m\right] \\ & + x\Gamma_i m D_i \Gamma_i t_\gamma - \left(\frac{x}{1-x}\right)\Gamma_i m D_i t_\gamma \Gamma_i \Big\} \end{aligned} \tag{3.20}$$

In Eqs. (3.19) and (3.20) we introduced

$$D_\alpha = \left[\mu_\alpha^2(1-x) + m^2x\right]^{-1}$$
$$D_i = \left[M_i^2(1-x) + m^2x\right]^{-1} \tag{3.21}$$

We remind the reader that t_α, Γ_i, m, D_α, and D_i are matrices whose indices label fermion species.

As we previously mentioned, the very small experimental limits on $\mu \to e\gamma$ or $\mu \to 3e$ decay amplitudes (of order 10^{-8} in branching ratio), restrict the kind of gauge theory experimentally allowed. If we suppose mixing angles such as ϕ (introduced in Section II) are not extremely small (a supposition supported by the relatively large value of θ_c, along with some assumption of lepton-quark parallelism), then the $\mu \to e\gamma$ branching ratio needs suppression by a factor other than the square of the gauge-coupling constant $\left(\sim \frac{\alpha}{\pi}\right)$. Such a small factor may be attained by assuming that gauge boson masses are much greater than fermion masses. Then several interesting conditions may be obtained from Eqs. (3.19) and (3.20) regarding the degree to which processes such as $\mu \to e\gamma$ are suppressed. Since all μ_α^2 in (3.19) are positive,

$$D_\alpha = \left[\mu_\alpha^2(1-x)\right]^{-1} - \frac{m^2x}{\mu_\alpha^2(1-x)\left[\mu_\alpha^2(1-x) + m^2x\right]} \tag{3.22}$$

where the second term on the right corresponds to two mass insertions on the internal fermion line. Then, the conditions that the leading order terms vanish, leaving corrections suppressed (in amplitude) by a factor $\sim m^2/\mu^2$ are as follows:

(i) Models for which $\ell_1 \to \ell_2\gamma$ proceeds only via photon emission from virtual charged massive gauge bosons $\left(\text{so that } t_\alpha D_\alpha t_\gamma t_\alpha = \bar{t}_\alpha m D_\alpha t_\gamma t_\alpha = \overline{m}_\alpha D_\alpha t_\gamma \mu_\alpha^{-2} = 0\right)$

will have $F_1^{\mathrm{Gauge}} = O\left(m\mu_\alpha^{-2}\right)$ unless

$$\left[m t_\alpha t_\alpha \mu_\alpha^{-2} + \bar{t}_\alpha \bar{t}_\alpha \mu_\alpha^{-2} m\right]_{\ell_2 \ell_1} = 0 \qquad (3.23a)$$

and

$$\left[\bar{t}_\alpha m t_\alpha \mu_\alpha^{-2}\right]_{\ell_2 \ell_1} = 0 \qquad (3.23b)$$

where $[\ ]_{\ell_2 \ell_1}$ means the element of the enclosed matrix connecting ℓ_1 to ℓ_2. If these conditions are met, then $F_1^{\mathrm{Gauge}} = O\left(m^3 \mu_\alpha^{-4}\right)$, since the remaining terms, involving $\bar{m}_\alpha m \mu_\alpha^{-2} D_\alpha m_\alpha t_\gamma$ and so on, are manifestly of this order. A sufficient condition that Eqs. (3.23) be met is that separate ℓ_1- and ℓ_2-number conservation is respected by all gauge couplings.

(ii) Models for which $\ell_1 \to \ell_2 \gamma$ also may proceed via photon emission from a virtual charged fermion (so that $t_\alpha D_\alpha t_\gamma t_\alpha$, etc., do not vanish) will have $F_1^{\mathrm{Gauge}} = O\left(m\mu_\alpha^{-2}\right)$ unless

$$\left[m t_\alpha t_\gamma t_\alpha \mu_\alpha^{-2} + \bar{t}_\alpha t_\gamma \bar{t}_\alpha \mu_\alpha^{-2} m\right]_{\ell_2 \ell_1} = 0 \qquad (3.24a)$$

and

$$\left[\bar{t}_\alpha m t_\gamma t_\alpha \mu_\alpha^{-2}\right]_{\ell_2 \ell_1} = 0 \qquad (3.24b)$$

as well as Eqs. (3.23a) and (3.23b). If conditions (3.24) are met, the resulting contribution to $\ell_1 \to \ell_2 \gamma$ is $O\left(m^3 \mu_\alpha^{-4} \ln\left(\mu_\alpha^2 / m^2\right)\right)$, where the logarithm is an infrared singularity at zero fermion mass.

(iii) Since, in general, there are no very large lower limits on the Higgs scalar masses (comparable to $\mu_\alpha \gtrsim 50$ GeV), it is difficult to make model-independent estimates of the strength of F_1^{Higgs}. Usually, $\Gamma_i = O(e\, m_{\mathrm{lepton}}/\mu)$, but with the models currently entertained, m_{lepton}/μ may range from about

0.002 (for $m_{\text{lepton}} = 100$ MeV) to 0.05 or greater. In the former case, considered by two of us,[7] F_1^{Higgs} is a negligible contribution to $\mu \to e\gamma$, while two-loop graphs in which the Higgs scalar couples only once to the fermion are not. In the Cheng-Li model with more than one Higgs doublet, however $m_{\text{lepton}}/\mu \sim 0.05$ is quite reasonable, and such a model could very well give an unacceptably large $\mu \to e\gamma$ rate, unless Higgs masses are large, say $M_i > \mu_\alpha \gtrsim 60$ GeV. The most general statement we can make is that model-builders should exercise great caution with the Higgs sector insofar as rare processes such as $\mu \to e\gamma$ are concerned.

IV. CALCULATION OF THE $\mu \to e\gamma$ RATE IN THE MODIFIED CHENG-LI MODEL

In the preceding section we have derived the amplitude for decay of a muon into an electron and a photon, to one-loop order of an arbitrary unified gauge model of weak and electromagnetic interactions. To demonstrate the use of the general formulae, Eqs. (3.18)-(3.20), we compute this amplitude in the $SU(2) \times U(1)$ model of Cheng and Li, modified as in Section II. We assume that the model contains just one Higgs doublet or, if more than one doublet is present, that Higgs couplings to leptons are so small that their contribution to one-loop order is negligible.

For this model, only the diagrams in Figs. 1a-d contribute to the amplitude

$$\mathscr{T} = \epsilon_\lambda(q) \langle e^-(p-q)|j^\lambda|\mu^-(p)\rangle = \epsilon_\lambda(q)\,\bar{u}_e(p-q)\,\frac{i}{2}\sigma^{\lambda\mu} q_\mu F_1 u_\mu(p) \tag{4.1}$$

where $\sigma^{\lambda\mu} = -\frac{i}{2}\left[\gamma^\lambda, \gamma^\mu\right]$. The transition moment F_1 is given by those terms in Eq. (3.19) with the charge matrix t_γ (or $t_\gamma m$) on the extreme right, namely

$$\begin{aligned} F_1 = \frac{ieg^2}{8\pi^2}\int_0^1 dx(1-x)^2 \Bigg\{ & (2-x)\left[m t_\alpha \frac{1}{\mu^2(1-x)+m^2 x} t_\alpha + \bar{t}_\alpha \frac{1}{\mu^2(1-x)+m^2 x}\bar{t}_\alpha m\right] \\ & - 4\bar{t}_\alpha \frac{m}{\mu^2(1-x)+m^2 x} t_\alpha \\ & + \frac{1}{2}x\left[m\bar{m}_\alpha \frac{1}{\mu^2\left[\mu^2(1-x)+m^2 x\right]} m_\alpha + m_\alpha \frac{1}{\mu^2\left[\mu^2(1-x)+m^2 x\right]}\bar{m}_\alpha m\right] \\ & + \left(\frac{x}{1-x}\right) m_\alpha \frac{m}{\mu^2\left[\mu^2(1-x)+m^2 x\right]} m_\alpha \Bigg\}_{e\mu} \end{aligned} \tag{4.2}$$

In Eq. (4.2), we have put $t_\gamma = -e$, the electron charge; g is the weak SU(2) gauge coupling constant, and μ the mass of charged weak vector boson (W). t_α and $\bar{t}_\alpha = \gamma_0 t_\alpha \gamma_0$ are matrices representing the SU(2) generators on the fermions in a basis in which the fermion mass matrix m is γ_5-free, the summed index α

takes the values 1 and 2, and

$$\begin{aligned} m_\alpha &= \gamma_0\left[t_\alpha, \gamma_0 m\right] = \bar{t}_\alpha m - m t_\alpha \quad , \\ \overline{m}_\alpha &= \gamma_0 m_\alpha \gamma_0 = t_\alpha m - m\bar{t}_\alpha \quad . \end{aligned} \tag{4.3}$$

Finally, the subscript $e\mu$ on the bracket in Eq. (4.2) labels the desired element of the enclosed matrix.

To compute the various terms in Eq. (4.2), suppose m has been diagonalized by putting

$$\psi_L \equiv \begin{pmatrix} \ell_e^- \\ \ell_\mu^- \\ a_e^o \\ a_\mu^o \\ c_e^o \\ c_\mu^o \end{pmatrix}_L = U_L \begin{pmatrix} e^- \\ \mu^- \\ \nu_e \\ \nu_\mu \\ N_1 \\ N_2 \end{pmatrix}_L \tag{4.4}$$

$$\psi_R = \begin{pmatrix} \ell_e^- \\ \ell_\mu^- \\ b_e^o \\ b_\mu^o \end{pmatrix} = U_R \begin{pmatrix} e^- \\ \mu^- \\ N_1 \\ N_2 \end{pmatrix}_R$$

In the basis (2.3), with ℓ_e=e and ℓ_μ=μ, the unitary transformations $U_{L,R}$ are

$$U_{L,R} = \begin{pmatrix} U_{L,R}^- & 0 \\ 0 & U_{L,R}^o \end{pmatrix}$$

$$U_L^- = U_R^- = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$U_L^o = \begin{pmatrix} 1-\epsilon_1 & \epsilon_3 & \epsilon_4 & \epsilon_5 \\ \epsilon_3 & 1-\epsilon_2 & \epsilon_6 & \epsilon_7 \\ \epsilon_4' & \epsilon_6' & 1-\epsilon_8 & \epsilon_{10} \\ \epsilon_5' & \epsilon_7' & \epsilon_{10}' & 1-\epsilon_9 \end{pmatrix}$$

$$U_R^o = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \tag{4.5}$$

The t_α in the diagonal-mass basis are given by

$$\begin{aligned} t_\alpha &= t_\alpha^R \left(\frac{1+\gamma_5}{2}\right) + t_\alpha^L \left(\frac{1-\gamma_5}{2}\right) \\ t_\alpha^{R,L} &= U_{R,L}^{-1} \frac{\tau_\alpha^{R,L}}{2} U_{R,L} \end{aligned} \tag{4.6}$$

where τ_α^R are essentially Pauli matrices which couple $(\ell_e^-)_R$ to $(b_e^o)_R$ and $(\ell_\mu^-)_R$ to $(b_\mu^o)_R$, with similar definitions for τ_α^L.

It is a good approximation to set $m_e=0$; this allows us to drop terms with m on the extreme left. Then, for $\mu^- \to e^-\gamma$, the surviving terms in Eq. (4.2) are:

$$\begin{aligned} \left[\bar{t}_\alpha \frac{1}{\mu^2(1-x)+m^2x} \bar{t}_\alpha m\right]_{e\mu} &= \frac{1}{2}\left(\frac{1-\gamma_5}{2}\right)\left[t_-^R \frac{1}{\mu^2(1-x)+m^2x} t_+^R\right] m_\mu \\ &= -\frac{1}{2}\left(\frac{1-\gamma_5}{2}\right) \cdot \frac{x m_\mu \Delta M^2 \sin\phi\cos\phi}{\left[\mu^2(1-x)+M_1^2x\right]\left[\mu^2(1-x)+M_2^2x\right]} \end{aligned} \tag{4.7}$$

where $t_\pm = t_1 \pm it_2$, $\Delta M^2 = M_2^2 - M_1^2$, and we have kept only those terms which are of leading order in $m_\mu/M_{1,2}$. Continuing,

$$\left[\bar{t}_\alpha \frac{m}{\mu^2(1-x)+m^2x} t_\alpha\right]_{e\mu} = \frac{1}{2}\left(\frac{1-\gamma_5}{2}\right)\left[t_-^R \frac{m}{\mu^2(1-x)+m^2x} t_+^L\right]_{e\mu}$$

$$= -\frac{1}{2}\left(\frac{1-\gamma_5}{2}\right)\frac{xm_\mu \Delta M^2 \sin\phi\cos\phi}{\left[\mu^2(1-x)+M_1^2x\right]\left[\mu^2(1-x)+M_2^2x\right]} \tag{4.8}$$

Next

$$\left[m_\alpha \frac{1}{\mu^2\left[\mu^2(1-x)+m^2x\right]}\bar{m}_\alpha m\right]_{e\mu} = \left[\bar{t}_\alpha \frac{m}{\mu^2\left[\mu^2(1-x)+m^2x\right]}(t_\alpha m - m\bar{t}_\alpha)\right]_{e\mu}$$

$$= -\frac{1}{2}\left(\frac{1-\gamma_5}{2}\right)\frac{m_\mu\left[xm_\mu^2 \Delta M^2 \sin\phi\cos\phi + \mu^2(1-x)\,\Delta M^2 \sin\phi\cos\phi\right]}{\mu^2\left[\mu^2(1-x)+M_1^2x\right]\left[\mu^2(1-x)+M_2^2x\right]} \tag{4.9}$$

and finally,

$$\left[m_\alpha \frac{m}{\mu^2\left[\mu^2(1-x)+m^2x\right]}m_\alpha\right]_{e\mu} = \left[\bar{t}_\alpha \frac{m^2}{\mu^2\left[\mu^2(1-x)+M^2x\right]}(\bar{t}_\alpha m - mt_\alpha)\right]_{e\mu}$$

$$= \frac{1}{2}\left(\frac{1-\gamma_5}{2}\right)\frac{m_\mu\left[\mu^2(1-x)\,\Delta M^2 \sin\phi\cos\phi - \mu^2(1-x)\,\Delta M^2 \sin\phi\cos\phi\right]}{\mu^2\left[\mu^2(1-x)+M_1^2x\right]\left[\mu^2(1-x)+M_2^2x\right]}$$

$$= 0 \tag{4.10}$$

We remark in passing that the Cheng-Li model with just one Higgs doublet satisfies the conditions discussed in Section III for strong suppression of $\mu \to e\gamma$. In particular, Eqs. (3.23) become

$$\left[t_- t_+\right]_{e\mu} \equiv \left[\sum_\alpha t_\alpha t_\alpha\right]_{e\mu} = 0 \tag{4.11}$$

and

$$\left[t_- m\, t_+\right]_{e\mu} \equiv \left[\sum_\alpha t_\alpha m\, t_\alpha\right]_{e\mu} = 0 \tag{4.12}$$

where the sums extend over all gauge indices. Equation (4.11) is a consequence of the fact that separate μ- and e-number conservation is violated in this model only by mass terms. If $m_e = m_\mu$, lepton fields could be redefined so that separate lepton-number conservation was restored. Equation (4.11) states that the gauge couplings themselves do not violate separate conservation. This requires fermion mass insertions (the second term on the right in Eq. (3.22)), and strong suppression of $\mu \to e\gamma$ results. Similarly, Eq. (4.12) states that there is no divergent, off-diagonal mass term in the e-μ self-energy matrix. This is expected since there is no counterterm to remove such a divergence (in the basis in which m is diagonal). Again, mass insertions are required for a nonvanishing result in Eq. (4.8).

Putting Eqs. (4.7)-(4.10) into Eq. (4.2), we find, to leading order in M_i^2/μ^2,

$$F_1 = \frac{ie\, g^2 m_\mu\, \Delta M^2 \sin\phi\, \cos\phi}{64\pi^2 \mu^4} \left(\frac{1-\gamma_5}{2}\right)(6-1)$$

$$= \frac{5ie\, G_F m_\mu\, \Delta M^2 \sin 2\phi}{16\sqrt{2}\, \pi^2 \mu^2} \left(\frac{1-\gamma_5}{2}\right) \tag{4.13}$$

where $G_F/\sqrt{2} = g^2/8\mu^2$ is the Fermi coupling and 6 comes from $\mu_L^- \to e_R^-\gamma$, -1 from $\mu_R^- \to e_R^-\gamma$. Let

$$\epsilon = \frac{\Delta M^2 \sin 2\phi}{2\mu^2} \tag{4.14}$$

Then the branching ratio for $\mu \to e\gamma$ in this model is

$$B(\mu \to e\gamma) = \frac{\Gamma(\mu \to e\gamma)}{\Gamma_\mu} = \frac{75\alpha}{32\pi}\, \epsilon^2 \tag{4.15}$$

where $\Gamma_\mu \cong G_F^2 m_\mu^5/192\pi^3$. For $\Delta M^2 \sin\phi\, \cos\phi \sim 1\ \mathrm{GeV}^2$ and $\mu \sim 60$ GeV, $B \cong 5 \times 10^{-10}$ which is to be compared with the present experimental upper limit of 2.2×10^{-8}.

V. OTHER EXPERIMENTAL CONSEQUENCES

Our modification of the Cheng-Li model has a number of other experimental consequences:

(a) The mixing of neutrinos with heavy neutral leptons in the upper members of the left-handed lepton doublets produces a small but nonnegligible violation of electron-muon universality. From Eqs. (2.4) and (2.5), we see that the Fermi coupling constants G_e, G_μ, and $G_{\mu e}$ measured respectively in electronic semileptonic processes, muonic semileptonic processes, and $\mu \to e\nu_\mu\bar{\nu}_e$ are related to the "true" Fermi coupling G by

$$\left(\frac{G_e}{G}\right)^2 = (1-\epsilon_1)^2 + \epsilon_3^2 \cong 1 - m_e^2\left(\frac{\cos^2\phi}{M_1^2} + \frac{\sin^2\phi}{M_2^2}\right)$$

$$\left(\frac{G_\mu}{G}\right)^2 = (1-\epsilon_2)^2 + \epsilon_3^2 \cong 1 - m_\mu^2\left(\frac{\sin^2\phi}{M_1^2} + \frac{\cos^2\phi}{M_2^2}\right) \tag{5.1}$$

$$\frac{G_{\mu e}}{G} = \frac{G_e G_\mu}{G^2}$$

Neglecting terms of order m_e^2/M_i^2, we have then

$$G_\mu = G_{\mu e}$$

$$\frac{G_e}{G_{\mu e}} \cong 1 + \frac{m_\mu^2}{2}\left(\frac{\sin^2\phi}{M_1^2} + \frac{\cos^2\phi}{M_2^2}\right) \tag{5.2}$$

That is, for neutral lepton masses M_i of order 1 GeV, we would find that the Fermi coupling constant of beta decay should be about 0.5% greater than would be expected (after radiative corrections and the Cabibbo angle are taken into account) from the rate for $\mu \to e\nu_\mu\bar{\nu}_e$. At present, using a Cabibbo angle θ_c with $\sin\theta_c = 0.229 \pm 0.003$, and taking into account only <u>nuclear</u> Coulomb effects, it appears[8] that the Fermi coupling in beta decay is greater than expected from $\mu \to e\nu_\mu \bar{\nu}_e$ by 1.04±0.08%. However, according to the best current estimates,[9] the "inner" radiative corrections in nuclear beta decay and $\mu \to e\nu_\mu \bar{\nu}_e$ would increase the Fermi coupling of beta decay relative to that in $\mu \to e\nu_\mu \bar{\nu}_e$ by 1.01%. This would leave no room for an additional 0.5% enhancement of beta decay relative to $\mu \to e\nu_\mu \bar{\nu}_e$ due to the mixing effects discussed here.

There are many ways of resolving this discrepancy. There might be a new heavy quark with charge -1/3; if this quark appears in a linear combination along with the s and d quarks as the bottom member of the weak doublet containing the u quark, then the factor "$\cos\,\theta_c$" appearing in the beta decay coupling would be less than would be expected from the "$\sin\,\theta_c$" measured in $\Delta S=1$ semileptonic decays. Also, the calculations of radiative corrections in nucleon beta decay may be invalidated by effects of the strong interactions. And of course, if $1/M_1^2$ is of order $(2\ \mathrm{GeV})^{-2}$ instead of $(1\ \mathrm{GeV})^{-2}$, the effect of ν-N mixing is reduced to 0.1%, well within present experimental uncertainties.

In considering radiative corrections to μ-e universality, one must take into account possible contributions due to new leptons and/or quarks. In the present model, all such new contributions are at most of order $m_\mu/M_{1,2}$ times as large as those in the standard model, and so do not affect our conclusions.

(b) The classic two-neutrino experiment[10] should show a finite but very small probability for muon-number nonconservation. If ν is the neutrino produced in the decay $\pi \to \mu\nu$, then the probability of producing an electron rather than a muon in ν-nucleon collisions is

$$\frac{\sigma(\nu \to e)}{\sigma(\nu \to \mu)} = \left| \frac{(1-\epsilon_2)\epsilon_3 + (1-\epsilon_1)\epsilon_3}{(1-\epsilon_2)^2 + \epsilon_3^2} \right|^2$$

$$\cong 4\epsilon_3^2 \cong m_e^2 m_\mu^2 \cos^2\phi\, \sin^2\phi \left(\frac{1}{M_1^2} - \frac{1}{M_2^2}\right)^2 \qquad (5.3)$$

Even if M_1 or M_2 were as small as 500 MeV, this probability would be less than 10^{-8}.

(c) The heavy neutral leptons can be produced by the neutral currents in neutrino-nucleon collisions. At energies sufficiently far above threshold, the ratio of the cross section for N-production to that for "ordinary" neutral current processes is

$$\frac{\sigma(\nu \to N_1) + \sigma(\nu \to N_2)}{\sigma(\nu \to \nu)} \cong |\epsilon_6|^2 + |\epsilon_7|^2 = m_\mu^2 \left(\frac{\cos^2\phi}{M_1^2} + \frac{\sin^2\phi}{M_2^2} \right) \qquad (5.4)$$

For M_1 and M_2 of order 1 GeV, this is about 1%. Alternative production mechanisms are e^+e^- colliding beams, electroproduction via high energy muon beams, or semileptonic decays of heavy quarks.[2]

(d) We also note that the coupling of the neutral intermediate boson Z^o to electrons and muons is purely vector in this model. This would eliminate the leading partiy-violating effects of neutral currents in heavy atoms, though not in hydrogen. However, we can also consider an extended version of the Cheng-Li models, with multiplets

$$\begin{pmatrix} a_i^o \\ a_i^- \end{pmatrix}_L \quad \begin{pmatrix} b_i^o \\ b_i^- \end{pmatrix}_R \quad \begin{pmatrix} c_i^o \\ c_i^- \end{pmatrix}_L \quad d_{iR}^- \qquad (5.5)$$

Such a theory would contain two neutrinos, two massive neutral leptons, and four massive charged leptons, including two new heavy charged leptons as well as the e^- and μ^-. If either of the d_{iR}^- contain an appreciable e^- component, then the coupling of the Z^o to the e^- would have an appreciable axial-vector part.

Of course, neither this model nor the original Cheng-Li model puts any constraints on the quark multiplet structure. If all the quarks are in left-handed doublets, then neutrino-nuclear neutral current processes would exhibit precisely the same parity violation as in the original SU(2) $\otimes$ U(1) model.

(e) Finally, the correction to $a_\mu = \frac{g-2}{2}$ for the muon in this model is small. From a diagram with emission and absorption of a charged W and an intermediate N, we find that "right-right" diagrams (analogous to Fig. 1) contribute a term equal to that of the "left-left" diagram with an intermediate neutrino. This "left-left" term has been calculated[11] to be well below the present experimental limit.[12] "Left-right" diagrams are enhanced by a factor M_N/m_μ from the mass insertion in intermediate lines, but suppressed by a factor $\lesssim m_\mu/M_N$ at the W-μ_L-N_L vertex. The net result from the W-exchange graphs is

$$a_\mu^W = -\frac{G_F m_\mu^2}{6\sqrt{2}\,\pi^2} \quad , \tag{5.6}$$

which is -2/5 the W-contribution of the standard model.[11] The contribution from Z-exchange graphs is found to be

$$a_\mu^Z = \frac{G_F\, m_\mu^2 \left(1 - 2\sin^2\theta_W\right)^2}{6\sqrt{2}\,\pi^2} \quad . \tag{5.7}$$

Here, $\tan\theta_W = g'/g$, the ratio of U(1) to SU(2) gauge coupling constants. For the model with only one Higgs doublet, there is no contribution to a_μ from physical Higgs mesons.

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FIGURE CAPTION

1. Graphs contributing to the transition magnetic moment for $\ell_1 \rightarrow \ell_2 + \gamma$ in an arbitrary R_ξ gauge. Internal wavy lines are weak gauge bosons and dashed lines are Higgs mesons.

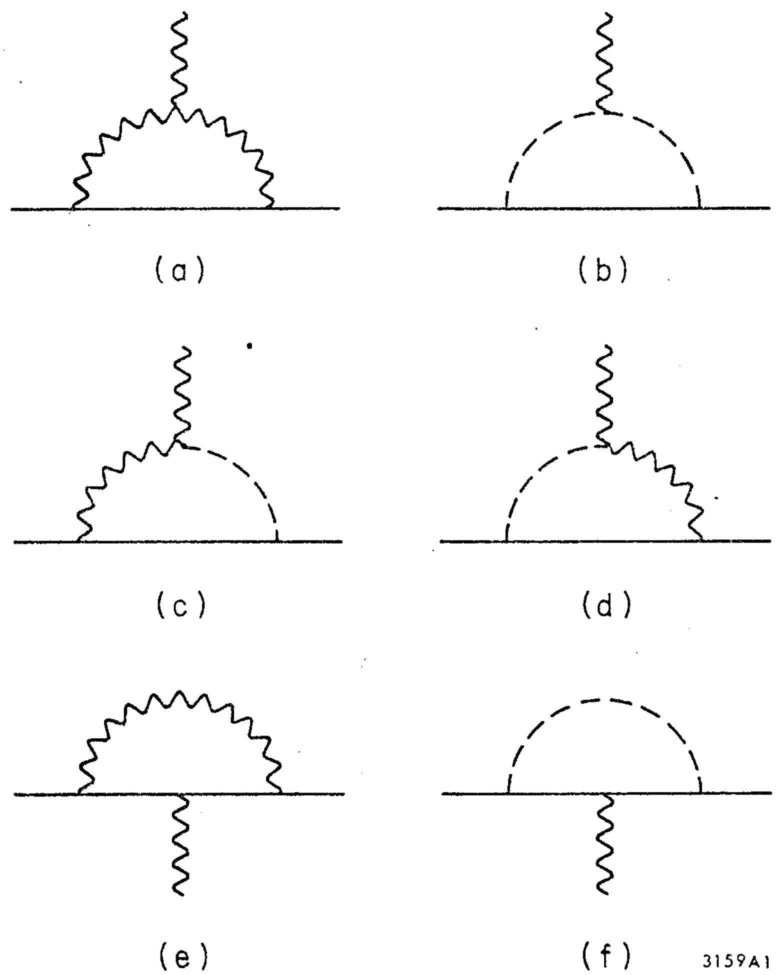

Fig. 1