SLAC-PUB-1924
April 1977
(T)

A TWO-COMPONENT SIGMA MODEL AND MODIFIED GOLDBERGER-TREIMAN RELATION*

P. Y. Pac

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

and

Department of Physics
Seoul National University, Seoul 151, Korea

ABSTRACT

A two-component SU(2) × SU(2) nonlinear σ-model with a general symmetry-breaking term is presented in which an SU(2) symmetry of internal discrete transformations is introduced. In a redefined PCAC relation this model gives a modified Goldberger-Treiman relation with a correction factor. The estimated value of the axial vector coupling constant g_A in neutron β-decay is in good agreement with experiment.

(Submitted to Phys. Rev. D)

*Work supported by the Energy Research and Development Administration and SNU-AID Graduate Basic Sciences Project.

Nowadays the validity of the Goldberger-Treiman (GT) relation[1] is properly understood as a consequence of a slightly broken SU(2) × SU(2) chiral symmetry with the pion as the Nambu-Goldstone boson.[2,3] With the recent experimental data[4,5,6] the corrections to the GT relation are about 6% ($\Delta_{N\pi}$(exp) = 0.06 ± 0.02). It was shown that in the unsubtracted dispersion treatment continuum contributions from 3π, $\rho\pi$, or $\sigma\pi$ states are too small to explain these corrections.[7,8] The most attractive candidate to enhance the corrections has been a heavy pion, the π' (which is not a Goldstone boson).[7,9] This two-component theory of PCAC was also used in the study of $\pi^0 \rightarrow 2\gamma$ decay[10] and generalized to many heavy bosons.[11] The possibilities of hadronic symmetry-breaking due to weak and electromagnetic interactions have also been studied in connection with these corrections.[12,13] In spite of all these efforts the understanding of these corrections still remains unsatisfactory.

In this article we present a two-component SU(2) × SU(2) nonlinear σ-model with the general symmetry-breaking term[14] in the tree approximation. Here the two components form an SU(2) discrete symmetry doublet. We first introduce the SU(2) symmetry of discrete transformations in the context of the nonlinear realization of the SU(2) × SU(2) σ-model. Then the usual GT relation is derived in the one-component theory of our model where the PCAC relation is redefined. This one-component theory is then extended to the two-component case. It is shown that the two-component theory gives a modified GT relation with a correction factor (a function of m_N, f_π, $G_{N\pi}$, and a characteristic constant of this model γ^2) to the usual GT relation, and that the estimated value of g_A (the axial vector coupling constant in neutron β-decay) is in very good agreement with experiments.

The SU(2) × SU(2) nonlinear σ-model which provides a realization of chiral symmetry in terms of the fundamental pion field alone is given by the condition

$$\sigma^2 + \pi^2 = \frac{1}{(2a)^2} = f_\pi^2 . \tag{1}$$

a is a constant with the dimension of length and f_π the pion decay constant. As pointed out by Weinberg,[15] the simplest nonlinear realization which rationalizes the relation between π^α and σ is as follows:

$$\pi^\alpha = \frac{\phi^\alpha}{a^2\phi^2 + 1} , \quad (\alpha = 1, 2, 3) \tag{2}$$

$$\sigma = \frac{-1}{2a} \cdot \frac{a^2\phi^2 - 1}{a^2\phi^2 + 1} , \tag{3}$$

where ϕ^α is the fundamental pion field. Eq. (1) is invariant under the chiral gauge transformations in the $(\vec{\pi}, \sigma)$-representation:

$$\pi^\alpha \to \pi^\alpha - \Lambda^\alpha \sigma , \tag{4}$$

$$\sigma \to \sigma + \Lambda^\alpha \pi^\alpha , \tag{5}$$

which are expressed as

$$\phi^\alpha \to \phi^\alpha - \frac{1}{2a}(1 + a^2\phi^2)\Lambda^\alpha \tag{6}$$

in the $\vec{\phi}$-representation, Λ^α being an infinitesimal constant vector component.

To begin, let us consider a R(a) symmetry which contains the following gauge transformations in the $\vec{\phi}$-representation:

$$R_1(a)\phi^\alpha(x)R_1^{-1}(a) = \frac{1}{a^2\phi^\alpha(x)} , \tag{7}$$

$$R_2(a)\phi^\alpha(x)R_2^{-1}(a) = -\phi^\alpha(x) , \tag{8}$$

$$R_3(a)\phi^\alpha(x)R_3^{-1}(a) = -\frac{1}{a^2\phi^\alpha(x)} \tag{9}$$

with the SU(2) commutation relations

$$[R_k(a), R_\ell(a)]_- = 2i\epsilon_{k\ell m} R_m(a) , \qquad (k,\ell,m = 1,2,3) \tag{10}$$

and

$$[R_k(a), R_\ell(a)]_+ = 2\delta_{k\ell} . \tag{11}$$

Then from Eqs. (2) and (3) we obtain the following transformation properties in the $(\vec{\pi},\sigma)$-representation:

$$R_1: \begin{cases} R_1(a)\sigma R_1^{-1}(a) = -\sigma \\ R_1(a)\pi^\alpha R_1^{-1}(a) = \pi^\alpha \end{cases} , \tag{12}$$

$$R_2: \begin{cases} R_2(a)\sigma R_2^{-1}(a) = \sigma \\ R_2(a)\pi^\alpha R_2^{-1}(a) = -\pi^\alpha \end{cases} , \tag{13}$$

$$R_3: \begin{cases} R_3(a)\sigma R_3^{-1}(a) = -\sigma \\ R_3(a)\pi^\alpha R_3^{-1}(a) = -\pi^\alpha \end{cases} . \tag{14}$$

Such R_ℓ's form an SU(2) symmetry of discrete transformations in the $(\vec{\pi},\sigma)$-representation. This symmetry commutes with the isotopic SU(2) subgroup of the internal O(4) symmetry in the $(\vec{\pi},\sigma)$-representation. In fact, we have

$$[R_k, R_\ell]_- = 2i\epsilon_{k\ell m} R_m , \tag{15}$$

$$[R_k, R_\ell]_+ = 2\delta_{k\ell} . \tag{16}$$

The chiral symmetric Lagrangian density in the linear realization is invariant under the R-symmetry. It is to be noted that the chiral gauge transformations, Eqs. (4) and (5), only commute with the total discrete transformation operator R_3. For later use we introduce the R_3 doublet, $(\vec{\pi}_1, \sigma_1)$ and $(\vec{\pi}_2, \sigma_2)$ obeying the following transformation properties:

$$R_3(\vec{\pi}_1, \sigma_1)R_3^{-1} = (\vec{\pi}_1, \sigma_1) \ , \tag{17}$$

$$R_3(\vec{\pi}_2, \sigma_2)R_3^{-1} = (-\vec{\pi}_2, -\sigma_2) \ . \tag{18}$$

Here we assume the R_3 transformation property of $\bar{\psi}\psi$ is the same as the σ_1.

Next we start with the one-component SU(2) × SU(2) chiral invariant Lagrangian density:[16]

$$L = -\frac{1}{2}\left[(\partial_\mu \sigma)^2 + (\partial_\mu \pi)^2\right] - \bar{\psi}\gamma_\mu \partial_\mu \psi - G_{N\pi}\bar{\psi}(\sigma - i\gamma_5 \tau^\alpha \pi^\alpha)\psi \ . \tag{19}$$

The nucleon mass and the pion mass μ_π are generated by the following general symmetry-breaking term[14]

$$a\sigma + b\bar{\psi}\psi \qquad (a, b > 0) \tag{20}$$

with the nonlinear constraint condition

$$\sigma^2 + \pi^2 = f_\pi^2 \ . \tag{21}$$

We choose a, b, and σ to be

$$a = f_\pi \mu_\pi^2 \equiv \epsilon \ , \tag{22}$$

$$b = f_\pi G_{N\pi} - m_N \ , \tag{23}$$

and

$$\sigma = f_\pi \sqrt{1 - \pi^2/f_\pi^2} \ , \tag{24}$$

where ϵ is the symmetry-breaking parameter in the sense of Dashen[14] and m_N the nucleon mass. In this broken chiral system we then have the following vector and axial vector currents:

$$V_\mu^\alpha = (\vec{\pi} \times \partial_\mu \vec{\pi})_\alpha + \bar{\psi} i\gamma_\mu \tfrac{1}{2}\tau^\alpha \psi \ , \tag{25}$$

$$A_\mu^\alpha = (\sigma\partial_\mu \pi^\alpha - (\partial_\mu \sigma)\pi^\alpha) + \bar{\psi} i\gamma_\mu \gamma_5 \tfrac{1}{2}\tau^\alpha \psi \ , \tag{26}$$

and their derivatives

$$\partial_\mu V^\alpha_\mu = 0 \ , \tag{27}$$

$$\partial_\mu A^\alpha_\mu = f_\pi \mu_\pi^2 \pi^\alpha - b\bar{\psi} i\gamma_5 \tau^\alpha \psi . \tag{28}$$

Using the expansion of σ (Eq. (24)) in terms of π^2, the total Lagrangian density is rewritten as

$$L_{tot} = -\tfrac{1}{2}(\partial_\mu \pi)^2 - \tfrac{1}{2}\mu_\pi^2 \pi^2 - \bar{\psi}(\gamma_\mu \partial_\mu + m_N)\psi + G_{N\pi}\bar{\psi} i\gamma_5 \tau^\alpha \psi \pi^\alpha + \ldots \ , \tag{29}$$

where the nucleon mass m_N fixes the symmetry breaking constant b by Eq. (23). From the leading part (associated with one pion) of L_{tot} in Eq. (29), we get an approximation consistent with Eq. (28) by setting

$$A^\alpha_\mu \simeq f_\pi \partial_\mu \pi^\alpha + \bar{\psi} i\gamma_\mu \gamma_5 \tfrac{1}{2}\tau^\alpha \psi , \tag{30}$$

$$\partial_\mu(\bar{\psi} i\gamma_\mu \gamma_5 \tfrac{1}{2}\tau^\alpha \psi) \simeq m_N \bar{\psi} i\gamma_5 \tau^\alpha \psi , \tag{31}$$

and

$$(\Box - \mu_\pi^2)\pi^\alpha \simeq - G_{N\pi}\bar{\psi} i\gamma_5 \tau^\alpha \psi . \tag{32}$$

Then we rewrite Eq. (28) as

$$\partial_\mu\left(A^\alpha_\mu - \frac{b}{G_{N\pi}}\partial_\mu \pi^\alpha\right) = \partial_\mu\left(\left(f_\pi - \frac{b}{G_{N\pi}}\right)\partial_\mu \pi^\alpha + \bar{\psi} i\gamma_\mu \gamma_5 \tfrac{1}{2}\tau^\alpha \psi\right) = \left(f_\pi - \frac{b}{G_{N\pi}}\right)\mu_\pi^2 \pi^\alpha , \tag{33}$$

where we have used Eq. (32). Multiplying by a factor $f_\pi G_{N\pi}(f_\pi G_{N\pi} - b)^{-1}$ on both sides of Eq. (33), this relation has the standard PCAC expression[17] of

$$\partial_\mu A^\alpha_{\mu,\mathrm{eff}} = f_\pi \mu_\pi^2 \pi^\alpha \ , \tag{34}$$

where $A^\alpha_{\mu,\mathrm{eff}}$ is given by

$$A^\alpha_{\mu,\mathrm{eff}} = f_\pi \partial_\mu \pi^\alpha + g_{A,GT}\bar{\psi} i\gamma_\mu \gamma_5 \tfrac{1}{2}\tau^\alpha \psi , \tag{35}$$

and the axial vector coupling constant $g_{A,GT}$ is

$$g_{A,GT} = \frac{f_\pi G_{N\pi}}{m_N} = \frac{f_\pi G_{N\pi}}{f_\pi G_{N\pi} - b} . \tag{36}$$

Eq. (36) is just the celebrated GT relation. Using the redefined PCAC relation (Eq. (34)), this result can be easily confirmed by the one-pion pole dominance approximation in the unsubtracted dispersion treatment or by the axial vector current conservation method, under the on- and off-shell smoothness hypotheses.[18] It is to be noted that in our model the axial vector current conservation does not correspond to exact chiral symmetry ($\partial_\mu A_\mu^\alpha = 0$, $\mu_\pi^2 = 0$, and $b = 0$; $g_{A,GT} = 1$). Hence the redefined PCAC relation in Eq. (34) should be reinterpreted as the consequence of deviations from the redefined (or partially) exact chiral symmetry ($\partial_\mu A_\mu^\alpha = 0$, $\mu_\pi^2 = 0$, and $b = f_\pi G_{N\pi} - m_N > 0$; $g_{A,GT} = \frac{f_\pi G_{N\pi}}{m_N}$), which is consistent with current algebra approach.

Next, restricting to the tree approximation, we proceed with the two-component SU(2) × SU(2) chiral invariant Lagrangian density:

$$L = -\frac{1}{2}\sum_{n=1}^{2}[(\partial_\mu \sigma_n)^2 + (\partial_\mu \pi_n)^2] - \bar{\psi}\gamma_\mu \partial_\mu \psi - \sum_{n=1}^{2} G_{N\pi_n}\bar{\psi}(\sigma_n - i\gamma_5 \tau^\alpha \pi_n^\alpha)\psi , \tag{37}$$

$$(G_{N\pi_n} = G_{N\pi_n}(-\mu_n^2)).$$

Here we have identified the total discrete symmetry doublet $(\vec{\pi}_1, \sigma_1)$ and $(\vec{\pi}_2, \sigma_2)$ as $(\vec{\pi}_1 \equiv \vec{\pi}, \sigma_1 \equiv \sigma)$ and $(\vec{\pi}_2 \equiv \vec{\pi}', \sigma_2 \equiv \sigma')$, respectively. The general symmetry breaking terms are given by

$$\sum_{n=1}^{2} a_n \sigma_n + b\bar{\psi}\psi \qquad (a_n > 0) \tag{20'}$$

with the conditions

$$\sigma_n^2 + \pi_n^2 = f_n^2 \qquad (n = 1, 2) . \tag{21'}$$

Now let us choose a_n and σ_n as follows:

$$a_2 = f_2\mu_2^2 = f_1\mu_1^2 = \epsilon \ , \tag{22'}$$

$$\sigma_n = f_n\sqrt{1 - \pi_n^2/f_n^2} \ , \tag{24'}$$

where $b = f_\pi G_{N\pi} - m_N$ has been fixed in Eq. (23). We observe that the R_3-symmetry breaking terms in L_{tot} are

$$- G_{N\pi_2}\bar{\psi}(\sigma_2 - i\gamma_5\tau^\alpha\pi_2^\alpha)\psi + a_2\sigma_2 \ ,$$

which guarantees $G_{N\pi_1} \neq G_{N\pi_2}$ and $\mu_1^2 \neq \mu_2^2$.

Then, the vector and axial vector currents of this system are

$$V_\mu^\alpha = \sum_n(\vec{\pi}_n \times \partial_\mu\vec{\pi}_n)_\alpha + \bar{\psi}i\gamma_\mu\frac{1}{2}\tau^\alpha\psi \ , \tag{38}$$

$$A_\mu^\alpha = \sum_n[(\sigma_n\partial_\mu\pi_n^\alpha - (\partial_\mu\sigma_n)\pi_n^\alpha] + \bar{\psi}i\gamma_\mu\gamma_5\tfrac{1}{2}\tau^\alpha\psi \ , \tag{39}$$

whence

$$\partial_\mu V_\mu^\alpha = 0 \ , \tag{40}$$

$$\partial_\mu A_\mu^\alpha = \sum_n f_n\mu_n^2\pi_n^\alpha - b\bar{\psi}i\gamma_5\tau^\alpha\psi \ . \tag{41}$$

Using the expansion of σ_n (Eq. (24')) in terms of π_n^2, the total Lagrangian density is rewritten as

$$L_{tot} = -\frac{1}{2}\sum_n(\partial_\mu\pi_n)^2 - \frac{1}{2}\sum_n\mu_n^2\pi_n^2 - \bar{\psi}(\gamma_\mu\partial_\mu + m_N') + \sum_n G_{N\pi_n}\bar{\psi}i\gamma_5\tau^\alpha\psi\pi_n^\alpha + \ldots, \tag{42}$$

where the shifted nucleon mass m_N' is given by

$$m_N' = \sum_n f_n G_{N\pi_n} - b \ . \tag{43}$$

Just as in the one-component theory we take the approximation which is consistent with Eq. (41):

$$A^{\alpha}_{\mu} \simeq \sum_{n} f_n \partial_{\mu} \pi^{\alpha}_{n} + \bar{\psi} i \gamma_{\mu} \gamma_5 \tfrac{1}{2} \tau^{\alpha} \psi , \tag{44}$$

$$\partial_{\mu}(\bar{\psi} i \gamma_{\mu} \gamma_5 \tfrac{1}{2} \tau^{\alpha} \psi) \simeq m'_N \bar{\psi} i \gamma_5 \tau^{\alpha} \psi , \tag{45}$$

and

$$(\Box - \mu^2_n) \pi^{\alpha}_{n} \simeq -G_{N\pi_n} \bar{\psi} i \gamma_5 \tau^{\alpha} \psi . \tag{46}$$

Thus the divergence of axial vector current can be taken in the standard PCAC form:

$$\partial_{\mu} A^{\alpha}_{\mu,\,\mathrm{eff}} = \sum_{n=1}^{2} f_n \mu^2_n \pi^{\alpha}_{n} , \tag{47}$$

with

$$A^{\alpha}_{\mu,\,\mathrm{eff}} = \sum_{n} f_n \partial_{\mu} \pi^{\alpha}_{n} + g_A \bar{\psi} i \gamma_5 \gamma_{\mu} \frac{1}{2} \tau^{\alpha} \psi , \tag{48}$$

$$g_A = \frac{\sum_{n} f_n G_{N\pi_n}}{\sum_{n} f_n G_{N\pi_n} - b} , \tag{49}$$

or

$$g_A = \frac{\gamma^2 f_{\pi} G_{N\pi}}{\gamma^2 f_{\pi} G_{N\pi} - b} = \frac{\gamma^2 f_{\pi} G_{N\pi}}{(\gamma^2 - 1) f_{\pi} G_{N\pi} + m_N} , \tag{51}$$

where

$$\gamma^2 = 1 + \left(f_2 G_{N\pi_2} / f_1 G_{N\pi_1} \right) . \tag{51}$$

Eq. (50) is just the desired modified GT relation. This g_A can be expressed in terms of the corrections $\Delta_{N\pi}$

$$g_A = (1 - \Delta_{N\pi}) g_{A,\,GT} , \tag{52}$$

where

$$\Delta_{N\pi} = \Delta_{N\pi}(m_N, f_{\pi}, G_{N\pi};\ \gamma^2) = \frac{(\gamma^2 - 1)(f_{\pi} G_{N\pi} - m_N)}{(\gamma^2 - 1)\, f_{\pi} G_{N\pi} + m_N} . \tag{53}$$

Using the recent experimental numbers for m_N, f_π, $G_{N\pi}$, and g_A (or $\Delta_{N\pi}$), we obtain the characteristic constant[19] of this model

$$\gamma^2 = \frac{\pi^2}{8}\left(= \sum_{r=1}^{\infty} \frac{1}{(2r-1)^2}\right) \simeq 1.2337\ldots. \tag{54}$$

Conversely, if we postulate γ^2 as the one given in Eq. (54), then the estimated value of g_A (or $\Delta_{N\pi}$) is in excellent agreement with data (see Table I).

From Eqs. (22'), (51), and (54) we have

$$f_{\pi_2} = \frac{\mu_1^2}{\mu_2^2} f_{\pi_1} , \tag{55}$$

$$G_{N\pi_2} = (\gamma^2-1)\frac{\mu_2^2}{\mu_1^2} G_{N\pi_1} . \tag{56}$$

Both Eqs. (55) and (56) suggest that there exists the heavy pion, $\pi_2 = \pi'$, obeying the bounds:

$$\mu_{\pi'}^2 \geq (3\mu_\pi)^2 , \tag{57}$$

$$G_{N\pi'} \geq (\gamma^2-1)9G_{N\pi} \simeq 2.1\, G_{N\pi} , \tag{58}$$

$$f_{\pi'} \leq \frac{1}{9} f_\pi . \tag{59}$$

In conclusion, our results indicate that the corrections to the GT relation are almost covered by the effect from the heavy pion π'. Using the redefined PCAC relation (Eq. (47)), the modified GT relation (Eq. (50)) can be easily obtained by summing up the effect from the heavy pion π'-pole dominance in the unsubtracted dispersion treatment,[20] or by the axial vector current conservation method, under the on- and off-shell smoothness hypotheses. It is to be noted that a shift in the experimental numbers has been all in the direction of reducing the experimental values of $\Delta_{N\pi}$. In fact, the value of $g_{A,\mathrm{exp}}$ has increased with time and the $NN\pi$ coupling constant $G_{N\pi}$ has tended to decrease with time. The estimated values for g_A with variant $G_{N\pi}$'s in the one- and two-component theories are summarized in Table I.

The author would like to thank Professor S. D. Drell for his hospitality at SLAC and Professor C. W. Kim, Dr. M. Hirayama, and Dr. M. Weinstein for discussions. He is much indebted to Dr. C. Tze for reading this manuscript. He also wishes to express his appreciation to the Center of Theoretical Physics and Chemistry, Seoul, where this work was initiated.

REFERENCES

1. M. L. Goldberger and S. B. Treiman, Phys. Rev. 111, 354 (1958).
2. R. Dashen, Phys. Rev. 183, 1245 (1969); R. Dashen and M. Weinstein, Phys. Rev. 183, 1261 (1969).
3. H. Pagels, Phys. Rep. 16, 219 (1975).
4. Particle Data Group, "Review of Particle Properties," Rev. Mod. Phys. 48, No. 2, Part II, S1 (1976). $g_A = -(1.25 \pm 0.009)$.
5. H. Pilkuhn et al., Nucl. Phys. B 65, 460 (1972). $G^2_{N\pi}/4\pi = 14.64^{+0.54}_{-0.72}$, where $G_{np\pi^+} = -\sqrt{2}\, G_{N\pi}$.
6. H. Pagels and A. Zepeda, Phys. Rev. D 5, 3262 (1972). $\sqrt{2}f_\pi = (0.93251 \pm 0.00144)\mu_{\pi^+}$, calculated with the values of G_V given by R. H. Blin-Stoyle and J. M. Freeman, Nucl. Phys. A 150, 369 (1970).
7. H. Pagels, Phys. Rev. 179, 1337 (1969); H. Pagels and A. Zepeda, Phys. Rev. D 5, 3262 (1972).
8. T. P. Cheng and R. Dashen, Phys. Rev. Lett. 26, 594 (1971); A. G. Höhnler, H. P. Jakob, and R. Strauss, Phys. Lett. 35B, 445 (1971); H. F. Jones and M. D. Scadron, Phys. Rev. D 11, 174 (1975).

9. C. Michael, Phys. Rev. 182, 1913 (1969); R. A. Coleman and J. W. Moffat, Phys. Rev. 186, 1635 (1969).

10. S. D. Drell, Phys. Rev. D 7, 2190 (1973); J. M. Newmeyer and S. D. Drell, Phys. Rev. D 8, 4070 (1973).

11. C. A. Domingues, to be published in Phys. Rev. D.

12. M. Weinstein, Phys. Rev. D 7, 1854 (1973).

13. B. de Wit, Phys. Rev. D 9, 3399 (1974); B. de Wit, R. Maciejko, and J. Smith, Preprint ITP-SB 76-53.

14. R. Dashen, "Lectures on Chiral Symmetry Breaking," in Proc. Int. Sch. of Physics "Enrico Fermi" (Academic Press, New York, 1972), p. 204.

15. S. Weinberg, Phys. Rev. 166, 1568 (1968).

16. M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960); B. W. Lee, Chiral Dynamics (Gordon and Breach Science Publishers Inc., New York, 1972).

17. S. Adler and R. Dashen, Current Algebras and Applications to Particle Physics (W. A. Benjamin, Inc., New York, 1967).

18. S. Treiman, R. Jackiw, and D. Gross, Lectures on Current Algebra and its Applications (Princeton University Press, Princeton, N.J., 1972).

19. Since $\gamma^2 f_\pi G_{N\pi} - b \simeq f_\pi G_{N\pi}/\beta^2$ with experimental numbers, we have a simple relation for g_A:

$$g_A \simeq \beta^2 \gamma^2 = 1.2518\ldots ,$$

or

$$g_A \simeq \gamma^2 = 1.2337\ldots,$$

where

$$\beta^2 = \frac{\pi^4}{96} = \sum_{r=1}^{\infty} \frac{1}{(2r-1)^4} \quad .$$

20. In fact, we have

$$2m'_N(0)g_A(0) = \lim_{q^2\to 0} 2\sum_n \frac{f_n\mu_n^2 G_{N\pi_n}}{q^2+\mu_n^2} \quad ;$$

hence

$$g_A(0) = \frac{\gamma^2 f_\pi G_{N\pi}(0)}{\gamma^2 f_\pi G_{N\pi}(0) - b(0)} \quad .$$

TABLE I

Estimated numbers of g_A in the one- and two-component theories $\left(\gamma^2 = \frac{\pi^2}{8}\right)$

$G^2_{N\pi}/4\pi$	$-G_{N\pi}$	$-g_{A,GT} = g_{A,1}$	$-g_{A,2} = g_A$	$\Delta_{N\pi}$	$-g_{A,exp}$
13.90	13.22	1.296	1.228	0.052	
14.00	13.26	1.300	1.230	0.054	
14.30	13.41	1.315	1.241	0.056	
14.64 [Ref. 5]	13.56	1.330	1.252	0.059	1.25±0.009 [Ref. 4]
15.00	13.73	1.346	1.264	0.061	
15.20	13.82	1.355	1.270	0.063	

$m_N = \frac{1}{2}(m_p+m_n) = 6.72\ \mu_{\pi^+}$, $\sqrt{2}\, f_\pi = 0.932\ \mu_{\pi^+}$ [Ref. 6], $\mu_{\pi^+} = 139.7$ MeV.