# ON MASS FORMULAE FOR MESONS AND THE QUARK MODEL* 

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#### Abstract

A mass formula for mesons analogous to the classical Gell-MannOkubo approach is constructed in the quark model, for both the $\operatorname{SU}(3)$ and $\operatorname{SU}(4)$ groups, emphasizing the symmetry breaking of the annihilation terms. Using the present meson masses, mixing angles and quark parameters are calculated for the pseudoscalar and vector multiplets in a quadratic formula. The trivial extension of $\operatorname{SU}(3)$ to $\operatorname{SU}(4)$ is shown to fail.


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[^0]
## I. INTRODUCTION

The success of $\mathrm{SU}(3)$ symmetry in describing mass relations and mixing angles in hadron physics is well known. ${ }^{1}$ With the enlargement which charm has brought, it is natural to try to broaden the mass formalism to $\mathrm{SU}(4)$, with the symmetry breaking pattern chosen so that the $S U(3)$ subgroup is broken in the usual way. This is implemented by assuming that the $\mathrm{SU}(4)$ breaking mass term is composed of two fifteen-plet tensors, one of which is an $\mathrm{SU}(3)$ singlet and the other an $\mathrm{SU}(3)$ octet, isospin singlet. ${ }^{2}$ To be sure this enlargement is meaningful, all the previous good $\mathrm{SU}(3)$ results have to be maintained, while at the same time the new phenomena are explained.

In this paper we study this question in the domain of the $0^{-+}$and $1^{--}$meson multiplets, using a mass formula developed in the quark model. This work is new because on one hand, mass formulae have not been calculated so far for these multiplets with the presently known values of the meson masses; the existing analysis used certain assumptions for charmed masses (or equivalently assumptions for the reduced matrix element), 2,3 and therefore the mixing angles obtained were not reliable. On the other hand, while mass formulae in the quark model have been much discussed ${ }^{4,5}$ a detailed treatment of the explicit symmetry breaking that the data forces to appear in the annihilation terms has not been given.

In Section II we present this simple quark model formalism and its connection with the usual Gell-Mann-Okubo approach. In Section III are collected the numerical results obtained for the mixing angles and quark parameters for both $0^{-+}$and $1^{--}$multiplets using a quadratic mass formula. Finally in Section IV we state our conclusions.

## II. MASS FORMULA IN THE QUARK MODEL

First of all we shall review the usual formalism. The SU(3) Gell-MannOkubo mass formula is obtained ${ }^{6}$ by supposing that the mass (mass ${ }^{2}$ ) ${ }^{7}$ operator is a sum of two $\operatorname{SU}(3)$ tensors, the leading one is a singlet and the other an octet (isospin singlet).

$$
\begin{equation*}
M^{2}=M_{(0)}^{2}+M_{(8)}^{2} \tag{1}
\end{equation*}
$$

Using the Wigner-Eckart theorem, the masses inside a meson octet can be related by means of two unknown parameters $O$ and $D$, so that with the notation ${ }^{8}$ we have

$$
\begin{align*}
& <1\left|M^{2}\right| 1>=O+D \\
& \left.<\frac{1}{2}\left|M^{2}\right| \frac{1}{2}\right\rangle=O-\frac{1}{2} D  \tag{2}\\
& <8\left|M^{2}\right| 8>=O-D
\end{align*}
$$

and therefore

$$
\begin{equation*}
3 M_{8}^{2}+M_{1}^{2}=4 M_{1 / 2}^{2} \tag{3}
\end{equation*}
$$

The nonexistence of a physical isospin singlet candidate fulfilling this mass relation suggests that the breaking mass operator $M_{(8)}^{2}$ connects the $18>$ and $10>$ states so that the real and symmetric mass matrix

$$
\left.<M^{2}\right\rangle=\left(\begin{array}{cc}
\langle 8| M^{2}|8\rangle & \left.<8\left|M^{2}\right| 0\right\rangle  \tag{4}\\
\langle 0| M^{2}|8\rangle & \left.<0\left|M^{2}\right| 0\right\rangle
\end{array}\right)=\left(\begin{array}{cc}
O-D & E \\
E & S
\end{array}\right)
$$

has nonnegligible off-diagonal elements. Therefore for the description of any nonet, four independent reduced mass matrix elements are required. These are fixed (except the E sign) on imposing the eigenvalues to be the physical values of the mass ${ }^{2}$ of the chosen mesons. Once $\left\langle\mathrm{M}^{2}\right\rangle$ is known, the mixing angle $\theta$ between physical and mathematical states is also fixed (except for its sign).

In the $\operatorname{SU}(4)$ case the logical extension of the above indicates ${ }^{2,3}$ that the operator is composed of three pieces with different SU(4) transformation properties: the first is a singlet, the second is a fifteen-plet [SU(3) singlet] and the third is a fifteen-plet [ $S U(3)$ octet, isospin singlet]

$$
\begin{equation*}
M^{2}=M_{(s)}^{2}+M_{(f, s)}^{2}+M_{(f, 0)}^{2} \tag{5}
\end{equation*}
$$

Again the use of the Wigner-Eckart theorem permits one to express the different mass ${ }^{2}$ matrix elements as a linear function of unknown reduced matrix elements. With the convention ${ }^{9}$ we would have ${ }^{3}$

$$
\begin{align*}
& \langle 1| M^{2}|1\rangle=m+3 m_{1}-9 m_{2} \\
& \left.<\frac{1}{2}\left|M^{2}\right| \frac{1}{2}\right\rangle=m+9 m_{1}-9 m_{2} \\
& \langle d| M^{2}|d\rangle=m+3 m_{1}-3 m_{2}  \tag{6}\\
& \langle f| M^{2}|f\rangle=m+9 m_{1}-3 m_{2} \\
& \langle 8| M^{2}|8\rangle=m+11 m_{1}-9 m_{2} \\
& \langle q| M^{2}|q\rangle=m+4 m_{1}
\end{align*}
$$

From which relations corresponding to the classical ones in Eq. (3) can be obtained. Following the same philosophy as in the $\operatorname{SU}(3)$ case, if the physical states are not $\operatorname{SU}(4)$ eigenstates then the real and symmetric mass matrix

$$
\left\langle M^{2}\right\rangle=\left(\begin{array}{ccc}
\left.<8\left|M^{2}\right| 8\right\rangle & \left.<8\left|M^{2}\right| q\right\rangle & \left.<8\left|M^{2}\right| s\right\rangle  \tag{7}\\
\left.<q\left|M^{2}\right| 8\right\rangle & \left.<q\left|M^{2}\right| q\right\rangle & \left.<q\left|M^{2}\right| s\right\rangle \\
\left.<s\left|M^{2}\right| 8\right\rangle & \left.<s\left|M^{2}\right| q\right\rangle & \left.<s\left|M^{2}\right| s\right\rangle
\end{array}\right)=\left(\begin{array}{ccc}
m+11 m_{1}-9 m_{2} & -2 \sqrt{2} m_{1} & A \\
-2 \sqrt{2} m_{1} & m+4 m_{1} & B \\
A & B & m_{0}
\end{array}\right)
$$

has off-diagonal elements which are not negligible.

So the description of every sixteen-plet mass matrix involves six independent reduced matrix elements that are fixed (except for the sign of A and B) by imposing that the physical mass ${ }^{2}$ values be the eigenvalues. The knowledge of $<\mathrm{M}^{2}>$ allows us to calculate the three mixing angles $\theta, \beta$ and $\alpha$ (except for their absolute sign) that exist for this case.

Now that the mass formulae in $\operatorname{SU}(3)-\operatorname{SU}(4)$ have been reviewed, we turn to develop an equivalent formalism in the quark model. This model has been frequently used in hadron spectroscopy either as an alternative parametrization ${ }^{4}$ of unitary symmetry or with some dynamical ingredient. ${ }^{5}$ However, working in the first context, the analysis has an advantage with respect to the purely group theoretic one in that the reduced matrix elements have a direct physical meaning. This also illuminates the parameters obtained in a dynamical approach. In addition the use of the quark language makes the extension to bigger groups immediate, since most of the parameters (except the two new ones) represent the same physical entities.

The quark mass parametrization is attained by expressing each meson in terms of its quark flavor content and writing each meson matrix element as a sum of quark-antiquark mass matrix elements.

This is usually done using two kinds of parameters as an expression of the two dynamical effects present in the bound $q \bar{q}$ system. This is shown pictorially in Fig. 1 and Fig. 2. The first will be referred to as scattering and the second as annihilation diagrams. The consideration of the second ones is qualitatively suggested by the idea that "gluons" are the gauge bosons responsible for the forces that bind quarks together. It is clear that the meson mass difference due to the quark mass difference $\left(m_{s} \neq m_{u}\right)$ is included in the $\delta$ parameter. However the correspondence of the annihilation terms with the physical parameters is not so intuitive.

With the mass assignment implied in Fig. 1 and Fig. 2 and the condition that $\lambda_{u d}=\lambda_{u u}=\lambda_{d d}$ it is obvious that $|u \bar{d}\rangle,|d \bar{u}\rangle, \frac{1}{\sqrt{2}}|u \bar{u}-d \bar{d}\rangle,|u \bar{s}\rangle,|s \bar{u}\rangle,|d \bar{s}\rangle$ and $\mid s \bar{d}>$ are eigenstates of the mass matrix with eigenvalues $\mathscr{P}$ for the first three and $\mathscr{P}+\delta$ for the last four.

The other two states $18>$ and $10>$ that complete the nonet are not (in principle) eigenstates but they span a two dimensional space where to diagonalize the mass operator.

$$
\begin{align*}
\left\langle M^{2}\right\rangle & \equiv\left(\begin{array}{ll}
\langle 8| \mathrm{M}^{2}|8\rangle & \langle 8| \mathrm{M}^{2}|0\rangle \\
\langle 0| \mathrm{M}^{2}|8\rangle & \langle 0| \mathrm{M}^{2}|0\rangle
\end{array}\right)= \\
& =\left(\begin{array}{ll}
\mathrm{P}+\frac{4}{3} \delta+\frac{2}{3}\left(\lambda_{\mathrm{uu}}+\lambda_{\mathrm{ss}}-2 \lambda_{\mathrm{us}}\right) & \frac{-\sqrt{2}}{3}\left(2 \delta-2 \lambda_{\mathrm{uu}}+\lambda_{\mathrm{us}}+\lambda_{\mathrm{ss}}\right) \\
\frac{-\sqrt{2}}{3}\left(2 \delta-2 \lambda_{\mathrm{uu}}+\lambda_{\mathrm{us}}+\lambda_{\mathrm{SS}}\right) & \mathrm{P}+\frac{2}{3} \delta+\frac{1}{3}\left(4 \lambda_{\mathrm{uu}}+4 \lambda_{\mathrm{us}}+\lambda_{\mathrm{ss}}\right)
\end{array}\right) \tag{8}
\end{align*}
$$

Clearly the Gell-Mann-Okubo relation (3) is obtained if the annihilation terms fulfill

$$
\begin{equation*}
\lambda_{u u}+\lambda_{s \mathrm{~s}}=2 \lambda_{\mathrm{us}} \tag{9}
\end{equation*}
$$

Taking this condition as valid, it is clear that in the $\operatorname{SU}(3)$ exact symmetry limit $\left(\delta=0, \lambda_{\mathrm{uu}}=\lambda_{\mathrm{us}}=\lambda_{\mathrm{ss}}\right.$ ) all the octet members have an equal mass ${ }^{2} \mathrm{P}$, being the annihilation terms responsible of the mass difference between the two different SU(3) representations.

One naive temptation would be ${ }^{4,10}$ to attach the $\operatorname{SU}(3)$ symmetry breaking only to scattering terms with the $\delta$ parameter, holding as symmetric the annihilation terms. This ansatz leads to good qualitative results but fails quantitatively.

In fact the pattern of symmetry breaking in annihilation terms that corresponds exactly to the (1) group assumption is as follows

$$
\begin{equation*}
\lambda_{\mathrm{uu}}=\mathrm{P}^{\prime} \quad \lambda_{\mathrm{us}}=\mathrm{P}^{\prime}+\delta^{\prime} \quad \lambda_{\mathrm{ss}}=\mathrm{P}^{\prime}+2 \delta^{\prime} \tag{10}
\end{equation*}
$$

With that parametrization there is a perfect parallelism between both approaches. The correspondence in parameters is

$$
\begin{array}{ll}
P=O+D & P^{\prime}=\frac{1}{3} S-\frac{1}{3} O-\frac{2}{3} D+\frac{\sqrt{2}}{3} E \\
\delta=-\frac{3}{2} D & \delta^{\prime}=D-\frac{1}{\sqrt{2}} E \tag{11}
\end{array}
$$

and the new (8) mass matrix would be

$$
<\mathrm{M}^{2}>=\left(\begin{array}{ll}
\mathrm{P}+\frac{4}{3} \delta & -\frac{\sqrt{2}}{3}\left(2 \delta+3 \delta^{\prime}\right)  \tag{12}\\
-\frac{\sqrt{2}}{3}\left(2 \delta+3 \delta^{\prime}\right) & \mathrm{P}+\frac{2}{3} \delta+3\left(\mathrm{P}^{\prime}+\frac{2}{3} \delta^{\prime}\right)
\end{array}\right)
$$

After this, ${ }^{11}$ the extension $\mathrm{SU}(3) \rightarrow \mathrm{SU}(4)$ in quark parameters is trivial, it is depicted in Fig. 3. Now we shall have six independent parameters $\mathrm{P}, \delta, \Delta$, $\mathrm{P}^{\prime}, \delta^{\prime}, \Delta^{\prime}$ and the $3 \times 3$ mass matrix will be

$$
<\mathrm{M}^{2}>=\left(\begin{array}{lll}
\mathrm{P}+\frac{4}{3} \delta & -\frac{\sqrt{2}}{3} \delta & -\sqrt{\frac{2}{3}}\left(\delta+2 \delta^{\prime}\right)  \tag{13}\\
-\frac{\sqrt{2}}{3} \delta & \mathrm{P}+\frac{1}{6} \delta+\frac{3}{2} \Delta & \frac{1}{2 \sqrt{3}}\left(\delta+2 \delta^{\prime}-3 \Delta-6 \Delta^{\prime}\right) \\
-\sqrt{\frac{2}{3}}\left(\delta+2 \delta^{\prime}\right) & \frac{1}{2 \sqrt{3}}\left(\delta+2 \delta^{\prime}-3 \Delta-6 \Delta^{\prime}\right) & \mathrm{P}+4 \mathrm{P}^{\prime}+\frac{1}{2}(\delta+\Delta)+2\left(\delta^{\prime}+\Delta^{\prime}\right)
\end{array}\right)
$$

Being P, $\mathrm{P}^{\prime}, \delta$ and $\delta^{\prime}$ the same that in the $\mathrm{SU}(3)$ case because they are just the same. The new table of correspondence is

$$
\begin{array}{ll}
P=m+3 m_{1}-9 m_{2} & P^{\prime}=\frac{1}{4}\left(m_{0}-m\right)+3 m_{2}+\sqrt{\frac{1}{6}} A+\frac{1}{2 \sqrt{3}} B \\
\delta=6 m_{1} & \delta^{\prime}=-\frac{\sqrt{3}}{2 \sqrt{2}} A-3 m_{1}  \tag{14}\\
\Delta=6 m_{2} & \Delta^{\prime}=-\frac{1}{\sqrt{3}} B-\frac{1}{2 \sqrt{6}} A-3 m_{2}
\end{array}
$$

As a final remark we think it is worth stressing that in the quark model the symmetry breaking in both the scattering and the annihilation terms is qualitatively identical, and in each extension the mass matrix of every multiplet is increased with two new parameters. We would have $P$ and $P^{\prime}$ in $S U(2), P, P^{\prime}$, $\delta$ and $\delta^{\prime}$ in $\operatorname{SU}(3), \mathrm{P}, \mathrm{p}^{\prime}, \delta, \delta^{\prime}, \Delta$ and $\Delta^{\prime}$ in $\mathrm{SU}(4)$, etc. Being apparent that a restriction OZI rule ${ }^{13}$ like would imply the cancellation of all primed parameters and therefore automatic ideal mixing would be attained.

It is also interesting to remark that the extension to wider symmetries like $\operatorname{SU}(6)$ where $\operatorname{SU}(3)$ and spin are tied together, establishes new conditions over the quark parameters of both $1^{--}$and $0^{-+}$nonets. Specifically the assumption that $\operatorname{SU}(6)$ is broken by a thirtyfive-plet tensor ${ }^{14}$ would imply $\left(\mathrm{M}_{\mathrm{K}^{*}}^{2}-\mathrm{M}_{\rho}^{2}=\mathrm{M}_{\mathrm{K}}^{2}-\mathrm{M}_{\pi}^{2}, \mathrm{M}_{\omega}^{2}=\mathrm{M}_{\rho}^{2}, 2 \mathrm{M}_{\mathrm{K}^{*}}^{2}=\mathrm{M}_{\rho}^{2}+\mathrm{M}_{\phi}^{2}\right)$ and so we would have

$$
\begin{equation*}
{ }^{(\delta)_{V}}={ }^{(\delta)^{2}} \quad\left(\mathrm{P}^{\prime}\right)_{\mathrm{V}}=\left(\delta^{\prime}\right)_{\mathrm{V}}=0 \tag{15}
\end{equation*}
$$

so that the ideal mixing of the vector nonet or equivalently the OZI rule consequences in the mass matrix are not but just the effect of this specific $\operatorname{SU}(6)$ breaking assumption.

## III. MIXING ANGLES AND QUARK PARAMETERS

Once $\left\langle\mathrm{M}^{2}\right\rangle$ is known in $\mathrm{SU}(3)$, the mixing angle $\theta$ between physical and mathematical states

$$
\begin{align*}
& \left|\lambda_{1}\right\rangle=\mid 8>\cos \theta-10>\sin \theta  \tag{16}\\
& \left|\lambda_{2}\right\rangle=\mid 8>\sin \theta+10>\cos \theta
\end{align*}
$$

is fixed except its sign. The well known result one obtains for $|\theta|$ is $10.9^{\circ}$ for the $\left(\pi, \mathrm{K}, \eta, \eta^{\circ}\right) 0^{-+}$nonet and $40.9^{\circ}$ for the $\left(\rho, \mathrm{K}^{*}, \phi, \omega\right) 1^{--}$vector one, being in good agreement with experiment. The uncertainty respect to the $\theta$ sign is rather clearly removed in both cases mainly on considering decay processes ${ }^{1}$
which are sensitive to the quark flavor content of the wave function. It results in $+40+9^{\circ}$ for the vectors (almost ideal mixing) and $-10.9^{\circ}$ for the pseudoscalars.

In the $\operatorname{SU}(4)$ case we would have

$$
\left|\Lambda_{1}\right\rangle=\cos \theta|8>-\sin \theta \sin \beta| q>-\sin \theta \cos \beta|\mathrm{s}\rangle
$$

$\left.\left|\Lambda_{2}\right\rangle=\sin \theta \cos \alpha|8>+(\cos \theta \sin \beta \cos \alpha+\cos \beta \sin \alpha)| q\right\rangle+(\cos \theta \cos \beta \cos \alpha-\sin \beta \sin \alpha)|\mathrm{s}\rangle$
$\left|\Lambda_{3}>=-\sin \theta \sin \alpha\right| 8>+(\cos \beta \cos \alpha-\cos \theta \sin \beta \sin \alpha)|q\rangle+(-\sin \beta \cos \alpha-\cos \theta \cos \beta \sin \alpha)|\mathrm{s}\rangle$

With this parametrization $\theta$ is again the measure of the $\operatorname{SU}(3)$ octet $\leftrightarrow$ singlet mixing, and the $\beta$ and $\alpha$ interplay establishes the $|\mathrm{c} \overline{\mathrm{c}}\rangle$ content of the physical states.

To obtain the value of these angles we shall use the present meson masses. Our actual imput will be in $\mathrm{GeV}^{2}$ as follows

$$
\begin{align*}
\mathrm{M}^{2}(\pi) & =0.019 & \mathrm{M}^{2}(\rho) & =0.588 \\
\mathrm{M}^{2}(\mathrm{~K}) & =0.246 & \mathrm{M}^{2}\left(\mathrm{~K}^{*}\right) & =0.795 \\
\mathrm{M}^{2}(\eta) & =0.301 & \mathrm{M}^{2}(\phi) & =1.040 \\
\mathrm{M}^{2}\left(\eta^{\prime}\right) & =0.917 & \mathrm{M}^{2}(\omega) & =0.612 \\
\mathrm{M}^{2}(\eta \mathrm{c}) & =7.560 & \mathrm{M}^{2}(\psi) & =9.579 \\
\mathrm{M}^{2}(\mathrm{D}) & =3.490 & \mathrm{M}^{2}\left(\mathrm{D}^{*}\right) & =4.040 \tag{18}
\end{align*}
$$

The set of results is

$$
\begin{array}{lll}
\theta_{\mathrm{V}}=\mp 44^{\circ} & \beta_{\mathrm{V}}=\mp 41^{\circ} & \alpha_{\mathrm{V}}= \pm 2^{\circ} \\
\theta_{\mathrm{P}}= \pm 10^{\circ} & \beta_{\mathrm{P}}=\mp 38^{\circ} & \alpha_{\mathrm{P}}= \pm 5^{\circ} \tag{19}
\end{array}
$$

Evidently the only possible meaningful choice of signs is

$$
\begin{array}{lll}
\theta_{\mathrm{V}}=+44^{\circ} & \beta_{\mathrm{V}}=+41^{\circ} & \alpha_{\mathrm{V}}=-2^{\circ}  \tag{20}\\
\theta_{\mathrm{P}}=-10^{\circ} & \beta_{\mathrm{P}}=+38^{\circ} & \alpha_{\mathrm{P}}=-5^{\circ}
\end{array}
$$

because on one hand it is near the good $\mathrm{SU}(3)$ old results, and on the other the alternative sign set leads in both cases to meaningless too important $|c \bar{c}\rangle$ impurities in the old mesons incompatible with charm phenomenology.

But that does not mean that (20) is satisfactory. It does establish a rather clear decoupling between new and old physics

$$
\begin{align*}
|\eta\rangle & =0.98|8\rangle+0.18|0\rangle-0.02|\mathrm{c} \overline{\mathrm{c}}\rangle \\
\left|\eta^{\prime}\right\rangle & =-0.18|8\rangle+0.98|0\rangle-0.05|\mathrm{c} \overline{\mathrm{c}}\rangle  \tag{21}\\
\left|\eta_{c^{\prime}}\right\rangle & =-0.02|8\rangle-0.05|0\rangle-0.99|\mathrm{c} \overline{\mathrm{c}}\rangle
\end{align*}
$$

and

$$
\begin{align*}
& |\phi\rangle=-0.09|u \bar{u}+d \bar{d}\rangle-0.99|\mathrm{~s} \bar{s}\rangle+0.13|c \bar{c}\rangle \\
& |\omega\rangle=0.69|\bar{u}+d \bar{d}\rangle-0.16|\mathrm{~s} \bar{s}\rangle-0.10|c \bar{c}\rangle  \tag{22}\\
& |\psi\rangle=-0.09|\bar{u}+d \bar{d}\rangle-0.12|\mathrm{~s} \bar{s}\rangle-0.98|c \bar{c}\rangle
\end{align*}
$$

But especially for the vector multiplet, ${ }^{15,16}$ the $|c \bar{c}\rangle$ content of old mesons and simultaneous $|u \bar{u}+d \bar{d}\rangle$ content of $|\psi\rangle$ is too important and incompatible with its narrowness.

So the situation looks like the $\operatorname{SU}(4)$ extension of $\operatorname{SU}(3)$ in mass formulae rather spoils its good results, and that can mean that the (5) tensor assignment is poor or simply that we have to await for new extensions $\operatorname{SU}(4) \rightarrow \mathrm{SU}(?)$ which will restore $\operatorname{SU}(3)$ results and at the same time will explain charm and the coming future.

Not only are the mixing angles susceptible to criticism but also the quark parameters obtained in our alternative approach. Among them the annihilation
terms provide a nice chance to observe the running coupling constant quark-gluon-behavior as predicted by QCD. ${ }^{17,12}$ Its consequences would be qualitatively something like

$$
\begin{align*}
& \lambda_{\mathrm{uu}}>\lambda_{\mathrm{us}}>\lambda_{\mathrm{ss}} \\
& \lambda_{\mathrm{uu}}>\lambda_{\mathrm{uc}}>\lambda_{\mathrm{cc}}  \tag{23}\\
& \lambda_{\mathrm{ss}}>\lambda_{\mathrm{sc}}>\lambda_{\mathrm{cc}}
\end{align*}
$$

Our numerical results for the annihilation parameters are

| $\mathrm{P}^{\prime}$ | $\delta^{\prime}$ | $\Delta^{\prime}$ | $\lambda_{\mathrm{uu}}$ | $\lambda_{\mathrm{us}}$ | $\lambda_{\mathrm{ss}}$ | $\lambda_{\mathrm{uc}}$ | $\lambda_{\mathrm{sc}}$ | $\lambda_{\mathrm{cc}}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{-+}$ | 0.31 | -0.08 | 0.11 | 0.31 | 0.23 | 0.15 | 0.42 | 0.34 | 0.53 |
| $1^{--}$ | 0.45 | 0.02 | 0.14 | 0.45 | 0.47 | 0.49 | 0.59 | 0.61 | 0.73 |

It is necessary to remark that the parameters obtained for the $1^{--}$multiplet are very sensitive to the mass input; ${ }^{3}$ changing $M_{\rho}$ one standard deviation (from 0.76 to 0.78 GeV ) may result in drastic changes of (24), so that with the $\rho$ mass uncertainty clear consequences can not be drawn. In the $0^{-+}$multiplet we see that the $\Delta^{\prime}$ contributions break clearly the pattern of inequalities expected from QCD. That possibly will be another question solved and understood in next extensions.

## IV. CONCLUSIONS

On one hand our analysis of the Gell-Mann Okubo formula in the quark language has shown that there must exist a definite pattern of $\operatorname{SU}(3)$ (SU(4)) breaking in the annihilation mass terms identical to the one existing for the scattering terms.

On the other hand the conclusion drawn by the numerical results of Section III is that the simple (5) $\mathrm{SU}(3) \rightarrow \mathrm{SU}(4)$ extension in quadratic mass formulae looks inconsistent with present phenomenology because it alters slightly good old $\operatorname{SU}(3)$ results and specially because it contradicts abruptly charm phenomenology. In addition we have shown that it implies a set of values for the annihilation mass terms, which do not exhibit a qualitative agreement with QCD expectations.

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S. Okubo, Prog. Theor. Phys. 27, 949 (1962).
7. In actual calculations for mesons in Section III we shall always assume a quadratic formula.
8. $|1>\equiv 18 ; T=1 ; \quad \mathrm{Y}=0\rangle$
$|8\rangle \equiv 18 ; \mathrm{T}=0 ; \mathrm{Y}=0>$
$\left.\left|\frac{1}{2}>\equiv 18 ; \mathrm{T}=\frac{1}{2} ; \mathrm{Y}= \pm 1>\quad\right| 0\right\rangle \equiv 10 ; \mathrm{T}=0 ; \mathrm{Y}=0>$
9. $|1\rangle,\left|\frac{1}{2}\right\rangle,|8\rangle$ now have the same previous $\operatorname{SU}(3)$ quantum numbers and are in a fifteen-plet of $\mathrm{SU}(4)$, and

$$
\begin{array}{ll}
|\mathrm{d}\rangle \equiv \mid 15 ; 3^{*} ; \mathrm{T}=\frac{1}{2} ; \mathrm{Y}=-\frac{1}{3}> & \\
|\mathrm{f}\rangle \equiv\left|15 ; 3^{*} ; \mathrm{T}=0 ; \mathrm{Y}=\frac{2}{3}\right\rangle & |\mathrm{S}\rangle \equiv 15 ; 0 ; \mathrm{T}=0 ; \mathrm{Y}=0\rangle \\
& \equiv 0 ; \mathrm{T}=0 ; \mathrm{Y}=0\rangle
\end{array}
$$

10. H. Fritzsch and P. Minkowsky, Il Nuovo Cimento 30A, No. 3, 393 (1975).
11. From this analysis it is obvious that the value of $\lambda_{u u}, \lambda_{u s}$ and $\lambda_{s \mathrm{~s}}$ is not a consequence of both eigenvalues and mixing angles as it is calculated in Ref. 12. They are obtained imposing just the first.
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## FIGURE CAPTIONS

1. -SU(3) "scattering" mass diagrams and their effective parametrization. Gluon lines have been omitted. $\mathrm{t} \equiv \mathrm{u}, \mathrm{d}$.
2. $\operatorname{SU}(3)$ "annihilation" mass diagrams and their effective parametrization. All possible gluon connections and insertions have been omitted. $\mathrm{q}, \mathrm{q}^{\prime} \equiv \mathrm{u}, \mathrm{d}, \mathrm{s}$.
3. SU(4) mass diagrams and their effective paramctrization. Gluon lines have been omitted again. $\quad t, t^{\prime} \equiv \mathrm{u}, \mathrm{d}$.


Fig. 1


Fig 2


$$
\rightarrow \frac{c}{\square} \equiv
$$

$$
\equiv P+2 \triangle
$$

$$
\rightarrow P^{\prime}+2 \Delta^{\prime}
$$

S

$$
\rightarrow \frac{c}{\square} \equiv P^{\prime}+\delta^{\prime}+\triangle^{\prime}
$$

Fig. 3


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