SLAC-PUB-1921
April 1977
(T/E)

LIMITS ON A π-μ ATOM ANOMALY

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ABSTRACT

The decay rate of K_L^o into $\pi\mu$ atoms provides a test for the existence of a possible short range anomalous interaction between the pion and muon. For an elastic interaction, the effect would show up in the $\pi\mu$ wave function of the Coulomb bound state, and we examine mechanisms that could alter the wave function at the origin. An inelastic interaction that could lead to depletion of $\pi\mu$ atoms through the Konopinski-Mahmoud allowed transition $\pi^+\mu^- \to \pi^- e^+$ is also considered. Limits on these interactions are computed from precise atomic measurements of energy levels in muonic atoms and from the measured muon g-2 value and capture rates. These limits are then related to recent observations on the formation of $\pi\mu$ atoms in K_L^o decay.

(Submitted to Phys. Rev. D.)

*Work supported by the Energy Research and Development Administration.

INTRODUCTION

It has been reported[1] that $\pi\mu$ atoms have been observed in the decay of K_L^o. The rate of formation of these atoms has been calculated,[2] and it depends on the $K_{\mu 3}$ decay parameters and the wave function of the $\pi\mu$ atom evaluated at the origin. The $K_{\mu 3}$ form factors are well known,[3] so measuring the rate of atom formation determines the square of the wave function $|\psi_{\pi\mu}(0)|^2$. A discrepancy with the usual hydrogenic wave function at the origin would indicate an interaction between the pion and muon in addition to the known Coulomb force. Interest in this problem has been stimulated by the recent report that a suppression of a factor five to eight in the formation rate of $\pi\mu$ atoms is not, at present, inconsistent with experiment.[4]

Independent of the $\pi\mu$ experiment, however, one can place severe limits on any modification of the wave function at the origin. In particular, an effective short range $\pi\mu$ interaction leads to a short range μ-nucleon interaction via two pion exchange and thereby causes a shift in the energy levels of muonic atoms. Measurements of the atomic transitions place an upper bound on the strength of such an interaction. This, in turn, places severe limits on any modification of the $\pi\mu$ wave function from the hypothesized short range interaction.

Another constraint, though less model-independent, is provided by the anomalous magnetic moment of the muon. Suppose for example that the π and μ interact via resonance formation in addition to their normal photon exchange. In a dispersion calculation of the muon's magnetic moment there would be an additional contribution to the absorptive part in the timelike region starting at the two-pion threshold (Fig. 1). The present agreement of the experimental value for $\left(\frac{g-2}{2}\right)_\mu$ with theory to an accuracy of 23 ppm[5] severely constrains the possible strength of such an anomalous $\pi\mu$ interaction. We show that it cannot alter $\psi_{\pi\mu}(0)$ by more than two parts per million and is therefore limited to a completely negligible effect on the rate of $\pi\mu$ atom formation.

Another possibility we consider is that the atoms are produced as expected, but decay through an inelastic channel before they have a chance to be detected. In connection with the several lepton schemes,[6] we examine upper limits on the reaction $\pi^+\mu^-$ (atom) $\to \pi^- e^+$ provided by an experimental search[7] for the reaction $\mu^- Z \to e^+ (Z-2)$. We conclude that the rate is far too small to be of significance here.

Some of the approximations we make involve off-shell extrapolation of the physical πN scattering amplitude. However, all our conclusions are insensitive to many orders of magnitude change in our calculations.

CALCULATION OF $K_L \to \pi\mu$ ATOM + $\bar{\nu}_\mu$

We follow Nemenov[2] in writing down the amplitude for atom formation from $K_{\mu 3}$ decay. Using the standard phenomenological weak Lagrangian, the covariant amplitude[8] for $K_{\mu 3}$ decay is

$$\mathcal{M}(K_L \to \pi^+\mu^-\bar{\nu}_\mu) = \frac{G_F \sin\theta_c}{\sqrt{2}} < \pi^+ | V_1^{\alpha^+} | K_L > \bar{u}_\mu \gamma_\alpha (1-\gamma_5) v_{\nu_\mu} \tag{1}$$

where $V_1^{\alpha^+}$ is the $\Delta S = \Delta Q$ weak vector current.[9]

Since the binding energy of the 1s state of the $\pi\mu$ atom is 1.6 keV, the atom is a nonrelativistic system, and we are justified to consider it as a superposition of one pion, one muon momentum eigenstates:

$$|\pi\mu \text{ atom, momentum } Q> = \sum_{\substack{\text{relative} \\ \text{momentum } q}} \left| \pi\left(\frac{m_\pi}{m_\pi + m_\mu} Q + q\right)\right\rangle \left| \mu\left(\frac{m_\mu}{m_\pi + m_\mu} Q - q\right)\right\rangle \psi(q) .$$

where $\psi(q)$ is the momentum space wave function of the atom. By the rules of quantum mechanics, the amplitude for decay into an atom is

$$\mathcal{M}(K_L \to \text{atom} + \bar{\nu}_\mu) = \sum_q \mathcal{M}\left(K_L \to \pi\left(\frac{m_\pi}{m_\pi+m_\mu}Q+q\right), \mu\left(\frac{m_\mu}{m_\pi+m_\mu}Q-q\right), \bar{\nu}_\mu\right) \psi^*(q) \quad .$$

After squaring and summing over spin, we obtain the total rate in the K_L rest frame

$$\Gamma(K_L \to \text{atom} + \bar{\nu}_\mu) = \frac{1}{2m_K} \frac{G_F^2 \sin^2\theta}{2} \left\{(f_+ + f_-)(m_\pi + m_\mu) + (f_+ - f_-)m_\pi\right\}^2$$

$$\times \frac{m_\mu\left[m_K^2 - (m_\pi+m_\mu)^2\right]^2}{2\pi m_K^2(m_\pi+m_\mu)} \frac{1}{2m_r} \sum_n |\psi_n(0)|^2 \qquad (2)$$

where $f_\pm$ are the $K_{\mu 3}$ form factors,[3] m_r is the reduced mass of the $\pi\mu$ system, and the sum n over all spatial quantum numbers (only s states contribute) is proportional to $\sum_n \frac{1}{n^3} \cong 1.2$. Numerically, this yields a branching ratio

$$\frac{\Gamma(K_L \to \text{atom} + \bar{\nu}_\mu)}{\Gamma(K_L \to \pi\mu\bar{\nu}_\mu)} = 5.1\times 10^{-7} \quad .$$

ELASTIC CHANNEL

1. Atomic Measurement

We use the measurement of the $2P_{3/2} - 2S_{1/2}$ transition in singly ionized muonic helium $(\mu^- \, {}^4He)^+$ as a test for an anomalous $\pi\mu$ interaction. The measurement was performed by Bertin et al.,[10] with the result that experiment and theory agree to within 0.008 eV. We now suppose that there is an additional contribution to the $(\mu^- \, {}^4He)^+$ energy levels due to a new short range $\pi\mu$ interaction where the pions connect the muon to the nucleus (Fig. 2). We take the effective $\pi\pi\mu\mu$ vertex to be pointlike since, on an atomic scale, the effective potential will be a Dirac delta function anyway.

The calculation of the graph goes as follows:

i) We take as our effective vertex $\frac{\lambda}{m_\pi}\bar{u}(p')\,O u(p)$, where p and p' are the momenta of the incoming and outgoing muons, respectively, m_π is the mass of the pion, λ is the (dimensionless) strength of the interaction, and O is an arbitrary matrix in spin and isotopic spin.

ii) There is a loop integral over intermediate pion momenta k.

iii) We assume that the scale of the amplitude is set by low energy πN scattering, and the (slightly) off shell πN vertex is approximated by a constant off mass shell extrapolation of the Chew-Low scattering amplitude[11]:

$$\sqrt{4\omega_k \omega_{k'}}\; t(k',k) = -4\pi \sum_\alpha P_\alpha(k',k)\, e^{i\delta_\alpha(k)} \frac{\sin\delta_\alpha(k)}{k^3} \tag{3}$$

where the sum α is over spin-isospin channels, and P_α is the corresponding projection operator normalized by

$$\sum_{q\text{-isospin}} \int d\Omega_q\, P_{\alpha'}(k',q)\, P_\alpha(q,k) = 4\pi \vec{q}^{\,2}\, \delta_{\alpha'\alpha} \tag{4}$$

The 3-3 resonance will dominate the sum over channels. Our covariant matrix element is, then:

$$\mathcal{M} = \frac{\lambda}{m_\pi}\bar{u}(p+q)Ou(p)\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_\pi^2+i\epsilon}\left[\sqrt{4\omega_k\omega_{k'}}\; t(k',k)\right]\frac{1}{(k+q)^2-m_\pi^2+i\epsilon}\,. \tag{5}$$

Evaluation is straightforward, since (3) has no dependence on the 0^{th} component of momentum. Also, since the momentum transferred to the Coulomb-bound muon is small on the scale of a pion mass, we may approximate $\omega_{k'} \cong \omega_k$.

If, for simplicity, we assume that O=1 in Eq. (5) and hence the muon couples to the isoscalar component of the two-pion system, we simply add equal contributions from all intermediate charged pion states. This leads to an effective potential between the muon and the ^{4}He nucleus:

$$\delta V \cong \frac{\lambda}{2m_\pi^2}\,\delta^3(\vec{x}) \tag{6}$$

We now appeal to the experimental agreement with theory, and ask the question, how large can λ be without disturbing this agreement. Since we are speaking of an energy difference in $(\mu^- \, {}^4\mathrm{He})^+$ of 0.01 eV, compared to the binding energy of 2.8 keV for the 2s state, we are justified in using first order perturbation theory:

a) $\delta E(2s\text{ state}) = \langle\psi_{2s}|\delta V|\psi_{2s}\rangle = \frac{\lambda}{2m_\pi^2}\,|\psi_{2s}(0)|^2$

b) $\delta E(2p\text{ state}) = 0$

c) $\delta E(2p - 2s) < 10^{-2}$ eV from experiment

setting an upper bound

$$\lambda \lesssim \frac{2\pi}{\alpha^3}\left(\frac{m_\pi}{m_\mu}\right)^2\left(\frac{10^{-2}\text{ eV}}{m_\mu}\right) \tag{7}$$

Going back to the $\pi\mu$ atom, we calculate, again using first order perturbation theory, the change in the wave function of the 1s state:

$$\frac{\delta\psi_{1s}(0)}{\psi_{1s}(0)} \cong -\frac{\lambda\alpha}{\pi}\left(\frac{m_\mu}{m_\pi + m_\mu}\right)^2 \sum_{n=2}^{\infty} \frac{1}{n^3 - n} \tag{8}$$

or

$$\delta\psi_{1s}^2(0)/\psi_{1s}^2(0) \lesssim 0.6 \times 10^{-6} \quad .$$

This microscopically small value for the change in the wave function is not, however, our upper bound. The ^{4}He nucleus has an equal number of protons and neutrons. If the coupling of the muon current to the pion current is isovector in character,[12] i.e., the operator $O = \gamma_\mu \vec{\tau}$, we would get a cancellation of muon-proton and muon-neutron potentials for the ^{4}He nucleus. We must then go to first order in the fine structure constant α to take into account the following Coulomb corrections to charge symmetry:

i) There are corrections of order α to the π^+p vs π^-n scattering amplitudes. We estimate this by using a nonrelativistic Coulomb barrier.

ii) The absolute strength of the $\mu^-\pi^-$ vertex will be diminished and that of the $\mu^-\pi^+$ vertex will be enhanced due to the Coulomb repulsion of the former, and Coulomb attraction for the latter. Again, these corrections are of order α, and they are estimated by a Coulomb barrier.

iii) Nuclear polarization. As a result of the Coulomb interaction, the ground state wave functions of the muon and the helium nucleus distort each other so that the muon can circulate around the charge center rather than the mass center of the nucleus.[13] The extent of this distortion depends on the polarizability of the nucleus. The polarizable limit will hold when the characteristic energy differences between the excited and ground states of the nucleus are small compared to the energies of the Fourier components of the muon at the nuclear surface taking part in the interaction, i.e., $\Delta E_{exc} R \ll 1$.[14] This condition is satisfied for ^{4}He, where $\Delta E \cong 20$ MeV. For a short-range μp interaction, $\Lambda\delta^3(\vec{r}_\mu - \vec{r}_p)$, and a short-range μn interaction of opposite sign, $-\Lambda\delta^3(\vec{r}_\mu - \vec{r}_n)$, we have for a μ-nucleus potential

$$V_{\mu\text{-He}}(x) = \Lambda\int d^3r_1 \ldots d^3r_4 \,|\Psi_N(r_1 \ldots r_4)|^2 \left[\delta^3(x-r_1)+\delta^3(x-r_2)-\delta^3(x-r_3)-\delta^3(x-r_4)\right]$$

where $r_1 \ldots r_4$ are the two proton and two neutron coordinates, respectively, and Ψ_N is the nuclear wave function. The first order shift in energy of the bound muon is $\int d^3x\, \psi_\mu^*(x)\, V_{\mu\text{-He}}(x)\, \psi_\mu(x)$, where $\psi_\mu(x)$ is the muon wave function centered about $\frac{1}{2}(\vec{r}_1 + \vec{r}_2)$, i.e., the charge center of the nucleus. We find the μp interaction will be felt more strongly than the μn interaction of opposite sign, due to the greater spread of the neutron vs proton wave functions with respect to the orbiting muon. The magnitude of this correction goes as the nuclear size divided by the atomic radius a_0, and the sign of the correction is the same as the sum from points i) and ii) above.

Taking all these corrections, we arrive at an effective potential for muonic helium

$$\delta V \cong \frac{4\lambda}{15 m_\pi^2} \delta^3(\vec{x}) \left[\alpha + \frac{4}{\pi^{3/2}} \frac{R}{a_0}\right]$$

$$\cong \frac{\lambda}{2 m_\pi^2} \delta^3(x) \times \alpha \tag{9}$$

Thus, the upper bound for the coupling constant is increased over isoscalar coupling by the factor α^{-1}, as is the limit on the change in the $\pi\mu$ wave function

$$\frac{\delta\psi_{1s}^2(0)}{\psi_{1s}^2(0)} < 0.6 \times 10^{-6} \times 137 = 8.2 \times 10^{-5} .$$

This is our final bound on the possible suppression of the $\pi\mu$ wave function from exotic mechanisms. At present, this effect would be unobservable.

2. $\underline{(g-2)_\mu}$

We assume, as in Section 1, an effective π-μ interaction, only this time with the interaction mediated by a resonance μ^* of mass $m^* >> m_\mu$ (Fig. 1a). Take Yukawa coupling for the $\pi\mu\mu^*$ vertex, with strength g. This leads to an additional contribution to g-2 for the muon from the triangle graph in Fig. 1b:

$$\mathcal{M} = -ieg^2 \bar{u}(p') \left[\int \frac{d^4k}{(2\pi)^4} \frac{i(\not{k}+m^*)}{k^2 - m^{*2} + i\epsilon} \frac{i}{(p'-k)^2 - m_\pi^2 + i\epsilon} (p'+p-2k)_\mu \times \frac{i}{(p-k)^2 - m_\pi^2 + i\epsilon} \right] u(p) \quad .$$

Carrying through the calculation for the magnetic moment, we find

$$\delta\left(\frac{g-2}{2}\right)_\mu \cong \frac{g^2}{16\pi^2} \frac{m_\mu}{m^*} \quad . \tag{10}$$

The quantity must be bounded by the experimental uncertainty of 23 ppm,[5] and this gives an upper bound on the ratio g^2/m^*. This is precisely the ratio that enters into the nonrelativistic potential generated by the graph in Fig. 1a. Lowest order perturbation theory gives the negligible bound $\delta\psi^2_{\pi\mu}(0)/\psi^2(0) < 1.8 \times 10^{-6}$ for the 1s state.

3. Inelastic Channel

From the experimental upper bound[7] on the reaction $\mu^- + Cu^{65} \to e^+ + Co^{64} + n$, we shall calculate an order of magnitude upper bound on the strength of an effective four point coupling for the reaction $\pi^+\mu^- \to \pi^- e^+$. From this effective coupling, we can estimate an upper bound on the decay rate of the $\pi\mu$ atom through this channel. We shall see that this rate is too small by seven orders of magnitude to explain a possible anomaly of a factor of five to eight in the production rate of $\pi\mu$ atoms from $K_{\mu 3}$ decay.

Recall that the Mahmoud-Konopinski prescription assigns an additive lepton number $K=+1$ for $e^-, \nu_e, \mu^+, \bar{\nu}_\mu$; $K=-1$ for the corresponding antiparticles, and zero for all other particles.[6] This lepton scheme is consistent with all known observations. In particular, it allows for the reaction $\mu^- + {}_{29}Cu^{65} \to e^+ + {}_{27}Co^{64} + n$, as a test to distinguish the above scheme from the usual assignment of separate lepton numbers for the electron and muon. The result of an experimental search[7] for this reaction was to set an upper limit on the ratio

$R(\mu^- Z \to e^+(Z-2))/R(\mu^- Z \to \nu_\mu Z) < 2.6 \times 10^{-8}$ (90% confidence).

If, in fact, there were an effective $\pi\pi\mu e$ vertex then by two pion exchange its strength would set the scale for the rate of such a process, $\mu^- Z \to e^+(Z-2)$, as shown in the Feynman diagram in Fig. 3. It is this graph that we now estimate.

For convenience, we take the semileptonic $\pi\pi\mu e$ vertex to be the product of a vector leptonic current and a conserved vector pion current, with a coupling constant λ of dimension $(\text{mass})^{-2}$:

$$\lambda \bar{e}(\not{p} + \not{p}')\mu \ . \tag{11}$$

The double charge exchange collision:

$$\pi^-(\text{off mass shell}) + Z \rightarrow \pi^+(\text{off shell}) + (Z-2)$$

is a two-step process.[15] First, the π^- scatters off of a proton resulting in a π^o and a neutron. The intermediate π^o then collides with another proton resulting in another neutron and a π^+:

$$\pi^- + p \rightarrow \pi^o + n$$
$$\quad\quad\quad\quad \llcorner\!\rightarrow + p \rightarrow \pi^+ + n \quad .$$

To calculate this part of the graph, we first make the approximation that the intermediate pions propagate "close" to their mass shell; "close" being on the scale of the 3-3 resonance. Within this approximation, we shall again use for the πN vertex the static Chew-Low t matrix, smoothly extrapolated off the pion mass shell.

For the nucleons, we neglect the effect of nuclear binding energy during the collision; i.e., we use the impulse approximation. The protons and neutrons move independently in a common harmonic oscillator potential, and the size parameter of the well, $\sqrt{\hbar/M_p\omega}$, is adjusted to fit the copper nucleus.

Within these approximations, we can now write the T matrix for the reaction[17] $\mu^- + {}_{29}Cu^{65} \rightarrow e^+ + {}_{27}Co^{65}$:

$$T_{fi} = \int_{\omega_R-\Gamma/2}^{\omega_R+\Gamma/2} \frac{d\omega}{2\pi} \int \frac{d^3q}{(2\pi)^3}\frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}(2\pi)^3\delta^3(\vec{p}_f-\vec{p}_i+\vec{p}-\vec{p}')\left(\omega^2-\omega_{p'}^2+i\epsilon\right)^{-1}$$

$$\times \langle Co^{65}| \sum_{n\neq n'} \left[\sqrt{4\omega_{p'}\omega_q}\, t_{n'}(p',q)\right]\left(\omega^2-\omega_q^2+i\epsilon\right)^{-1}\left[\sqrt{4\omega_q\omega_p}\, t_n(q,p)\right]|Cu^{65}\rangle$$

$$\times \left(\omega^2-\omega_p^2+i\epsilon\right)^{-1}\left\{\lambda\sqrt{\frac{m_e m_\mu}{E_f E_i}}\,\bar{v}_f(\not{p}+\not{p}')\,u_i\right\} \tag{12}$$

where ω is the common 0^{th} component of the pion loop. In the static approximation for the πN interaction, nuclear recoil is neglected. ω_R = the pion energy that excites the 3-3 resonance (330 MeV); Γ = full width of the 3-3 resonance; $\sum_{n \neq n'}$ is over all nucleons; $t_n(p, q)$ is the Chew-Low t matrix for scattering from a nucleon at position $\vec{x}_n$. By translation invariance, $t_n(p, q) = e^{-i(\vec{p}-\vec{q})\cdot\vec{x}_n} t(p, q)$, where $t(p, q)$ is the Chew-Low amplitude given by (3). Thus, our nuclear matrix element is an integral over nucleon space, spin, and isospin coordinates.

The estimate of this integral is straightforward, and, for forward scattering, we have

$$T(\text{fwd}) \simeq \frac{1}{6(2\pi)^3} \left(\frac{\omega_R}{k_R}\right)^2 \lambda$$

where

$$k_R = \left(\omega_R^2 - m_\pi^2\right)^{1/2} \quad .$$

The total cross section is

$$\sigma(\mu^- Z \to e^+(Z-2)) \simeq \frac{\lambda^2}{72(2\pi)^2} \left(\frac{\omega_R}{k_R}\right)^4 E_e^2 \tag{13}$$

where E_e is the energy of the emerging positron.

The rate of meson capture in copper[17] is $5.7 \times 10^6/\text{sec} = 2.7 \times 10^{-17} m_\pi$. For copper, this is equivalent to a cross section

$$\sigma(\mu^- Z \to \nu_\mu Z) \simeq \frac{\pi}{(Z\alpha)^3 m_\mu^3} \times 2.7 \times 10^{-17} m_\pi$$

From experiment $\sigma(\mu^- \to e^+)/\sigma(\mu^- \to \nu_\mu) < 2.6 \times 10^{-8}$, which leads to the bound

$$(\lambda m_\pi^2) < 36(2\pi)^8 \left(\frac{m_\pi}{m_\mu}\right)^3 \left(\frac{k_R}{\omega_R}\right)^4 \frac{1}{(Z\alpha)^3} (2.7\times 10^{-17})(2.6\times 10^{-8})$$

$$= 1.0\times 10^{-14} \quad . \tag{14}$$

We now calculate the decay rate for the π-μ atom through this channel (Fig. 4). Taking the on shell amplitude for $\pi^+(k_1)\,\mu^-(p_1) \to \pi^-(k_2)\,e^+(p_2)$ to be given by (11), $\lambda\bar{v}_e(p_2)(\not{k}_1+\not{k}_2)\,u_\mu(p_1)$, we calculate the decay rate for the atom in the i^{th} state to be:

$$\Gamma(\pi^+\mu^- \,\text{atom(state i)} \to \pi^- e^+) = \frac{1}{2(m_\pi+m_\mu)} \frac{\lambda^2(m_\mu+2m_\pi)^4 m_\pi^2}{8\pi m_\pi(m_\pi+m_\mu)^2} |\psi_i(0)|^2 \tag{15}$$

which, using (14), gives a partial lifetime $1/\Gamma > .32$ sec. This is seven orders of magnitude greater than the known lifetime of the pion and therefore would have a negligible effect on the atom's lifetime.

CONCLUSION

We have examined and ruled out candidates for a possible anomaly in the formation rate of $\pi\mu$ atoms in K_L decay. For a $\pi\mu$ potential interaction, we use measurements of the energy levels in muonic atoms to strongly limit its effect on $\pi\mu$ atoms. For the case of $\pi\mu$ resonance formation, the accuracy of the muon g-2 value makes an effective bound. Furthermore, in the lepton scheme that allows for $\pi^+\mu^-$ atom decay into π^-e^+, experimental bounds on $\mu^- \to e^+$ double charge exchange with nuclei rule out this mode as having an effect on the atom's lifetime. Therefore, it is very difficult - if an anomaly is experimentally confirmed - to make it compatible with previous experimental results involving the $\pi\mu$ system.

ACKNOWLEDGEMENT

I wish to thank Sidney Drell for suggesting this investigation and for many helpful discussions.

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FIGURE CAPTIONS

1. (a) $\pi\mu$ resonance; (b) $(g-2)_\mu$.
2. Effective μ–nucleon interaction.
3. $\mu^- Z \rightarrow e^+(Z-2)$.
4. $\pi^+\mu^-$ atom $\rightarrow \pi^- e^+$.

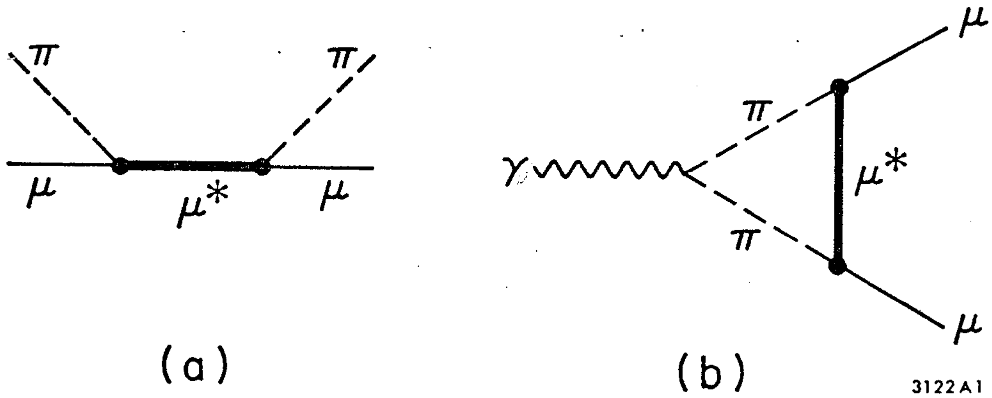

Fig. 1

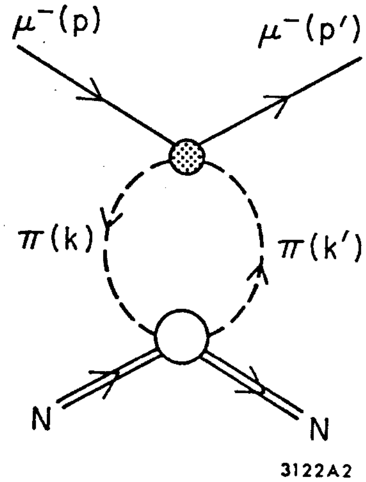

Fig. 2

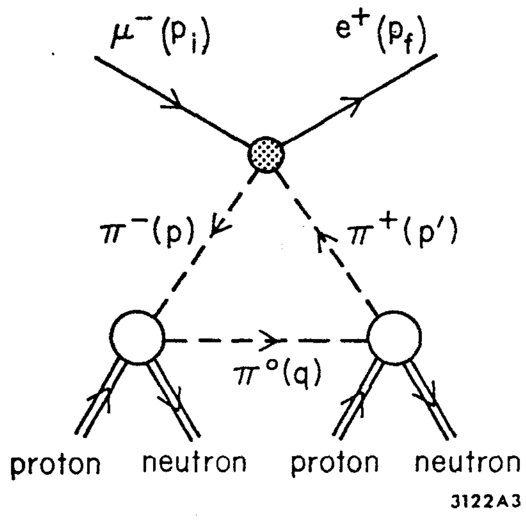
$\mu^-(p_i)$
$e^+(p_f)$
$\pi^-(p)$
$\pi^+(p')$
$\pi^\circ(q)$
proton
neutron
proton
neutron
3122A3

Fig. 3

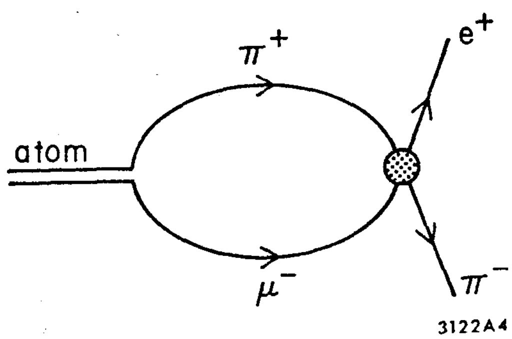
π^+
e^+
atom
μ^-
π^-
3122A4

Fig. 4