**BUNCH LENGTHENING DUE TO POTENTIAL WELL DISTORTION FROM CYLINDRICAL CAVITIES WITH BEAM PORTS**

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**Introduction**

The effect on bunch shape of potential well distortion arising from the interaction between a bunched beam and cylindrical cavities without beam ports has been computed previously, 1,2,3 In addition to the fact that the effect of beam apertures is not taken into account, these computations are subject to the further restriction that both the cavity height (gap length) and the bunch length must be less than one-half the cavity radius. In this paper, the effect on bunch shape of potential well distortion from cavities having beam port apertures and a radius comparable to the gap length is considered.

**Basic Expressions and Computational Method**

The expression for the current distribution in the bunch, as modified by potential well distortion, can be written in normalized form as 2,3

\[ I'(t) = K \exp \left[ \frac{-L^2}{2\sigma^2} - \frac{Q}{\sigma^2} \int_0^\infty s(\tau) I'(t-\tau) d\tau \right] \]  

where \( I'(t) = I(t)/I_0 \) and \( I_0 = \sqrt{2\pi} \sigma \) is the peak current for the unperturbed RF voltage. Here \( \sigma \) is the unperturbed bunch length in units of time, \( Q \) is the charge in the bunch and

\[ G = \frac{Q}{\sqrt{2\pi} V} \]

is a measure of the strength of the potential well distortion, where \( V \) is the slope of the unperturbed RF voltage. The constant \( K \) in Eq. (1) is chosen to conserve total charge so as to satisfy

\[ \int I'(t) dt = \frac{Q}{\sigma} \]

The wake function \( s(\tau) \) in Eq. (1) gives the response of the cavity at time \( \tau \) to a unit current step at \( \tau = 0 \). It is the integral of the impulse wake potential \( w(\tau) \), which gives the energy loss (or gain) at time \( \tau \) behind a unit charge impulse. The dimensions of \( s(\tau) \) are ohms, while \( w(\tau) \) is expressed in volts per unit charge.

The method by which the impulse wake is computed is described elsewhere. 4,5 It should be repeated here that in this calculation any contribution to the wake function due to the scalar potential arising from the charge in the cavity has been ignored. The relative contribution to the wake of the missing scalar potential term is not known. It was, however, found \(^4\) that good agreement is obtained between the results of a bench measurement of the energy loss as a function of time within a charge distribution passing along a wire on the axis of the cavity, and the loss computed using a wake in which the scalar potential contribution is not included.

The impulse wake potential is computed from the parameters of the cavity modes, following the procedure described in Ref. 5, by a program WAKEFIELD. The computer function \( w(\tau) \) is then fed to a second program, BUNCH, which integrates the impulse wake to obtain the step response wake \( s(\tau) \). Examples of \( s(\tau) \) for several different cavities are shown in Fig. 1. Note first of all that for all three cavities \( s(\tau) = 0 \). This is a consequence of the fact that the total energy loss, even for a bunch with \( = 0 \), is finite if a cavity has beam apertures. The finite energy loss in turn implies that the impulse wake \( w(\tau) \) is finite at \( \tau = 0 \). The integral of \( w(\tau) \) will then be zero at \( \tau = 0 \). Thus \( ds/d\tau \) must be positive over at least some range near \( \tau = 0 \). For sufficiently short bunches the cavity therefore looks capacitive, and bunch shortening is expected. Longer bunches encounter an inductive wake, where \( ds/d\tau \) is negative. The dashed curve in Fig. 1 shows the step response for a parallel-plate gap with no beam.

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Fig. 2. Bunch current distribution for cavity C, Fig. 1, for σ=75 ps and G=40 ps²/ohm.

Fig. 3. Bunch current distribution for cavity C, Fig. 1, for σ=75 ps and G=120 ps²/ohm.

Fig. 4. Bunch current distribution for cavity C, Fig. 1, for σ=75 ps and G=500 ps²/ohm.

Fig. 5. Bunch current distribution for cavity C, Fig. 1, for σ=240 ps and G=120 ps²/ohm.

Table I

<table>
<thead>
<tr>
<th>G (ps²/ohm)</th>
<th>T¹max (ps)</th>
<th>FWHM (ps)</th>
<th>σ rms (ps)</th>
<th>k (V/pC)</th>
<th>K</th>
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<td>.358</td>
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<tr>
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<td>76</td>
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<td>.337</td>
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<td>112</td>
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<tr>
<td>500</td>
<td>0.73</td>
<td>87</td>
<td>196</td>
<td>.046</td>
<td>1.2×10⁴</td>
</tr>
</tbody>
</table>

σ=76 ps

σ=240 ps

Note that in all the curves the center of charge has shifted to earlier (negative) times. This time shift is a measure of the total energy lost to the cavity. If the center of charge shifts by time t, where \( t = \frac{\int f(t)dt}{\sqrt{2\pi} \sigma} \), then the loss parameter k, expressed in volts per unit charge, is given by \( k = \frac{\langle V/Q \rangle}{\sqrt{2\pi} G} \).

Table I shows how k and other parameters of interest vary as a function of G for cavity C at two different bunch lengths. Note in particular that with increasing G the full-width-half-maximum (FWHM) bunch length decreases, but that the rms bunch length increases for the 75 ps case. The variation in the loss parameter k is consistent with the change in \( \sigma_{rms} \). The loss decreases as \( \sigma_{rms} \) increases, but not with the change in the FWHM bunch length.

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References