SLAC-PUB-1909
PEP-241
March 1977
(A)

BUNCH LENGTHENING DUE TO POTENTIAL WELL DISTORTION FROM CYLINDRICAL CAVITIES WITH BEAM PORTS*

K.L.F. Bane and P. B. Wilson
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

Introduction

The effect on bunch shape of potential well distortion arising from the interaction between a bunched beam and cylindrical cavities without beam ports has been computed previously.[1,2,3] In addition to the fact that the effect of beam apertures is not taken into account, these computations are subject to the further restriction that both the cavity height (gap length) and the bunch length must be less than one-half the cavity radius. In this paper, the effect on bunch shape of potential well distortion from cavities having beam port apertures and a radius comparable to the gap length is considered.

Basic Expressions and Computational Method

The expression for the current distribution in the bunch, as modified by potential well distortion, can be written in normalized form as[2,3]

$$I'(t) = K \exp\left[-\frac{t^2}{2\sigma^2} - \frac{G}{\sigma^3}\int_0^\infty s(\tau)\, I'(t-\tau)\, d\tau\right] \qquad (1)$$

where $I'(t) = I(t)/I_p$ and $I_p = Q/(\sqrt{2\pi}\,\sigma)$ is the peak current for the unperturbed (G=0) Gaussian bunch. Here σ is the unperturbed bunch length in units of time, Q is the charge in the bunch and

$$G = \frac{Q}{\sqrt{2\pi}\,\dot{V}} \qquad (2)$$

is a measure of the strength of the potential well distortion, where $\dot{V}$ is the slope of the unperturbed RF voltage. The constant K in Eq. (1) is chosen to conserve total charge so as to satisfy

$$\int I'(t)\, dt = \sqrt{2\pi}\,\sigma \qquad (3)$$

The wake function $s(\tau)$ in Eq. (1) gives the response of the cavity at time τ to a unit current step at τ=0. It is the integral of the impulse wake potential $w(\tau)$, which gives the energy loss (or gain) at time τ behind a unit charge impulse. The dimensions of $s(\tau)$ are ohms, while $w(\tau)$ is expressed in volts per unit charge.

The method by which the impulse wake is computed from the frequencies and R/Q's of the normal modes of a cylindrical cavity with beam ports is described elsewhere.[4,5] It should be repeated here that in this calculation any contribution to the wake function due to the scalar potential arising from free charges in the cavity has been ignored. The relative contribution to the wake of the missing scalar potential term is not known. It was, however, found[4] that good agreement is obtained between the results of a bench measurement of the energy loss as a function of time within a charge distribution passing along a wire on the axis of the cavity, and the loss computed using a wake in which the scalar potential contribution is not included.

The impulse wake potential is computed from the parameters of the cavity modes, following the procedure described in Ref. 5, by a program WAKEFIELD. The computed function $w(\tau)$ is then fed to a second program, BUNCH, which integrates the impulse wake to obtain the step response wake $s(\tau)$. Examples of $s(\tau)$ for several different cavities are shown in Fig. 1. Note first of all that for all three cavities $s(0)=0$. This is a consequence of the fact that the total energy loss, even for a bunch with σ=0, is finite if a cavity has beam apertures. The finite energy loss in turn implies that the impulse wake $w(\tau)$ is finite at τ=0. The integral of $w(\tau)$ will then be zero at τ=0. Thus $ds/d\tau$ must be positive over at least some range near τ=0. For sufficiently short bunches the cavity therefore looks capacitative, and bunch shortening is expected. Longer bunches encounter an inductive wake, where $ds/d\tau$ is negative. The dashed curve in Fig. 1 shows the step response for a parallel-plate gap with no beam

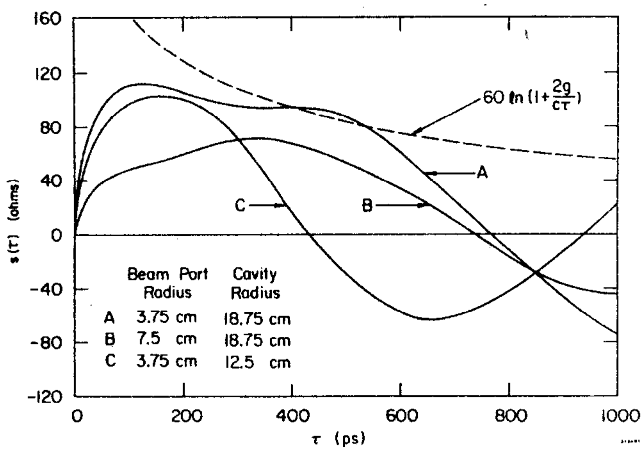

Fig. 1. The step response wake function $s(\tau)$ for three cavities with beam ports (solid curves) and for a parallel-plate gap without beam apertures (dashed curve). In all cases, the gap length is g = 22.5 cm.

apertures. This model, used in previous calculations,[1,2,3] is inductive for all values of τ.

Cavities A and B in Fig. 1 have the same outer radius but different beam port radii. Note that the cavity with the smaller beam port radius has a larger loss. Both response functions, however, fall to zero at nearly the same characteristic time. In the case of cavity C, which has a smaller outer radius, $s(\tau)$ crosses zero at an earlier time. The time to zero crossing is clearly related to the time it takes a signal to propagate to the outer radius, be reflected with a reversal of phase, and travel back to the axis.

After computing $s(\tau)$, BUNCH proceeds to solve for the current distribution defined by Eq. (1). For $t<-b$ the distribution is assumed to be the unperturbed Gaussian distribution. The time $-b$ is chosen such that $I'(-b)$ is very small, typically 10^{-6}. Starting at $t=-b$, the distribution is computed for small increments in time. The computation is simplified by the fact that, since $s(0)=0$, $I'(t)$ need not be known to evaluate the integral on the right-hand side of Eq. (1). Thus the current at each time t can be computed explicitly in terms of current values already found for earlier time intervals. After the distribution has been computed, the program calculates the normalized charge, given by the left-hand side of Eq. (3). If $\sqrt{2\pi}\,\sigma$ is not obtained, the constant K in Eq. (1) is adjusted by an iterative procedure until Eq. (3) is satisfied.

Results for the Current Distribution

In Figs. 2, 3 and 4 results for the current distribution are given for the case of cavity C in Fig. 1 for σ = 75 ps. As the parameter G increases, it is seen that the bunch at first becomes narrower and that the peak current increases. However, at larger G (see Fig. 4) the peak current begins to decrease, the current in the tail of the distribution increases, and the bunch tends to become broader and flatter. For still larger G, the bunch becomes even wider and flatter, with a short sharp spike of current at the leading edge. Recent measurements[6] indicate that the impedance of SPEAR is equivalent to that of about 100 of the cavities considered here. For an accelerating voltage of 1 MV in SPEAR, G=500 corresponds to a beam current of 20 mA. It is known however, that for this RF voltage and for σ = 75 ps the bunch becomes unstable at about 4 mA (the threshold for the "turbulent" bunch instability). Thus distributions for σ = 75 ps and G greater than about 100 are probably physically unrealizable.

Figure 5 shows that for σ = 240 ps there is a slight amount of bunch lengthening rather than bunch shortening at

*Supported by the Energy Research and Development Admin.

(Contributed to the 1977 Particle Accelerator Conference, Chicago, Illinois, March 16 - 18, 1977)

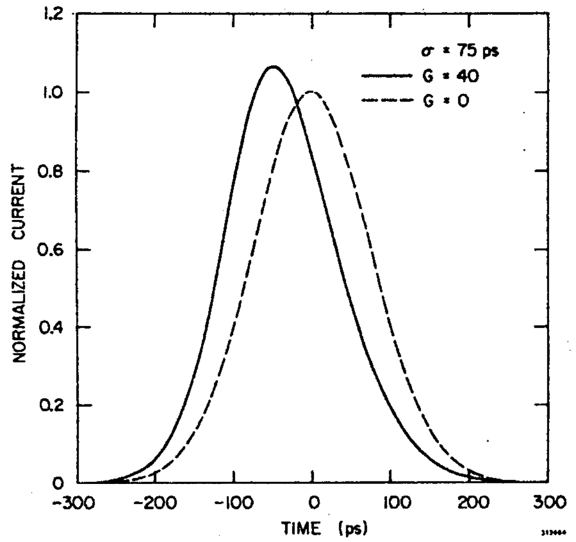

Fig. 2. Bunch current distribution for cavity C, Fig. 1, for $\sigma = 75$ ps and $G = 40$ ps^2/ohm.

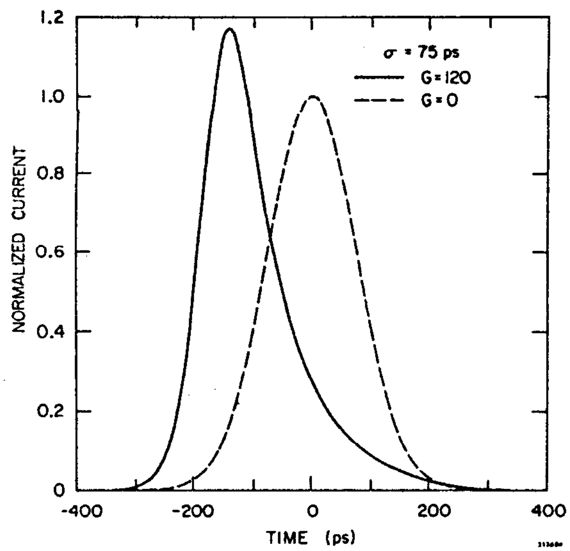

Fig. 3. Bunch current distribution for cavity C, Fig. 1, for $\sigma = 75$ ps and $G = 120$ ps^2/ohm.

Table I

	G (ps^2/Ω)	I'_{max}	FWHM (ps)	σ_{rms} (ps)	k (V/pC)	K
σ=75 ps	40	1.07	162	73	.358	2.3
	80	1.13	146	76	.360	6.5
	120	1.17	132	87	.337	21.9
	160	1.15	121	112	.281	66.4
	500	0.73	87	196	.046	1.2×10^4
σ=240 ps	40	.98	575	243	.024	1.05
	80	.97	586	246	.022	1.10
	120	.95	598	249	.020	1.14
	160	.94	610	252	.019	1.19
	500	.81	735	277	.009	1.66

low values of G. This behavior at long and short bunch lengths is that expected from the form of the wake in Fig. 1. For still larger G in the 240 ps case, the distribution simply becomes somewhat lower and broader. For the other cavities investigated in detail (cavities A and B in Fig. 1), the behavior as a function of G and σ is qualitatively similar to that for case of cavity C discussed here.

Note that in all the curves the center of charge has shifted to earlier (negative) times. This time shift is a

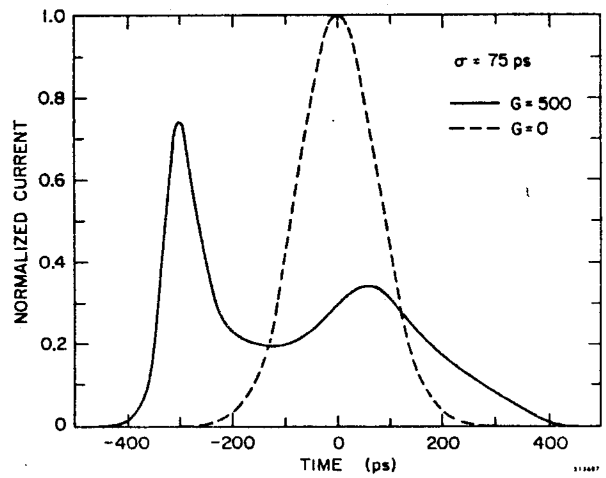

Fig. 4. Bunch current distribution for cavity C, Fig. 1, for $\sigma = 75$ ps and $G = 500$ ps^2/ohm.

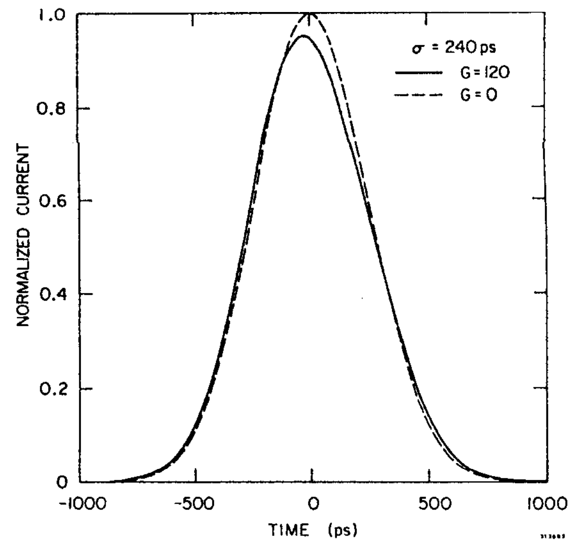

Fig. 5. Bunch current distribution for cavity C, Fig. 1, for $\sigma = 240$ ps and $G = 120$ ps^2/ohm.

measure of the total energy lost to the cavity. If the center of charge shifts by time $\bar{t}$, where $\bar{t} = \int t I'(t)dt/(\sqrt{2\pi}\,\sigma)$, then the loss parameter k, expressed in volts per unit charge, is given by $k = (\bar{V}/Q) = (\bar{t}/\sqrt{2\pi}\,G)$.

Table I shows how k and other parameters of interest vary as a function of G for cavity C at two different bunch lengths. Note in particular that with increasing G the full-width-half-maximum (FWHM) bunch length decreases, but that the rms bunch length increases for the 75 ps case. The variation in the loss parameter k is consistent with the change in σ_{rms} (the loss decreases as σ_{rms} increases), but not with the change in the FWHM bunch length.

The authors would like to thank E. Keil for helpful discussions and correspondence.

References

1. A. Papiernik, M. Chatard-Moulin, B. Jecko, Proc. 9th Int. Conf. on High Energy Accel., SLAC, 1974, p. 375.
2. P. Germain and H. G. Hereward, CERN/MPS/DL 75-5 (July 1975, unpublished).
3. E. Keil, SLAC PEP-126 (August 1975, unpublished).
4. P. B. Wilson, J. B. Styles, K.L.F. Bane, this conference.
5. P. B. Wilson, K.L.F. Bane, SLAC PEP-226A (March 1977, unpublished).
6. P. B. Wilson, R. Servranckx, A. P. Sabersky, J. Gareyte, G. E. Fischer, A. W. Chao, this conference.