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MAGNETIC FIELD QUALITY REQUIREMENTS FOR PEP*

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I. Introduction

In previous publications,[1,2] A. W. Chao, M. J. Lee, and P. M. Morton studied the field quality of the cell quadrupole magnets of PEP. With an improved formula, which takes into account the synchrotron oscillations, the field quality of the bending magnets and of the insertion quadrupole magnets is studied in this paper. An attempt is made to give a quality parameter. The instability prediction given by the betatron frequency shifts is compared with the instability prediction given by a particle tracing program.[3]

II. Betatron Frequency Shifts

The theory was developed in Ref. 1 to which we refer the reader for details and notation. In the following analysis $\Delta\nu_x$, $\Delta\nu_y$ are the betatron frequency shifts, x_e the equilibrium orbit, x_β y_β the betatron oscillation amplitudes, η β the dispersion and betatron functions, $\delta=\Delta p/p$ characterizes the energy of the particle, ΔB_x ΔB_y are the field imperfections, and $B\rho$ is the particle rigidity.

The equations of motion are

$$\begin{aligned} \frac{d^2x}{ds^2} + K_x(s) &= -\frac{\Delta B_y(x,y,s)}{B\rho} + \frac{\delta}{\rho} \\ \frac{d^2y}{ds^2} + K_y(s) &= \frac{\Delta B_x(x,y,s)}{B\rho} \end{aligned} \tag{1}$$

The field imperfections are usually given as Taylor series:

$$\Delta B_y = B_0 \sum a_{mn} x^m y^n$$
$$\Delta B_x = B_0 \sum b_{mn} x^m y^n$$

Solving Eqs. (1) as in Ref. 1 yields the following tune shifts

$$\Delta\nu_x = \sum_i \frac{\beta_{xi}\ell_i}{2\pi\rho}\left[\sum_{mn} a_{mn}\left(\sum_{k=1} C_k^m x_e^{m-k}(\sqrt{J_x\beta_x})^{k-1} I_{k+1}\right)(\sqrt{J_y\beta_y})^n I_n\right]$$

$$\Delta\nu_y = -\sum_i \frac{\beta_{yi}\ell_i}{2\pi\rho}\left[\sum_{mn} b_{mn}\left(\sum_{k=0} C^m x_e^{m-k}(\sqrt{J_x\beta_x})^{k} I_{k}\right)(\sqrt{J_y\beta_y})^{n-1} I_{n+1}\right] \tag{2}$$

where

$$C_k^m = \frac{m!}{k!\,(m-k)!} \qquad x_e(s) = n\sigma_e\eta(s)$$

$$\sqrt{J_x\beta_x} = n\sigma_x(s) \qquad \sqrt{J_y\beta_y} = n\sigma_y(s)$$

$$I_p = \frac{1}{2\pi}\int_0^{2\pi} \cos^p\theta\, d\theta = \begin{cases} 0 \text{ if } p \text{ is odd} \\ 1 \text{ if } p=0 \\ \dfrac{(p-1)(p-3)\ldots 1}{p(p-2)\ldots 2} \text{ if } p \text{ is even} \end{cases}$$

The magnetic fields of elements around the machine usually have a horizontal plane of symmetry. Then, to order M

$$\Delta B_y(s,y) = \sum_{m=0}^{M} \sum_{q=0}^{\left[\frac{M-m}{2}\right]} (-1)^q \frac{(m+2q)!}{m!\,(2q)!} a_{m+2q} x^m y^{2q}$$

$$\Delta B_x(x,y) = \sum_{m=0}^{M-1} \sum_{q=0}^{\left[\frac{M-m-1}{2}\right]} (-1)^q \frac{(m+2q+1)!}{m!\,(2q+1)!} a_{m+2q+1} x^m y^{2q+1}$$

With these expansions the frequency shifts become

$$\Delta\nu_x = \sum_i \frac{\beta_{xi}\ell_i}{2\pi\rho}\left[\sum_{m=1}^{M} \sum_{q=0}^{\left[\frac{M-m}{2}\right]} (-1)^q \frac{(m+2q)!}{(m)!\,(2q)!} a_{m+2q} \left(\sum_{p=0}^{\left[\frac{m-1}{2}\right]} C_{2p+1}^m x_e^{m-2p-1} (\sqrt{J_x\beta_x})^{2p} I_{2p+2}\right)(\sqrt{J_y\beta_y})^{2q} I_{2q}\right]$$

$$\Delta\nu_y = -\sum_i \frac{\beta_{yi}\ell_i}{2\pi\rho}\left[\sum_{m=0}^{M-1} \sum_{q=0}^{\left[\frac{M-m-1}{2}\right]} (-1)^q \frac{(m+2q+1)!}{m!\,(2q+1)!} a_{m+2q+1} \left(\sum_{p=0}^{\left[\frac{m}{2}\right]} C_{2p}^m x_e^{m-2p} (\sqrt{J_x\beta_x})^{2p} I_{2p}\right)(\sqrt{J_y\beta_y})^{2q} I_{2q+2}\right] \tag{3}$$

In PEP the synchrotron oscillation period is about 16 turns. Because of the relatively short value of this period a more realistic value for the tune shifts is obtained by averaging the results over the energy oscillations. The frequency shifts then become

$$\Delta\nu_x = \sum_i \frac{\beta_{xi}\ell_i}{2\pi\rho}\left[\sum_{n=0}^{\left[\frac{M-1}{2}\right]} \sum_{q=0}^{\left[\frac{M-2n-1}{2}\right]} (-1)^q \frac{(2n+2q+1)!}{(2n+1)!\,(2q)!} a_{2n+2q+1} \left(\sum_{p=0}^{n} C_{2p+1}^{2n+1} x_e^{2n-2p} I_{2n-2p} (\sqrt{J_x\beta_x})^{2p} I_{2p+2}\right)(\sqrt{J_y\beta_y})^{2q} I_{2q}\right]$$

$$\Delta\nu_y = -\sum_i \frac{\beta_{yi}\ell_i}{2\pi\rho}\left[\sum_{n=0}^{\left[\frac{M-1}{2}\right]} \sum_{q=0}^{\left[\frac{M-2n-1}{2}\right]} (-1)^q \frac{(2n+2q+1)!}{(2n)!\,(2q+1)!} a_{2n+2q+1} \left(\sum_{p=0}^{n} C_{2p}^{2n} I_{2n-2p} x_e^{2n-2p} (\sqrt{J_x\beta_x})^{2p} I_{2p}\right)(\sqrt{J_y\beta_y})^{2q} I_{2q+2}\right] \tag{4}$$

III. Basic Quality Requirements

It was decided rather arbitrarily, to set the upper limit of allowable frequency shifts to a value of 0.01. Stability checks using a particle tracing program confirmed that it was a reasonable choice.

Formulae (3) and (4) are rather tedious to compute and we felt there was a need to find a manageable parameter to orient the search for a satisfactory design. The following paragraphs outline the rationale leading to the choice of this parameter.

Consider the simplified version of (3) and (4) when $J_y=J_x=0$

$$\Delta\nu_x = \sum_i \frac{\beta_{xi}\ell_i}{4\pi\rho} \sum_{m=1}^{M} m a_m x_e^{m-1} \tag{5a}$$

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$$\Delta\nu_y = -\sum_i \frac{\beta_{yi}\ell_i}{4\pi\rho} \sum_{m=1}^{M} m a_m x_e^{m-1} \tag{5b}$$

$$\Delta\nu_x = \sum_i \frac{\beta_{xi}\ell_i}{4\pi\rho} \sum (2n-1)\, a_{2n-1} x_e^{2n-2} I_{2n-2}$$

$$\Delta\nu_y = -\sum_i \frac{\beta_{yi}\ell_i}{4\pi\rho} \sum (2n-1)\, a_{2n-1} x_e^{2n-2} I_{2n-1} \tag{6}$$

On the other hand, if $x_e=0$ and $J_y=0$, both sets reduce to:

$$\Delta\nu_x = \sum_i \frac{\beta_{xi}\ell_i}{4\pi\rho} \sum_p (2p-1)\, a_{2p-1} (\sqrt{J_x\beta_x})^{2p-2} I_{2p}$$

$$\Delta\nu_y = -\sum_i \frac{\beta_{yi}\ell_i}{4\pi\rho} \sum_p (2p+1)\, a_{2p+1} (\sqrt{J_x\beta_x})^{2p} I_{2p} \tag{7}$$

Formulae (5), (6) and (7) show that if the frequency shifts are to be kept small, one should reduce the local slope of the field and the odd terms in the field expansion. However, since the ring contains quadrupoles to control the tune and sextupoles to control the chromatic effects, we need only direct our concern to the terms of higher order. This leads to the definition of a residual asymmetry factor

$$\mathrm{RAF} = \left| \int_0^d \frac{\Delta B_y(x,0) - \Delta B_y(-x,0)}{B_0}\, dx \right| \tag{8}$$

where quadrupolar and sextupolar terms are omitted in the field expansion. This residual asymmetry factor can be used to compare different magnet designs. As defined, this parameter is not absolute and the upper limit that is suitable, varies with the ring parameters and the kind and number of magnets to which it refers. Nevertheless, it has proved useful in guiding our search.

IV. Bending Magnets

The main bending magnets are 5.4 m long, the curvature of the trajectory is 165 m, so that the sagitta of the central orbit over the magnet length is 22 mm. To determine the betatron frequency shifts the magnet was split into four partial magnets and the displacement a of the closed orbit was optimized. The maximum excursion of the particles in the bending magnets is 25 mm; this value was adopted for the parameter d defining RAF. Amongst the various steps in the design we chose three that illustrate the use of the residual asymmetry factor as a quality factor. The basic design parameters can be found in Figs. 1 and 2.

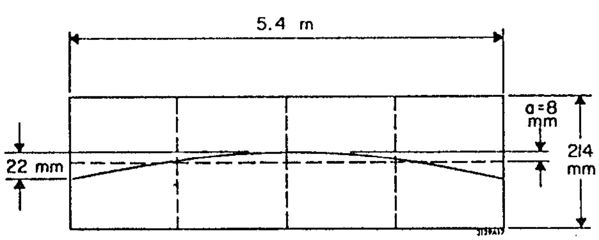

Fig. 1. Bending magnet and trajectory (top view).

Frequency shift values for the three cases are shown in Figs. 3, 4 and 5 together with the corresponding residual asymmetry factor. The results are self-explanatory.

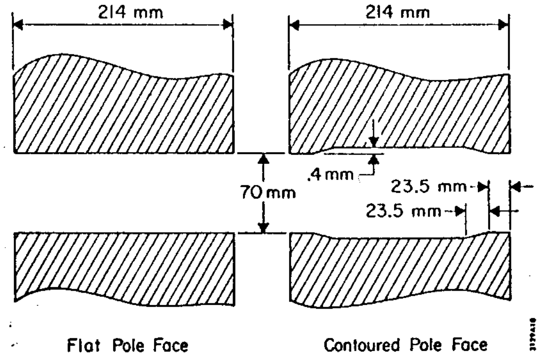

Fig. 2. Bending magnet pole faces.

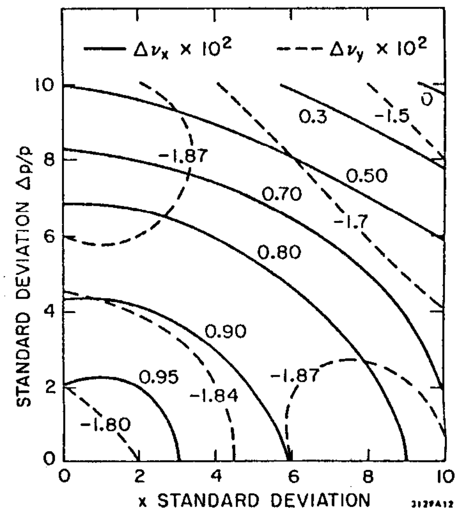

Fig. 3. Tune shifts for bending magnet with flat pole face. RAF $= 1.1 \times 10^{-4}$ m. d $= 0.025$ m.

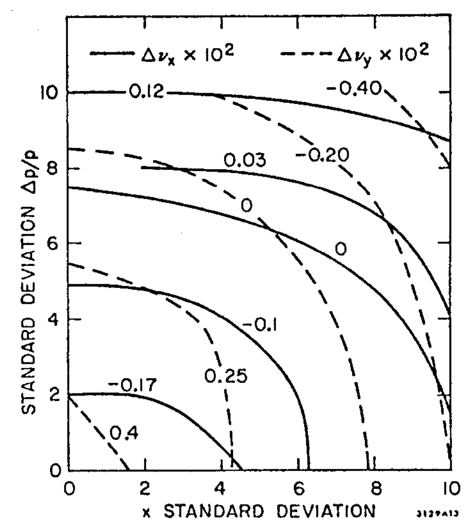

Fig. 4. Tune shifts for bending magnet with contoured pole face. RAF $= 4 \times 10^{-6}$ m. d $= 0.025$ m.

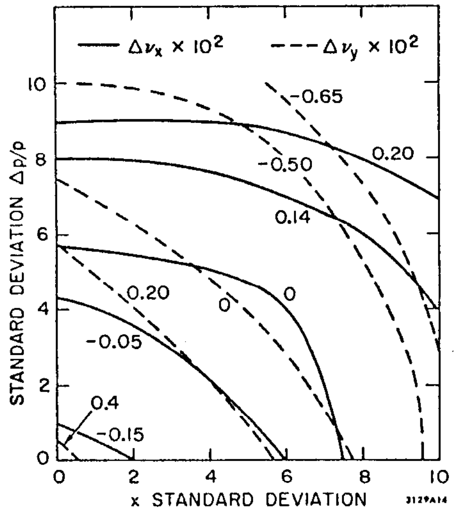

Fig. 5. Tune shifts for bending magnet with contoured pole face. RAF $=5.7\times10^{-6}$ m. d=0.025 m.

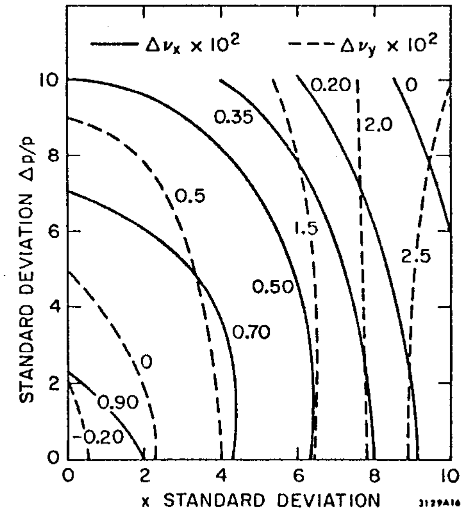

Fig. 7. Tune shifts for insertion quadrupoles with .002 in. assembly tolerance. RAF $=7\times10^{-7}$ m. d=0.08 m.

V. Insertion Quadrupoles

The insertion quadrupoles, in the present design, are 2 m and 1.5 m long with a bore of 80 mm and a pole width of 140 mm.

Figures 6 and 7 show the frequency shifts expected for the present design. The values indicate that the quality is marginal. A particle tracing run, whose results are given in Fig. 8 confirm the marginal quality of the quadrupole field. Work is being pursued to improve the quality of these magnets.

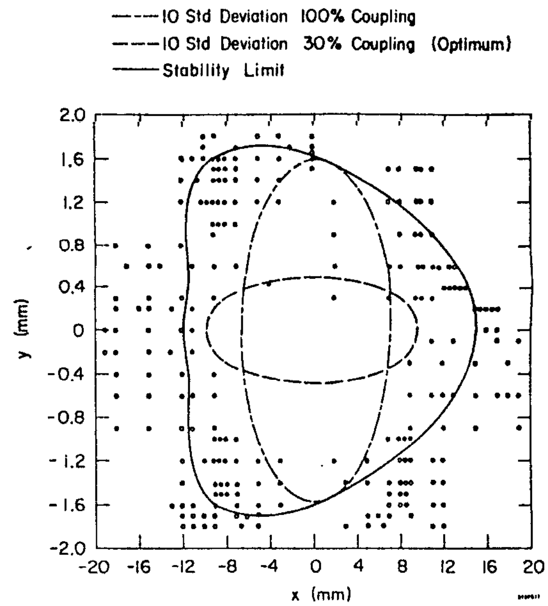

Fig. 8. Stability diagram in x, y for the case of Fig. 7.

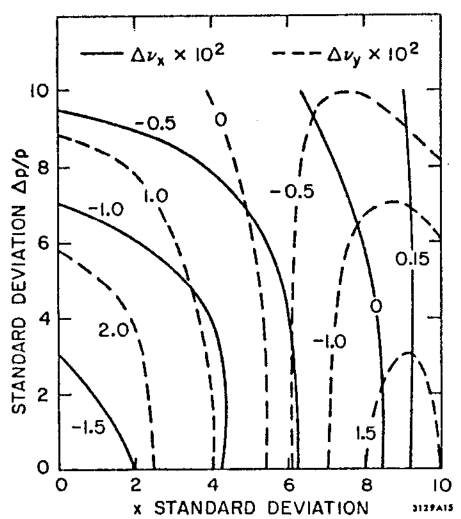

Fig. 6. Tune shifts for insertion quadrupoles. Ideal field RAF $=1.6\times10^{-8}$ m. d=0.08 m.

References

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2. A. W. Chao, M. J. Lee, and P. L. Morton, Stanford Linear Accelerator Center report PEP-132 (1974).
3. R. Servranckx, "Depart: A differential equation particle tracing program," Particle Accelerator, 1975, Vol. 6, pp. 83-93.