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THE SEARCH FOR HEAVY PARTICLES*

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ABSTRACT

Direct and indirect evidence for the existence of a new heavy quark b and of new heavy neutral leptons $\mathcal{N}_e$ and $\mathcal{N}_\mu$ can be sought in neutrino and e^+e^- scattering. These particles are expected to have right-handed currents. Discussion is given on the characteristics, production and decay of hadrons such as $b\bar{b}$, $u\bar{b}$ and $d\bar{b}$, and of the massive neutral leptons. Muon number violation with and without $\mathcal{N}_e$ and $\mathcal{N}_\mu$ is considered.

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I. INTRODUCTION

Interest in gauge theories of the weak and electromagnetic interactions[1] has led many people in the last few years to consider the possibility that new heavy particles, both hadrons and leptons, may exist and might have right-handed currents.[2] Two types of heavy particles will be considered here: (1) hadrons containing a quark, b,[3-7] of charge -1/3 with a right-handed coupling to u quarks, (2) massive neutral leptons,[4-7] $\mathcal{N}_e$ and $\mathcal{N}_\mu$, with right-handed couplings to e and μ.

Several experimental results have motivated recent interest in new heavy particles. Originally the anomalous behavior of charged-current antineutrino scattering hinted at the possibility of a $(u,b)_R$ coupling. However, more substantial evidence concerning this coupling may be found in dilepton ($\mu^+\mu^-$ and μ^+e^-) antineutrino data at high energies, soon to be reported. Among the best evidence for the existence of b quarks would be the observation of $b\bar{b}$ states in e^+e^- scattering at CESR, PETRA and PEP colliding beams (located at Cornell, DESY and SLAC, respectively).

The first possible evidence for the couplings $(\mathcal{N}_e, e)_R$ and $(\mathcal{N}_\mu, \mu)_R$ arose from the atomic parity violation experiments. These experiments appear (given present experimental and theoretical uncertainties) to be inconsistent with the standard four-quark, four-lepton model with left-handed couplings; but they appear to be consistent with models where the electron has similar left- and right-handed couplings. More attention for these couplings occurred when reports (discussed at this conference) were heard indicating that the decay $\mu \rightarrow e\gamma$ <u>may</u> have been observed. This decay would be expected in models with the above couplings. Also intriguing is the possibility that the reported $\mu^-\mu^-$ and $\mu^-\mu^+\mu^-$ events in neutrino scattering data (of which much more will soon be available) can be understood in terms of decays to $\mathcal{N}_e$ and $\mathcal{N}_\mu$. At PETRA and PEP there will be clear means of observing $\mathcal{N}_e$ and $\mathcal{N}_\mu$ and determining their mass and mixing.

Many factors enter into the analysis of the relevant data. Among these are the scaling violations expected in asymptotically free gauge theories, and the branching ratios to muons of hadrons with b quarks. Also relevant to the possibility of b quarks and $\mathcal{N}_e$ and $\mathcal{N}_\mu$ leptons with right-handed couplings are the various neutral-current neutrino interactions (νN deep inelastic, $\nu_\mu e$, $\bar{\nu}_\mu e$, $\bar{\nu}_e e$, νp, $\bar{\nu} p$, etc.). The presence of such couplings in gauge theories alters the expected behavior of neutral-current reactions (even at low energies).

The above subjects, which have been discussed by many different authors, will be summarized in this report. For simplicity and clarity, these topics will be discussed in the context of (only) three $SU(2) \times U(1)$ gauge models of the weak and electromagnetic interactions, shown in Table I. The Weinberg-Salam-Glashow-Iliopoulos-Maiani (WS-GIM) model[1,8] has no right-handed couplings. The CHYM model[3-6] has the $(u,b)_R$, $(\mathcal{N}_e, e)_R$ and $(\mathcal{N}_\mu, \mu)_R$ couplings, but no $(t,d)_R$ coupling. Other variations of the leptonic couplings are possible, and in particular, the versions[6] of this model found from the exceptional group E_7 have slightly different couplings. The vector model[7] has, in addition to the above couplings, the $(t,d)_R$ coupling. While the vector model is in some disrepute at the present time, it is useful for comparison purposes.

One is by no means limited to consideration of $SU(2) \times U(1)$ models. However, the larger groups can give more parameters and make it easier to fit data. Among alternative models are the $SU(2)_L \times SU(2)_R \times U(1)$ models of De Rujula, Georgi, Glashow[9] and Mohapatra, Sidhu.[10] But no discussion of these models is given here.[11]

Table I

		WS-GIM Model			
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$		$u_R \quad d_R \quad c_R \quad s_R$		
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$		$e_R \quad \mu_R$		
		CHYM Model			
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$b_L \quad g_L$	$\begin{pmatrix} u \\ b \end{pmatrix}_R$	$\begin{pmatrix} c \\ g \end{pmatrix}_R$	$d_R \quad s_R$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_U \\ U \end{pmatrix}_L$	$\begin{pmatrix} \mathcal{N}_e \\ e \end{pmatrix}_R$	$\begin{pmatrix} \mathcal{N}_\mu \\ \mu \end{pmatrix}_R$	$\begin{pmatrix} \mathcal{N}_U \\ U \end{pmatrix}_R$
		Vector Model			
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	$\begin{pmatrix} u \\ b \end{pmatrix}_R$	$\begin{pmatrix} c \\ s \end{pmatrix}_R$	$\begin{pmatrix} t \\ d \end{pmatrix}_R$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_U \\ U \end{pmatrix}_L$	$\begin{pmatrix} \mathcal{N}_e \\ e \end{pmatrix}_R$	$\begin{pmatrix} \mathcal{N}_\mu \\ \mu \end{pmatrix}_R$	$\begin{pmatrix} \mathcal{N}_U \\ U \end{pmatrix}_R$

II. EVIDENCE FOR b QUARK IN $\nu N \to \mu X$

A. b Quarks vs. Scaling Violation

The anomalous behaviors with energy of the ratio, R_c, of $\bar{\nu}$ to ν total cross sections and of the antineutrino average y (where $y \equiv (E_\nu - E_\mu)/E_\nu$) have been interpreted[3-5, 12-14] as evidence for the right-handed coupling of u quarks to a quark, b (of charge -1/3 and of mass 5-6 GeV). This argument has been clouded by the alternative possibility that the anomalies could be explained as scaling violations expected in asymptotically free gauge theories.[12-16] These anomalies have been well discussed in the literature and the old arguments will not be repeated here.

It would be useful to find tests of scaling violation which are independent of the $(u,b)_R$ coupling, and tests of the $(u,b)_R$ coupling which are independent of asymptotic freedom corrections. F. Martin and I[17] have sought such tests (as described below).

There are two types of scaling violation expected in asymptotically free theories. The dominant effect is the logarithmic change with Q^2 (and therefore with E) of the relative amounts of each type of quark, antiquark and gluons. The valence quarks (u and d) decrease and sea quarks ($\bar{u}$, $\bar{d}$, s, $\bar{s}$, c, $\bar{c}$, etc.) increase with Q^2. Since sea quarks are concentrated at small x, this causes the $\langle x \rangle$ to decrease with Q^2. A second effect (called shrinkage) is that the $\langle x \rangle$ of valence quarks decreases with Q^2; however, this effect (unlike the first) has little effect on R_c and $\langle y \rangle$.

B. Scaling Violation Independent of Model

These scaling violations can be measured experimentally by the quantity

$$R_x \equiv \frac{\sigma(x < 0.15)}{\sigma(x > 0.15)} \tag{2.1}$$

shown[17] in Fig. 1. Clearly for $E < 80$ GeV this quantity is independent of the $(u, b)_R$ coupling. If the secondary effect of decreasing $\langle x \rangle$ for valence quarks were included, the rise of R_x from 5 to 80 GeV would be about 30 percent greater than that shown on the curve labelled AF. In order for scaling violation to be a viable explanation of the antineutrino anomalies, R_x would have to rise even more quickly than as shown in Fig. 1 (with label "AF").

C. $(u, b)_R$ Independent of Scaling Violation

Are there tests which are independent of the asymptotic freedom corrections (or any similar scaling violations)? Since the sea contributions are concentrated at small x, any effects due to increasing sea can be eliminated by considering only events at large x (defined here as $x > 0.15$). In Fig. 2 the $\langle y \rangle$ for $x > 0.15$ is shown. As anticipated, the WS-GIM model gives little rise with E in contrast to the CHYM model (with a $(u, b)_R$ coupling) which shows a significant rise. Similarly, one could examine R_c at $x > 0.15$.

D. Tests with Dileptons

Significant tests for a quark, b, with coupling $(u, b)_R$ might be found in data for

$$\nu N \rightarrow \mu^- \mu^+ X$$

and (2.2)

$$\bar{\nu} N \rightarrow \mu^+ \mu^- X$$

(or with $\mu^- e^+$ and $\mu^+ e^-$). An especially useful quantity[17, 18] should be the following "ratio of ratios":

$$R_r \equiv \frac{\sigma(\bar{\nu} \rightarrow \mu\mu)}{\sigma(\bar{\nu} \rightarrow \mu\ )} \Big/ \frac{\sigma(\nu \rightarrow \mu\mu)}{\sigma(\nu \rightarrow \mu\ )} \qquad (2.3)$$

This ratio allows one freedom from knowing the branching ratio of charm to muons (assuming mesons and baryons are similar). However, some input is required on the relative $b \rightarrow \mu$ and $c \rightarrow \mu$ branching ratios, and this will be discussed shortly. Clearly the relative ν and $\bar{\nu}$ cross sections, which are difficult to determine, cancel out of R_r. While the separate ν and $\bar{\nu}$ ratios are very sensitive to the detection efficiency for the secondary (slow) muon, the ratio of ratios is found to be rather insensitive to this problem.[19,20] However, R_r should clearly not be calculated with cross sections from different experiments with different cuts. Somewhat more sensitivity (but still small) is found for detection efficiency for initial (fast) muons; however, this can be accounted for theoretically by model-independent means. There is some sensitivity to the amount of strange sea quarks relative to $\bar{u}$ and $\bar{d}$ sea quarks, but at higher Q^2 (and E), it is reasonable to assume SU(3) symmetry.

This ratio, R_r, is however very dependent on asymptotic freedom corrections as is shown[17] in Fig. 3 (where $s = \bar{s} = \bar{u} = \bar{d}$, $m_b = 5$ GeV, and $\Gamma(b \rightarrow \mu) = \Gamma(c \rightarrow \mu)$ are assumed). Since b production is a valence process, an increasing sea to valence ratio goes against dimuon production through b quarks. It should be emphasized that the $(u, b)_R$ prediction (labelled CHYM here) would decrease if $m_b > 5$ GeV and if $\Gamma(b \rightarrow \mu) < \Gamma(c \rightarrow \mu)$ whereas the WS-GIM prediction cannot easily be increased.

E. Branching Ratios of Y Mesons to Muons

In considering dilepton events (as discussed above), it is necessary to know the relative branching ratios of $b \rightarrow \mu$ and $c \rightarrow \mu$. This has been discussed by R. Cahn and S. Ellis,[18] and I will summarize their results. In this discussion I define the mesons containing b quarks as

$$Y^+ \equiv u\bar{b} \qquad Y^0 \equiv d\bar{b} \tag{2.4}$$

There are two decay modes possible for Y^+ and two for Y^o as shown in Fig. 4:

Modes (4a) and (4b): The width is

$$\Gamma(b \rightarrow u\,\mu\nu) = \frac{G^2 m_b^5}{192\pi^3} \quad . \tag{2.5}$$

With phase space (assuming simple three-body decays) included, the naive ratios for X in $b \rightarrow u + X$ are

$$\begin{array}{ccccc} e\nu & \mu\nu & U\nu & ud & cs \\ 1 & 1 & 0.5 & 3 & 1.5 \end{array} \tag{2.6}$$

(quarks have a factor 3 for three colors). However, one must remember that both c and U, formed in b decay, also decay into muons. Then given the branching ratio $B(U \rightarrow \mu) \approx 20\%$, one finds the branching ratio

$$B_{a,b}(b \rightarrow \mu) \approx B(c \rightarrow \mu) \tag{2.7}$$

This result can be obtained even if some of the above assumptions are relaxed.

Mode (4c): The widths for these annihilation diagrams are

$$\Gamma(Y^+ \rightarrow cs) = 9\,\frac{G^2 m_c^2}{2\pi}\,|\psi(0)|^2$$

$$\Gamma(Y^+ \rightarrow U\nu) = 3\,\frac{G^2 m_U^2}{2\pi}\,|\psi(0)|^2 \tag{2.8}$$

Since the width of the pseudoscalar Y^+, in this mode, is proportional to produced mass, the other possible products are negligible. Clearly we again find

$$B_c(b \rightarrow \mu) \approx B(c \rightarrow \mu) \tag{2.9}$$

Note that this mode is Cabibbo-suppressed for D^+ mesons.

Mode (4d): The width for this mode which has left-right couplings (unlike D_0 which has left-left couplings) is

$$\Gamma(Y^o \rightarrow u\bar{u}) = 2\,\frac{G^2 m_b^2}{\pi}\,|\psi(0)|^2 \tag{2.10}$$

Obviously

$$B_d(b \rightarrow \mu) = 0 \qquad (2.11)$$

In order to compare the relative rates, Cahn and Ellis needed to estimate the wave function at the origin $|\psi(0)|^2$. Since its dependence is on the reduced mass, one can compare with estimates of $|\psi(0)|^2$ for charmed mesons. Lane and Weinberg[21] use $|\psi(0)|^2 = 0.02$ and De Rujula, Georgi, Glashow[22] use $|\psi(0)|^2 = 0.55$. From these numbers one obtains (with $m_b = 5$ GeV)

$$\frac{B\left[(Y^+ + Y^0) \rightarrow \mu\right]}{B\left[(D^+ + D^0) \rightarrow \mu\right]} \approx \begin{cases} 0.97 \text{ (LW)} \\ 0.67 \text{ (DGG)} \end{cases} \qquad (2.12)$$

In conclusion, one finds that the b quark may have a slightly smaller branching ratio to muons than does the c quark.

III. NEUTRAL CURRENT NEUTRINO SCATTERING

Charged-current scattering provides direct tests for heavy particles. The neutral-current neutrino interactions, while presumably lacking such direct production, do allow one to examine the structure of the theory even at low energies. The neutral-current, deep-inelastic scattering gave the earliest evidence of problems for the vector model of weak interactions.

Deep-inelastic scattering through neutral currents has been widely discussed,[23] so only brief remarks will be added here. F. Martin and I have investigated[17] the effects of corrections expected in asymptotically free theories. To the extent that one considers only the ratios of neutral to charged currents

$$R_\nu \equiv \frac{\sigma(\nu N \rightarrow \nu X)}{\sigma(\nu N \rightarrow \mu X)} \qquad (3.1)$$

the effects tend in general to cancel. By including all energy dependent effects (asymptotic freedom corrections, experimental cuts, new currents, etc.) in theoretical calculations of the numerator and denominator of R_ν, we could determine the best $\sin^2\theta_W$ for each model (considering the three experiments which occur at different energies); this is shown later in this section.

With this determination of $\sin^2\theta_W$, we[17] could address the "problem" that rising $\sigma(\bar{\nu}N \to \mu X)/E$ and "constant" $R_{\bar{\nu}}$ (comparing three experiments) implies $\sigma(\bar{\nu}N \to \bar{\nu}X)/E$ must be rising (suggesting, perhaps, charm-changing neutral-currents). In fact there is no problem for the WS-GIM or CHYM models (see Fig. 5): (a) Any rise in $\sigma(\bar{\nu}N \to \mu X)$ due to asymptotic freedom corrections is approximately matched in $\sigma(\bar{\nu}N \to \bar{\nu}X)$; (b) Accounting for experimental cuts[24-26] would lower both high energy points by 20-30% (from values shown) so $R_{\bar{\nu}}$ is not really constant; (c) The error bars are large.

There are three other types of neutral-current experiments against which models can be tested. The νp elastic scattering experiments[27, 28] appear reasonably consistent[23, 29] with the WS-GIM and CHYM models and inconsistent with the vector model. In the next few months, greatly increased statistics and new calculations of background should make this experiment[30] a crucial test.

In the elastic scattering[31-33] of ν and e and in the atomic parity violation experiments,[34] the neutral currents also probe the weak interactions of the electron. ν_e elastic scattering, as discussed elsewhere,[23, 35] is consistent with the electron having (in addition to $(\nu_e, e)_L$) the couplings e_R or $(\mathcal{N}_e, e)_R$ but not $(E^+, E^o, e^-)_R$ or $(e^-, E^{--})_R$, etc. So the WS-GIM and CHYM models (as described in Section I) are consistent with present data.

The search for parity violation in atomic physics via weak neutral-currents is potentially a critical test of the electron coupling. Present experiments on bismuth still have large systematic uncertainties and some people[36] argue that the atomic-nuclear theory is also uncertain. If the above complaints are ignored, the apparent lack of parity violation in the Seattle and Oxford experiments[34] casts doubt upon the WS-GIM model and is suggestive of models (such as CHYM) where the electron's neutral-currents are pure vector, assuring no parity violation irrespective of $\sin^2 \theta_W$. Perhaps, more decisive will be the proposed parity-violation searches on hydrogen.[37]

The $\mathcal{N}_e$ and $\mathcal{N}_\mu$ suggested by νe elastic scattering and by the atomic parity-violation experiments are discussed in later sections.

All of these neutral-current results are summarized in Figs. 6-8.

IV. THE $b\bar{b}$ MESONS

A. Spectroscopy of $b\bar{b}$

An important test of the existence of a b quark will be the search for $b\bar{b}$ states in e^+e^- scattering. Let us define β to be the lowest-lying vector $b\bar{b}$ state (the equivalent of ψ). Eichten and Gottfried (EG)[38] have estimated the β, β', ... and $Y(\equiv u\bar{b})$ masses in a linear-Coulombic potential of the form

$$V(r) = -\frac{4}{3}\alpha_s\left(\frac{1}{r}\right) + \frac{r}{a^2} \tag{4.1}$$

(while I will consider only the case of b quarks and masses of 5-6 GeV, the work of EG is more general). From charm, EG found $a = 2.2\ \text{GeV}^{-1}$, $\alpha_s\,(m^2{=}10) = 0.2$ and $m_c = 1.35$ GeV. The β mass is calculated from

$$M_\beta(m_b) = 2m_b + E_0(m_b) + \Delta(m_b) \tag{4.2}$$

where E_0 is the ground state eigenvalue of the Schrödinger equation and Δ is the "zero of energy" term (approximated by EG as $\Delta(m_b) = \Delta(m_c)(m_c/m_b)$, $\Delta(m_c) = -225$ MeV). The result of this calculation is (for $m_b = 5$ GeV)

$$m_\beta = 10.3\ \text{GeV}, \quad m_{\beta'} = 10.7\ \text{GeV}, \quad m_{\beta''} = 11.05\ \text{GeV} \tag{4.3}$$

Similarly the mass of $Y \equiv u\bar{b}$ and $d\bar{b}$ can be calculated from

$$m_Y = m_D + (m_b - m_c) + \frac{3}{4}\left(1 - \frac{m_c}{m_b}\right)(m_{D*} - m_D) \tag{4.4}$$

The last term is, of course, a simple means of estimating the hyperfine contribution. EG find for $m_b = 5$ GeV

$$m_Y = 5.6 \text{ GeV} \tag{4.5}$$

It is now clear, since $2m_Y = 11.2$ GeV, that not only are β and β' below threshold for $Y\bar{Y}$ production, but also β'' (and β''' if $m_b \gtrsim 6$ GeV which is quite possible). In addition there are two sets of p-wave states below threshold. This is shown in Fig. 9 (from EG[38]). The enormous number of gamma and hadron decays which result are shown in Figs. 10 and 11 (also Ref. 38).

B. Can β Be Seen at the New Accelerators?

It will be quite difficult to see the β states (assuming they exist) in e^+e^- scattering[39] at the new accelerators PETRA (at DESY), CESR (at Cornell) and PEP (at SLAC). The reasons for this are: (a) the b charge is half the c charge; (b) Γ_{ee} is proportional to m_b^{-2} and therefore the integrated cross section is also; (c) the expected experimental resolution will be much worse than at SPEAR; (d) the production is proportional to m_β^{-2}.

The integrated area under a resonance in e^+e^- scattering is given by

$$\Sigma_\beta = \frac{6\pi^2}{m_\beta^2} B_{had} \Gamma_{ee} \tag{4.6}$$

B_{had} is the branching ratio to hadrons and should be close to 1.0. Γ_{ee} is proportional to

$$\Gamma_{ee} \propto \frac{|\psi(0)|^2}{m_b^2} Q^2 \tag{4.7}$$

Eichten and Gottfried[38] found $\Gamma_{ee}(\beta)$ by comparison with $\Gamma_{ee}(\psi)$. Using potential (4.1) (which gives $|\psi(0)|^2$ increasing by a factor of ten from c to b), they found

$$\Gamma_{ee}(\beta) = 0.7 \text{ keV} \tag{4.8}$$

which gives for $m_b = 5$ GeV

$$\Sigma_\beta = 130 \text{ nb-MeV} \tag{4.9}$$

(and $\Sigma_{\beta'} = 90$, $\Sigma_{\beta''} = 70$ nb-MeV). Compare this with $\Sigma_\psi = 10,000$ nb-MeV.

Background can be estimated as follows. If $R(e^+e^-) \approx 5.3$ at $\sqrt{s} = 10.3$ GeV, then the usual cross section is $\sigma_{had} = 4.3$ nb. The experimental resolution at PEP is

$$\Delta m = 16 \text{ MeV} \quad \text{at} \quad \sqrt{s} = 10.3 \text{ GeV} \tag{4.10}$$

giving a background area of

$$\sigma_{had}\,\Delta m = 70 \text{ nb-MeV} \tag{4.11}$$

From Eqs. (4.9) and (4.11) we see that signal to background is only 2 for β of mass 10.3 GeV (1.3 for β' and 1.0 for β'') in comparison with 250 for ψ above background.

Given that the expected luminosity for PEP and PETRA at $\sqrt{s} = 10$ GeV is the same as at SPEAR (10^{31} cm^{-2} sec^{-1}), one can compare with the present upper limits on Σ for narrow resonances.[40] For most of the lower energy region the limits are not good, but at higher energies much more stringent limits are set:

$\sqrt{s}$ (GeV)	Σ (nb-MeV)
3.2–5.4	1000
5.4–7.0	100
7.0–7.4	30

The conclusion (comparing with Eq. (4.9)) is then that if experiments at PEP (etc.) scan at the same level as the early SPEAR scans (the lower energies), they will definitely miss all β states (and even states of 2/3 charge quarks). However, if enough time for a careful and fine scan is alloted, then the β states can be found.

One can also estimate the maximum value (above background) which $R(e^+e^-)$ will reach, given the resolution (4.10). In the Breit-Wigner approximation

$$R_{max} \approx 169 \frac{\Gamma_{ee}(\mathrm{keV})}{\Delta(\mathrm{MeV})} B_{had} \approx 7 \tag{4.12}$$

compared to 300 for ψ.

In conclusion, experimentalists should be aware that the search for $\beta \equiv b\bar{b}$ states will be difficult.

C. <u>Branching Ratios for β</u>

Before considering β, let us consider the branching ratios[41] of ψ:

$$\Gamma(\psi \to \mathrm{hadrons}) = 59 \text{ keV} \tag{4.13}$$

$$\Gamma(\psi \to e^+e^-) = \Gamma(\psi \to \mu^+\mu^-) = 4.8 \text{ keV} \tag{4.14}$$

so the e^+e^- branching ratio is

$$B(\psi \to e^+e^-) = 7\% \tag{4.15}$$

The decay of ψ to hadrons can occur two ways: through gluons and through a photon. Given that $\Gamma_{ee} = 4.8$ keV and $R(e^+e^-) \approx 2.3$ near ψ, one can estimate

$$\Gamma(\psi \to \gamma \to \mathrm{hadrons}) \approx 11 \text{ keV} \tag{4.16}$$

so that

$$\Gamma(\psi \to \mathrm{gluons} \to \mathrm{hadrons}) \approx 48 \text{ keV} \tag{4.17}$$

For β we have from Eq. (4.8) $\Gamma_{ee}(\beta) = 0.7$ keV. Assuming $R(e^+e^-)$ is still approximately 5.3 near β we find[39]

$$\Gamma(\beta \to \gamma \to \mathrm{hadrons}) \approx 3.6 \text{ keV} \tag{4.18}$$

Following the arguments of Appelquist and Politzer,[42] the assumptions that gluon decay occurs through three gluons and that $\alpha_s \approx 0.2$ at ψ (both of which can be disputed) would lead to

$$\Gamma(\beta \to \mathrm{gluons} \to \mathrm{hadrons}) \approx 16 \text{ keV} \tag{4.19}$$

and

$$B(\beta \to e^+ e^-) \approx 4\% \quad . \tag{4.20}$$

V. MUON NUMBER VIOLATION

A. General Remarks

A brief discussion will be given here of three models (all in $SU(2) \times U(1)$) which lead to violations of muon number in processes such as $\mu \to e\gamma$, $K_L \to \mu e$, $\mu \to 3e$ and $\mu^- +$ nucleus $\to e^- +$ nucleus. Needless to say, recent interest in this subject has been sparked by the unofficial reports from SIN in Zurich that several events have been observed which look like $\mu \to e\gamma$ and are above the expected background. But it need not concern theorists (yet) whether this particular experiment has or has not observed $\mu \to e\gamma$. The question is whether we expect muon number violation. These reports have reminded us that such experiments provide us with an interesting tool for understanding the structure of the weak and electromagnetic interactions.[43]

B. Higgs Exchange

In context of the WS-GIM model (although it is applicable elsewhere) Bjorken and Weinberg[44] consider the interactions of leptons with Higgs scalars:

$$H = -g_1 \overline{\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}}_L \begin{pmatrix} \phi_1^+ \\ \phi_1^o \end{pmatrix} \mu_R^- - g_2 \overline{\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}}_L \begin{pmatrix} \phi_2^+ \\ \phi_2^o \end{pmatrix} \mu_R^-$$

$$-g_3 \overline{\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}}_L \begin{pmatrix} \phi_3^+ \\ \phi_3^o \end{pmatrix} e_R^- - g_4 \overline{\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}}_L \begin{pmatrix} \phi_4^+ \\ \phi_4^o \end{pmatrix} e_R^- + \text{H.C.} \tag{5.1}$$

where the ϕ_i are linear combinations (not necessarily independent) of several scalar fields of definite mass. Since the μ and e are defined as the physical states found in the diagonalization of the mass matrix, if there is only one Higgs doublet (as is sometimes assumed), then g_2 and g_3 must be zero. However, if there are more than one Higgs doublet, then in general it is possible that g_2 and/or g_3 are nonzero and virtual Higgs scalars will give physical transitions between μ and e such as shown in Fig. 12. Because the Higgs coupling to the light leptons are so weak, the two loop diagrams (Fig. 12b), in general, dominate one loop diagrams (Fig. 12a):

$$\frac{1 \text{ loop}}{2 \text{ loops}} \approx \frac{2\pi}{\alpha}\left(\frac{m_\mu}{m_H}\right)^2 \tag{5.2}$$

Bjorken and Weinberg roughly estimate[44]

$$\frac{\mu \to e\gamma}{\mu \to e\nu\bar{\nu}} \lesssim 10^{-8} \tag{5.3}$$

where the present experimental limit is 2.2×10^{-8}. The decay $\mu \to 3e$ occurs by a very small tree graph. $K_L \to \mu e$ is forbidden in lowest order (or one would get strangeness-changing neutral-currents). They also predict

$$\frac{\mu^- N \to e^- N}{\mu^- N \to \nu N'} \sim 4 \times 10^{-9} \tag{5.4}$$

where N is a nucleus and the experimental limit is 1.6×10^{-8}.

C. Mixing Between $\mathcal{N}_e$ and $\mathcal{N}_\mu$—

In the context of models such as the CHYM and vector models (see Section I), Cheng and Li[45] considered the mixing of massive neutral leptons which have right-handed couplings to the electron and muon. Greater detail of their work is given in T. P. Cheng's talk at this conference, but a brief summary follows. In analogy with the Cabibbo mixing of the d and s quarks, they suggest

$$\begin{aligned} \mathcal{N}_e' &= \mathcal{N}_e \cos\phi + \mathcal{N}_\mu \sin\phi \\ \mathcal{N}_\mu' &= -\mathcal{N}_e \sin\phi + \mathcal{N}_\mu \cos\phi \end{aligned} \tag{5.5}$$

Then clearly if one considers the simple one-loop diagram of Fig. 13, there will be a GIM-like cancellation. The cancellation is not complete to the extent that $\mathcal{N}_e$ and $\mathcal{N}_\mu$ have unequal masses; the amplitude for this $\mu \to e\gamma$ process is proportional to

$$\cos\phi \, \sin\phi \left(m^2_{\mathcal{N}_\mu} - m^2_{\mathcal{N}_e}\right) \tag{5.6}$$

Bjorken, Lane and Weinberg[46] argue that the Higgs couplings which give masses and lead to the above mixing also cause small but finite mixing of the left-handed singlet parts of $\mathcal{N}_e$ and $\mathcal{N}_\mu$ with ν_e and ν_μ. This mixing is order $m_\mu/m_{\mathcal{N}}$. There are, as a result, left-right diagrams in addition to the right-right diagram, Fig. 13. In amplitude they find if right-right terms give +1, left-right terms give -6; but essential features of the Cheng-Li work remain unchanged. If the term Eq. (5.6) is 1 GeV^2, then

$$\frac{\mu \to e\gamma}{\mu \to e\nu\bar{\nu}} \sim 10^{-9} \tag{5.7}$$

The processes $K_L \to \mu e$, $\mu \to 3e$, $\mu^- N \to e^- N$ are allowed, but below present experimental limits.

D. <u>Mixing Between ν_e, ν_μ and $\mathcal{N}_U$</u>

Glashow[47] and Fritzsch[48] have shown that muon number can be violated without right-handed currents and with only one Higgs doublet. They propose that the charged heavy lepton has a left-handed coupling to a massive neutral lepton which can mix with ν_e and ν_μ

$$\begin{pmatrix} \nu'_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu' \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \mathcal{N}'_U \\ U \end{pmatrix}_L \tag{5.8}$$

Decays such as $\mu \to e\gamma$ occur then in the same fashion as proposed by Cheng and Li[45] (see above) where Δm^2 is replaced with $m^2_{\mathcal{N}_U}$.

The mixed states can be written as:

$$\nu'_e = \nu_e \cos\theta + \mathcal{N}_U \sin\theta$$

$$\nu'_\mu = \nu_\mu \cos\phi + (-\nu_e \sin\theta + \mathcal{N}_U \cos\theta)\sin\phi \tag{5.9}$$

$$\mathcal{N}'_U = (\mathcal{N}_U \cos\theta - \nu_e \sin\theta)\cos\phi - \nu_\mu \sin\phi$$

Both angles can be shown to be small by the need to maintain universality (seen through μ and β decay) and by the lack of ν_e in ν'_μ (muon neutrinos do not produce electrons in scattering). If $\mu \to e\gamma$ is observed at the $10^{-10} - 10^{-9}$ level and is to be explained in this manner, then the smallness of the angles θ and ϕ requires that $\mathcal{N}_U$ have a much larger mass than U. As a consequence, the charged heavy lepton U can decay only through the mixing of $\mathcal{N}_U$ with ν_e and ν_μ.

It would be possible to rule out this method of muon number violation by measuring τ_U carefully, but the present limits on τ_U are not restrictive enough. Given the mixing of neutral leptons, the charged lepton U could be produced in ν_μ scattering although it would be difficult to observe at that level.

VI. OBSERVATION OF $\mathcal{N}_e$ AND $\mathcal{N}_\mu$

J. D. Bjorken and I have considered the production of massive neutral leptons, $\mathcal{N}_e$ and $\mathcal{N}_\mu$, which have right-handed couplings to e and μ (see the CHYM and vector models, Section I). Some of the following discussion originally appeared in Ref. 49 (Bjorken and Llewellyn Smith).[50]

A. Direct Production in Deep Inelastic Scattering

In the process μp or $ep \to (\mathcal{N}_\mu$ or $\mathcal{N}_e) +$ hadrons, as shown in Fig. 14, one could look for decays such as $\mathcal{N}_\mu \to \mu\mu\nu$ which would probably have a branching ratio of about 10%. Since this is a weak process, the highest energies are desirable. At E = 100 GeV, $\sigma \approx 10^{-36}$ cm^2 so such dimuons would be hard to detect unless strong cuts are made such as: (a) require missing energy (the neutrino); (b) require that the muon pair have large $p_\perp$ (since this is unlikely in other sources of muons); (c) observe m $(\mu\mu)$; (d) use the Pais-Treiman relations[51] for p_{μ^+} vs. p_{μ^-}.

The process $\nu N \to \mathcal{N} +$ hadrons shown in Fig. 14 occurs because of the mixing described by Bjorken, Lane and Weinberg.[46] Since the mixing is proportional to $m_\mu/m_{\mathcal{N}}$, the production of $\mathcal{N}$ (with subsequent decay to $\mu\mu\nu$) relative to the usual μ production is (if $m_{\mathcal{N}} \approx 2$ GeV):

$$\left(\frac{\mu}{\mathcal{N}_\mu}\right)^2 0.1 \approx 5\times 10^{-4} \tag{6.1}$$

compared to charm decay which gives dimuons at the 10^{-2} level. Perhaps with cuts as discussed above, this signal might be visible.

B. <u>Indirect Production in Neutrino Scattering</u>

The following process (also discussed by J. Rosner at this conference) is possible if $\mathcal{N}$ ($\equiv \mathcal{N}_e$ or $\mathcal{N}_\mu$) is light enough (see Fig. 15)

$$\begin{aligned} \nu N \to \mu^- &+ D + \text{hadrons} \\ &\;\;\llcorner\!\to K\,\mu^+\mathcal{N} \\ &\qquad\quad\;\llcorner\!\to \mu^-\,(\mu^+\nu) \end{aligned} \tag{6.2}$$

For F production (which may be 10% of D production) the same process occurs, but without the K. According to the reported estimates of Rosner, an $\mathcal{N}$ mass of 1.15 GeV maximizes the branching ratio for $F \to \mu\mathcal{N}$ (at about 20%).

Since $\mathcal{N}_e$ and $\mathcal{N}_\mu$ mix, one doesn't know whether the production will be in association with a μ (as shown) or with an e. Similarly, the decay can be to μ or to e. And, of course, the virtual W (at the final decay) gives $\mu\nu$ only 20% of the time. Since counter experiments do not distinguish electrons from hadrons, if the D decay is to $Ke\mathcal{N}$ (with $\mathcal{N} \to \mu X$), a "same sign dimuon" could be observed. Otherwise a trimuon event can result, and at 20% of that level quadramuons (although muon detection efficiency hurts the chances of seeing such events). If $\mathcal{N}$ is light enough, it is possible that $B(D \rightarrow K\mu\mathcal{N}) \approx 5\%$ and that (e.g.) $B(\mathcal{N} \rightarrow \mu X) \approx 30\%$ so that

$$\frac{\mu^-\mu^-}{\mu^+\mu^-} \approx \frac{1}{10} \tag{6.3}$$

which is the rate observed experimentally.[52] The trimuon rate (corrected for efficiency) could be as large.

In trimuon events the two secondary muons could have a relatively low invariant mass (less than 2 GeV). The secondary muon in same-sign dimuons should be relatively slow.

If the $\mathcal{N}_e$ and $\mathcal{N}_\mu$ are too heavy for D or F decay, then perhaps they occur as products of $u\bar{b}$ and $d\bar{b}$ decay. In any case the subject of $\mu^-\mu^-$ and trimuon events is an interesting one to pursue, since there is no obvious explanation of them at this time.

C. Direct Production in e^+e^- Scattering

As energy increases, the weak interactions begin to become competitive with the electromagnetic interactions. Then the process (see Fig. 16)

$$\begin{array}{l} e^+e^- \rightarrow \nu_e \bar{\mathcal{N}}_e \\ \qquad\qquad\quad \hookrightarrow e^+\pi^- \quad \text{or} \quad \mu^+\pi^- \end{array} \tag{6.4}$$

should be considered (it is allowed in models such as CHYM and vector). The cross section to a good approximation is

$$\sigma\left(e^+e^- \rightarrow \bar{\mathcal{N}}_e\nu_e \ \text{ or } \ \mathcal{N}_e\bar{\nu}_e\right) = \frac{G^2 s}{\pi} \tag{6.5}$$

At PEP or PETRA

$$\sigma(s = 1000\ \text{GeV}^2) = 3 \times 10^{-35}\ \text{cm}^2 \tag{6.6}$$

and since luminosity will be 10^{32} cm^{-2} sec^{-1}, they should produce ten $\mathcal{N}_e \nu_e$ events per hour. The branching ratio to the $e\pi$ mode can be taken from charged lepton estimates in Thacker and Sakurai[53] and Tsai.[54] This ratio varies with mass giving about 70% for 0.6 GeV, 30% for 1 GeV, 8% for 2 GeV, and 4% for 3 GeV.

Even at SPEAR it is conceivable that such events could have been produced. If $m_{\mathcal{N}} \leq 1$ GeV, then at SPEAR

$$B\sigma \approx 0.25 \times 10^{-36} \text{ cm}^2 \tag{6.7}$$

Since the total running at high SPEAR energies is 20×10^{36} cm^{-2}, there would be five such events of $e^+e^- \to \nu e \pi$ or $\nu \mu \pi$.

Ordinarily one would think there is no possibility to separate five events from background; 40,000 charged heavy leptons have probably been produced but only 100 seen (and indirectly); and many charmed mesons were produced before their discovery. However, these events are quite unique. One could first examine only two charged prong events in which both prongs go in the same general direction. Next require that the pair's momentum equal that of the beam. Then since this is a three particle event, one can reconstruct and find that the missing particle is massless. At this point little background should remain, and one can eliminate most by separating $\pi^+\pi^-$, $\mu^+\mu^-$ and e^+e^- from the sample. If any events remain, the $e\pi$ or $\mu\pi$ pairs should have the same invariant mass (that of $\mathcal{N}_e$).

Similarly the process $e^+e^- \to Z^o_{\text{virtual}} \to \mathcal{N}_e \overline{\mathcal{N}}_e$ is allowed but the cross section is smaller, and the signatures of the decay modes are not quite so distinct. However, an important point is that this process is allowed in a greater range of theories than is $\nu_e \overline{\mathcal{N}}_e$ production. At PEP and PETRA the resulting decay product $e\pi$, $\mu\pi$, $\mu\mu\nu$, $\mu e\nu$ (etc.), will be produced in two distinct jets (one for each $\mathcal{N}$); within each jet one could make cuts on invariant mass.

D. Indirect Production in e^+e^- Scattering

As discussed in Section VI.B, $\mathcal{N}_e$ and $\mathcal{N}_\mu$ can be the decay products of D, F, Y ($\equiv u\bar{b}$) and U (heavy charged leptons) if the $\mathcal{N}$ masses are light enough. For the heavy leptons U produced in e^+e^- scattering at high energies, the products of each decay will be in jets and therefore easier to find. The resulting events could contain 4μ, 4e, 3μ 1e, etc. Or one could again look for $e\pi$ and $\mu\pi$ pairs. While production of $\mathcal{N}_e$ and $\mathcal{N}_\mu$ in this fashion could be greater than through Z (as above), the signal is confused by the added presence of accompanying hadrons or neutrinos.

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FIGURE CAPTIONS

1. The energy dependence of R_x, the ratio of small x to large x cross sections, for antineutrino scattering. The calculated R_x are shown for the cases with and without asymptotic freedom (AF) corrections. Effects of shrinkage (discussed in text) are not included. For the CHYM model, $m_b = 5$ GeV.

2. The average value of y for $x > 0.15$ and for all x in antineutrino scattering. The efficiencies and cuts of the HPWF experiment (1976) are included. For the CHYM model, $m_b = 5$ GeV. In all cases asymptotic freedom corrections are incorporated.

3. The antineutrino dimuon to single muon ratio divided by that ratio for neutrinos. The solid (dashed) curves include (exclude) asymptotic freedom corrections. For the CHYM model, it is assumed $m_b = 5$ GeV, $B(b \to \mu) = B(c \to \mu)$, and s quarks $= \bar{u}$ quarks.

4. The decay modes of $Y^+ \equiv u\bar{b}$ and $Y^o \equiv d\bar{b}$.

5. The ratio $R_{\bar{\nu}}$ of neutral to charged current antineutrino scattering. The curves include asymptotic freedom corrections. The point at $E = 2$ GeV from Gargamelle[26] has been corrected for experimental cuts, since at $E = 2$ GeV it is model-independent (assuming scaling). The points at $E \approx 40$ GeV (HPWF[24]) and at $E \approx 50$ GeV (CF[25]) have not been corrected. In these models, these points would be lowered by 20-30% by correcting for cuts.

6. The allowed values of $\sin^2 \theta_W$ for various neutral-current experiments. The lines show the regions, for the WS-GIM model, within one standard deviation (two for νe experiments) of experiment. The bottom six experiments are the deep-inelastic neutrino experiments where theory includes asymptotic freedom corrections and experimental cuts.

7. The same as Fig. 6, but for the CHYM model.

8. The same as Fig. 6, but for the vector model.

9. The splitting between the lowest vector $b\bar{b}$ state and the radial excitations and p-wave states, as a function of m_b. The region where $Y\bar{Y}$ production can occur is shaded ($Y \equiv u\bar{b}$ and $d\bar{b}$). This figure was taken from Ref. 38.

10. The allowed gamma transitions for $\beta(\equiv b\bar{b})$ states if $m_b = 5$ GeV. This figure was taken from Ref. 38.

11. The allowed hadronic transitions for $\beta(\equiv b\bar{b})$ states if $m_b = 5$ GeV. This figure was taken from Ref. 38.

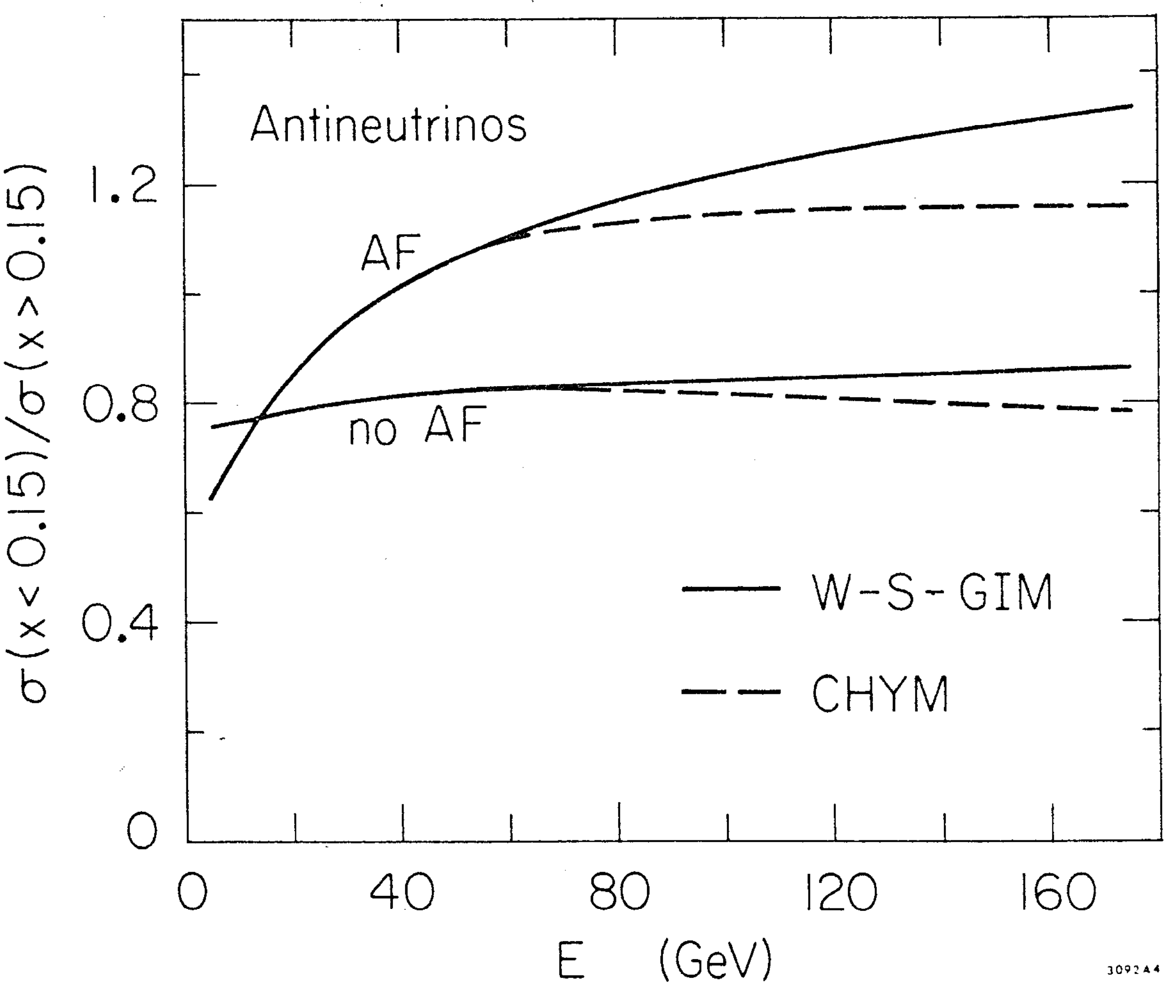
12. One (a) and two (b) loop diagrams in which virtual Higgs exchange leads to the decay $\mu \to e\gamma$. This figure was taken from Ref. 44.

13. One of the diagrams in which $\mathcal{N}_e$ and $\mathcal{N}_\mu$ exchange leads to the decay $\mu \to e\gamma$. This approach was suggested by Cheng and Li.[45]

14. Direct production of $\mathcal{N}_\mu$ in either muon or neutrino deep-inelastic scattering off a nucleon.

15. Multiple muon production in neutrino-nucleon scattering through the production of a charmed meson D which decays into a neutral heavy lepton $\mathcal{N}$. For F meson production and decay, simply remove the K meson.

16. The direct production in e^+e^- scattering of ν_e and $\bar{\mathcal{N}}_e$ by W exchange. The $\bar{\mathcal{N}}_e$ then decays into $e^+\pi^-$.



Antineutrinos
AF
no AF
W-S-GIM
CHYM
$\sigma(x<0.15)/\sigma(x>0.15)$
1.2
0.8
0.4
0
0
40
80
120
160
E (GeV)
3092A4

Fig. 1

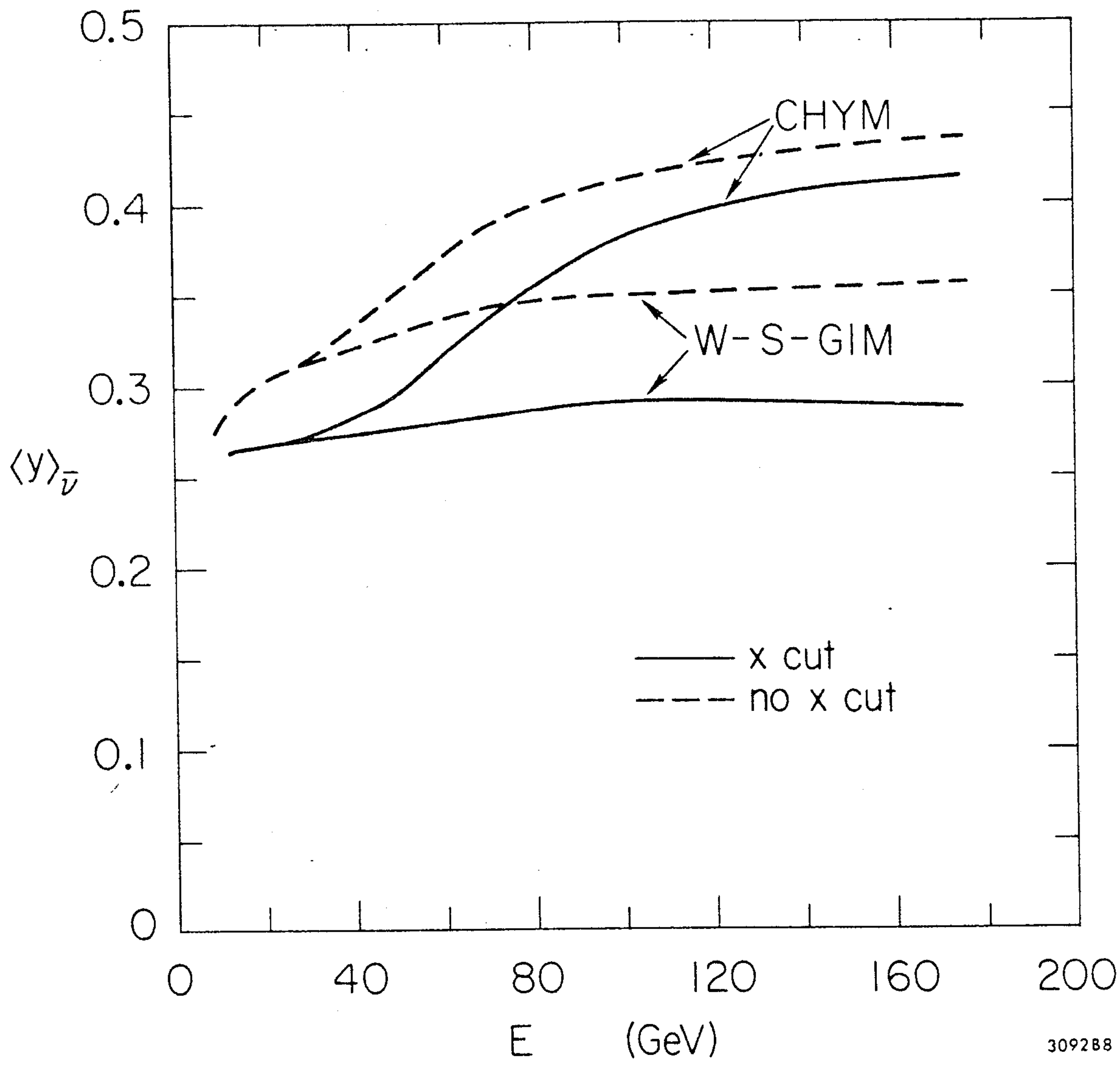
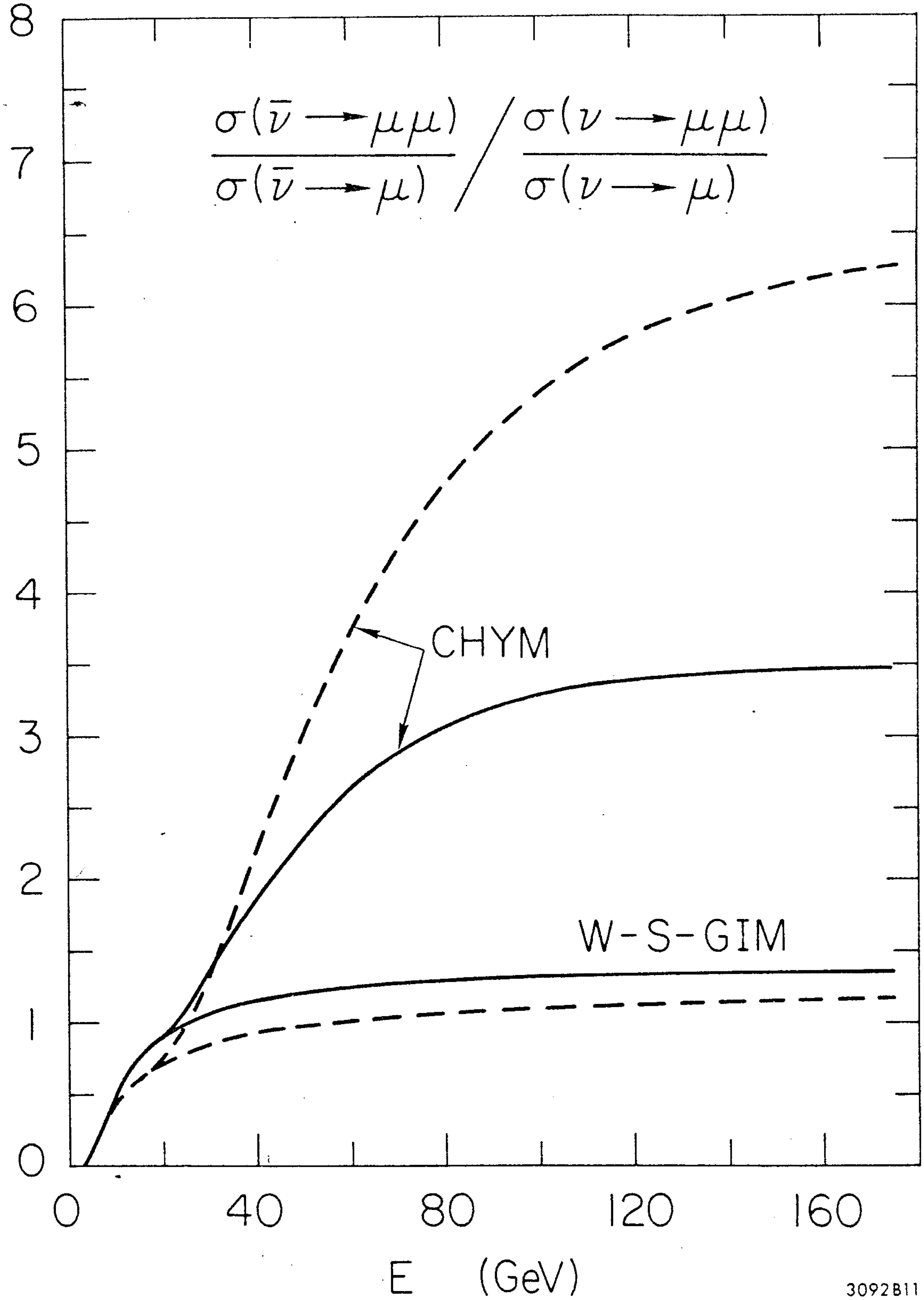

0.5
0.4
0.3
0.2
0.1
0
⟨y⟩$_{\bar{\nu}}$
CHYM
W-S-GIM
x cut
no x cut
0
40
80
120
160
200
E (GeV)
309288

Fig. 2



$\dfrac{\sigma(\bar{\nu} \to \mu\mu)}{\sigma(\bar{\nu} \to \mu)} \Big/ \dfrac{\sigma(\nu \to \mu\mu)}{\sigma(\nu \to \mu)}$
CHYM
W-S-GIM
0
1
2
3
4
5
6
7
8
0
40
80
120
160
E (GeV)
3092B11

Fig. 3

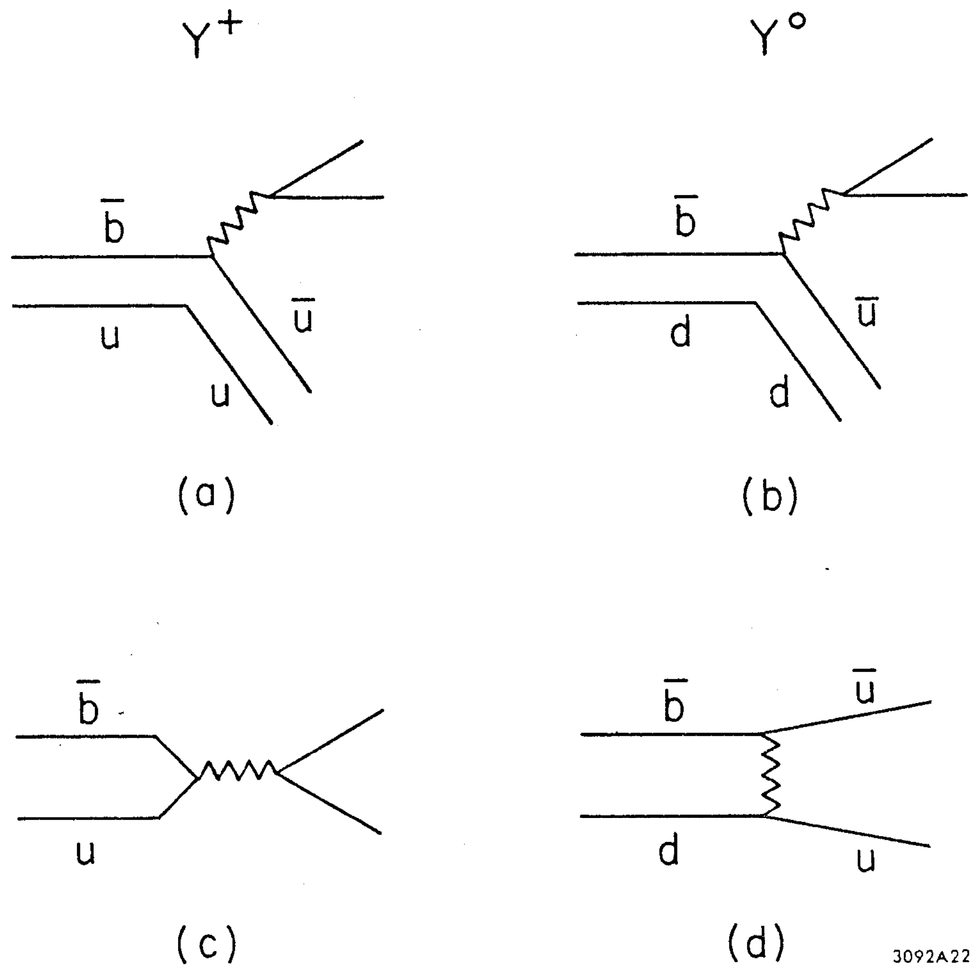
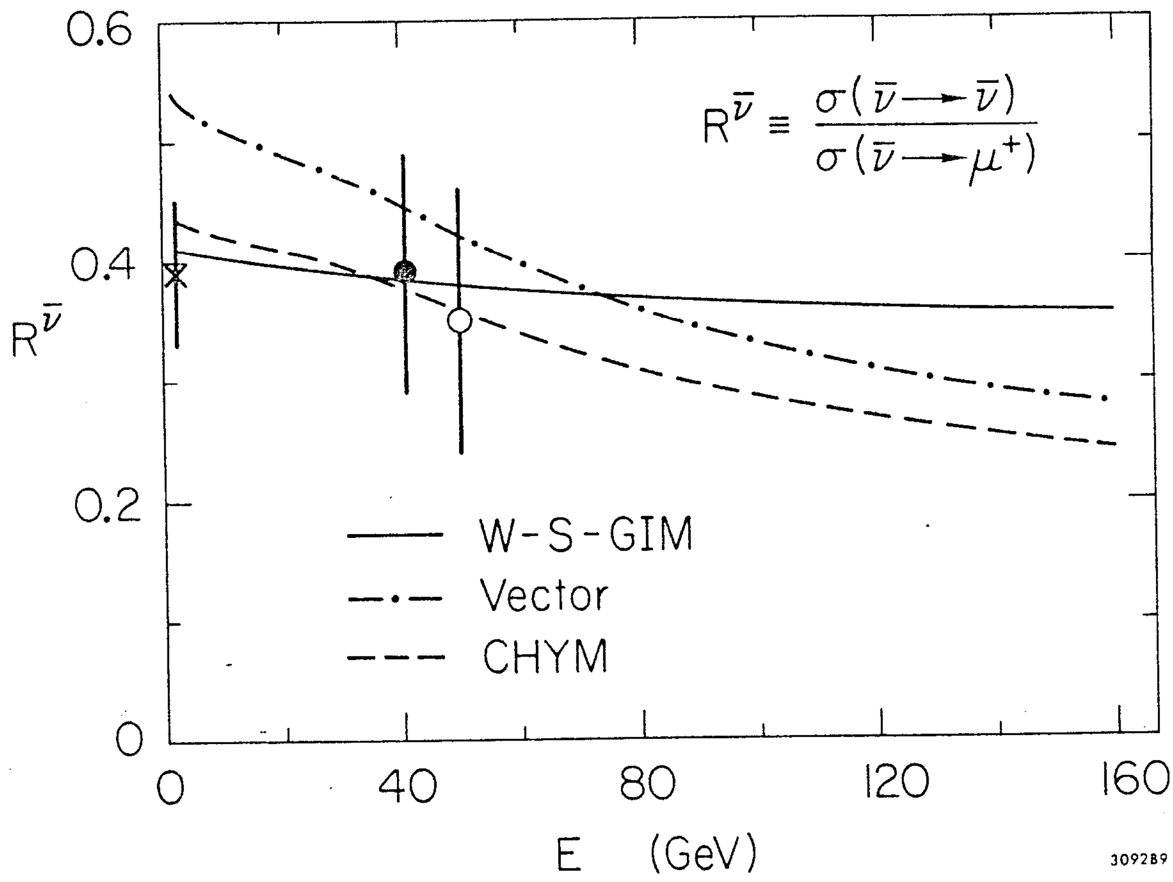


Fig. 4



0.6
0.4
0.2
0
$R^{\bar{\nu}} \equiv \frac{\sigma(\bar{\nu} \to \bar{\nu})}{\sigma(\bar{\nu} \to \mu^+)}$
$R^{\bar{\nu}}$
W-S-GIM
Vector
CHYM
0
40
80
120
160
E (GeV)
309289

Fig. 5

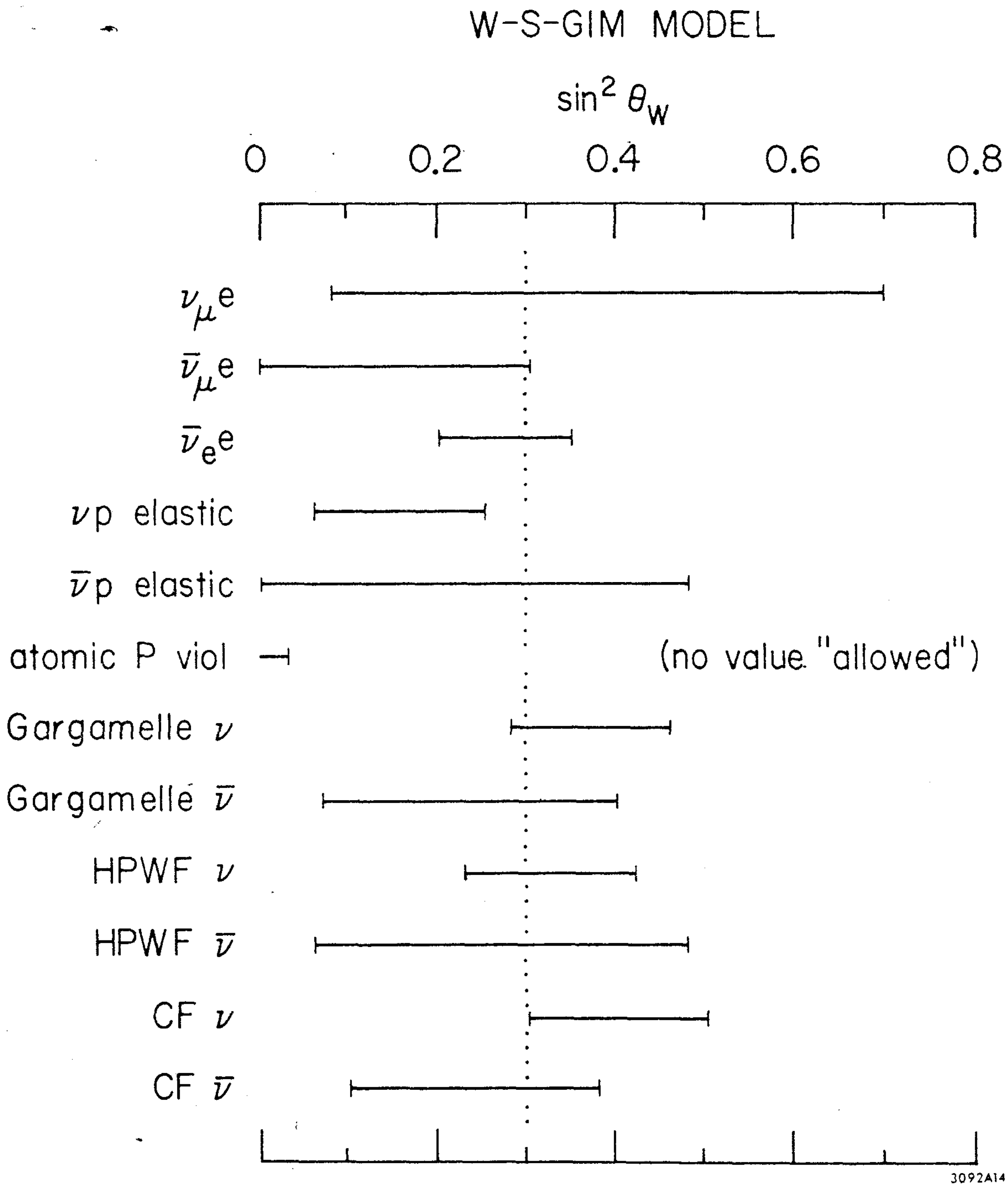

W-S-GIM MODEL
$\sin^2\theta_W$
0
0.2
0.4
0.6
0.8
$\nu_\mu e$
$\bar{\nu}_\mu e$
$\bar{\nu}_e e$
νp elastic
$\bar{\nu} p$ elastic
atomic P viol
(no value "allowed")
Gargamelle ν
Gargamelle $\bar{\nu}$
HPWF ν
HPWF $\bar{\nu}$
CF ν
CF $\bar{\nu}$
3092A14

Fig. 6

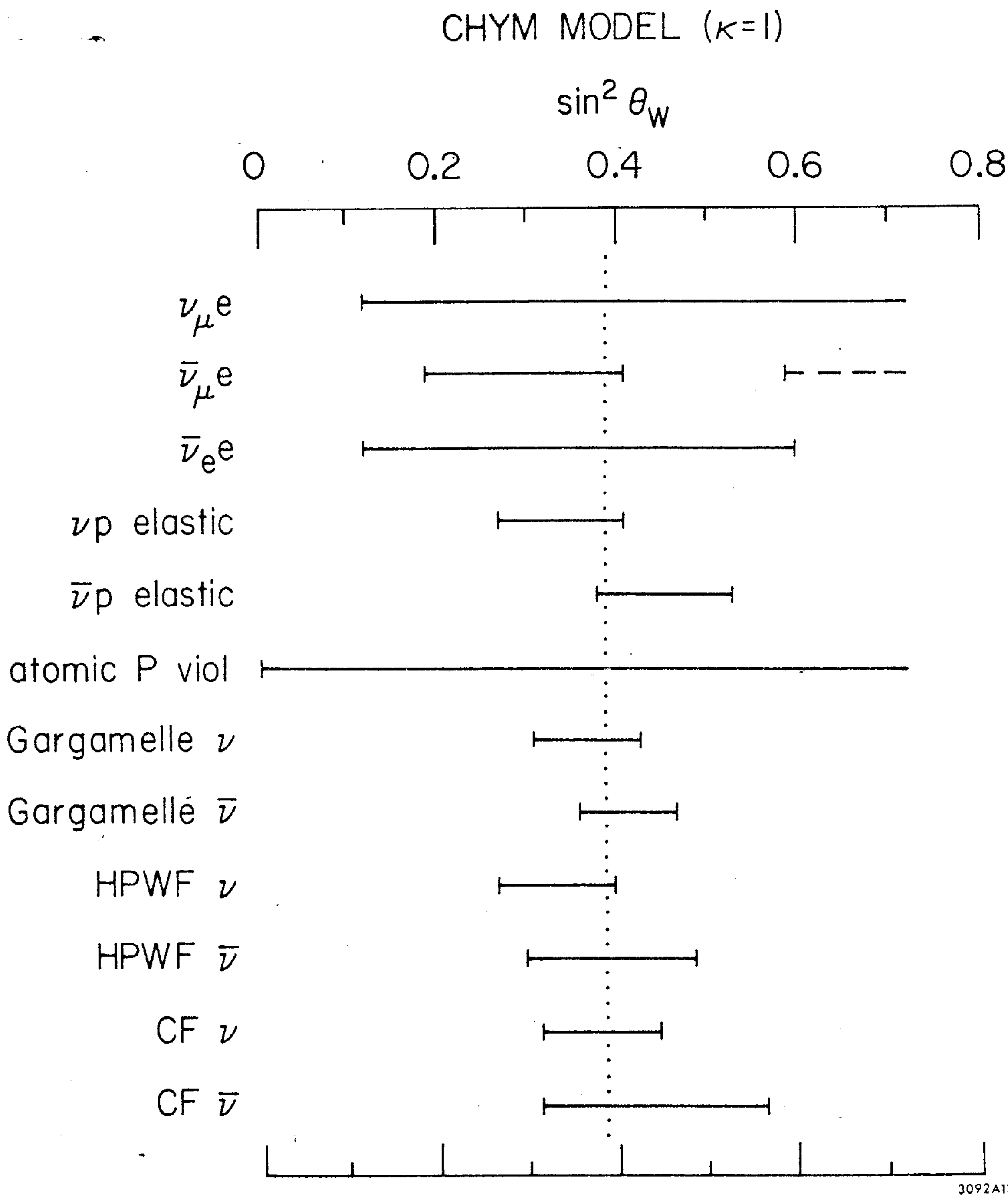
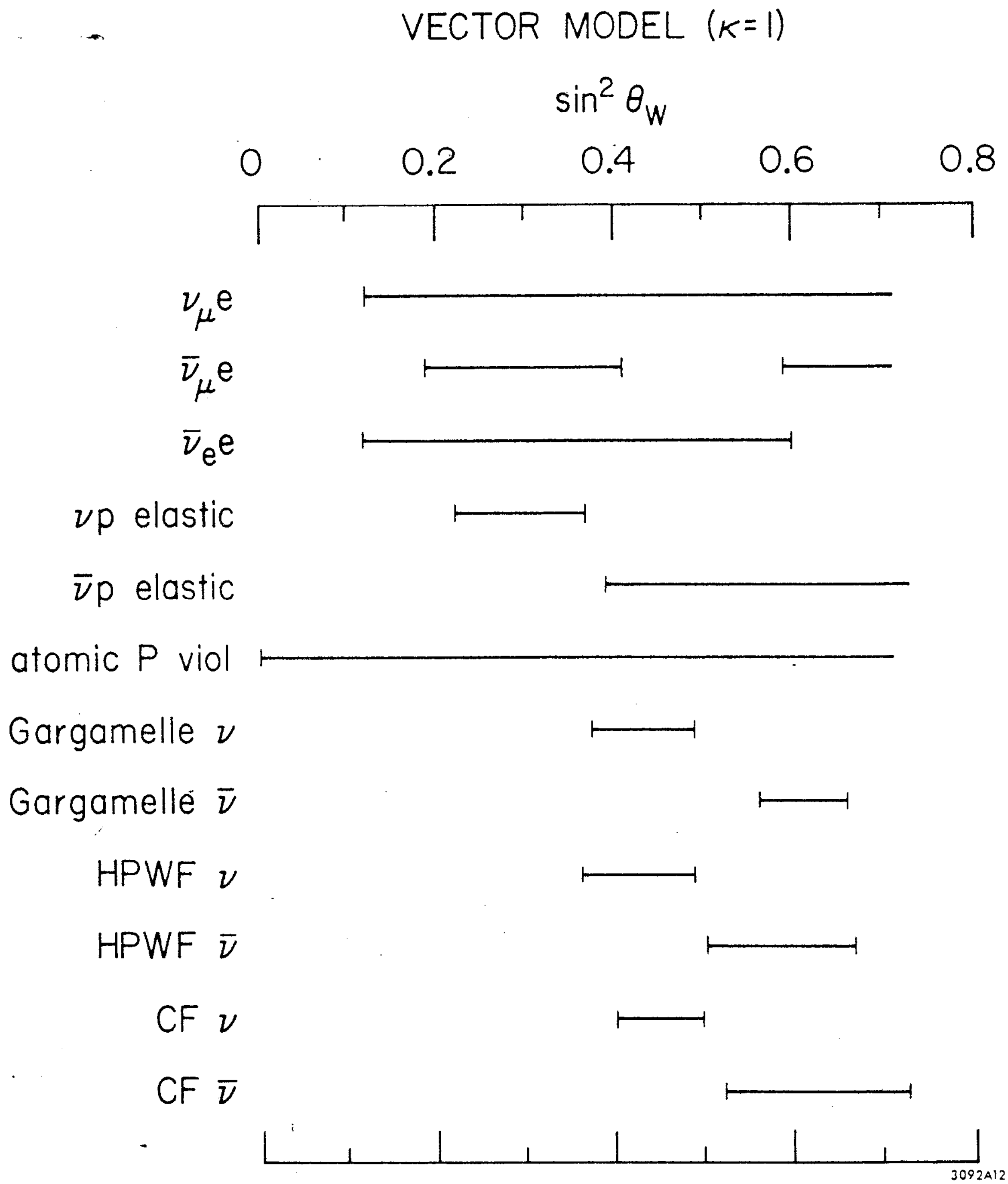
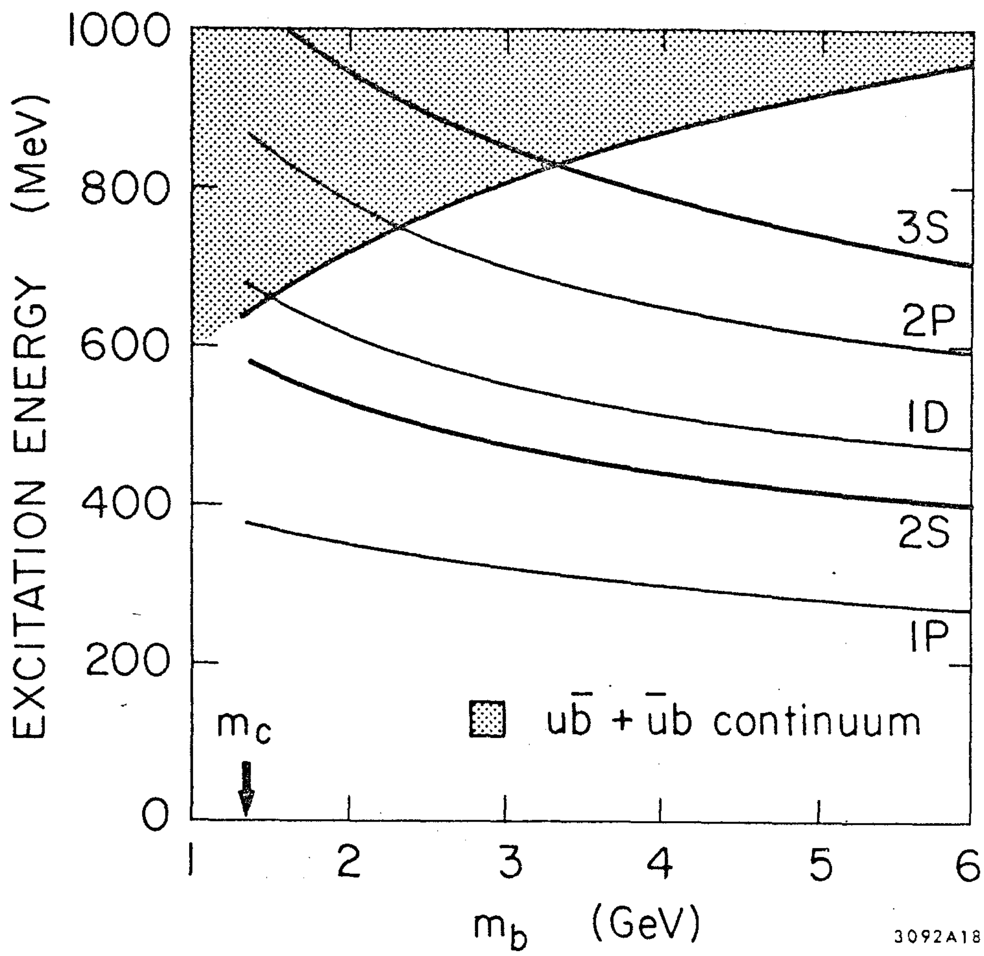

CHYM MODEL (κ=1)
$\sin^2\theta_W$
0
0.2
0.4
0.6
0.8
$\nu_\mu e$
$\bar{\nu}_\mu e$
$\bar{\nu}_e e$
νp elastic
$\bar{\nu} p$ elastic
atomic P viol
Gargamelle ν
Gargamelle $\bar{\nu}$
HPWF ν
HPWF $\bar{\nu}$
CF ν
CF $\bar{\nu}$
3092A13

Fig. 7



VECTOR MODEL (κ=1)
$\sin^2\theta_W$
0
0.2
0.4
0.6
0.8
$\nu_\mu e$
$\bar{\nu}_\mu e$
$\bar{\nu}_e e$
νp elastic
$\bar{\nu}$p elastic
atomic P viol
Gargamelle ν
Gargamelle $\bar{\nu}$
HPWF ν
HPWF $\bar{\nu}$
CF ν
CF $\bar{\nu}$
3092A12

Fig. 8



1000
800
600
400
200
0
EXCITATION ENERGY (MeV)
3S
2P
1D
2S
1P
m_c
$u\bar{b} + \bar{u}b$ continuum
1
2
3
4
5
6
m_b (GeV)
3092A18

Fig. 9

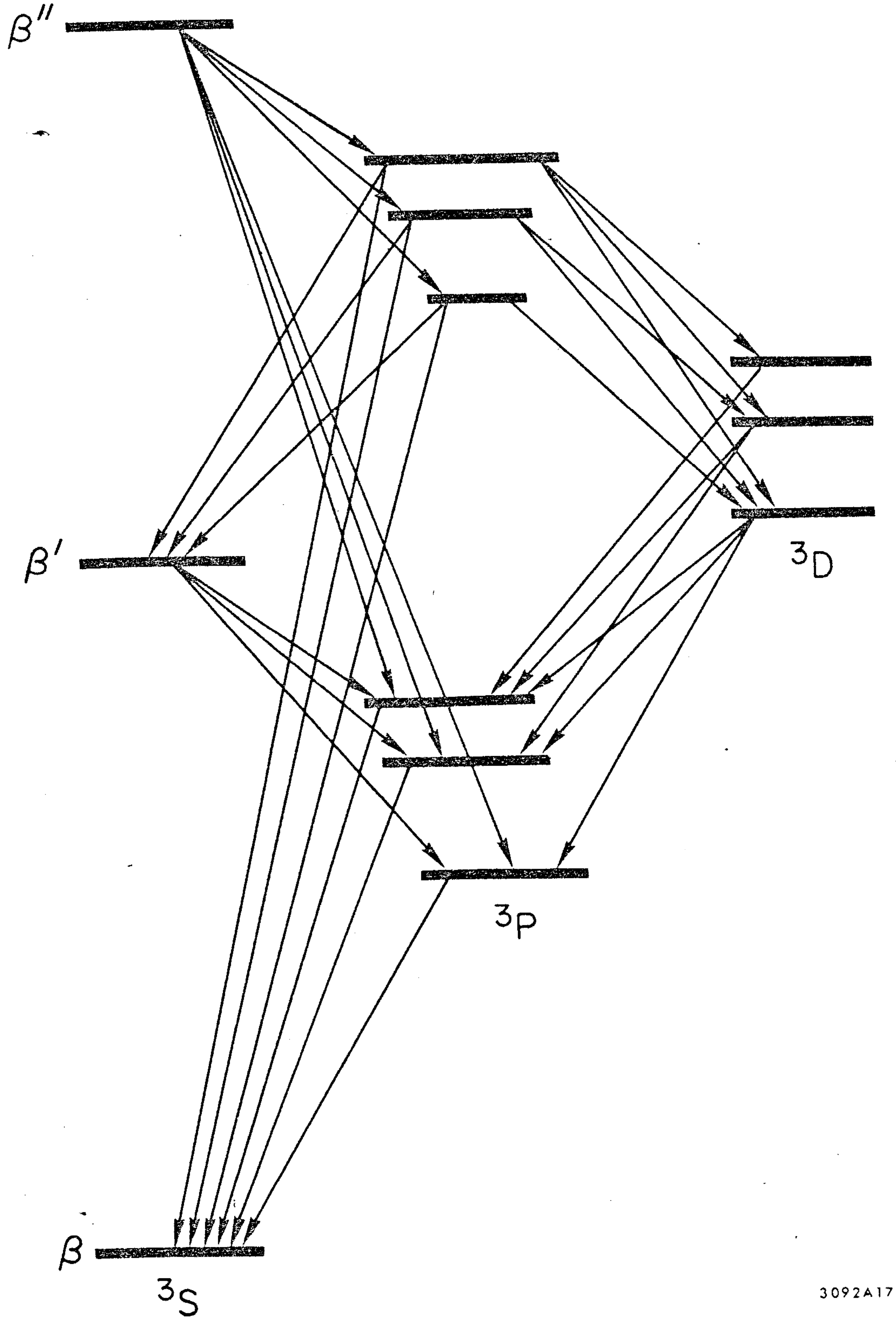
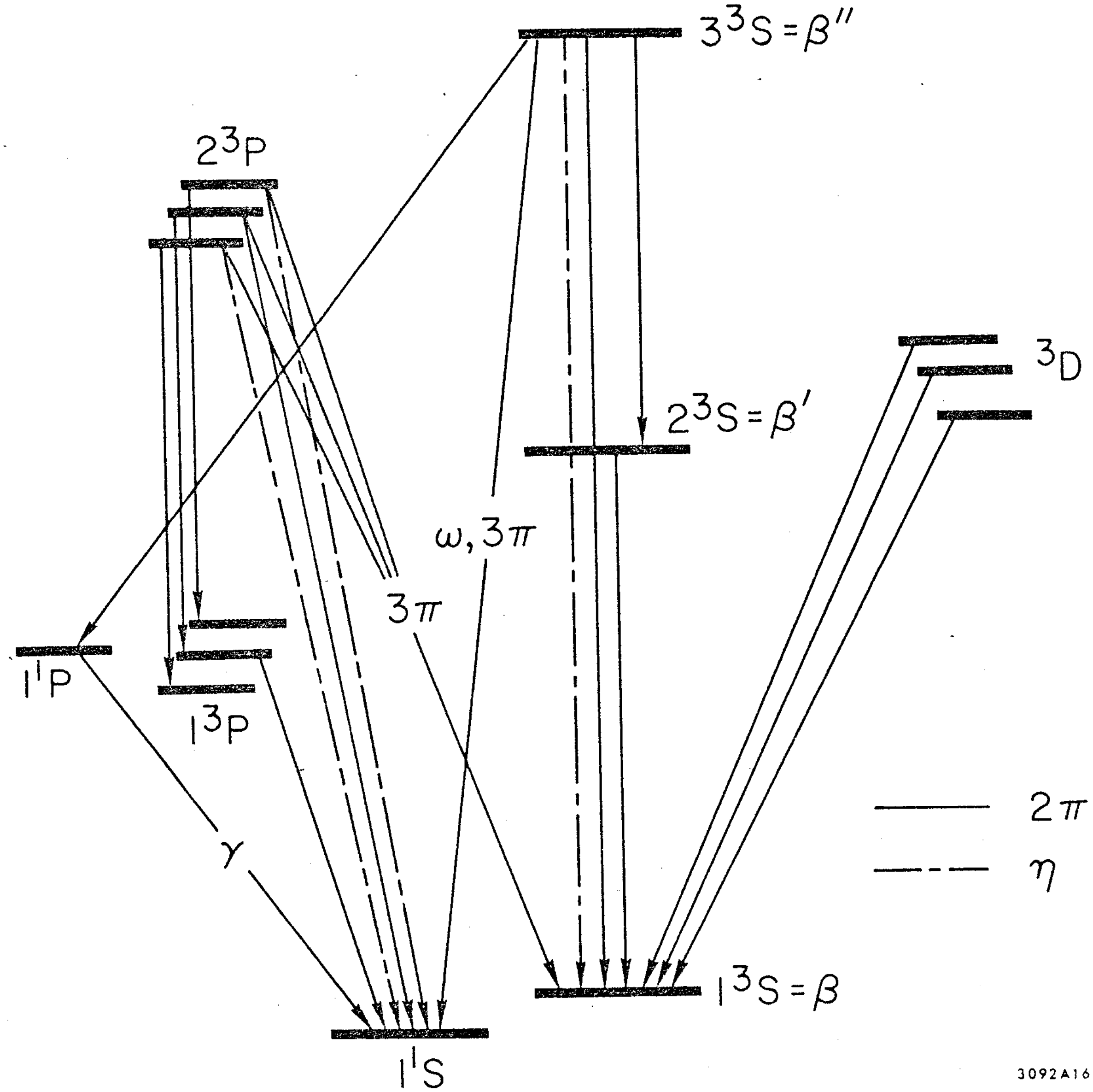
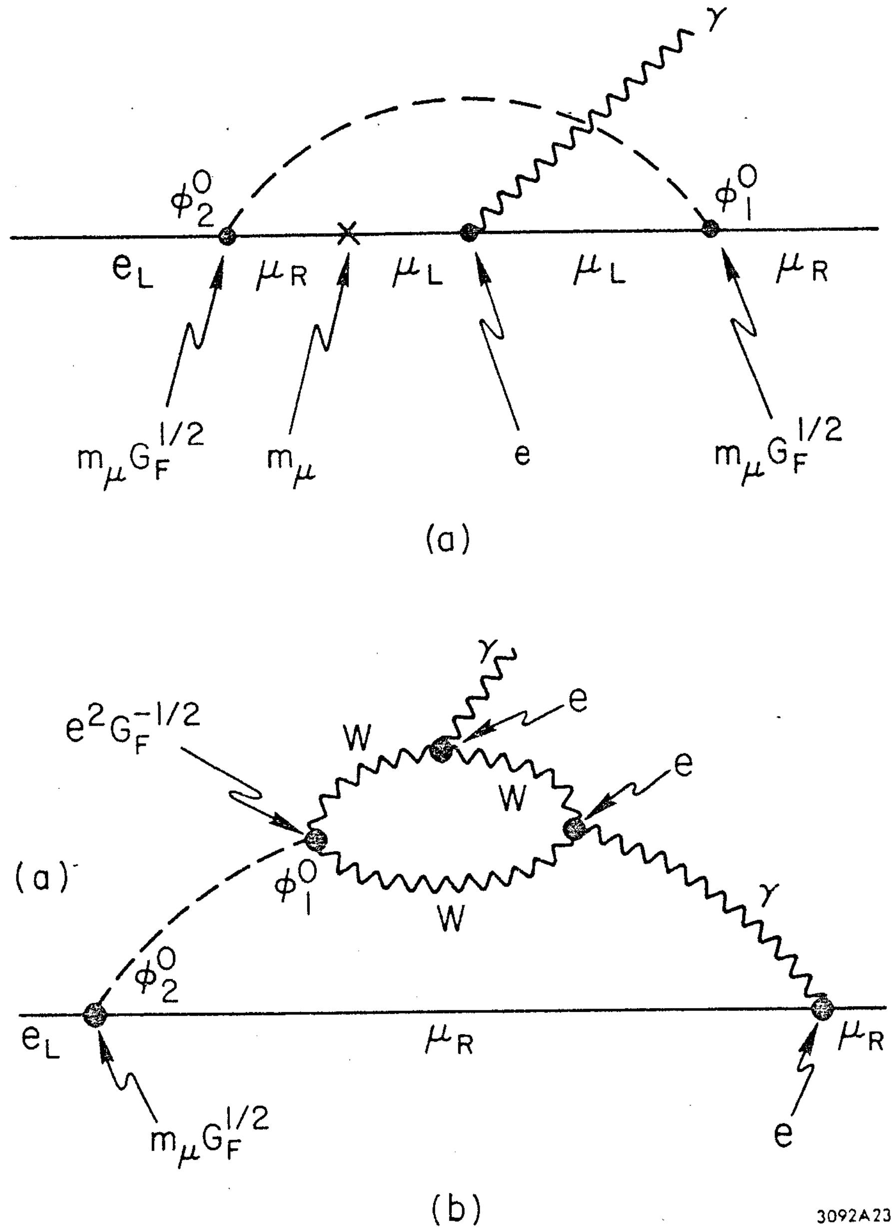


Fig. 10



$3^3S = \beta''$
2^3P
^3D
$2^3S = \beta'$
$\omega, 3\pi$
3π
1^1P
1^3P
γ
2π
η
$1^3S = \beta$
1^1S
3092A16

Fig. 11



γ
ϕ_2^0
ϕ_1^0
e_L
μ_R
μ_L
μ_L
μ_R
$m_\mu G_F^{1/2}$
m_μ
e
$m_\mu G_F^{1/2}$
(a)
γ
e
$e^2 G_F^{-1/2}$
W
W
e
(a)
ϕ_1^0
W
γ
ϕ_2^0
e_L
μ_R
μ_R
$m_\mu G_F^{1/2}$
e
(b)
3092A23

Fig. 12

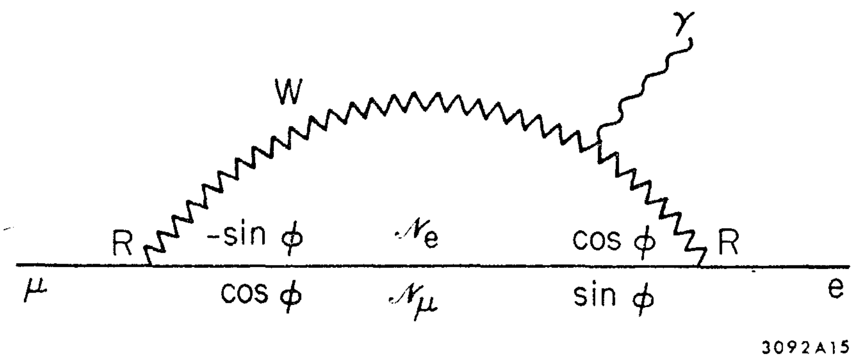


Fig. 13

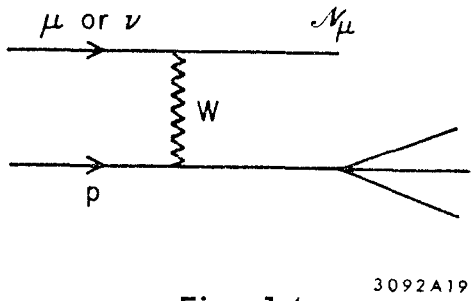


Fig. 14

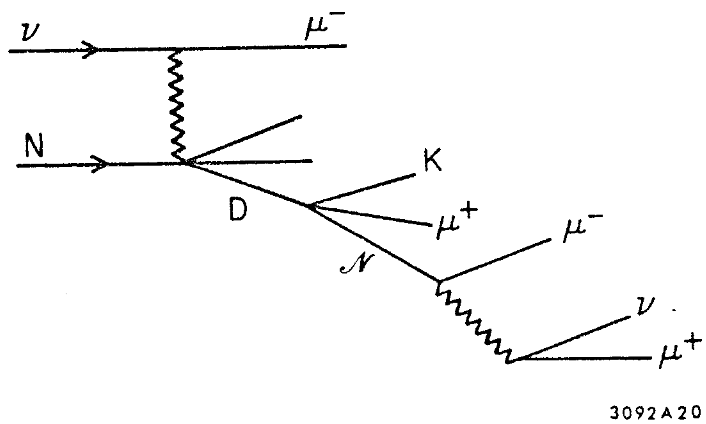


Fig. 15

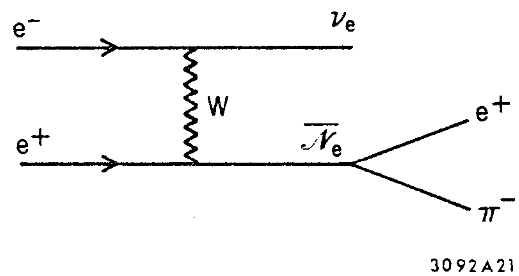


Fig. 16