STUDY OF K $\pi$ SCATTERING USING THE REACTIONS<br><br>P．Estabrooks<br>McGill University，Montreal，Canada<br>R．K．Carnegie Carleton University，Ottawa，Canada<br>A。D．Martin<br>University of Durham，Durham City，England<br>W。M．Dunwoodie，T．A．Lasinski，and D．W．G．S．Leith Stanford Linear Accelerator Center<br>Stanford University，Stanford，California 94305


#### Abstract

High statistics data for the reactions $K^{ \pm} p \rightarrow K^{ \pm} \pi^{+} n$ at 13 GeV have been ob－ tained in a spectrometer experiment performed at SLAC。For each reaction，a t －dependent parametrization of the production amplitudes provides information on both the $K \pi$ mass dependence of the production mechanisms and on $K \pi$ scat－ tering．Knowledge of the t－dependence then allows a calculation of the $K \pi$ par－ tial wave amplitudes for $K \pi$ masses from 0.7 to 1.9 GeV 。 Besides the leading $J^{P}=1^{-}, 2^{+}$，and $3^{-}$resonances at $M_{K \pi}=.896,1.433$ ，and 1.78 GeV ，there is evidence in two of the four possible partial wave solutions for a broad P－wave resonant－like structure in the region of 1700 MeV 。 The $\mathrm{K}^{+} \pi^{+}$reaction is dom－ inated by S－wave scattering with a total cross section of 5.2 mb 。The $I=\frac{1}{2} S-$ wave magnitude rises slowly and smoothly to a maximum near 1400 MeV ，but then decreases rapidly between 1400 and 1600 MeV ．This structure may pos－ sibly be associated with an $S$－wave resonance near 1500 MeV 。 Both the $\mathrm{I}=\frac{1}{2} \mathrm{~S}$－ wave below 1400 MeV and the $\mathrm{I}=3 / 2 \mathrm{~S}$－wave are well described by an effective range parametrization．


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## I．INTRODUCTION

We have performed a spectrometer experiment to study the reactions

$$
\begin{align*}
& K^{-} p \rightarrow K^{-} \pi^{+} n  \tag{1}\\
& K^{+} p \rightarrow K^{+} \pi^{+} n \tag{2}
\end{align*}
$$

at 13 GeV with high statistics and good $\mathrm{K}^{+} / \mathrm{K}^{-}$relative normalization。By iso－ lating the $\pi$ exchange contributions to these reactions and then performing a $K \pi$ partial wave analysis，we have obtained $\mathrm{K} \pi$ scattering amplitudes from 0.7 to 1.9 GeV in $\mathrm{K} \pi$ mass．The $\mathrm{I}=3 / 2 \mathrm{~K} \pi$ scattering obtained directly from the analysis of reaction（2）is found to be dominated by the S－wave．The analysis of reaction（1）yields a unique solution for $\mathrm{M}_{\mathrm{K} \pi}<1.46 \mathrm{GeV}$ which includes the $1^{-} \mathrm{K}^{*}(890)$ and $2^{+} \mathrm{K}^{*}(1420)$ resonances and an $\mathrm{I}=1 / 2 \mathrm{~S}$－wave that rises smoothly to a maximum near 1400 MeV 。 However，for higher $\mathrm{K} \pi$ mass we find several possible solutions which we classify in terms of the zero structure of the scattering amplitude。 Each solution contains a broad spin parity $3^{-}$reso－ nance；${ }^{1}$ however，the solutions differ in the structure of the nonleading partial waves．

This experiment also includes data on the $\Delta^{++}$recoil reactions

$$
\begin{aligned}
& \mathrm{K}^{+} \mathrm{p} \rightarrow \mathrm{~K}^{+} \pi^{-} \Delta^{++} \\
& \mathrm{K}^{-} \mathrm{p} \rightarrow \mathrm{~K}^{-} \pi^{-} \Delta^{++}
\end{aligned}
$$

which were obtained simultaneously with the neutron recoil data。A $K \pi$ partial wave analysis，using the data from all four reactions，is in progress．

## II．THE EXPERIMENT

The experiment was performed at SLAC using 13 GeV RF separated $K^{ \pm}$ beams incident on a 1 m hydrogen target．A forward wire spark chamber spec－ trometer system ${ }^{2}$ was used to detect the outgoing $K$ and $\pi$ and events
corresponding to reactions（1）and（2）were selceted by requiring that the miss－ ing mas opposite the $\mathrm{K} \pi$ system lie in the range $0.75<\mathrm{MM}<1.05 \mathrm{GeV}$ 。A multicell Cerenkov counter provided $K / \pi$ identification for reaction（2）．For reaction（1） $\mathrm{K} / \pi$ identification was not necessary．Instead events that were am－ biguous with a forward $\mathrm{K}^{\circ}, \Lambda$ ，or the reactions $\mathrm{K}^{-} \mathrm{p} \rightarrow \phi \Lambda, \rho \Lambda$ were explicitly rejected．Events were also rejected if the $\pi n$ invariant mass was less than 2.0 GeV．Finally，for both reactions a direct experimental subtraction was made to correct for events associated with the small $\mathrm{Kp} \rightarrow \mathrm{K} \pi \Delta^{\circ}$ background appear－ ing in the neutron missing mass cut．${ }^{3}$ The experimental data samples contain 51,000 and 14,400 events，respectively，for reactions（1）and（2）．In addition， very large data samples from $K^{ \pm} \rightarrow \pi^{ \pm} \pi^{+} \pi^{-}$beam decays were obtained simul－ taneously with the $\mathrm{K} \pi$ data and provide a direct measurement of the $\mathrm{K}^{+} / \mathrm{K}^{-}$rel－ ative normalization which is known to $\pm 2 \%$ 。

A maximum likelihood fitting procedure is used to correct the observed $K \pi$ data for the effects of the spectrometer acceptance，event selection criteria， and other factors．This yields acceptance corrected reaction cross sections and the spherical harmonic moments， $\mathrm{Y}_{\mathrm{LM}}$ ，of the $\mathrm{K} \pi$ angular distribution as a function of $K \pi$ mass and four momentum transfer，$t^{\prime}\left(=t-t_{\text {min }}\right)$ 。The resulting $K \pi$ mass spectra and the $t$ channel unnormalized $M=0$ moments are presented in Figs． 1 and 2 as a function of $\mathrm{K} \pi$ mass for the small momentum transfer re－ gion，$\left|t^{\prime}\right|<0.15 \mathrm{GeV}^{2}$ ，for reactions（1）and（2）respectively．The errors shown are the statistical errors only．The $\mathrm{K}^{+} \pi^{+} \mathrm{n}$ cross section is substantial； it rises smoothly from threshold to a broad maximum near 1.6 GeV ．The fits to the $\mathrm{K}^{+} \pi^{+} n$ data were performed with $L, M \leq 2$ 。Separate fits indicated that higher L moments were not required to describe these data。 Indeed the $\mathrm{L}=1$ and $\mathrm{L}=2$ moments in Fig． 2 are small，indicating that the $\mathrm{K}^{+} \pi^{+}$system is
predominantly in an $S$ wave. For the $\mathrm{K}^{-} \pi^{+} \mathrm{n}$ data, the maximum $L$ value used in the fits increased from $\mathrm{L}=2$ to $\mathrm{L}=6$ with increasing $\mathrm{K} \pi$ mass, as indicated in Fig. 1 with $\mathrm{L} \leq 4$ moments required to describe the $\mathrm{K}^{*}(1420)$ region and the $\mathrm{L}=6$ moments necessary above 1.6 GeV . The prominent features of the $\mathrm{K}^{-} \pi^{+}$ distributions are due to the $1^{-} K^{*}(890)$ and $2^{+} K^{*}(1420)$ mesons. In addition, as reported previously, ${ }^{1}$ these data have been combined with $\mathrm{K}^{+} \mathrm{p} \rightarrow \mathrm{K}^{+} \pi^{-} \Delta^{++}$data from this experiment to clearly demonstrate the presence of a broad spinparity $3^{-} \mathrm{K} *$ resonance at $\sim 1.78 \mathrm{GeV}$ 。

## III. $\mathrm{K} \pi$ PRODUCTION

The extraction of $\mathrm{K} \pi$ scattering amplitudes from these data requires isolating the $\pi$ exchange contribution to the production amplitude。Data ${ }^{4}$ at 4 GeV for the line reversed reactions

$$
\begin{align*}
& K^{-} p \rightarrow \bar{K}^{*}(890) n  \tag{3}\\
& K^{+} n \rightarrow K^{*}(890) p
\end{align*}
$$

have provided valuable information about the $\mathrm{K}^{*}(890)$ production mechanisms. In Ref. 5 it was shown that these two reactions could be simply described in terms of strongly exchange degenerate $\pi-\mathrm{B}$ and $\mathrm{A}_{2}-\rho$ Regge exchanges and 'cuts' (nonevasive contributions, absorptive corrections, ...) which have simple tchannel structure. We have shown ${ }^{6}$ that, for $K^{*}(890)$ and $K^{*}(1420)$ production in reaction (1) at 13 GeV , a good description of the momentum transfer dependence of the data is also provided by this simple exchange model. Moreover, the $\mathrm{M}_{\mathrm{K} \pi}$ dependence of the ratios of non-pi/pi exchanges was seen to be almost identical to that observed in $\pi \mathrm{N} \rightarrow \rho \mathrm{N}$ compared with $\pi \mathrm{N} \rightarrow \mathrm{fN}^{7}$

For each $\mathrm{K} \pi$ mass interval we parametrize the t -dependence of the amplitudes $L_{\lambda \pm}$ for production of a $K \pi$ state with angular momentum $L, t$-channel helicity $\lambda$, by natural ( + ) or unnatural ( - ) parity exchange by

$$
\begin{align*}
& \mathrm{L}_{0}=\mathrm{g}_{\mathrm{L}} \frac{\sqrt{-\mathrm{t}}}{\mu^{2}-\mathrm{t}} \\
& \mathrm{~L}_{1-}=\frac{1}{\sqrt{2}} \mathrm{~g}_{\mathrm{L}} \sqrt{\mathrm{~L}(\mathrm{~L}+1)} \gamma_{\mathrm{C}} \\
& \mathrm{~L}_{1+}=\frac{1}{\sqrt{2}} \mathrm{~g}_{\mathrm{L}} \sqrt{\mathrm{~L}(\mathrm{~L}+1)}\left[\gamma_{\mathrm{C}}+2\left\{\begin{array}{|c|}
(-\mathrm{i} \mid \\
1
\end{array} \gamma_{\mathrm{A}}\left|\mathrm{t}^{\prime}\right|\right]\right.  \tag{4}\\
& \mathrm{L}_{\lambda_{ \pm}}=0, \quad \lambda \geq 2 .
\end{align*}
$$

For partial wave analysis，we are only concerned with the small $t, e_{\circ} g_{\circ},|t| \mid$ $<0.2 \mathrm{GeV}^{2}$ region．The simplified parametrization of Eq．（4）is a good small ｜t｜approximation to the more general description used in Ref。6。The relation of $g_{L}$ to the $K \pi$ scattering amplitudes，$a_{L}$ ，is given by

$$
g_{L}=\mathscr{N}_{\mp} \frac{M_{K \pi}}{\sqrt{q}} a_{L} e^{b\left(t-\mu^{2}\right)}
$$

where the $\mp$ subscript on $\mathscr{N}$ refers to the charge of the beam particle and $\dagger$

$$
\begin{align*}
& a_{L}=\sqrt{2 \mathrm{~L}+1} \epsilon_{L} \sin \delta_{L} e^{i \delta} L \text { for elastic scattering } \\
& a_{L}=\left|a_{L}\right| e^{i \delta} L  \tag{5}\\
& \text { for inelastic scattering. }
\end{align*}
$$

The normalization constant， $\mathscr{N}_{-}$，is determined by requiring the $\mathrm{K}^{-} \pi^{+} \mathrm{P}$ wave in the 900 MeV region to be an elastic Breit－Wigner resonance。 The $\mathrm{K}^{+}$normali－ zation constant， $\mathscr{N}_{+}$，is then fixed by the relative $\mathrm{K}^{-}$and $\mathrm{K}^{+}$experimental nor－ malizations and the value of $\mathscr{N}_{-}$．We have also calculated $\mathscr{N}_{+}$from the absolute experimental normalization and the Chew－Low equation。 ${ }^{8}$ This gives a value

[^1]some $10 \%$ larger than that obtained from $\mathrm{K}^{-} / \mathrm{K}^{+}$comparison and the require－ ment that the $\mathrm{K}^{*}(890)$ be elastic．

The parametrization of Eq。（4）provides a good description of the $t$－ dependence of the moments，$N\left\langle\mathrm{Y}_{\mathrm{M}}^{\mathrm{L}}\right\rangle$ ，of the $\mathrm{K}^{-} \pi^{+}$angular distribution in all $M_{K \pi}$ bins．In Fig． 3 both the data and the results of the fits are shown for－$t^{\prime}$ $<0.2 \mathrm{GeV}^{2}$ for the $0.94<\mathrm{M}_{\mathrm{K} \pi}<1.0 \mathrm{GeV}$ bin．We have checked that performing the fits for $-t^{\prime}<0.3 \mathrm{GeV}^{2}$ does not significantly affect the results for the pa－ rameters．

In Fig．4，the values of the parameterst $b, \gamma_{C}, \gamma_{A}$ ，describing the $t-$ dependence are plotted as functions of $\mathrm{K}^{-} \pi^{+}$mass．For the $\mathrm{K}^{+} \pi^{+}$reaction，only the S －wave is significantly nonzero and so b is the only reliably determined $\mathrm{pa}-$ rameter．Its values are represented by the open circles in Fig． 4 and are seen to be in agreement with the $\mathrm{K}^{-} \pi^{+}$results（solid points）．We note that the $\rho-\mathrm{A}_{2}$ and cut contributions become increasingly important with decreasing $K \pi$ mass．

In the course of describing the t －dependence of the $\mathrm{K} \pi$ production using Eqs． （4），we also obtain results for the $K \pi$ scattering amplitudes．The $I=\frac{3}{2} S$－wave phase shifts，$\delta_{S}^{3}$ ，obtained by imposing elastic unitarity in the fits to the t－ dependence of the $\mathrm{K}^{+} \pi^{+}$moments are shown as open circles in Fig。5．The ex－ otic P and D waves were found to be consistent with zero，$\delta_{\mathrm{P}}^{3}=\delta_{\mathrm{D}}^{3}=0$ ．The $\mathrm{I}=$ $\frac{1}{2}$ phase shifts，$\delta_{\mathrm{S}}^{1}$ and $\delta_{\mathrm{P}}^{1}$ ，in the elastic region， $\mathrm{M}_{\mathrm{K}^{-} \pi^{+}}<1.3 \mathrm{GeV}$ ，are shown by the open circles in Fig。6。

IV． $\mathrm{K} \pi$ PARTIAL WAVES FROM A SINGLE t BIN ANALYSIS

## A．Method of Analysis

Consideration of the moments of the $K \pi$ angular distribution in a broad $t$ bin $\left(0<-t<0.15 \mathrm{GeV}^{2}\right)$ permits a more detailed investigation of the $M_{K \pi}$ structure $\dagger$ These were obtained from fits in which $\epsilon_{\mathrm{S}}$ of Eq．（5）was left as a free param－ eter while $\epsilon_{P}$ was fixed equal to 1.0 。
of the $\mathrm{K} \pi$ partial wave amplitudes．Such $\mathrm{K} \pi$ moments are shown in Figs． 1 and 2 as functions of $\mathrm{K} \pi$ mass．Calculation of the $\mathrm{K} \pi$ partial waves from these t － averaged moments is straightforward，given the $t$－dependence of Eqs。（4）with $b, \gamma_{C}, \gamma_{A}$ known．Here we assume that these parameters are given by low order polynomial fits to the values shown in Fig。4，in particular，

$$
\begin{align*}
& \mathrm{b}=2.36 \\
& \gamma_{\mathrm{C}}=-1.27+2.41 \Delta_{\mathrm{M}}-1.44 \Delta_{\mathrm{M}}^{2}  \tag{6}\\
& \gamma_{\mathrm{A}}=7.3-8.66 \Delta_{\mathrm{M}}
\end{align*}
$$

where $\Delta_{M} \equiv M_{K \pi}-0.895$ 。 These values correspond to the curves shown in Fig．4．To verify that the $\mathrm{K} \pi$ partial waves are unaffected by this smoothing of the mass dependence of the parameters describing the $t$ structure of the mo－ ments，we repeated the $\mathrm{K}^{-} \pi^{+}$analysis of the previous section with $\mathrm{b}, \gamma_{\mathrm{C}}, \gamma_{\mathrm{A}}$ fixed as in Eq．（6）and found no significant difference between the resulting phase shifts and those of the previous section．

The $K \pi$ moments can now be expressed in terms of the partial wave ampli－ tudes，$a_{L}$ ，and $t$ integrals，$\left\langle L_{\lambda}^{\prime} L_{\mu}^{*}\right\rangle$ ，

$$
\begin{equation*}
\left\langle\mathrm{L}_{\lambda}^{\prime} \mathrm{L}_{\mu}^{*}\right\rangle \equiv \frac{\int_{-0}^{0} d 5 \mathrm{dt}^{\prime} \mathrm{L}_{\lambda}^{\prime}\left(\mathrm{t}^{\prime}\right) \mathrm{L}_{\mu}^{*}\left(\mathrm{t}^{\prime}\right)}{0.15 \mathrm{a}_{\mathrm{L}^{\prime} \mathrm{a}} \mathrm{~L}^{*}} \tag{7}
\end{equation*}
$$

Thus in each $M_{K \pi}$ interval，since the $\left\langle L_{\lambda}^{\prime} L_{\mu}^{*}\right\rangle$ can be calculated explicitly，the $\mathrm{a}_{\mathrm{L}}$ can be determined。
B． $\mathrm{K}^{+} \pi^{+}$Results
For $\mathrm{K}^{+} \pi^{+}$，we assume elastic unitarity so that $\left|a_{S}^{K^{+}} \pi^{+}\right|$is just $\left|\sin \delta_{S}^{3}\right|_{\text {。 }}$ The results for $\delta_{S}^{3}$ are shown as the solid points in Fig． 5 and can be seen to be in excellent agreement with those（open circles）obtained from the $t$－dependent fits of the previous section．Here also the exotic P and D wave phase shifts
are found to be always smaller than three degrees for $M_{K \pi}<1.8 \mathrm{GeV}$ 。
We have fitted $\delta_{S}^{3}$ to an effective range form

$$
\begin{equation*}
q^{2 L+1} \cot \delta_{L}^{2 I}=\frac{1}{a_{L}^{2 I}}+\frac{1}{2} r_{L}^{2 I} q^{2} \tag{8}
\end{equation*}
$$

We find，not surprisingly，that $\delta_{S}^{3}$ is well described by Eq。（8）with the low en－ ergy $K \pi$ parameters

$$
\begin{aligned}
& \mathrm{a}_{\mathrm{S}}^{3}=(-1.03 \pm 0.02) \mathrm{GeV}^{-1} \\
& \mathrm{r}_{\mathrm{S}}^{3}=(-0.94 \pm 0.19) \mathrm{GeV}^{-1}
\end{aligned}
$$

These yield the curve shown in Fig。5．To lowest order in $q^{2}$ ，this corresponds to a $\mathrm{K}^{+} \pi^{+}$total cross section of 5.2 mb 。

## C．Elastic $\mathrm{K}^{-} \pi^{+}$Results

In $\mathrm{K}^{-} \pi^{+}$scattering，inelastic channels become important for $\mathrm{M}_{\mathrm{K} \pi}>1.3 \mathrm{GeV}_{0}$ Therefore we only impose elastic unitarity for $\mathrm{M}_{\mathrm{K} \pi}<1_{0} 2 \mathrm{GeV}$ 。 The $\mathrm{I}=\frac{3}{2} \mathrm{P}$ and D wave phase shifts are assumed to be zero while the $S$ wave is given by the ef－ fective range curve of Fig。 $5 . \dagger$ In the region from 1.2 to 1.3 GeV ，we require the $P$ and $I=\frac{1}{2}$ S waves to be elastic but allow some inelasticity in the $D$ wave which is，in fact，small in this mass region．Since the $P$ wave is small（and not well determined）here，forcing it to be elastic is not overly restrictive．Thus， the imposition of elastic unitarity for $S$ and $P$ waves is，in practice，simply a prescription for determining the overall phase．The results for $\delta_{\mathrm{S}}^{1}$ and $\delta_{\mathrm{P}}^{1}$ in the region $0.7<\mathrm{M}_{\mathrm{K} \pi}<1.30 \mathrm{GeV}$ are shown by the solid points in Fig。6，where they can be seen to be in good agreement with the（extrapolated in t）results （open circles）of the previous section．The curve on the $\delta_{\mathrm{P}}^{1}$ plot represents a Breit－Wigner fit to the phase shifts
$\dagger$ These are the exotic phase shift values which were also used in the t－dependent $\mathrm{K}^{-} \pi^{+}$analysis of the previous section．

$$
\begin{gathered}
\frac{{ }_{\mathrm{a}}^{P}}{} \\
\sqrt{3}
\end{gathered}=\sin \delta_{P}^{1} e^{i \delta_{P}^{1}=\frac{M_{R} \Gamma}{M_{R}^{2}-M_{K \pi}^{2}-i M_{R} \Gamma}} \begin{aligned}
& \Gamma=\left(\frac{q}{q_{R}}\right)^{3} \Gamma_{R}\left[\frac{1+\left(q_{R} R\right)^{2}}{1+(q R)^{2}}\right]
\end{aligned}
$$

from 0.8 to 1.0 GeV . The resulting resonance parameters are

$$
\begin{align*}
\mathrm{M}_{\mathrm{R}} & =895.8 \pm 0.5 \mathrm{MeV} \\
\Gamma_{\mathrm{R}} & =51.7 \pm 0.9 \mathrm{MeV}  \tag{9}\\
\mathrm{R} & =5.6 \pm 1.2 \mathrm{GeV}^{-1}
\end{align*}
$$

The curve on the $S$ wave plot represents the result of an effective range fit of the form of Eq. (8) to $\delta_{S^{\circ}}^{1}$ The resulting low energy $\mathrm{K} \pi$ parameters are

$$
\begin{align*}
& \mathrm{a}_{\mathrm{S}}^{1}=2.4 \pm 0.1 \mathrm{GeV}^{-1} \\
& \mathrm{r}_{\mathrm{S}}^{1}=-1.7 \pm 0.3 \mathrm{GeV}^{-1} \tag{10}
\end{align*}
$$

We note that our value for $\mathrm{a}_{\mathrm{S}}^{1} / \mathrm{a}_{\mathrm{S}}^{3}=-2.3$, in excellent agreement with the current algebra prediction ${ }^{9}$ of -2 , although the individual scattering lengths are twice as large as would be expected from current algebra. There is another possible value of the $S$ wave magnitude and phase relative to $\delta_{\mathrm{P}}^{1}$ but it is in such violent disagreement with unitarity that there can be no question that it is unphysical. Nonetheless, it is of course impossible, as in any phase shift analysis, to rule out rapid $180^{\circ}$ changes in the phase shifts between any two neighboring mass bins.

## D. $\underline{K}^{-} \pi^{+}$Partial Waves in the Inelastic Region

In mass regions where the $K \pi$ partial waves are inelastic, it is obviously no longer sensible to impose elastic unitarity. Without information on the total cross section or on inelastic channels, the data determine only the magnitudes
and relative phases of the different partial waves. Fortunately, in nonexotic channeds such as $\mathrm{K}^{-} \pi^{+}$, the existence of resonances in the leading partial waves, coupled with unitarity and continuity requirements, provides strong constraints on this phase. There is also a problem of discrete ambiguities in the inelastic region. This is most readily apparent as an indeterminacy in the signs of the imaginary parts of the (Barrelet) ${ }^{10}$ zeros, $z_{i}$, of the scattering amplitude and, in fact, the signs of $\operatorname{Im} z_{i}$ provide a convenient ${ }^{11}$ means of distinguishing and classifying the various solutions for the partial waves. Moreover, only when one of the $\operatorname{Im} z_{i}$ approaches zero is it possible to change from one solution to another. In the $\mathrm{K}^{-} \pi^{+}$case, since elastic unitarity allows only one solution up to about 1.3 GeV , the possibility of discrete ambiguities does not arise until one of the $\operatorname{Im} z_{i}$ approaches zero. From Fig. 7, where the real and imaginary parts of the zeros have been plotted (for one choice of sign for $\operatorname{Im} z_{i}$ ), it is apparent that this cannot occur before $M_{K \pi} \sim 1.45 \mathrm{GeV}$. However, in the region from 1.4 to 1.5 GeV , both $\operatorname{Im} \mathrm{z}_{1}$ and $\operatorname{Im} \mathrm{z}_{2}$ could change sign so that, although below 1.4 GeV there is only one possible solution, above 1.5 GeV there are four which can be specified by the signs of $\operatorname{Im} z_{1}$ and $\operatorname{Im} z_{2}$, as indicated in Table $I_{0} \dagger$

In Fig. 8 we present the magnitudes and phases of the $\mathrm{K}^{-} \pi^{+}$partial waves for all four solutions. Although the solution is unique for $M_{K \pi}<1.5 \mathrm{GeV}$, for clarity we show each solution in the mass interval 1.3 to 1.9 GeV , and include the magnitudes and phases for the full mass range with solution $A$ 。 The $K^{-} \pi^{+} \mathrm{S}$ and $P$ partial waves below 1.3 GeV were calculated from the $I=1 / 2$ and $I=3 / 2$ phase shifts shown in Figs. 5 and 6. In this $M_{K \pi}$ region the imposition of elastic unitarity determines the overall phase. Above 1.3 GeV , the data determine †We have chosen to label the $\mathrm{K}^{-} \pi^{+}$solutions in the same way as the $\pi \pi$ solutions of Ref. 11.

TABLE I
The definition of the four possible $\mathrm{K}^{-} \pi^{+}$partial wave solutions according to the signs of the $\operatorname{Im} z_{i}$ at $M_{~}^{\text {K }}$ $=1.6 \mathrm{GeV}$ 。 The existence of the $\mathrm{K}^{*}(1420) \stackrel{\dot{1}}{\text { res }}$ ． $\mathrm{K}_{\mathrm{K}} \pi$ requires $\operatorname{Im} \mathrm{z}_{2}<0$ below about 1.5 GeV and the $K^{*}(1780)$ resonance requires $\operatorname{Im} z_{3}<0$ everywhere。

| Solution | $\operatorname{Im} z_{1}(1.6)$ | $\operatorname{Im~z}$ |
| :---: | :---: | :---: |
| 2 | $(1.6)$ |  |
| A | - | - |
| B | + | - |
| C | - | + |
| D | + | + |

only the relative phases of the different partial waves．For $1.3<\mathrm{M}_{\mathrm{K} \pi}<1.6$ $G e V$ ，the $S, P$ ，and $F$ wave phases are shown relative to the（absolute）$D$ wave Breit－Wigner phase which was obtained by fitting the $D$ wave magnitude from 1.3 to 1.5 GeV to a Breit－Wigner form（curve on Fig．8），yielding resonance parameters of

$$
\begin{aligned}
\mathrm{M} & =(1433 \pm 4) \mathrm{MeV} \\
\Gamma & =(94 \pm 10) \mathrm{MeV} \\
\mathrm{x} & =0.50 \pm 0.02 \\
\mathrm{R} & =(4.0 \pm 4.0) \mathrm{GeV}^{-1}
\end{aligned}
$$

Above 1.6 GeV the phases are measured relative to $\delta_{F^{\circ}}$ For plotting purposes， the $F$ wave phase has been chosen to correspond to that of a resonance ${ }^{1}$ of mass 1.75 GeV and width of about 250 MeV 。 We emphasize that at each mass value the choice of overall phase is arbitrary and all values provide identical values of $\mathrm{d} \sigma_{K \pi} / \mathrm{d} \Omega$ 。

## E．Discussion of Results

In the region above 1.5 GeV ，all four solutions yield very similar results for the magnitude of the leading $3^{-}$partial wave．The solutions differ in the amplitude and phase behavior of the $S, P$ ，and $D$ partial waves．In solutions $B$ and $D$ ，there is a broad bump in $\left|a_{P}\right|$ ，peaking at about 1700 MeV ．Particularly in solution $B$ ，there is a rapid increase in $\delta_{P}$ in the same region，indicative of resonance－like behavior．Comparison of $\left|a_{P}\right|$ at the peak with its unitarity limit of $\sqrt{3}$ allows us to estimate the branching ratio of the＇resonance＇into $\mathrm{K} \pi$ as about $40 \%$ ．In contrast，the analogous calculation for the F wave gives a $\mathrm{K} \pi$ branching fraction of only $15 \%$ 。

In the region below 1.6 GeV ，in addition to the $\mathrm{I}^{-} \mathrm{K}^{*}(890)$ and $2^{+} \mathrm{K}^{*}(1420)$ resonances，there is a steady rise of the $S$ wave magnitude to a peak at about 1420 MeV ，followed by a fairly rapid drop．This structure，which is common to all four solutions，is more apparent in Fig．9，where the data for the $S$ wave magnitude and phase $\dagger$ are compared with the elastic region effective range fits of Sections IVB and IVC and their extrapolation into the region $M_{K \pi}>1.2 \mathrm{GeV}$ 。 The excellent agreement between curves and data points over the entire mass range from threshold to 1.42 GeV contrasts sharply with the marked deviation of the data above 1.45 GeV from the effective range form．This S －wave struc－ ture could conceivably be attributed to the opening of an inelastic channel（e．g．， $\eta^{\prime} \mathrm{K}, \mathrm{Q} \pi, \ldots$ ）and／or to an S－wave resonance in the 1500 MeV region．

We wish to point out the remarkable similarity between the $\mathrm{K}^{-} \pi^{+}$partial waves of Fig。 8 and the $\pi^{-} \pi^{+}$partial waves in the inelastic region（see，for ex－ ample，Fig． 5 of Ref．11）．This similarity extends to the amplitude and phase

[^2]variations of all partial waves in each of the corresponding $K \pi$ and $\pi \pi$ solutions．

## V．CONCLUSIONS

In summary，then，data from the same experiment on reactions（1）and（2） have allowed not only calculation of the $\mathrm{K}^{-} \pi^{+}$partial waves but also a reliable determination of the exotic $\mathrm{I}=3 / 2$ S－wave phase shift。 For $\mathrm{K}^{-} \pi^{+}$in the inelas－ tic region，we have presented all possible ambiguous partial wave solutions classified in terms of their zeros．Data on the reaction ${ }^{12} \mathrm{~K}^{+} \mathrm{p} \rightarrow \mathrm{K}^{+} \pi^{-} \Delta^{++}$in conjunction with the data on reaction（1）will make possible a more detailed in－ vestigation of this intriguing inelastic region．

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## FIGURE CAPTIONS

1．The（unnormalized）moments of the $\mathrm{K}^{-} \pi^{+}$angular distribution in the $\mathrm{t}^{\prime}$ range $0<-t^{\prime}<0.15 \mathrm{GeV}^{2}$ as a function of $M_{K \pi}$ ．
2．The（unnormalized）moments of the $\mathrm{K}^{+} \pi^{+}$angular distribution in the $t^{\prime}$ ． range $0<-\mathrm{t}^{8}<0.15 \mathrm{GeV}^{2}$ as a function of $\mathrm{M}_{\mathrm{K}} \pi^{\text {。 }}$
3．The moments（unnormalized）of the $\mathrm{K}^{-} \pi^{+}$angular distribution as a function of $t^{9}$ in the mass range $0.94<\mathrm{M}_{\mathrm{K} \pi}<1.0 \mathrm{GeV}$ 。 The curves represent the t－dependent amplitude parametrization described in the text．

4．The $\mathrm{M}_{\mathrm{K} \pi}$ dependence of the parameters of Eq．（4）describing the $t$－ dependence of the $\mathrm{K} \pi$ production amplitudes．The open circles refer to $\mathrm{K}^{+} \pi^{+}$production，the solid points to $\mathrm{K}^{-} \pi^{+}$production．The curves are the polynomial fits used in the single $t$ bin analysis．

5．The $I=3 / 2$ S－wave phase shift．The open circles were obtained by extrap－ olating the production amplitudes to $\mathrm{t}=\mu^{2}$ ，the solid points are from the single $t$ bin analysis．The curve represents an effective range fit with $\mathrm{a}_{\mathrm{S}}^{3}$ $=-1.03 \mathrm{GeV}^{-1}$ and $\mathrm{r}_{\mathrm{S}}^{3}=-0.94 \mathrm{GeV}^{-1}$ 。

6．The $\mathrm{I}=\frac{1}{2} \mathrm{~K} \pi$ phase shifts in the elastic region．Open circles represent re－ sults from extrapolating to $t=\mu^{2}$ ，solid points come from the single $t$ bin analysis．The input values of $\delta_{\mathrm{S}}^{3}$ were taken from the effective range curve of Fig．5．The curve through $\delta_{\mathrm{P}}^{1}$ represents a Breit－Wigner fit to the points from 0.8 to 1.0 GeV ，yielding resonance parameters $\mathrm{M}_{\mathrm{R}}=0.896$ $\mathrm{GeV},-\Gamma=0.052 \mathrm{GeV}$ ，and $\mathrm{R}=5.6 \mathrm{GeV}^{-1}$ ．The curve through $\delta_{\mathrm{S}}^{1}$ repre－ sents an effective range fit（for $M<1.2 \mathrm{GeV}$ ）with $\mathrm{a}_{\mathrm{S}}^{1}=2.4 \mathrm{GeV}^{-1}$ and $\mathrm{r}_{\mathrm{S}}^{1}$ $=-1.7 \mathrm{GeV}^{-1}$ ．

7．The（complex）zeros of the $\mathrm{K}^{-} \pi^{+}$scattering amplitude as calculated from partial waves of Figs． 6 and 8．The solid points（open circles）represent
the real（imaginary）parts of the zeros．Either $\operatorname{Im} z_{1}$ and／or $\operatorname{Im} z_{2}$ could change sign at $M_{K \pi} \sim 1.5 \mathrm{GeV}$ 。
8．The magnitudes and phases of the $K^{-} \pi^{+}$partial waves．The curves for $\left|\mathrm{a}_{\mathrm{D}}\right|$ and $\delta_{\mathrm{D}}$ represent Breit－Wigner fits to $\left|\mathrm{a}_{\mathrm{D}}\right|$ with $\mathrm{M}=1.433 \mathrm{GeV}$ ， $\Gamma=.094 \mathrm{GeV}, \mathrm{x}=0.50$ ，and $\mathrm{R}=4.0 \mathrm{GeV}^{-1}$ 。 The open circles are the re－ sults of $t$－dependent fits，the solid points come from the single $t$ bin anal－ ysis．Below 1.3 GeV ，the overall phase is fixed by elastic unitarity．For $1.3<\mathrm{M}_{\mathrm{K} \pi}<1.6 \mathrm{GeV}, \delta_{\mathrm{D}}$ is chosen to have the Breit－Wigner phase shown by the curve；for $\mathrm{M}_{\mathrm{K} \pi}>1.6 \mathrm{GeV}, \delta_{\mathrm{F}}$ is chosen to have the values shown by the errorless points．Below about 1.5 GeV ，all four solutions are identi－ cal．Above 1.5 GeV ，all solutions have $\operatorname{Im} \mathrm{z}_{3}<0$ ，while in $\mathrm{A}, \operatorname{Im} \mathrm{z}_{1}<0$ and $\operatorname{Im} z_{2}<0$ ；in $B, \operatorname{Im} z_{1}>0$ ， $\operatorname{Im} z_{2}<0$ ；in $C, \operatorname{Im} z_{1}<0, \operatorname{Im} z_{2}>0$ ；and in $\mathrm{D}, \operatorname{Im} \mathrm{z}_{1}>0, \operatorname{Im} \mathrm{z}_{2}>0$ 。
9．The magnitudes and phases of the $K^{-} \pi^{+} S$ and $P$ partial waves of solution B． The choice of overall phase in the inclastic region is the same as that of Fig．8．The S－wave curves correspond to effective range fits to the $I=$ $1 / 2$ and $\mathrm{I}=3 / 2$ S－wave phase shifts for $\mathrm{M}_{\mathrm{K} \pi}<1.2 \mathrm{GeV}$ 。 The curves shown above 1.2 GeV are extrapolations of these fits．The P－wave curves corre－ spond to a Breit－Wigner resonance fit to the phase shifts for $0.8<M_{\mathrm{K} \pi}<$ 1.0 GeV and the extrapolation of this fit outside that $\mathrm{M}_{\mathrm{K} \pi}$ region．


Fig. 1


Fig. 2


Fig. 3


Fig. 4


Fig. 5


Fig. 6

$M_{k \pi}(G \mathrm{GeV})$


Fig. 7


Fig. 8


Fig. 9


[^0]:    ＊Work supported by the Energy Research and Development Administration and the National Research Council of Canada．

[^1]:    $\dagger$ For $K^{-} \pi^{+}$scattering，we use $a_{L} \equiv a_{L}^{1}+\frac{1}{2} a_{L}^{3}$ where the superscripts refer to twice the $K \pi$ isospin and the $a_{L}^{2 I}$ are given by Eq。（5）for elastic scattering。 The isospin Clebsch－Gordan coefficient of $\frac{2}{3}$ has been absorbed into the defini－ tion of $\mathscr{N}_{-}$。

[^2]:    $\dagger$ For $\mathrm{M}_{\mathrm{K} \pi}>1.5 \mathrm{GeV}$ ，where the four solutions differ we have chosen to show solution B．The（arbitrary）choice of overall phase is the same as that of Fig．8．

