THEORETICAL MODELS FOR ANGULAR DISTRIBUTION
OF MASSIVE MUON PAIR PRODUCED IN HADRONIC COLLISIONS

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#### Abstract

We point out that accurate measurements of the angular distri－ butions of direct muons produced in hadronic collisions will have important consequences for various theoretical models for such processes．A number of models based on concepts of quark－partons and gluons are discussed within the context of the preliminary data available at present．


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[^0]Production of massive $\mu$ pairs in hadronic collisions has been studied extensively recently both experimentally and theoretically. It has already provided insight into dynamics of various theoretical models based on the concepts of quark-partons and gluons. Ideally one would like to have data for $Q^{2}$ (mass of dimuon) ${ }^{2}, \mathrm{p}_{\|}$(longitudinal momentum of the dimuon), $\mathrm{p}_{\mathrm{T}}$ (transverse momentum) distributions along with the angular distribution of the muons in a range of values of each of these variables. It will be some time in the future before such complete high statistics data become available. But already some preliminary data for angular distributions averaged over the other variables have been obtained。 ${ }^{1}$ In the past several authors have considered $Q^{2}, p_{\|}$and $\mathrm{p}_{\mathrm{T}}$ distributions of the dimuon in various models, but the angular distributions of muons have not been considered. In the present note we discuss various theoretical models for dimuon production, with special emphasis on the angular distributions. We show that even from such averaged data some interesting conclusions follow. Magnitudes of the cross sections and $p_{T}$ distributions, etc., for some of the models are being considered and will be reported separately. ${ }^{2}$ We hope to present more detailed analysis of angular distributions when complete data are available in the future. One of the purposes of this work is to point out that accurate measurements of such distributions will have very important consequences for various theoretical models, since predictions differ considerably from one model to another.

The preliminary results which follow from a recent experiment ${ }^{1}$ give the muon angular distribution in the dimuon rest frame as $1+\alpha \cos ^{2} \theta$, where the angle $\theta$ is measured with respect to the incident beam axis. There are considerable variations in the measured $\alpha$ values. But they are found to be consistent with 1 (although could be different from 1) for continuum $\mu$-pair in the
$Q \approx 1.9-2.3 \mathrm{GeV}$ region. In the $J / \psi$ resonance region $(Q=3.1 \mathrm{GeV})$, however, $\alpha \alpha$ is found to be consistent with zero ( $\alpha=-0.26 \pm 0.19$ for incident protons, $\alpha=0.26 \pm 0.25$ for incident pions). If the future analyses uphold this conclusion, it will already be an indication that production mechanism for continuum $\mu$ pairs production is entirely different from that of $J / \psi$ production.

The complete angular distribution of the muon is given by the expression ${ }^{3}$ $\mathrm{W}(\theta, \phi)=1-\rho_{11} \sin ^{2} \theta-\rho_{00} \cos ^{2} \theta+\rho_{1,-1} \sin ^{2} \theta \cos 2 \phi+\sqrt{2} \operatorname{Re} \rho_{10} \sin ^{2} \theta \cos \phi$
where the $\rho^{\prime}$ s are the density matrix elements. The angles $(\theta, \phi)$ are measured with respect to some convenient axes. In the Gottfried-Jackson (G-J) frame the $z$-axis is along the direction of the incident beam in the dimuon rest frame. In the helicity frame it is opposite to the direction of the momentum of the recoiling missing mass in the dimuon rest frame. Averaging over the azimuthal angle $\phi, \mathrm{W}(\theta, \phi)$ can be written as

$$
\begin{equation*}
\mathrm{W}(\theta)=1+\frac{3 \rho_{11}-1}{1-\rho_{11}} \cos ^{2} \theta \tag{2}
\end{equation*}
$$

where we have used the normalization condition $\rho_{00}+2 \rho_{11}=1$. Hence

$$
\begin{equation*}
\alpha=\frac{3 \rho_{11}-1}{1-\rho_{11}} \tag{3}
\end{equation*}
$$

In general $\rho_{11}$ ( and hence $\alpha$ ) will be a function of other kinematic variables like $\mathrm{p}_{\|}, \mathrm{p}_{\mathrm{T}}$, etc. At present, however, data only give the average value of $\alpha$. If $\mathrm{m}_{1}, \mathrm{~m}_{2}$; and $Q$ are the masses of the incident particle, target particle, and the dimuon, and $m_{x}$ is the missing mass, the crossing angle $\chi$ which relates the G-J frame ( t -channel) to the s -channel helicity frame is given by

$$
\begin{equation*}
\cos \chi=\left\{\left(s+Q^{2}-m_{x}^{2}\right)\left(t+Q^{2}-m_{1}^{2}\right)+2 Q^{2} \Delta\right\} / D \tag{4}
\end{equation*}
$$

where

$$
\begin{equation*}
\Delta=m_{x}^{2}-m_{2}^{2}+m_{1}^{2}-Q^{2} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
D=\left\{\left(Q+m_{1}\right)^{2}-t\right\}^{\frac{1}{2}}\left\{\left(Q-m_{1}\right)^{2}-t\right\}^{\frac{1}{2}}\left\{s-\left(Q+m_{x}\right)^{2}\right\}^{\frac{1}{2}}\left\{s-\left(Q-m_{x}^{2}\right)^{\frac{1}{2}}\right\}^{2} \tag{6}
\end{equation*}
$$

Here $s$ and $t$ are the usual Mandelstam variables for the process

$$
m_{1}+m_{2} \rightarrow Q+M_{x}
$$

Then $\rho_{11}^{\mathrm{t}(\mathrm{G}-J)}$ is related to $\rho_{11}^{\mathrm{s}}$ by

$$
\begin{equation*}
\rho_{11}^{\mathrm{t}}=\frac{\left(1-2 \rho_{11}^{\mathrm{s}}\right)}{2} \sin ^{2} \chi+\frac{\rho_{11}^{\mathrm{s}}\left(1+\cos ^{2} \chi\right)}{2} \tag{7}
\end{equation*}
$$

assuming that $\rho_{1,-1}$ and $\rho_{1,0}$ are zero。
Next, we consider various quark-parton and gluon models. Throughout this work we assume, for simplicity, that the quark-partons are on mass-shell. If the off-shell effects are significant, they could modify some of the conclusions. ${ }^{4}$ These will be considered in the future.

In the following, we will write equations explicitly for the case where the $\mu$-pair is produced through an intermediate heavy photon. Equations for the case of $\psi$ can be readily obtained by using Breit-Wigner form instead of $1 / Q^{2}$.

## I. DRELL-YAN (D-Y) MECHANISM WITH LIGHT (NONCHARMED) QUARKS AND POINT COUPLINGS

Drell-Yan mechanism ${ }^{5}$ is the most popular model for this process. Here, as shown in Fig. 1, a quark (parton) from one hadron annihilates an antiquark (antiparton) from the other hadron. If neither beam nor target contains valency antiquark, the antiquark is supposed to come from the sea of $q \bar{q}$ pairs. It is often stated that the distribution is of the form $1+\cos ^{2} \theta$ 。 Here we consider some more exact expressions. Let $k_{1}, k_{2}, p_{1}, p_{2}$, and $Q$ be the four momenta
of the $q-\bar{q}$ and $\mu^{+}-\mu^{-}$pairs and the intermediate heavy photon or $\psi$ particle． Then the matrix element for pointlike $\left(\gamma_{\mu}\right)$ couplings is given by

$$
\begin{equation*}
\mathrm{M}=\overline{\mathrm{v}}\left(\mathrm{k}_{2}\right) \gamma^{\mu} \mathrm{u}\left(\mathrm{k}_{1}\right) \frac{1}{\mathrm{Q}^{2}} \overline{\mathrm{u}}\left(\mathrm{p}_{1}\right) \gamma_{\mu} \mathrm{v}\left(\mathrm{p}_{2}\right) \tag{8}
\end{equation*}
$$

Squaring and summing over the spins，one finds

$$
\begin{equation*}
|M|^{2} \propto 1+\frac{4 m^{2}}{Q^{2}}+\beta^{2} \cos ^{2} \theta \tag{9}
\end{equation*}
$$

where

$$
\begin{equation*}
\beta^{2}=1-4 m^{2} / Q^{2} \tag{10}
\end{equation*}
$$

is the square of the velocity of the quarks in the c．m．system．$\theta$ is the angle between $\underset{\sim}{k_{1}}$ and $\mathrm{R}_{1}$ in the same system。 We have neglected the lepton masses but keep quark mass（ m ）nonzero．Setting $\mathrm{m}=0$ ，one gets the well－known Drell－Yan result of $1+\cos ^{2} \theta$ ．So，for very light quarks，one should have this distribution for relatively large value of $Q^{2}$ 。This should be true for both heavy photon and the $\psi$－meson if the production mechanism is the Drell－Yan process．For very low values of $Q^{2}$ ，the value of $\alpha$ will be sensitive to the quark－mass $m$ ．If the magnetic moment of quark is assumed to be the same as that of the proton，a typical value of $\mathrm{m}=0.336 \mathrm{GeV}$ is obtained．In the $\rho(\omega)$ mass region this gives $\alpha=\frac{1-4 \mathrm{~m}^{2} / \mathrm{Q}^{2}}{1+4 \mathrm{~m}^{2} / \mathrm{Q}^{2}}=0$ 。14．On the other hand，if $\mathrm{m}=$ $\mathrm{m}_{\rho / 2}, \alpha=0$ ．So，for light quarks，the distribution should be essentially iso－ tropic．In fact，experimentally it has been found that，at NAL energies，for $\rho(\omega)$ production the distribution is close to being isotropic．${ }^{6}$ We note in pass－ ing that，for pion exchange（for $\rho$－production）and rho exchange（for $\omega$－produc－ tion），the distribution should be $\sin ^{2} \theta$ and $1+\cos ^{2} \theta$ ，respectively．At low en－ ergies（11•2 GeV／c $\pi$ beam）results consistent with OPE + absorption were found．${ }^{3}$ But at higher energies the quark models presumably should work better．

## II．D－Y MODEL WITH HEAVY QUARKS

As we noted above，for $\mathrm{m} \approx \frac{\mathrm{Q}}{2}, \alpha=\beta^{2} \approx 0$ ．Now，the mass of a charmed quark is believed to be about half the mass of the $\psi$－meson．So，in such a case， one would automatically get isotropic distribution in $\psi$ production．Some authors have already considered the $Q^{2}, p_{\Pi}$ ，and $p_{T}$ distribution in such models。 ${ }^{7}$ The number of charmed quark c－ $\bar{c}$ pairs in the sea associated with nucleons and pions is presumably very small．But this is compensated by a large coupling of cc to $\psi$ ．In $q \bar{q}$ model，that coupling is Zweig－forbidden and hence small。 In c $\bar{c}$ model one has to arrange carefully so that excessive numbers of charmed particles are not produced（in agreement with the experiments）and there are some diffi－ culties．But，on the whole，at present the model is not ruled out．The iso－ tropic angular distribution will give strong support to such a model if other pre－ dicted distributions could keep up with the experiments．

III．D－Y MODEL WITH STRUCTURE IN THE qqu COUPLINGS TO $\gamma$ AND $\psi$
Scaling observed in the electroproduction puts strong restrictions on the structure one could allow in $q \bar{q} \gamma$ vertex．However，as suggested by Drell， Chanowitz，${ }^{8}$ and West，${ }^{9}$ one can introduce a $\sigma_{\mu \nu}$ term with a form factor to de－ scribe the anomalous magnetic moment of the quark if it exists．It can be ar－ gued that even if the fundamental quark vertex is pointlike（ $\gamma_{\mu}$ type），renormali－ zation effects due to exchanges of gluons could produce a $\sigma_{\mu \nu}$ term analogous to the anomalous magnetic moment term for the elementary electron．${ }^{9}$ Now with two form factors $F_{1}\left(Q^{2}\right)$ and $F_{2}\left(Q^{2}\right)$ Eq．（8）becomes

$$
\begin{equation*}
\mathrm{M}=\overline{\mathrm{v}}\left(\mathrm{k}_{2}\right)\left\{\gamma^{\mu} \mathrm{F}_{1}\left(\mathrm{Q}^{2}\right)+\mathrm{i} \sigma^{\mu \nu} \mathrm{Q}_{\nu} \mu_{Q} \mathrm{~F}_{2}\left(\mathrm{Q}^{2}\right)\right\} \mathrm{u}\left(\mathrm{k}_{1}\right) \frac{1}{Q^{2}} \overline{\mathrm{u}}\left(\mathrm{p}_{1}\right) \gamma_{\mu} \mathrm{v}\left(\mathrm{p}_{2}\right) \tag{11}
\end{equation*}
$$

Leptonic vertex has been kept $\gamma_{\mu}$ type。For $q \bar{q} \gamma$ case $\mu_{Q}$ is the anomalous mag－ netic moment of the quark．Squaring and spin averaging lead to（for small m
and large $Q^{2}$ )

$$
\begin{equation*}
-|M|^{2} \propto F_{1}^{2}\left(Q^{2}\right)\left(1+\beta^{2} \cos ^{2} \theta\right)+F_{2}^{2}\left(Q^{2}\right) \mu_{Q}^{2}\left(1-\beta^{2} \cos ^{2} \theta\right) \tag{12}
\end{equation*}
$$

This gives the distribution function

$$
\begin{equation*}
W(\theta) \propto 1+\frac{\beta^{2}\left(1-\mu_{Q^{2}}^{2}\right)}{\left(1+\mu_{Q^{2}}^{2} Q^{2}\right)} \cos ^{2} \theta \tag{13}
\end{equation*}
$$

where we have set $\frac{F_{2}\left(Q^{2}\right)}{F_{1}\left(Q^{2}\right)}=1$. Now West ${ }^{9}$ found that values of $\mu_{Q}$ of 0.1 to $0.2 \mathrm{GeV}^{-1}$ are consistent with electroproduction, $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation data, and also with the quark model assumption that the quark magnetic moment be the same as that of the proton (with quark mass $\approx 1 / 3$ proton mass)。 Taking $\beta=1$, $\mathrm{Q}=3.1 \mathrm{GeV}$, this gives $\alpha=0.83$ and 0.44 for $\mu_{\mathrm{Q}}=0.1$ and $0.2 \mathrm{GeV}^{-1}$, respectively。 $\alpha=0$ would require $\mu_{\mathrm{Q}}=0.32 \mathrm{GeV}^{-1}$. Also the value of $\alpha$ will decrease as $Q^{2}$ increases. As $Q^{2}$ increases the distribution changes from $1+\cos ^{2} \theta$ to 1 and then to $1-\cos ^{2} \theta$. This is for the case of heavy photons. In principle the hadronic $q \bar{q} \psi$ vertex could be completely different from $q \bar{q} \gamma$ vertex and it is possible that $q \bar{q} \gamma$ vertex may be pointlike and $q \bar{q} \psi$ vertex has a structure. In the latter case there is essentially no restriction on the value of $\mu_{\mathrm{Q}}$. It seems that such models may be arranged so that they are not in contradiction with existing experiments and could even be forced on us by future experiments, although the elegance and simplicity of parton picture will be somewhat lost.

## IV. D-Y MODEL WITH SPIN ZERO PARTONS (OR QUARKS ?)

This is not a very attractive hypothesis but we consider it for the sake of completeness. The relevant matrix element is given by

$$
\begin{equation*}
\mathrm{M}=\overline{\mathrm{u}}\left(\mathrm{p}_{1}\right)\left(k_{1}-\not k_{2}\right) \mathrm{v}\left(\mathrm{p}_{2}\right) \tag{14}
\end{equation*}
$$

Then the spin-averaging leads to

$$
\begin{equation*}
|M|^{2} \propto 1-\frac{4 m^{2}}{Q^{2}}-\beta^{2} \cos ^{2} \theta \tag{15}
\end{equation*}
$$

For light quarks, this gives $1-\cos ^{2} \theta$ distribution, as is well known. It should be noted that electroproduction data do favor spin $1 / 2$ partons.

In all the four $D-Y$ models, the distribution has been calculated with respect to the direction of the quark-antiquark three momenta in the $c . m$. system of the dimuon. Relationship of this axis to the beam axis will depend on the dynamical details of the model (i.e., probability functions for the quarks to have different momenta). We consider two simple cases in view of the crudeness of the present data which are given with respect to the incident beam axis and averaged over all $\mathrm{p}_{\|}$and $\mathrm{p}_{\mathrm{T}}$ values of the dimuon. In the first case we assume that the quarkantiquark have essentially only longitudinal momenta, the transverse momenta being extremely small. Then the values of $\alpha$ are essentially the same as the ones given above, even with respect to the beam axis. In the second case, we assume that the $q-\bar{q}$ are essentially moving in the same direction as the heavy photon (or $\psi$ or dimuon) they produce. Then we use Eqs. (3), (4), (5), (6), and (7) to obtain $\alpha$ relative to the beam axis. The results are averaged over $p_{\|}$and $\mathrm{p}_{\mathrm{T}}$ to obtain $\bar{\alpha}$. As suggested by the experimental fits, ${ }^{1}$ the weight factor $\left(1-\mathrm{X}_{\mathrm{F}}\right)^{4} \mathrm{e}^{-2 \mathrm{p}} \mathrm{T}_{\left(\mathrm{X}_{\mathrm{F}}=2 p_{\|} / \sqrt{s}\right) \text { is used. It turns out that in this case the value of } \alpha}$ does change substantially by such averaging. For example, $\alpha=1$ leads to $\bar{\alpha}=$ 0.36. On the other hand, however, it seems unlikely that any amount of averaging can lead from $\alpha=1$ to $\bar{\alpha} \approx 0$ 。

So far we have considered quark-antiquark annihilation models. Next in order of complexity is the Constituent Interchange Model of Blankenbecler, Brodsky, and Gunion, ${ }^{10}$ in which a quark from one hadron scatters on a hadron (meson) emitted by the other hadron and produces a photon or a vector meson and anything else。

## V。 CONSTITUENT INTERCHANGE MODEL（CIM）

This model has been quite successful in the large $\mathrm{p}_{\mathrm{T}}$ region，but application to the entire $\mathrm{p}_{\mathrm{T}}$ range is only currently being made。 ${ }^{2}$ For the sake of definite－ ness we assume that，as in Fig．2，the dominating hard scattering subprocess is the one in which a quark is scattered by a $\pi$ meson to produce a quark and a di－ muon．The final quark then combines with the rest of the hadronic states which are not detected．This process is analogous to $\pi \mathrm{N} \rightarrow \gamma \mathrm{N}$ or $\rho \mathrm{N}$ ．We consider the dominant contribution to be given by an electric Born model。 Our treatment will be somewhat similar to the one given by Sachrajda and Blankenbecler，who con－ sider s－channel pole term for this process。 ${ }^{11}$ Let $\mathrm{p}, \mathrm{q}, \mathrm{k}$ ，and Q be the four momenta of the initial quark，initial meson，final quark，and the dimuon，respec－ tively。Then $s^{\prime}=(p+q)^{2}, t^{\prime}=(p-k)^{2}, u^{\prime}=(p-Q)^{2}$ 。 In the gauge invariant electric－ Born model there are two quark（ $s^{8}$ and $u^{\prime}$ ）pole terms and one pion（ $t^{\prime}$ ）pole。 （See Fig．3．）The matrix element is given by

$$
\begin{equation*}
T\left(s^{\prime}, t^{\prime}, u^{\prime}\right)=\operatorname{eg} \bar{u}(k) \gamma_{5}[] u(p) \tag{16}
\end{equation*}
$$

where

$$
[]=A \frac{2 \epsilon \cdot \mathrm{k}+\left(\gamma^{\circ} \epsilon\right)(\gamma \cdot Q)}{s^{\prime}-\mathrm{m}_{\mathrm{q}}^{2}}+\mathrm{B} \frac{2 \epsilon \cdot \mathrm{p}-\left(\gamma^{\circ} \mathrm{Q}\right)(\gamma \cdot \epsilon)}{\mathrm{u}^{\prime}-\mathrm{m}_{\mathrm{q}}^{2}}+\mathrm{C} \frac{2 \epsilon \cdot \mathrm{q}-\epsilon \circ \mathrm{k}}{\mathrm{t}^{\prime}-\mathrm{m}_{\pi}^{2}}
$$

$m_{q}$ and $m_{\pi}$ are the masses of the quark and the meson respectively．The values of constants $\mathrm{A}, \mathrm{B}, \mathrm{C}$ depend on the charge states under consideration．e is pro－ ton charge and $g$ is $q q \pi$ coupling constant．The values of $A, B, C$ are easily found to be（in this order）

$$
\begin{aligned}
& \pi^{\mathrm{o}} \mathrm{u} \rightarrow \gamma \mathrm{u}\left(\frac{2}{3}, \frac{2}{3}, 0\right) \\
& \pi^{\mathrm{o}} \mathrm{~d} \rightarrow \gamma \mathrm{~d}\left(-\frac{1}{3},-\frac{1}{3}, 0\right) \pi^{+} \mathrm{d} \rightarrow \gamma \mathrm{u}\left(\frac{2}{3},-\frac{1}{3}, 1\right) \sqrt{2} \\
& \pi^{-} \mathrm{u} \rightarrow \gamma \mathrm{~d}\left(\frac{1}{3},-\frac{2}{3}, 1\right) \sqrt{2}
\end{aligned}
$$

$u$ and $d$ refer to the up and down quarks with charges $2 / 3$ and $-1 / 3$ ，respec－ tively．From Eq．（16）one can read off the Ball amplitudes for the process and obtain the s－channel helicity amplitudes $H_{\lambda_{f}}^{\lambda}, \lambda_{i}\left(s^{\prime}, t^{\prime}\right) 。 \lambda, \lambda_{f}$ ，and $\lambda_{i}$ are the helicities of the off－shell photon（or $\psi$ meson）and final and initial quarks，re－ spectively。 ${ }^{12}$ The nonzero $H_{\lambda_{f}}{ }^{\lambda}, \lambda_{i}$ are as follows：

$$
\begin{align*}
& \mathrm{H}_{+-}^{\mathrm{o}}\left(\mathrm{~s}^{\prime}, \mathrm{t}^{\prime}\right)=-\frac{\mathrm{Q} \sqrt{-\mathrm{t}^{\prime}}}{\mathrm{m}_{\mathrm{q}}}\left[\frac{\mathrm{~A}}{2\left(\mathrm{~s}^{\prime}-\mathrm{m}_{\mathrm{q}}^{2}\right)}+\frac{\mathrm{C}}{2\left(\mathrm{t}^{\prime}-\mathrm{m}_{\pi}^{2}\right)}\right]  \tag{17}\\
& \mathrm{H}_{+-}^{1}\left(\mathrm{~s}^{\prime}, \mathrm{t}^{\prime}\right)=-\frac{1}{\sqrt{2} \mathrm{~m}_{\mathrm{q}}}\left[\frac{\mathrm{As}^{\prime}}{2\left(\mathrm{~s}^{\prime}-\mathrm{m}_{\mathrm{q}}^{2}\right)}-\frac{\mathrm{Bs}^{\prime}}{2\left(\mathrm{u}^{\prime}-\mathrm{m}_{\mathrm{q}}^{2}\right)}-\frac{\mathrm{At}^{\prime}}{4\left(\mathrm{~s}^{\prime}-\mathrm{m}_{\mathrm{q}}^{2}\right)}-\frac{\mathrm{Bt}^{\prime}}{4\left(\mathrm{u}^{\prime}-\mathrm{m}_{\mathrm{q}}^{2}\right)}-\frac{\mathrm{Ct}^{\prime}}{\mathrm{t}^{\prime}-\mathrm{m}_{\pi}^{2}}\right]  \tag{18}\\
& \mathrm{H}_{+-}^{-1}\left(\mathrm{~s}^{\prime}, \mathrm{t}^{\prime}\right)=-\frac{\mathrm{t}^{\prime}}{\sqrt{2 m_{q}}}\left[\frac{\mathrm{~A}}{4\left(\mathrm{~s}^{\prime}-\mathrm{m}_{\mathrm{q}}^{2}\right)}+\frac{\mathrm{B}}{4\left(\mathrm{u}^{\prime}-\mathrm{m}_{\mathrm{q}}^{2}\right)}+\frac{\mathrm{C}}{\mathrm{t}^{\prime}-\mathrm{m}_{\pi}^{2}}\right] \tag{19}
\end{align*}
$$

By squaring these，the s－channel density matrix elements are obtained．These will be useful in future．However，the present data give distribution with re－ spect to the beam axis（Gottfried－Jackson frame）．Hence we obtain the ampli－ tudes $G$ by crossing，

$$
\begin{equation*}
G_{\lambda_{f}}^{m}, \lambda_{i}\left(s^{8}, t^{\prime}\right)=\sum_{\lambda} d_{m \lambda}^{1}(\chi) H_{\lambda_{f}}^{\lambda}, \lambda_{i}^{\left(s^{\prime}, t^{8}\right)} \tag{20}
\end{equation*}
$$

$d_{m \lambda}^{1}(\chi)$ is the usual d－function of the $\gamma(\psi)$ crossing angle $\chi$ given by Eq。（4）with the replacement

$$
\begin{equation*}
\mathrm{m}_{1} \rightarrow \mathrm{~m}_{\pi}, \mathrm{m}_{2} \rightarrow \mathrm{~m}_{\mathrm{q}}, \text { and } \mathrm{m}_{\mathrm{x}} \rightarrow \mathrm{~m}_{\mathrm{q}} \text { 。 } \tag{21}
\end{equation*}
$$

In terms of G＇s the G－J（t－channel）density matrix elements are given by

$$
\begin{align*}
& \rho_{00}\left(s^{8}, t^{8}\right)=\left|G_{0}\right|^{2} /\left(\left|G_{0}\right|^{2}+\left|G_{1}\right|^{2}+\left|G_{-1}\right|^{2}\right) \\
& \rho_{11}\left(s^{8}, t^{t}\right)=\left(\left|G_{1}\right|^{2}+\left|G_{-1}\right|^{2}\right) / 2\left(\left|G_{0}\right|^{2}+\left|G_{1}\right|^{2}+\left|G_{-1}\right|^{2}\right) \tag{22}
\end{align*}
$$

etc．Then the angular distribution for $\gamma$ or $\psi \rightarrow \mu^{+} \mu^{-}$is obtained from Eq。（2）。

Now $\rho_{11}$ is a function of $s^{\prime}$ and $t^{\prime}$ 。 To compare with experiment we have to average over these variables. In CIM the basic equation for total inclusive cross section of $\gamma($ or $\psi$ ) is given by
$Q_{0} \frac{d^{3} \sigma}{d^{3} Q}=\int_{0}^{1} d x_{1} \int_{0}^{1} d x_{2} G_{q / H}\left(x_{1}\right) G_{\pi / H}\left(x_{2}\right) \theta\left(s^{\prime}-s_{t h}^{\prime}\right) Q_{0} \frac{d^{3} \sigma}{d^{3} Q}(p+q \rightarrow k+Q) 。$
We have neglected the transverse momentum distributions of incoming quark and meson. $G_{q / H}$ and $G_{\pi / H}$ arc the probabilities of finding the quark and the $\pi$ meson in the two hadrons with the fraction $\mathrm{x}_{1}$ and $\mathrm{x}_{2}$ of the longitudinal momenta. Then $s^{\prime}=x_{1} x_{2} s, s$ being the square of the total $c: m$. energy of the two incoming hadrons. In the quark-meson $c . m$. system we have

$$
\begin{align*}
& t^{\prime}=t_{\min }-2|q||Q|\left(1-\cos \theta_{c}\right)  \tag{24}\\
& u^{\prime}=2 m_{q}^{2}+m_{\pi}^{2}+Q^{2}-s^{\prime}-t^{\prime} \tag{25}
\end{align*}
$$

where $|\underset{\sim}{q}|,|\underset{\sim}{Q}|$, and $\theta_{c}$ are, respectively, the incoming quark three momentum, $\gamma(\psi)$ three momentum, and the scattering angle in this system. To produce experimentally observed rapid falloff in $p_{T}$, we multiply the integrand by a phenomenological factor $e^{-2 p} T$ as before. ${ }^{1}$ The G-functions are determined from the electroproduction and neutrino data. For example, for the up-quark in proton, we have

$$
\mathrm{G}_{\mathrm{q} / \mathrm{p}^{(\mathrm{x})}}=\frac{0.2(1-\mathrm{x})^{7}}{\mathrm{x}}+\frac{1.89(1-\mathrm{x})^{7}}{\sqrt{\mathrm{x}}}+ \begin{cases}90.2 \mathrm{x}^{3 / 2} \mathrm{e}^{-7.5 \mathrm{x}} & \mathrm{x} \leq 0.35  \tag{26}\\ 5(1-\mathrm{x})^{3} & \mathrm{x} \geq 0.35\end{cases}
$$

For $G_{\pi / p}$ one can consider various cases. At one extreme we can take it to be the same as the sea-quark distribution in proton, $\mathrm{i}_{\mathrm{o}} \mathrm{e}_{\mathrm{O}}$,

$$
\begin{equation*}
\mathrm{G}_{\pi / \mathrm{p}}(\mathrm{x})=0.2(1-\mathrm{x})^{5} / \mathrm{x} \tag{27}
\end{equation*}
$$

At the other extreme it can be taken to be the same as the one for the up-quark in the proton. It should be emphasized that since we are interested only in the
averaged angular distribution，and not the actual magnitude of the cross sec－ tion，such differences are not crucial here。 Actually we have verified numeri－ cally that this is so．The average values of $\alpha$ obtained using Eq。（23）are given below with values of $A, B, C$（for different charged states）in the bracket

$$
\begin{align*}
\alpha & =0.55\left\{\left(\frac{2}{3}, \frac{2}{3}, 0\right\}\right. \\
\alpha & =0.36\left\{\left(\frac{2}{3},-\frac{1}{3}, 1\right) \sqrt{2}\right\}  \tag{28}\\
\alpha & =0.48\left\{\left(\frac{1}{3},-\frac{2}{3}, 1\right) \sqrt{2}\right\} .
\end{align*}
$$

It is also interesting to note that part of the $u$－channel diagram is the Drell－Yan $q \bar{q}$ annihilation process．In fact it is amusing to note that，if we ignore gauge in－ variance and set s and t pole terms as zero，the value of $\alpha$ obtained is 0.35 ， which is extremely close to the Drell－Yan averaged value 0．36．

The above results show the range of variation with different charged states． With a full isospin treatment of the CIM model，some average results in this range can be expected．So CIM model does lead to values of $\alpha$ smaller than 1.

VI．CIM MODEL WITH MESON－MESON SCATTERING SUBPROCESS
In this model，which was considered by Chu and Koplik，${ }^{13}$ a meson is emitted by each hadron and they produce a heavy photon in analogy with the D－Y process．We do not consider this mechanism in detail but merely point out that if the mesons are spinless，the angular distribution will be similar to the case （IV）considered above．Next，we consider gluon models．

## VII．GLUON MODELS

Such a model for $\psi$ production has been considered by Ellis，Einhorn，and Quigg ${ }^{14}$ and Carlson and Suaya．${ }^{15}$ In this model，as shown in Fig．4，one gluon is emitted by each of the hadrons to produce an even charge conjugation state $\chi$ 。 This in turn decays into $\psi$ and $\gamma_{0} \psi$ can then decay into muons．These authors
did not consider implications for the angular distributions of the muons. It is clear that, if the intermediate state $\chi$ has a spin zero (as one of the $\chi$ states should have), the decay products of $\psi$ cannot have any correlation with the gluon axis or incident hadron-axis. Thus isotropic distribution will be obtained. If the intermediate state $\chi$ has a spin different from zero (e.g., 2), the muons will have some angular correlation with the gluon axis and hence the beam axis. On the basis of charmonium model it can be argued that the spin 2 intermediate state will be relatively suppressed as compared to the spin 0 intermediate state. Thus, if the value of $\alpha \approx 0$ is confirmed, it will lend support to the model with spin zero $\chi$ intermediate state. Of course the overall predictive power of the gluon models seems to be somewhat less than the quark-parton models.

In the present work we have discussed various theoretical models for angular distribution of muons produced in hadronic collisions. Different models differ considerably in their predictions. So good data on such distributions will provide a critical test for various models. Angular distribution data in both helicity and Gottfried-Jackson frames and in different $p_{\|}$and $p_{T}$ bins should turn out to be very useful for this purpose. Experimental data of this kind will have important consequences for the concepts of quark substructure of hadrons.

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Fig. 1
Drell-Yan model.


Fig 2
Constituent Interchange Model (CIM)。

(a)

(b)
(c)


Fig. 3
(a) s-channel quark pole for CIM.
(b) u-channel quark pole for CIM.
(c) t-channel meson pole for CIM.


Fig. 4

Gluon model with intermediate state $\chi$.


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