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CORRECTIONS TO THE TWO-GLUON ANNIHILATION RATES OF PARACHARMONIUM STATES*

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ABSTRACT

The effect of a color anomalous magnetic moment and of the finite size of the charmonium system on the annihilation rate of paracharmonium into two gluons is considered. Although these effects can reduce this rate by a factor of two from its lowest order value, there remains a large discrepancy for one of these states between the theoretical and experimental values.

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The reader who has been following the developments in quantum chromodynamics (QCD), and in particular the conventional charmonium interpretation of the psion family,[1,2] should be aware by now of its long-standing tradition of ups and downs as experimental data has accumulated. The step in R above 4 GeV, the presence of P waves, and even more detailed questions such as the E1 transition matrix elements are now consistent with predictions of the model.[3] Nonetheless, as expected, new puzzles have emerged.

One of these puzzles involves the paracharmonium states that correspond to the $\psi(3095)$ and $\psi'(3684)$. There are two candidates for these states, the X(2830) and the χ (3455), respectively. However, there are problems involved with identifying each of these states as the pseudoscalars of charmonium.

The X(2830) is observed[4] at DESY through the decay $\psi \to \gamma X$ and $X \to \gamma\gamma$ for which the branching ratio product BR $(\psi \to \gamma X)$ BR $(X \to \gamma\gamma)$ is given by $(1.2 \pm .5) \times 10^{-4}$. This state is naturally identified with the $^1S_0$ ground state of charmonium, the η_c. When we combine the DESY results with the SPEAR upper limit[5] of 1.7% for BR$(\psi \to \gamma X)$, we infer that BR$(X \to \gamma\gamma) > 7 \times 10^{-3}$. This is to be compared with the QCD prediction[6] of 1.4×10^{-3}. Thus there is about a factor of five disagreement. It should be noted that this branching ratio should be one of the most accurate charmonium predictions since it is independent of a knowledge of the wave function of the bound state.

A more serious problem concerns the $\chi(3455)$. Chanowitz and Gilman (C-G)[7] addressed the problem of quantum-number assignment for the four experimentally observed χ states between the ψ/J and the ψ'. The identification of the $\chi(3415)$ and the $\chi(3545)$ as corresponding to the $^3P_0$ and $^3P_2$ charmonium levels is strongly supported by the data. For the other two states the situation is more fluid. Nonetheless C-G conclude that the most plausible assignment is $\chi(3505)$: $^3P_1$ and

$\chi(3455)$: $^1S_0$, the last one being the η_c', a radial excitation of the $^1S_0$ (2830), the η_c. If this identification is correct, C-G conclude that an upper limit to the total decay width of the η_c' of about 70 keV can be inferred. This is in disagreement with the QCD result[1]

$$\Gamma(^1S_0 \to \text{hadrons}) = \frac{8\pi\alpha_s^2}{3m^2}|\psi(0)|^2 = \left(\frac{3}{2}\right)\left(\frac{\alpha_s}{\alpha}\right)^2 \Gamma\left(^3S_1 \to \ell\bar{\ell}\right) \tag{1}$$

that gives $\Gamma(\eta_c' \to \text{hadrons}) \sim 2\text{-}3$ MeV.

In light of the above-mentioned results, we have examined some possible corrections to the lowest order QCD results. One of the ideas that we have pursued follows from a suggestion due to Schnitzer,[8] who proposes a modification to the effective quark-gluon vertex of the form

$$\Gamma_\mu = \left(\gamma_\mu + \frac{i\kappa_c}{2m}\sigma_{\mu\nu}q^\nu\right)\frac{\lambda}{2} \quad . \tag{2}$$

He finds that with an anomalous color magnetic moment $\kappa_c \sim 1$ that the spacing of the P levels as well as the large S-state hyperfine splittings in charmonium can be roughly accounted for. The source of this anomalous Pauli term can be understood within the context of perturbation theory,[9] where it is found that the contribution of the graph shown in Fig. 1 to κ_c is infrared divergent and can be parametrized as

$$\kappa_c \simeq \frac{3\alpha_s}{4\pi}\ln\left(\frac{m^2}{q^2}\right) \quad . \tag{3}$$

In the small distance regime that is relevant for the two-gluon annihilation rate, the effective coupling constant $\alpha_s \sim 0.2$ and the natural cutoff in q^2 is given by R^{-2}, with R the radius of the bound state. This gives $\kappa_c \sim 0.2$. Since the usual Schwinger-like diagram is infrared finite, its contribution to κ_c can be neglected. Presumably in the large distance regime appropriate to Schnitzer's calculation, the running coupling constant becomes of order unity, justifying his ansatz.

Using Eq. (2) in the calculation of the matrix elements shown in Fig. 2, we obtain for the annihilation rate for ${}^1S_0$ into two gluons in the nonrelativistic limit the result

$$\Gamma = \Gamma^{(0)}[1 - 4\kappa_c + 5\kappa_c^2 - 2\kappa_c^3 + \tfrac{1}{4}\kappa_c^4] \quad , \tag{4}$$

where $\Gamma^{(0)}$ is given in eq. (1). Using the value $\kappa_c \sim 0.2$ in eq. (4) gives a reduction in the hadronic width of the ${}^1S_0$ levels of about 60%. It is clear that if this last equation is to be taken as an expansion in powers of α_s, there are many other graphs that will contribute to $0(\alpha_s^2)$.

Since only the annihilation rate into gluons is affected by this anomalous color moment coupling, the effect of this correction is to increase the ratio $\Gamma(\eta_c \to \gamma\gamma)/\Gamma(\eta_c \to \text{had})$. For $\kappa_c \sim 0.2$ this increases ratio by a factor of two, in closer agreement with experiment.

We have also examined the effect of the finite size of the charmonium system on the annihilation rate into two gluons. In the nonrelativistic limit the transition amplitude is given by

$$\int d^3p\, \phi(\vec{p})\mathcal{M}(p, P, k_1, k_2) \ , \tag{5}$$

where $\phi(\vec{p})$ is the Schrödinger wave function of the bound state and $\mathcal{M}(p, P, k_1, k_2)$ is the usual two-gluon annihilation amplitude of fig. 2, calculated in the center-of-mass system of the bound state $(P = (M, \vec{0}))$. As is well known, the zero-order result arises from expanding $\mathcal{M}$ in powers of $\vec{p}/m$ and retaining the lowest nonvanishing term. For S states this gives

$$\mathcal{M}(\vec{p} = 0, \vec{k})\int d^3p\, \phi(\vec{p}) \ , \tag{6}$$

where $\vec{k} = \vec{k}_1$ in the center of mass. This amplitude is proportional to the wave function at the origin in coordinate space. The resulting decay width is given in Eq. (1).

The two-gluon annihilation rate for P states is obtained from the O(p/m) term in the series. Naively, therefore, one would expect the widths of these P states to be small in comparison with the $^1S_0$ states. Calculations by Carlson and Suaya and by Barbieri, Gatto, and Kögerler[10] show this is not the case, but rather

$$\Gamma(^3P_0 \to 2\text{ gluons}) = \left(\frac{2}{3}\right) \cdot 9\,\frac{\alpha_s^2}{m^4}\left|\frac{d\phi}{dr}(0)\right|^2 \simeq 1.3\text{ MeV}$$

$$\Gamma(^3P_2 \to 2\text{ gluons}) = \frac{4}{15}\,\Gamma(^3P_0 \to 2\text{ gluons})\ .$$

These results are supported by the available experimental data.[11]

Since the orbital angular momentum barrier does not seem to significantly inhibit these hadronic decays, it makes sense to examine the effect of retaining terms of $O(p^2/m^2)$ on the decay of the S states. In a truly relativistic formulation based on the Bethe-Salpeter equation there would be additional p^2/m^2 corrections due to retardation and other relativistic effects that we have not included. The explicit form of the matrix element evaluated up to $O(p^2/m^2)$ is given by

$$\int d^3p\,\phi(\vec{p})\mathcal{M}(\vec{p}=0,\vec{k})\left[1+\frac{4(\vec{k}\cdot\vec{p})^2}{m^2M^2}-\frac{1}{2}\frac{\vec{p}^{\,2}}{m^2}\right]\ . \tag{7}$$

To estimate the size of this effect we have used Gaussian wave functions whose parameters have been determined from a variational calculation using a linear potential (see ref. [10]). We find that this correction amounts to a reduction of about 15% in the amplitude and therefore 30% in the rate. It should be noted that this correction does not affect the ratio $\Gamma(\eta_c \to 2\gamma)/\Gamma(\eta_c \to 2\text{gluons})$. Similar corrections to the widths of the P states are expected to be negligible $(O(p^3/m^3))$.

In summary, the following points should be made. The corrections to the $BR(\eta_c \to \gamma\gamma)$ due to an anomalous color magnetic moment push the theory and experiment toward closer agreement, but with the present 1.7% upper limit for $BR(\psi \to \gamma\eta_c)$ and the experimental uncertainties, the agreement is at best marginal. The color moment corrections can reduce the hadronic width for the η_c and η'_c by a factor of two, while the finite size of the charmonium system reduces the rate by about 30%. Neither of these effects alters the serious discrepancy between theory and experiment for the hadronic width of the η'_c candidate, the $\chi(3455)$. This discrepancy has led Harari[12] to propose that the $\chi(3455)$ is the $^1D_2$ state of charmonium. This interpretation also faces serious problems. The splitting between the $^3D_1(3774)$ and the $\chi(3554)$ would be larger than the S-wave hyperfine splitting in disagreement with the predictions of the Breit-Fermi Hamiltonian with a linear potential. Moreover, there are problems here too[6] with the small hadronic width of the χ.

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FIGURE CAPTIONS

1. Contribution to κ_c that is infrared divergent in second order. Wavy lines represent color gluons.
2. Feynman diagrams for the two-gluon (or two-photon) annihilation of a quark-antiquark bound state.

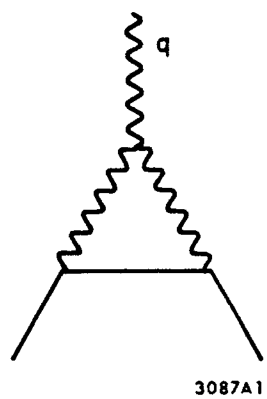

Fig. 1

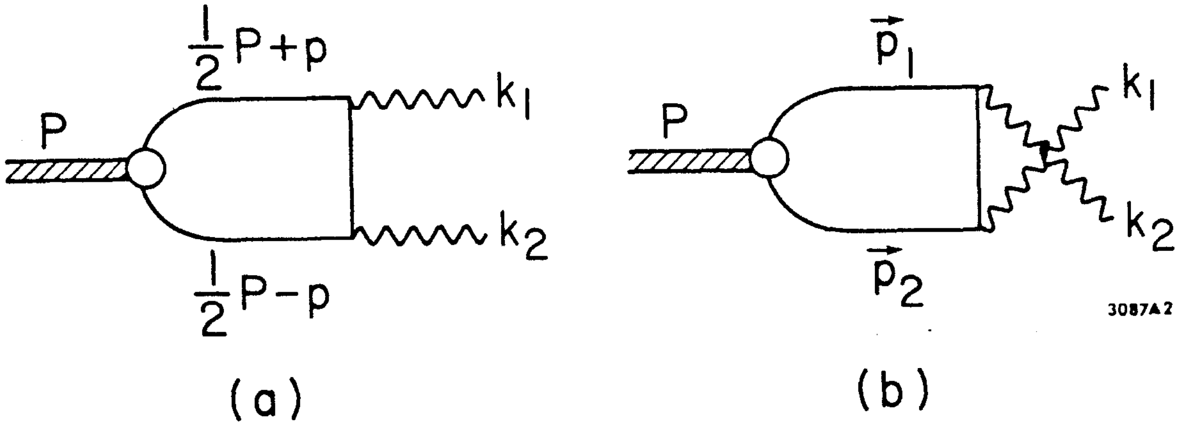

Fig. 2