# A TEST OF THE QUARK PARTON MODEL WITH DATA FROM ELECTROPRODUCTION OF PIONS* 

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#### Abstract

We derive some simple ratios obtained from $\pi^{ \pm}$yields from protons and neutrons. These ratios, when tested against data, confirm predictions of quark parton models.


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[^0]It has been pointed out by several authors ${ }^{1,2}$ that a measurement of the relative production of $\pi^{ \pm}$and $\pi^{\circ}$ mesons from lepton-nucleon collisions can be related to the parameters of quark parton models. The use of isotopic spin reflection decreases the number of unknowns to the point that one can measure these parameters. In this paper, we derive some new relations for relative yields and compare all these with recent experimental data on electroproduction.

Concentrating on only $\pi^{+}$and $\pi^{-}$, we write the following general expressions:

$$
\begin{align*}
& \pi_{p}^{+}=u_{p} q_{u}^{2} D_{u}^{+}+d_{p} q_{d}^{2} D_{d}^{+}+\beta  \tag{1a}\\
& \pi_{p}^{-}=u_{p} q_{u}^{2} D_{u}^{-}+d_{p} q_{d}^{2} D_{d}^{-}+\beta  \tag{1b}\\
& \pi_{n}^{+}=u_{n} q_{u}^{2} D_{u}^{+}+d_{n} q_{d}^{2} D_{d}^{+}+\beta  \tag{1c}\\
& \pi_{n}^{-}=u_{n} q_{u}^{2} D_{u}^{-}+d_{n} q_{d}^{2} D_{d}^{-}+\beta \tag{1d}
\end{align*}
$$

where $\pi_{i}^{ \pm} \equiv$ yield of $\pi^{ \pm}$mesons from a struck hadron, $i=p$ or $n$ (proton or neutron); ${ }^{3} u_{i}, d_{i} \equiv$ relative probability that a $u$ or d quark was present to strike; ${ }^{3}$ $q_{u(d)} \equiv$ charge of the $u(d)$ quark; $D_{u(d)}^{ \pm} \equiv$ the fragmentation function for quark $u(d)$ giving rise to a $\pi^{ \pm}$meson; $\beta \equiv$ contribution to the yield from the vacuum quarks. We assume, naturally, that this contribution is charge-and source-symmetric.
$\pi, u, D$, and $\beta$ can be differential in all experimental variables, which are, for the

1. electron: $E_{0}$-electron energy, $-q^{2}$-4-momentum transfer, and $\nu$ - electron energy loss (or a combination of these such as $\mathrm{x}=\mathrm{q}^{2} / 2 \mathrm{~m} \nu=1 / \omega$ or $\left.\mathrm{y}=\nu / \mathrm{E}_{0}\right)$.
2. hadron: in addition to the above and in the hadron center-of-mass system with the collision axis taken in the direction of the momentum of the
virtual photon, $\mathrm{z}=\mathrm{p}_{\ell} / \mathrm{p}_{\ell \text { max }}, \mathrm{p}_{\perp}^{2}=(\text { transverse momentum })^{2}$, and $\phi=$ azimuthal angle $(\phi=0$ is toward electron).

Isotopic spin reflection gives,

$$
\begin{array}{ll}
\mathrm{D}_{\mathrm{u}}^{+}=\mathrm{D}_{\mathrm{d}}^{-} \equiv \mathrm{D}_{1} & \mathrm{D}_{\mathrm{u}}^{-}=\mathrm{D}_{\mathrm{d}}^{+} \equiv \mathrm{D}_{2}  \tag{2}\\
\mathrm{u}_{\mathrm{p}}=\mathrm{d}_{\mathrm{n}} \equiv \mathrm{u}_{1} & u_{\mathrm{n}}=\mathrm{d}_{\mathrm{p}} \equiv \mathrm{u}_{2}
\end{array}
$$

We form the following sums and differences:

$$
\begin{align*}
& \mathrm{A}^{\prime}=\left(\pi_{\mathrm{p}}^{+}+\pi_{\mathrm{p}}^{-}\right)+\left(\pi_{\mathrm{n}}^{+}+\pi_{\mathrm{n}}^{-}\right)  \tag{3a}\\
& \mathrm{A}=\mathrm{A}^{\prime}-4 \beta  \tag{3b}\\
& \mathrm{~B}=\left(\pi_{\mathrm{p}}^{+}+\pi_{\mathrm{p}}^{-}\right)-\left(\pi_{\mathrm{n}}^{+}+\pi_{\mathrm{n}}^{-}\right)  \tag{3c}\\
& \mathrm{C}=\left(\pi_{\mathrm{p}}^{+}-\pi_{\mathrm{p}}^{-}\right)+\left(\pi_{\mathrm{n}}^{+}-\pi_{\mathrm{n}}^{-}\right)  \tag{3d}\\
& \mathrm{D}=\left(\pi_{\mathrm{p}}^{+}-\pi_{\mathrm{p}}^{-}\right)-\left(\pi_{\mathrm{n}}^{+}-\pi_{\mathrm{n}}^{-}\right) \tag{3e}
\end{align*}
$$

Note that the $\beta$ term cancels in the last three equations.
We also define a ratio,

$$
\begin{equation*}
R=A / A^{\prime} \tag{4}
\end{equation*}
$$

This ratio gives the fractional contribution to the $\pi$ yields from valence quarks. It could, in principle, be measured from the $y$ distributions in neutrino events, ${ }^{4,5}$ but this data is not yet sufficiently accurate for use.

Expressions A through D in terms of Eqs. (1) are all products of sums and differences of the numbers $q_{i}, u_{i}, D_{i}$; for example

$$
\begin{equation*}
B=\left(q_{u}^{2}-q_{d}^{2}\right)\left(u_{1}-u_{2}\right)\left(D_{1}+D_{2}\right) \tag{5}
\end{equation*}
$$

By taking appropriate ratios we can solve for three expressions relating measurable yields to functions of the fractions $q_{u}^{2} / q_{d}^{2}, u_{1} / u_{2}$, and $D_{1} / D_{2}$ alone. These
functions, however, include R.

$$
\begin{align*}
& \mathrm{r}_{1}^{\prime}=\mathrm{B} \cdot \mathrm{C} / \mathrm{A} \cdot \mathrm{D}=\mathrm{B} \cdot \mathrm{C} / \mathrm{R} \cdot \mathrm{~A}^{\prime} \cdot \mathrm{D}=\left(\mathrm{q}_{\mathrm{u}}^{2}-\mathrm{q}_{\mathrm{d}}^{2}\right)^{2} /\left(\mathrm{q}_{\mathrm{u}}^{2}+\mathrm{q}_{\mathrm{d}}^{2}\right)^{2}  \tag{6a}\\
& \mathrm{r}_{2}^{\prime}=\mathrm{B} \cdot \mathrm{D} / \mathrm{A} \cdot \mathrm{C}=\mathrm{B} \cdot \mathrm{D} / \mathrm{R} \cdot \mathrm{~A}^{\prime} \cdot \mathrm{C}=\left(\mathrm{u}_{1}-\mathrm{u}_{2}\right)^{2} /\left(\mathrm{u}_{1}+\mathrm{u}_{2}\right)^{2}  \tag{6b}\\
& \mathrm{r}_{3}^{\prime}=\mathrm{C} \cdot \mathrm{D} / \mathrm{A} \cdot \mathrm{~B}=\mathrm{C} \cdot \mathrm{D} / \mathrm{R} \cdot \mathrm{~A} \cdot \mathrm{D}=\left(\mathrm{D}_{1}-\mathrm{D}_{2}\right)^{2} /\left(\mathrm{D}_{1}+\mathrm{D}_{2}\right)^{2} \tag{6c}
\end{align*}
$$

We can also get three ratios each of which eliminates one of the three above fractions (Eq. (5)) and does not involve R.

$$
\begin{align*}
& r_{4}=D / C=\left[\left(u_{1}-u_{2}\right) /\left(u_{1}+u_{2}\right)\right]\left[\left(q_{u}^{2}+q_{d}^{2}\right) /\left(q_{u}^{2}-q_{d}^{2}\right)\right]  \tag{7a}\\
& r_{5}=D / B=\left[\left(q_{u}^{2}+q_{d}^{2}\right) /\left(q_{u}^{2}-q_{d}^{2}\right)\right]\left[\left(D_{1}-D_{2}\right) /\left(D_{1}+D_{2}\right)\right]  \tag{7b}\\
& r_{6}=B / C=\left[\left(u_{1}-u_{2}\right) /\left(u_{1}+u_{2}\right)\right]\left[\left(D_{1}+D_{2}\right) /\left(D_{1}-D_{2}\right)\right] \tag{7c}
\end{align*}
$$

Equation (7a) is a recasting of Eq. (4.11) in GRZ. ${ }^{2}$ A simple quark model would require $r_{1}^{\prime}$ to be independent of all variables, $r_{2}^{\prime}$ and $r_{4}$ to be dependent on $x$ and independent of the emitted hadron variables, and $r_{3}$ and $r_{5}$ to be independent of $x$. Equivalent expressions can be derived for neutrino interactions.

The experimental determination of $r_{i}$ requires good statistics and small systematic uncertainties. A recent electroproduction experiment allows their calculation. ${ }^{6}$ Particle identification was provided by a gas $\stackrel{V}{C}$ Cerenkov counter for a subset of the data $(\sim 1 / 6)$. We have corrected the total hadron samples by removing an average $K$ and $p$ contribution as measured in the identified subsample. For data with $x>.2$ there is insufficient data and we use the correction for smaller x ; we observe no appreciable change in this correction for x between .02 and .2. The error bars in the figures reflect statistical uncertainty as well as systematic uncertainties from beam normalization and K-p corrections. The data is summed over $p_{1}, .3<z<.85$, and $\phi$ excluding a region near the beam
pipe where background contamination might be possible. In Fig. 1 we plot $r_{1}\left(=\operatorname{Rr}_{\rightarrow-1}\right)$ and the value of $R_{1}=\left|q_{u} / q_{d}\right|$ (effective) derived from it assuming $R=1$. Our result is clearly compatible with $q_{u}=2 e / 3$ and $q_{d}=-e / 3$ for large $x .{ }^{7}$ Shown also is the deviation from 2 expected from vacuum quark contributions using two published parametrizations. ${ }^{8,9}$

From the above we feel justified in accepting the charge ratio as -2 and using this number in Eqs. (7a) and (7b) to find $u_{1} / u_{2}$ (i.e., $u_{v} / d_{v}$ ) and $\eta=\overline{\mathrm{D}}_{1} / \overline{\mathrm{D}}_{2}(0.3<\mathrm{z}<0.85)$. These are plotted in Figs. 2 and 3 respectively. We see that $u_{v} / d_{v}$ is not a constant with respect to $x$. It clearly falls below the expected average value of 2 at low x and is consistent with a number greater than 2 at large $\mathrm{x} .{ }^{10}$ We also note that $\eta$ is independent of $x$ even in the small x region where the vacuum quarks contribute appreciably to the cross section.

In summary, we have derived expressions which allow us to calculate the parameters of a general quark model directly from experimental data. We find that the ratio of up to down quark charge is consistent with -2 and that the ratio of the fragmentation functions, $D_{i}$, is independent of $x$. The majority to minority quark ratio is not a constant with x , falling well below 2 at small x .

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## FIGURE CAPTIONS

1. (a) A comparison of $r_{1}$, defined in the text, with data from Ref. 6. From $r_{1}$, we have also calculated the ratio, $R_{1}$. This is shown in (b). The solid line is the expectation from Ref. 9, the dashed line from Ref. 8.
2. A plot of $u_{v} / d_{v}\left(u_{1} / u_{2}\right)$ from the data of Ref. 6 and using Eq. (7a). We have set $q_{u}^{2} / q_{d}^{2}=4$. The solid (dashed) line is the expectation from Ref. $9(8)$.
3. A plot of $\eta$ as a function of $x . \eta$ is the ratio of $\bar{D}_{1}$ to $\bar{D}_{2}$, i.e., $D_{1}$ and $D_{2}$ averaged over z from $\mathrm{z}=0.3$ to 0.85 and over $\phi$ and $\mathrm{p}_{\perp}^{2}$.


Fig. 1


Fig. 2


Fig. 3


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