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A MECHANISM FOR NONCONSERVATION OF MUON NUMBER*

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ABSTRACT

We consider the possibility that muon number conservation is not a fundamental symmetry of nature. In simple $SU(2) \otimes U(1)$ gauge theories with several scalar boson doublets, muon number will still automatically be conserved by the intermediate vector boson interactions, but not by effects of virtual scalar bosons. The branching ratio for $\mu \to e\gamma$ is estimated to be of order $(\alpha/\pi)^3$. Other μ-e transition processes are also discussed.

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The stringent experimental upper limits on the rates of processes such as $\mu \to 3e$, $\mu + N \to e + N$, $\pi \to \nu_e + \mu$, $\nu_\mu + N \to e + N$, and $K_L \to \mu e$ appear to establish the separate conservation of muonic and electronic lepton numbers.[1] In this note we wish to explore the possibility that there is no such fundamental conservation law (or that it is spontaneously broken); that the above processes are automatically suppressed by the constraints imposed by a wide class of gauge theories; and that in fact these processes do occur, but at a level that is naturally superweak.[2]

In studying this problem, we work for definiteness in the familiar $SU(2) \otimes U(1)$ unified gauge theory. As usual, the leptons are taken to form two left-handed doublets with charges $(0, -1)$, and two right-handed singlets with charges -1.[3] The only scalar fields that can couple to these leptons are then doublets with charges $(+1, 0)$. For the moment, we impose no constraint on the numbers or coupling constants of these scalar doublets.

The vacuum expectation values of the neutral scalar bosons break $SU(2) \otimes U(1)$, and generate a 2×2 mass matrix connecting the two negative leptons, which in general is neither real nor diagonal. However, by subjecting the left- and right-handed leptons to independent unitary transformations, we can always reduce the charged-lepton mass matrix to a real diagonal form, without changing the form of the kinematic part $\bar{\psi}\gamma^\mu \partial_\mu \psi$ of the Lagrangian or the associated gauge interactions of leptons with photons and intermediate vector bosons.[4] The two charged leptons in this mass basis are identified as the observed muon and electron, and the neutrinos associated with e and μ in the two doublets are identified as ν_e and ν_μ, respectively. With these identifications, muon number is automatically conserved by all mass terms and gauge interaction terms in the Lagrangian.

The old analogy between muon number and strangeness is instructuve here. Strangeness is automatically conserved in the color gauge theory of strong interactions, for reasons much the same as applied above to muon number. Strangeness is not conserved in weak interactions, because the unitary operators needed to diagonalize the mass matrix of the charge -1/3 and charge +2/3 quarks are not the same. If ν_e and ν_μ are massless, then no unitary operator is needed to diagonalize their mass matrix, and the gauge interaction terms automatically conserve muon number.

But muon number is <u>not</u> automatically conserved by the interaction of leptons with the scalar bosons. In general, we can write these couplings in the form

$$\mathscr{L}_H = -g_1 \overline{\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L} \begin{pmatrix} \phi_1^+ \\ \phi_1^o \end{pmatrix} \mu_R^- - g_2 \overline{\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L} \begin{pmatrix} \phi_2^+ \\ \phi_2^o \end{pmatrix} \mu_R^-$$

$$-g_3 \overline{\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L} \begin{pmatrix} \phi_3^+ \\ \phi_3^o \end{pmatrix} e_R^- - g_4 \overline{\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L} \begin{pmatrix} \phi_4^+ \\ \phi_4^o \end{pmatrix} e_R^- + \text{H.C.} \qquad (1)$$

where ϕ_i are linear combinations, not necessarily independent, of an unknown number of scalar fields of definite mass. (A subscript L or R denotes multiplication with $(1-\gamma_5)/2$ or $(1+\gamma_5)/2$, respectively.[5]) Our choice of the lepton basis dictates that these linear combinations must be chosen so that

$$g_1 \langle\phi_1^o\rangle = m_\mu \qquad g_2\langle\phi_2^o\rangle = g_3\langle\phi_3^o\rangle = 0 \qquad g_4\langle\phi_4^o\rangle = m_e \qquad (2)$$

If the ϕ_i are all multiples of <u>one</u> elementary doublet, then (2) requires that $g_2=g_3=0$ so that muon number is conserved. But with more than one independent doublet, there is no reason why this should be the case. We may want to enforce strict masslessness for the electron, trusting to effects of some as yet unobserved weak interaction to produce the tiny electron mass. This requirement can be met quite naturally, by imposing some global symmetry which keeps either $(\nu_e, e^-)_L$ or e_R^- from having interactions with scalar bosons, so that either $g_2=g_4=0$ or $g_3=g_4=0$. However, there is no reason why g_2 and g_3 should both vanish. If g_2 or g_3 does not vanish, and if there is a $\phi_1^o - \phi_2^o$ or $\phi_1^o - \phi_3^o$ mixing, (either because muon number is not conserved at all or because muon conservation is spontaneously broken) then the effects of virtual scalar bosons will induce physical transitions between muons and electrons.

Let us consider how the process $\mu^- \rightarrow e^- + \gamma$ would arise in such a theory. The invariant matrix element is in general of the form[4] $(a + b\gamma_5)[\not{q}, \not{\epsilon}]$, where q and ϵ are the momentum and polarization four-vectors of the photon. The $\mu \rightarrow e\gamma$ rate is $(|a|^2 + |b|^2)m_\mu^3/2\pi$. (If $g_2=0$ or $g_3=0$, as suggested above, then $a=+b$ or $a=-b$, and the angular distribution for $\mu^\pm \rightarrow e^\pm\gamma$ is respectively $1 \pm \vec{s}_\mu \cdot \vec{p}_e/E_e$ or $1 \mp \vec{s}_\mu \cdot \vec{p}_e/E_e$.) In order to estimate a and b, we make the following assumptions: (I) The ϕ_i^+ and ϕ_i^o in Eq. (1) are linear combinations of a number of canonically normalized charged and neutral scalar fields of definite mass, with mixing coefficients that are all of order unity. Then $\langle\phi_1^o\rangle$ is of order $G_F^{-1/2}$, or 300 GeV. (II) The couplings g_2 and/or g_3 are of the same order as g_1, i.e., $m_\mu G_F^{1/2}$. (III) All gauge couplings are of order e, and all intermediate vector boson masses are of order $m_W \approx eG_F^{-1/2}$. (IV) The quartic scalar self-couplings are taken (somewhat arbitrarily) to be at most of order e^2, corresponding to scalar boson masses which are at most of order m_W. (V) Every loop in a Feynman diagram generates a factor of $(2\pi)^{-4}$ times the area $2\pi^2$ of a four-dimensional sphere, or $1/8\pi^2$.

It might at first be thought that the leading contributions to the $\mu \rightarrow e + \gamma$ decay would be the one-loop diagrams of the sort shown in Fig. 1. However, the scalar-boson couplings to leptons are so weak that these diagrams make a relatively small contribution

$$a \approx b \approx (8\pi^2)^{-1}\, e\left(m_\mu G_F^{1/2}\right)^2 m_\mu m_H^{-2} \qquad (3)$$

We find a larger contribution from two-loop graphs in which the scalar boson couples only once to leptons, the other coupling being to a heavy virtual particle—either a scalar boson or an intermediate vector boson. Typical graphs of this type are shown in Fig. 2. They all make a contribution of order

$$a \approx b \approx (8\pi^2)^{-2}\, e^3\left(m_\mu G_F^{1/2}\right)\left(e^2 G_F^{-1/2}\right) m_W^{-2} \qquad (4)$$

except that if $m_H \ll m_W$ then an extra factor of m_H^2/m_W^2 appears in some graphs, like Fig. 2c. The ratio of (3) and (4) is

$$\frac{1 \text{ loop}}{2 \text{ loops}} \approx \frac{m_\mu^2 G_F}{2\alpha^2}\left(\frac{m_W}{m_H}\right)^2 \approx \frac{2\pi}{\alpha}\left(\frac{m_\mu}{m_H}\right)^2 \qquad (5)$$

so two-loop terms dominate if $m_H > 3$ GeV. The rate of $\mu \rightarrow e + \gamma$ estimated from (4) is $\alpha^3 m_\mu^5 G_F^2/2^6\pi^6$. This is to be compared with the rate $m_\mu^5 G_F^2/192\pi^3$ of $\mu \rightarrow e + \nu + \bar{\nu}$; the branching ratio is roughly $3(\alpha/\pi)^3 \cong 4\cdot 10^{-8}$, close to the present upper limit[6] of 2.2×10^{-8}. Of course our calculation has been exceedingly rough; in particular the mixing among the Higgs bosons is unlikely to be precisely maximal, so the expected rate for $\mu \rightarrow e\gamma$ should be less than estimated here.

There are so many unknown parameters in the scalar masses and self-couplings that it does not seem worthwhile to attempt a detailed calculation of all the two-loop graphs. For illustration we consider only Fig. 2a, which dominates if $m_H \ll m_W$ and for at least some range of quartic self couplings. The W-loop can be approximated here by the amplitude[7] for scalar boson decay into

2γ. With $g_3 = g_4 = 0$, this graph yields a branching ratio

$$\frac{\mu \to e+\gamma}{\mu \to e+\nu+\bar{\nu}} = \frac{147}{16}\left(\frac{\alpha}{\pi}\right)^3 \left| \frac{g_2 \sum_i \xi_{2i} \langle \chi_i^o \rangle_0 \ln m_{Hi}^2}{g_1 \sum_i \xi_{1i} \langle \chi_i^o \rangle_0} \right|^2 \tag{6}$$

where ϕ_j^o is written as a sum of real canonically-normalized scalar fields χ_i^o of definite mass m_{Hi}, with coefficients ξ_{ji}. (Note that $\langle \phi_2^o \rangle_0 = \sum \xi_{2i} \langle \chi_i^o \rangle_0$ vanishes, so the numerator depends only on logarithms of mass ratios.) The coefficient of $(\alpha/\pi)^3$ is of order unity, confirming our previous rough estimate of the branching ratio.

What about other muon-nonconserving processes in this picture? The process $\mu \to e\gamma\gamma$ can be produced by graphs like Fig. 2a, in which the virtual photon is replaced by a second real one. However, this gives a rate which is less than the $\mu \to e\gamma$ rate by a factor of order $(\pi/\alpha)(m_\mu/m_H)^4$, so even if m_H is as small as 4 GeV, we expect $\mu \to e\gamma\gamma$ to be dominated by ordinary inner bremsstrahlung. The process $\mu \to 3e$ can go by a simple Higgs-exchange tree diagram. This gives a rate which is less than the $\mu \to e\gamma$ rate by a factor of order $(\pi/\alpha)^3 m_\mu^2 m_e^2/m_H^4$, so even with m_H as small as 4 GeV, we expect $\mu \to 3e$ to be dominated by ordinary Dalitz pairs from $\mu \to e\gamma$.

If the scalar fields ϕ_2^o or ϕ_3^o couple to quarks, there could also be semi-leptonic muon-nonconserving processes, such as $K_L \to \mu e$ or $K \to \pi\mu e$. However, we must take care not to allow neutral scalar-boson exchange to induce too large a K_L-K_S mass difference or $K_L \to 2\mu$ rate. It seems necessary to suppose[8] that only one scalar doublet couples to both $\bar{d}_R(u, d_c)_L$ and $\bar{s}_R(u, d_c)_L$; then neutral scalar couplings conserve strangeness, and $K_L \to \mu e$ and $K \to \pi\mu e$ are forbidden in lowest order. There will still be strangeness-conserving interactions—in particular, $\mu N \to eN$ will have an effective Fermi coupling of order

$m_N^* m_\mu G_F/m_H^2$, where m_N^* is that part of the nucleon mass which arises from "bare" quark masses rather than from the spontaneous breakdown of chiral $SU(2) \otimes SU(2)$. For a μ^- in a Bohr orbit around a nucleus $\mathcal{N}(A, Z)$, the coherent process $\mu^- \mathcal{N} \to e^- \mathcal{N}$ will be slower than the usual incoherent process $\mu^- \mathcal{N} \to \nu \mathcal{N}'$ by a factor of order $A^2 |F|^2 m_\mu^2 m_N^{*2}/Z m_H^4$, with F the elastic nuclear form factor for momentum transfer m_μ. The quantity $A^2|F|^2/Z$ reaches a maximum value of about 30 for nuclei near copper,[9] so if we (arbitrarily) set $m_N^* = 100$ MeV and $m_H = 30$ GeV, the ratio of $\mu^- \mathcal{N} \to e^- \mathcal{N}$ to $\mu^- \mathcal{N} \to \nu \mathcal{N}'$ would be of order 4×10^{-9}. The present upper limit[10] for this ratio in copper is 1.6×10^{-8}. A modest improvement in the precision of this experiment might yield interesting results.

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2. It would be disingenuous for us not to acknowledge that our interest in this question was kindled by an experiment now in progress at SIN (cf. "Physics Research in Switzerland," Catalog 1975 (Swiss Physical Society, Bern, 1975), p. 207), and by rumors of a positive signal. However, our considerations here do not depend on any assumptions about the eventual outcome of this experiment; indeed we believe that even if this measurement were to yield a null result, it would be worthwhile to push on to the greatest possible accuracy.
3. We choose here not to introduce unobserved leptons in the model. A very interesting $SU(2) \times U(1)$ model incorporating two new neutral leptons has recently been proposed by T. P. Cheng and L.-F. Li, to be published.
4. The argument here is identical to that used for purely electromagnetic interactions by G. Feinberg, P. Kabir, and S. Weinberg, Phys. Rev. Letters 3, 527 (1959).
5. We use the sign conventions of J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics (McGraw-Hill, New York, 1964).
6. S. Parker, H. L. Anderson, and C. Rey, Phys. Rev. B 133, 768 (1964); S. Korenchenko, B. Kostin, G. Mitsel-Makher, K. Nekrasov, and

V. Smirnov, Yad. Fiz. 13, 341 (1971) [Sov. J. Nucl. Phys. 13, 190 (1971)].

7. We use the 2γ decay amplitude calculated by J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B 106, 292 (1976). They show that the W-loop makes the dominant contribution, unless there exist charged fermions or scalar bosons with mass comparable to m_W, in which case these particles' contribution interferes destructively with the W contribution. Only the W-loop is included in Eq. (6).

8. S. L. Glashow and S. Weinberg, in preparation. Note that just the opposite assumption is being made here for leptons.

9. The process $\mu^- \mathcal{N} \rightarrow e^- \mathcal{N}$ was studied by S. Weinberg and G. Feinberg, Phys. Rev. Letters 3, 111 (1959).

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FIGURE CAPTIONS

1. A one-loop graph for $\mu \rightarrow e\gamma$.

2. Some two-loop graphs for $\mu \rightarrow e\gamma$.

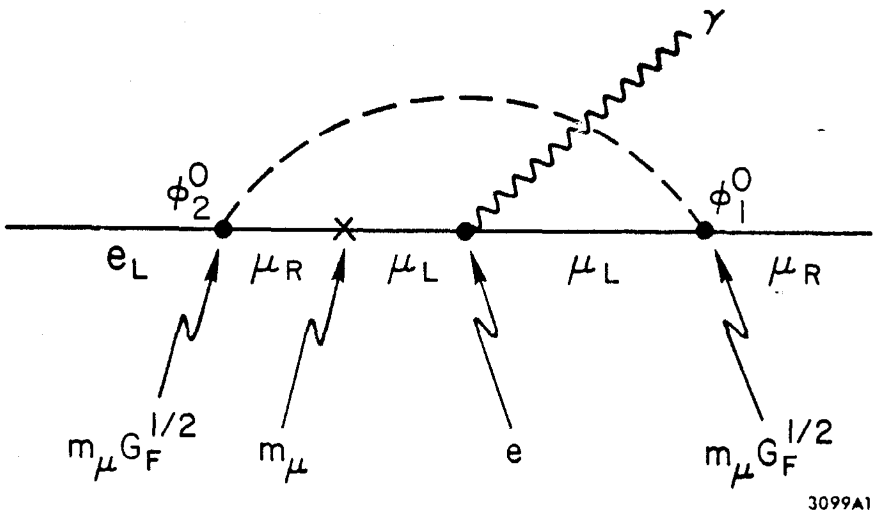

Fig. 1

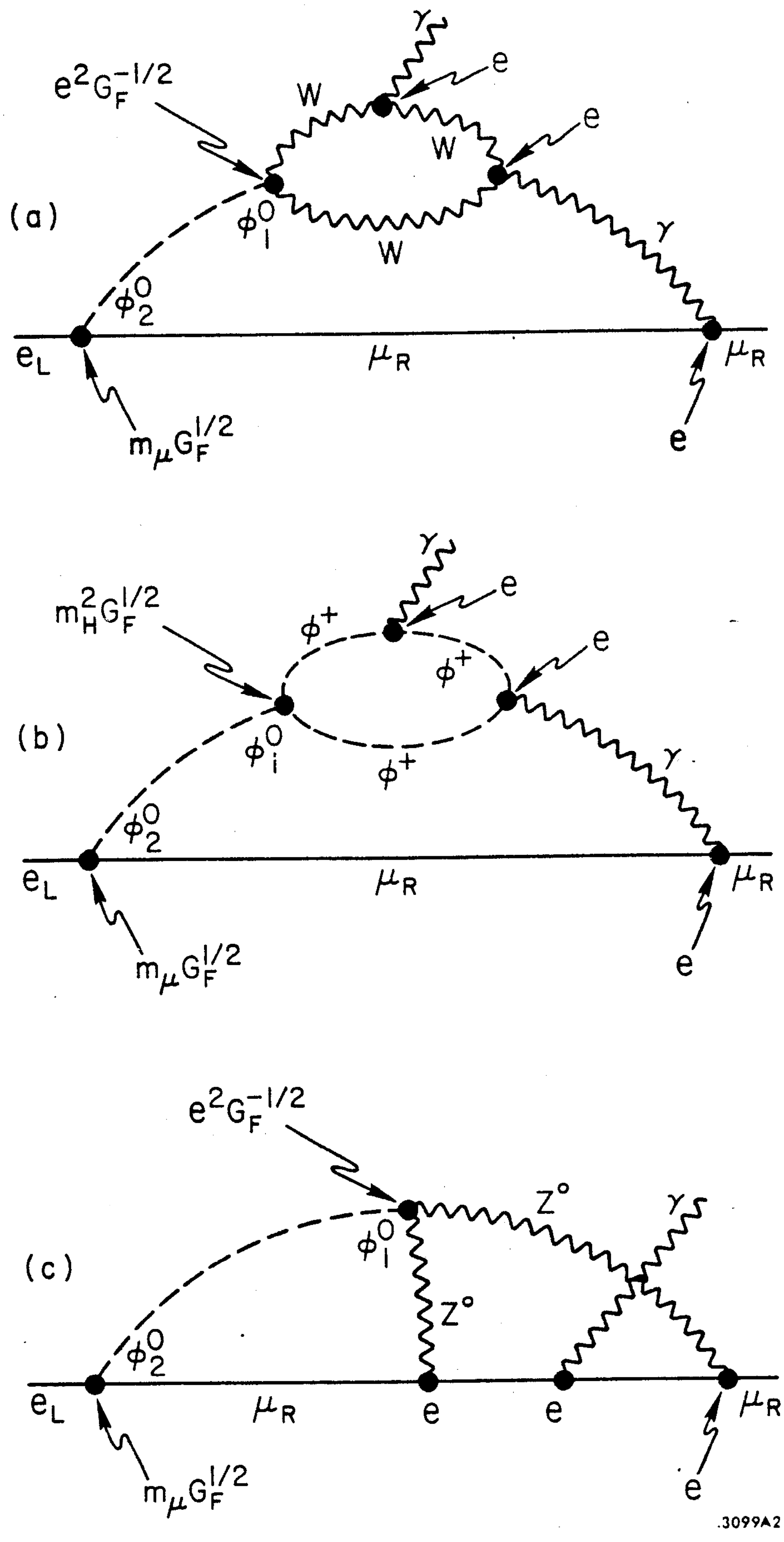
(a)
$e^2G_F^{-1/2}$
γ
e
W
W
e
W
ϕ_1^0
γ
ϕ_2^0
e_L
μ_R
μ_R
$m_\mu G_F^{1/2}$
e
(b)
$m_H^2G_F^{1/2}$
γ
e
ϕ^+
ϕ^+
e
ϕ_i^0
ϕ^+
γ
ϕ_2^0
e_L
μ_R
μ_R
$m_\mu G_F^{1/2}$
e
(c)
$e^2G_F^{-1/2}$
Z°
γ
ϕ_1^0
Z°
ϕ_2^0
e_L
μ_R
e
e
μ_R
$m_\mu G_F^{1/2}$
e
.3099A2

Fig. 2