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BOUND STATE SPECTROSCOPY OF SUPERHEAVY QUARKS*

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ABSTRACT

The transition to the exact positronium spectrum is studied as a function of quark mass. It is shown that, in the nonrelativistic potential approach, only for quarks as heavy as ~ 100 GeV might one expect to see the level structure originally attributed to charmonium. It is also noted that, in the hypothetical case that superheavy quarks exist, one may directly produce, via the weak current, their 1^{++} bound states in future e^+e^- experiments.

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By now it is clear that the very simple interpretation of the $c\bar{c}$ system as the QCD analogue of positronium with a pure Coulombic potential giving rise to the spectrum of the low-lying states in the ψ family has to be abandoned [1]. Instead, the confining potential plays a central role in determining the mass spectrum of these particles. In fact, many of the characteristics of the ψ spectroscopy can be successfully fitted by a linear potential added to the Coulomb potential [2]. As to the fine structure [3], the situation is still not well understood; specifically, the large hyperfine splittings between ψ and η_c and the large ratio for the splittings of the P-states $^3P_2 - {}^3P_1/{}^3P_1 - {}^3P_0$ constitute a major problem [4,5]. However, all this can be at least qualitatively understood by considering an effective potential which incorporates both, the confining long distance potential as well as the short distance Coulomb-like potential, and giving the quarks an anomalous magnetic color coupling to the gluons [6]. This effective quark-gluon magnetic moment should phenomenologically account for the strictly non-Abelian contribution to the nonrelativistic Breit Hamiltonian. Furthermore, this anomalous magnetic coupling should be attached to the confining potential, leaving the Coulomb interaction unaltered [6].

Yet, to test the underlying ideas on asymptotic freedom and color gauge theories, one would like to isolate the short distance one gluon exchange potential from the confining potential and check its consequences against experiment. Unfortunately, one should need quark masses as high as $m \gtrsim 15$ GeV in order to accomplish this goal. However, since the existence of additional flavors with heavier masses is being copiously discussed in the literature [7], it seems not to be an empty exercise to study this possibility, especially having experiments at PEP and PETRA in mind. Indeed, a very rich spectroscopy has already been predicted in case these heavy quarks exist [8].

In the context of the phenomenological approach just mentioned above we will study the transition to the "positronium" limit as a function of the quark mass. We therefore restrict ourselves to a nonrelativistic effective potential $V = -\frac{4}{3}\alpha_s \frac{1}{r} + Kr$, where all genuinely non-Abelian complications are supposed to be buried in the linear potential in the sense explained before.

Since, at masses of the order of 15 GeV, considerations about the ground state Bohr radius and the use of the parameters of ref. [2] give essentially Coulombic wave functions for the ground states of the $q\bar{q}$ system, we treat the linear potential as a perturbation (for masses as high as ~ 15 GeV) and find

$$R \equiv \left| \frac{\Delta E^{1s}_{\text{linear}}}{E^{1s}_{\text{Coulomb}}} \right| \xrightarrow[m \gg m_c]{} \frac{K}{\alpha_s^3 m^2} \tag{1}$$

where α_s is given by the familiar asymptotic freedom expression*

$$\alpha_s = \frac{\alpha_s(m_0)}{1 + \frac{21}{12\pi}\alpha_s(m_0)\ln\left(\frac{M}{m_0}\right)^2} \tag{2}$$

Here m_0 is a reference mass which we will take to be the $\psi(3.1)$ mass. M is the mass of a $q\bar{q}$ bound state and m is the q quark mass.

Taking $\alpha(m_\psi)$ to lie in the interval 0.2-0.4** and K to be 0.194 GeV^2, we then get from eq. (1) the results shown in table 1.

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*To be definite, we consider a typical model with 6 flavors.

**The original estimates of the ortho-para mass difference of ~ 30 MeV were obtained using a phenomenological $|\phi(0)|^2$ wave function at the origin and a value $\alpha_s \approx 0.2$-0.3 [1]. The use of the exact positronium fine structure spectrum, however, renders a factor ten smaller splitting for the same α_s values. Thus a $\alpha_s \approx 0.4$ will give the same values for the level spacings as the phenomenological $|\phi(0)|^2$ approach does. Since in the present work we do not want to underestimate the Coulomb potential below the commonly accepted limits, we will take the value of $\alpha_s \approx 0.4$ as the one beyond which we certainly do overestimate the 1/r potential.

Table 1

m (GeV)	R = 10%	R = 1%
$\alpha = 0.2$	24	99
$\alpha = 0.3$	15	69
$\alpha = 0.4$	10	55

We see that even at masses m ~ 10-25 GeV the linear potential is still 10% of the Coulomb potential. However formula (1) tells us that, apart from the logarithmic dependence in α_s, the limit is approached quadratically in mass. The actual variation with m is shown in fig. 1 for the particular value of $\alpha_s(m_\psi) = 0.4$.

In the region of quark masses under consideration ($m \gg m_c$) the linear potential can be considered as a perturbation of the 1/r potential so that the fine structure arising from the confining force will be negligible. On the other hand, since for a pure positronium atom the fine structure splittings grow linearly with the mass, there will be a region of masses where the Coulombic fine structure and the linear potential will compete, and, eventually, at even higher masses, the fine structure of the Coulomb interaction will overcome the contribution from the linear potential to the spectrum of levels. To see how this operates for the simplest case, we consider the distance of the c.o.g. of the 1S doublet to the unperturbed n=1 Bohr level and compare it to the $\Delta E^{1s}_{\text{linear}}$. We obtain

$$\left|\frac{\Delta E^{1s}_{\text{linear}}}{\Delta E^{1s}_{\text{fine}}}\right| \xrightarrow[m \gg m_c]{} \frac{243\,K}{496\,\alpha_s^5 m^2} \tag{3}$$

This behavior is depicted in fig. 2 for different values of α_s. For $\alpha_s = 0.4$ we see that the ratio in eq. (3) is about 25% for m ~ 75 GeV. Therefore, in order to see the true positronium-like spectrum one should go, at least, to masses of the order of 100 GeV and more.* This is really very far away from the original expectations [1]. The spectrum of lowest states for such a system is shown for completeness in fig. 3, although it has presently only an academic interest. This spectrum is exactly that of positronium except for the fact that we have subtracted out from actual positronium the contribution due to the one gluon annihilation processes which in the present case are suppressed by the usual color argument. This incidentally shifts the orthostate downwards and renders the degeneracy between the $^3S_1$ and $^1P_1$ states indicated in fig. 3. To get a feeling for the orders of magnitude of the level spacings we note that for a quark mass $m \sim 100$ GeV, the mass difference between the $n = 1$ $q\bar{q}$ state and its first radial excitation is approximately 650 MeV and the ortho-para mass splitting amounts to about 40 MeV.

Since for masses over 15 GeV the Coulomb potential should actually govern the bound state dynamics, we can use the hydrogen-like wave functions to compute the hadronic and leptonic widths of the lowest orthostate as a function of quark mass with the usual bound state calculation formulas. This gives

$$\Gamma_{e^+e^-}\left(^3S_1\right) = \frac{32}{27} e_q^2 \alpha^2 \alpha_s^3 m \tag{4}$$

$$\Gamma_{had}\left(^3S_1\right) = \frac{320(\pi^2-9)}{2187\pi} \alpha_s^6 m \tag{5}$$

where e_q stands for the charge of the new flavor. On fig. 4 we give both functions for the special choice $e_q = \frac{2}{3}$ and $\alpha_s(m_\psi) = 0.4$.

To conclude this brief discussion we will comment on the possibility of weak production of resonances in very high energy e^+e^- experiments. At energies of

*At this point we remind the reader of a recent speculation discussing the possibility of quark masses of the order of the W-boson mass [9].

PEP and PETRA one expects that weak neutral current effects will show up. Indeed at these energies the weak amplitudes may represent several percent of the electromagnetic ones. If in this energy region bound states of new heavy quarks happen to exist, one may hope to produce directly, via weak interaction, bound states other than the ones reached electromagnetically. Since the process we are describing is expected to conserve CP, the only resonances that could be reached via the neutral weak current would be those with even CP. Specifically, the 1^{++} $q\bar{q}$ bound state could be produced via the parity violating weak current. This production could, however, be substantially affected by the fact that in the nonrelativistic potential scheme the 1^{++} state would be a P wave state and consequently suppressed due to the vanishing wave function at the origin. A typically S-wave state like the 0^{-+}, on the other hand, although favored with a non-vanishing wave function, would be strictly CP forbidden.

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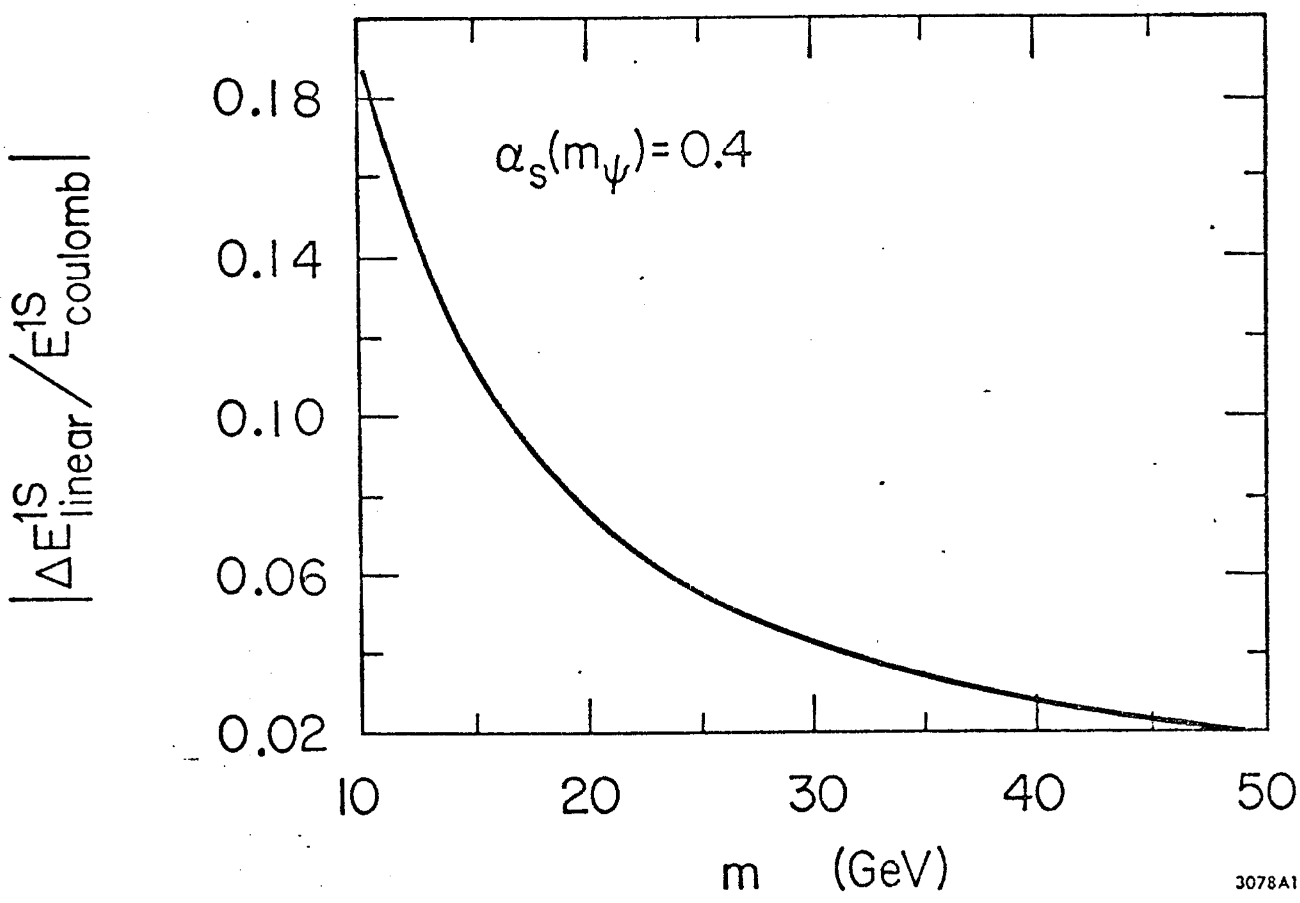
Figure Captions

1. The ratio $\left|\Delta E^{1s}_{\text{linear}}/E^{1s}_{\text{Coulomb}}\right|$ for the ground state 1s of the $q\bar{q}$ system as a function of quark mass m. The value of $\alpha_s(m_\psi) = 0.4$ has been taken.

2. The quantity $\left|\Delta E^{1s}_{\text{linear}}/\Delta E^{1s}_{\text{fine}}\right|$ as a function of quark mass for various values of $\alpha_s(m_\psi)$.

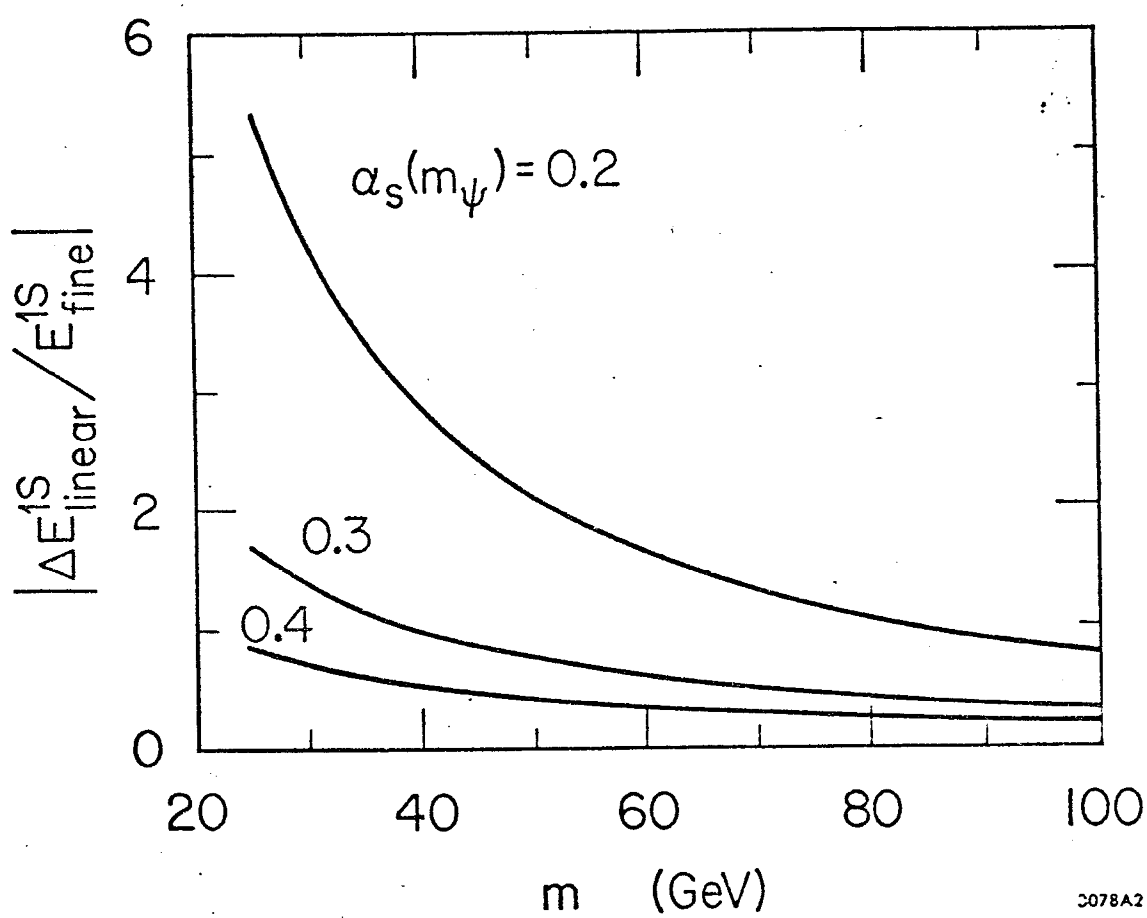
3. Positronium-like spectrum of the $q\bar{q}$ system as a function of mass. For each n, the origin of the energy axis is arbitrary and only differences between levels have physical significance.

4. Hadronic and leptonic widths of the n=1 $^3S_1$ bound state as a function of quark mass.



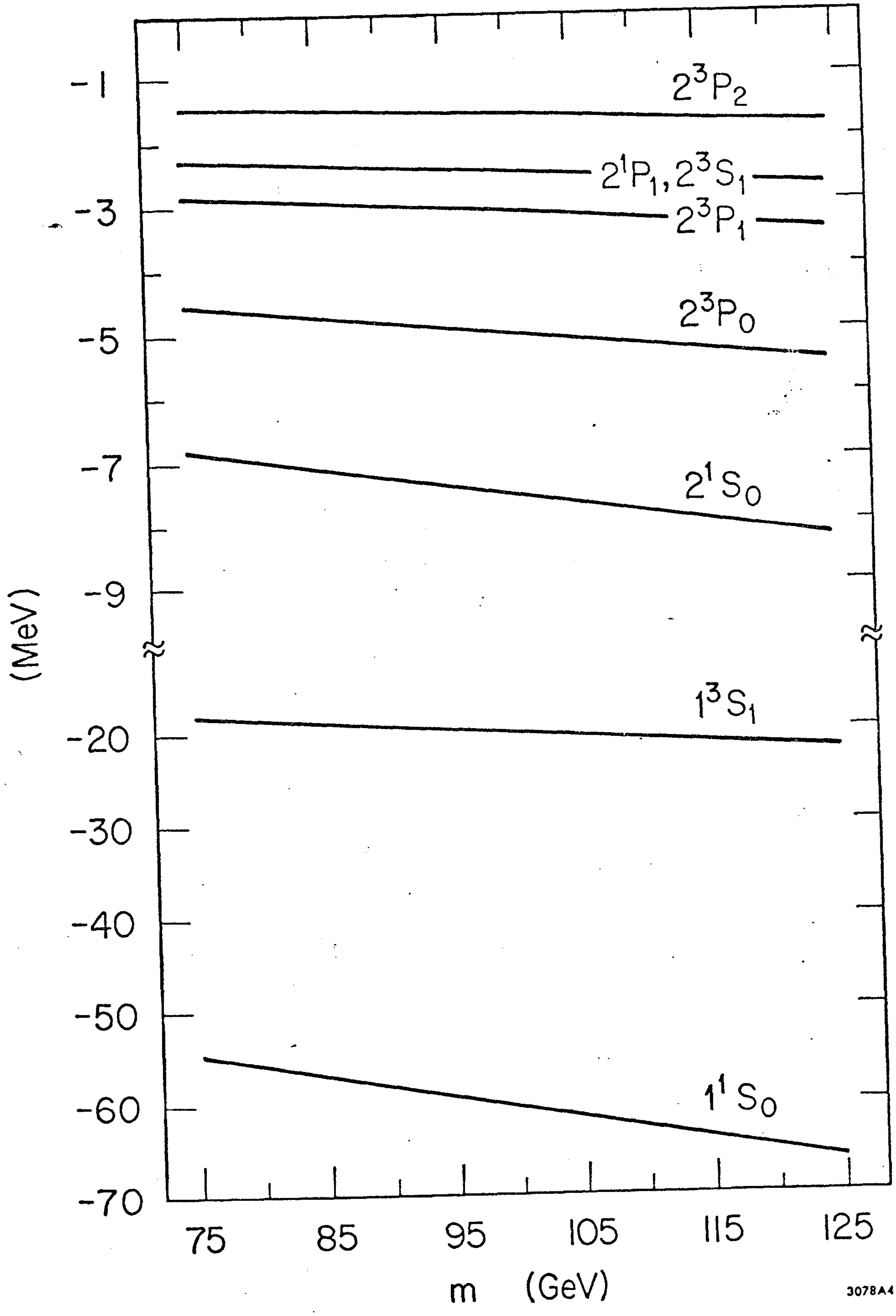
$\left|\Delta E^{1S}_{linear} / E^{1S}_{coulomb}\right|$
0.18
0.14
0.10
0.06
0.02
$\alpha_s(m_\psi) = 0.4$
10
20
30
40
50
m (GeV)
3078A1

Fig. 1



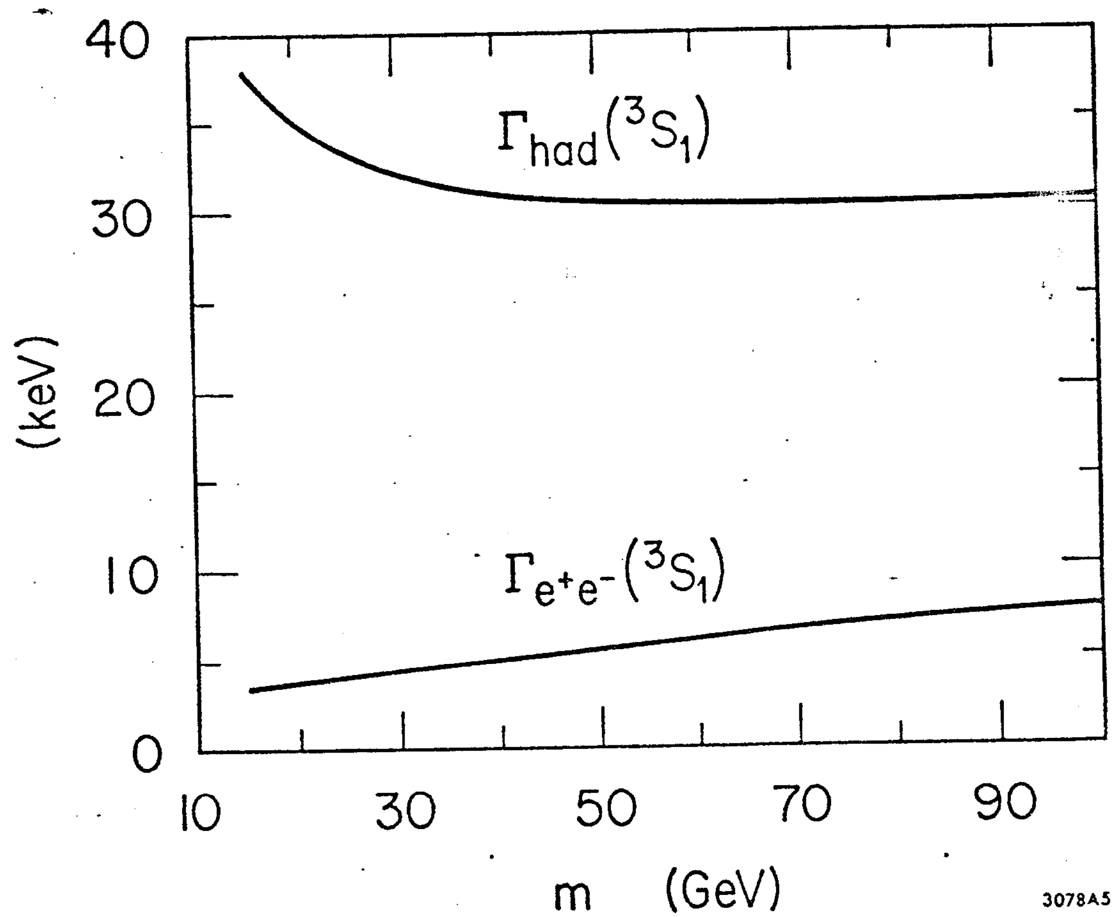
6
4
2
0
20
40
60
80
100
$\left|\Delta E^{1S}_{linear} / E^{1S}_{fine}\right|$
$\alpha_s(m_\psi) = 0.2$
0.3
0.4
m (GeV)
3078A2

Fig. 2



2^3P_2
$2^1P_1, 2^3S_1$
2^3P_1
2^3P_0
2^1S_0
1^3S_1
1^1S_0
-1
-3
-5
-7
-9
-20
-30
-40
-50
-60
-70
(MeV)
75
85
95
105
115
125
m (GeV)
3078A4

Fig. 3



40
30
20
10
0
(keV)
$\Gamma_{had}(^3S_1)$
$\Gamma_{e^+e^-}(^3S_1)$
10
30
50
70
90
m (GeV)
3078A5

Fig. 4