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GRAVITATIONAL RADIATION FROM SUPERNOVAS: QUALITATIVE PROPERTIES IN A SIMPLE MODEL*

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ABSTRACT

In a previous work we introduced a simple model to describe gravitational radiation from supernova explosions; the model consists of a point mass exploding into two point ejecta of equal mass moving back-to-back. In this work we study a variety of motions of the ejecta, corresponding to various scenarios for the supernova explosion, e.g., a single explosion, double explosion, implosion followed by explosion, etc. Our purpose is to illustrate the qualitative nature of the gravitational radiation field and the energy emitted as gravitational radiation.

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I. INTRODUCTION

In a previous work[1] we introduced a simple model to describe the gravitational radiation emitted in the most violent initial phase of a supernova explosion: a point mass explodes into two equal mass ejecta moving back-to-back with final velocity v.[2] We may summarize the results as follows, for nonrelativistic ejecta; with the linear motion of the ejecta described by $\xi(t)$ the gravitational radiation field $h_{\mu\nu}$ is "+" polarized,[3] with

$$h_{11} = \frac{-2\,G\,m\,\sin^2\theta}{r}\frac{d^2}{dt^2}(\xi^2) \quad , \tag{1.1}$$

other $h_{\mu\nu}$ equal to 0. The total energy emitted as a function of time is

$$\frac{dE}{d\Omega dt} = \frac{G m^2 \sin^4\theta}{4\pi}\left[\frac{d^3}{dt^3}(\xi^2)\right]^2 \quad , \tag{1.2}$$

and the energy spectrum is

$$\begin{aligned} \frac{dE}{d\Omega d\omega} &= \frac{G m^2 \sin^4\theta}{4\pi^2}\,\omega^2\,|f(\omega)|^2 \quad , \\ f(\omega) &= \text{Fourier transform of } \frac{d^2}{dt^2}(\xi^2) \quad . \end{aligned} \tag{1.3}$$

(We use units in which c=1.) Definitions of the various symbols are as follows: G is the gravitational constant, m is the mass of a single ejectum, r is the distance of the supernova from the observer on earth, θ is the angle between the line of motion of the ejecta and the earth-supernova line, and ω is the frequency of the radiation.

One example of an analytic function ξ describing a single pulse explosion was discussed in Ref. 1, and the quantities in Eqs. (1.1) to (1.3) evaluated analytically. Functions that lend themselves to such an analytic calculation are difficult to find, and do not allow one to study readily a variety of types of explosion, e.g., double pulse explosions, implosion-explosion pulses, etc. Thus we have developed a numerical procedure to study the qualitative features of more diverse types of explosion. Our procedure involves expressing the acceleration of the supernova ejecta, $d^2\xi/dt^2$, as a sum of 3 Gaussians of arbitrary amplitude, width, and position in time. The quantities in Eqs. (1.1) to (1.3) are then evaluated numerically and presented as graphs.[4] Clearly such a sum of Gaussians can qualitatively represent rather a large variety of ejecta motions.

II. CALCULATION

For convenience the Eqs. (1.1) to (1.3) can be recast with a dimensionless motion function ξ and dimensionless time variable τ by a simple rescaling: $t \rightarrow t/t_0 = \tau$, $\xi(t) \rightarrow vt_0\,\xi(\tau)$, $d\xi(t)/dt \rightarrow v d\xi(\tau)/d\tau$, with t_0 being the characteristic time scale of the explosion, and v the final ejecta velocity. Then

$$h_{11} = \frac{-2\,G\,m\,\sin^2\theta\,v^2}{r}\;\frac{d^2}{d\tau^2}\,(\xi^2) \tag{2.1}$$

$$\frac{dE}{d\Omega dt} = \frac{G\,m^2\,\sin^4\theta\,v^4}{4\pi t_0^2}\left[\frac{d^3}{d\tau^3}\,(\xi^2)\right]^2 \tag{2.2}$$

$$\frac{dE}{d\Omega d\omega} = \frac{G\,m^2\,\sin^4\theta\,v^4}{4\pi^2}\,w^2\,|f(w)|^2 \tag{2.3}$$

With this rescaling the final dimensionless velocity is $d\xi(\tau)/d\tau \rightarrow 1$; w is defined as $t_0\omega$, and it is easy to show that the asymptotic value of $d^2(\xi(\tau)^2)/d\tau^2$ is 2, and the limit of $w^2\,|f(w)|^2$ for $w \rightarrow 0$ is 4. The input acceleration function is written explicitly as

$$\frac{d^2\xi}{d\tau^2} = \sum_{i=1}^{3} A_i\, e^{-(\tau-\tau_i)^2/s_i^2} \tag{2.4}$$

In Figs. 1a and 1b the accelerations we have chosen are plotted. We have chosen in Fig. 1a a single pulse explosion (case 1), a double pulse explosion (case 2), and an implosion-explosion (case 3). In Fig. 1b we have chosen a sharper implosion-explosion (case 4), a broad and smooth triple explosion pulse (case 5), and finally an implosion-double explosion pulse (case 6).

In Figs. 2a and 2b the functions $d^2(\xi^2)/d\tau^2$, which are proportional to h_{11} in (2.1), are shown for the accelerations given in Figs. 1a and 1b. These are qualitatively similar, beginning at zero and ultimately rising to the asymptotic value 2. When an initial implosion is present (negative acceleration) a dip occurs before the final rise.

In Figs. 3a and 3b the functions $\left[d^3(\xi^2)/d\tau^3\right]^2$, proportional to the energy output in (2.2) are shown. The energy is typically emitted in several pulses with one being much larger than the others. The number of dips to zero in each case is equal to the number of maxima and minima in h_{11}.

In Figs. 4a and 4b the functions $w^2\,|f(w)|^2$, proportional to the energy spectrum in (2.3) are shown. All begin at a value 4 for w=0, and of course go to zero asymptotically. In general the more complex the acceleration the more wiggles and structure in the energy spectrum. In particular case 6 in Fig. 4b contains a surprising amount of structure in comparison to the others. In all cases the spectrum is broadband and extends to some characteristic frequency of order t_0^{-1} as discussed in Refs. 1 and 5. It should be emphasized that the absolute value of the energy for the various cases has no significance since only the final velocity is normalized and not the characteristic explosion time t_0. The area under the curves in time and frequency differ by a factor of π, as is evident in (2.2) and (2.3).

Acknowledgments

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2. S. Weinberg, Gravitation and Cosmology (Wiley, New York, 1972), Chapter 10.

3. C. W. Misner, K. Thorne, and J. A. Wheeler, Gravitation (Freeman, San Francisco, 1973), Part 8. R. J. Adler, M. Bazin, and M. M. Schiffer, Introduction to General Relativity (McGraw Hill, New York, 1975), 2nd ed., Section 9.5.

4. Calculations were carried out using the SLAC speakeasy package, with a Tektronix graphics output.

5. W. H. Press and K. Thorne, Ann. Rev. Astron. Astrophys. 10, 335 (1972).

FIGURE CAPTIONS

1. a) The acceleration of an ejectum expressed as a sum of Gaussians as in (2.4). Case 1 (——) is a single Gaussian with $A_1=1$, $\tau_1=5$, $s_1=1$. Case 2 (— · —) is a sum of two Gaussians with $A_1=1$, $\tau_1=3$, $s_1=1$, $A_2=1$, $\tau_2=5$, $s_2=1$. Case 3 (·····) is a sum of 2 Gaussians with $A_1=-1$, $\tau_1=4$, $s_1=1$, $A_2=3$, $\tau_2=6$, $s_2=1$. b) The same as a) but with different parameters. Case 4 (——) is a sum of 2 Gaussians with $A_1=-1$, $\tau_1=3$, $s_1=.5$, $A_2=2$, $\tau_2=6$, $s_2=1$. Case 5 (— · —) is a sum of 3 Gaussians with $A_1=1$, $\tau_1=3$, $s_1=1.5$, $A_2=1$, $\tau_2=5$, $s_2=1.5$, $A_3=1$, $\tau_3=7$, $s_3=1.5$. Case 6 (·····) is a sum of 3 Gaussians with $A_1=-1$, $\tau_1=3$, $s_1=.5$, $A_2=2$, $\tau_2=5$, $s_2=.5$, $A_3=2$, $\tau_3=7$, $s_3=.5$.
2. a) The functions $(\xi^2)''=d^2\xi^2(\tau)/d\tau^2$, proportional to the gravitational radiation field h_{11} in (2.1). The asymptotic values are 2. The cases 1 to 3 correspond to the accelerations shown in Fig. 1a. b) Same as a) but with cases 4 to 6 corresponding to the accelerations shown in Fig. 1b.
3. a) The functions $\left[(\xi^2)'''\right]^2=\left[d^3\xi^2(\tau)/d\tau^3\right]^2$, proportional to the energy output in Eq. (2.2). The cases are labeled as in Figs. 1a and 2a. b) Same as a), but with the 3 cases as shown in Figs. 1b and 2b.
4. a) The functions w^2 f(w), proportional to the energy spectrum in (2.3). The curves are labeled as in previous figures. b) Same as a) but for the cases 4 to 6.

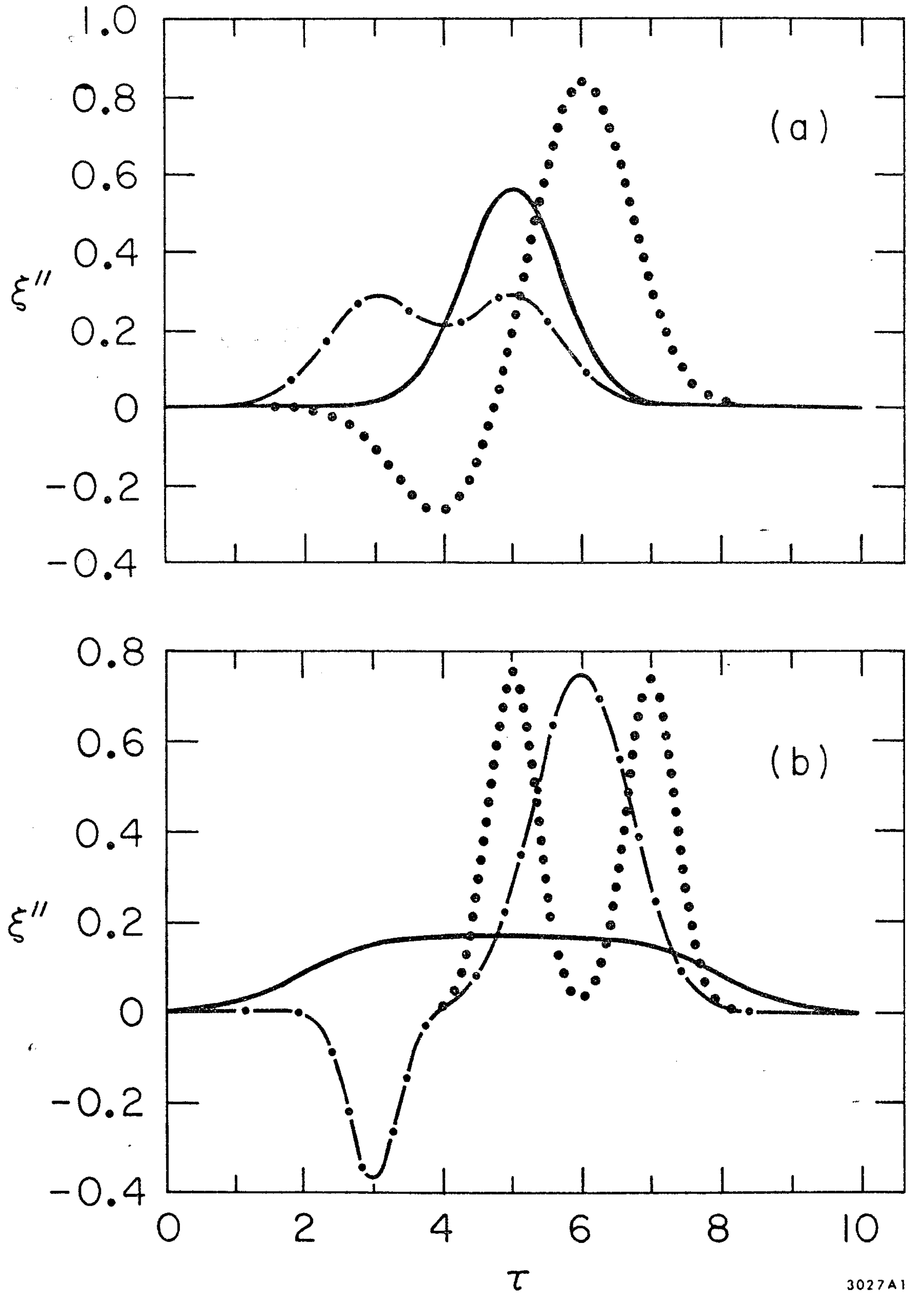


Fig. 1

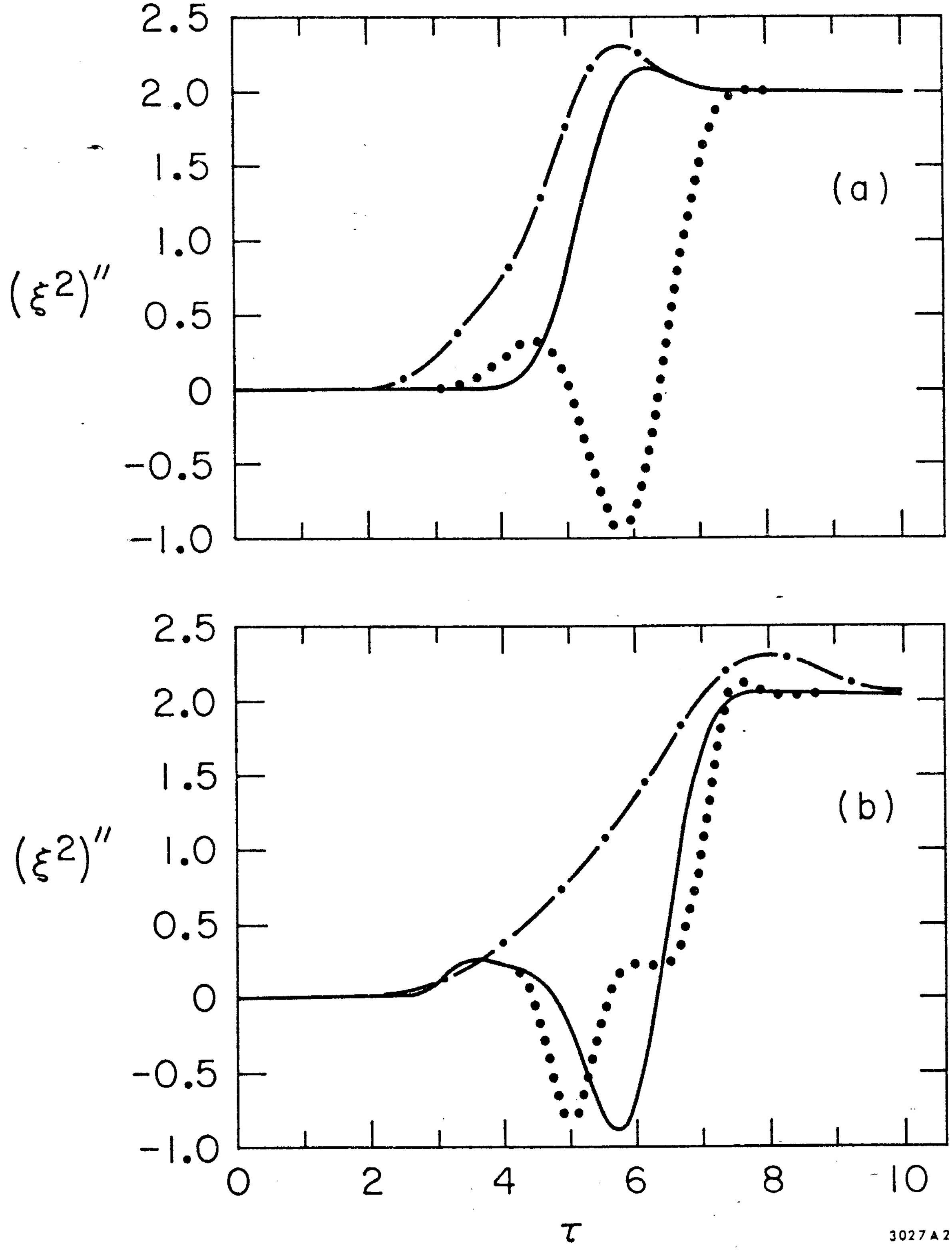
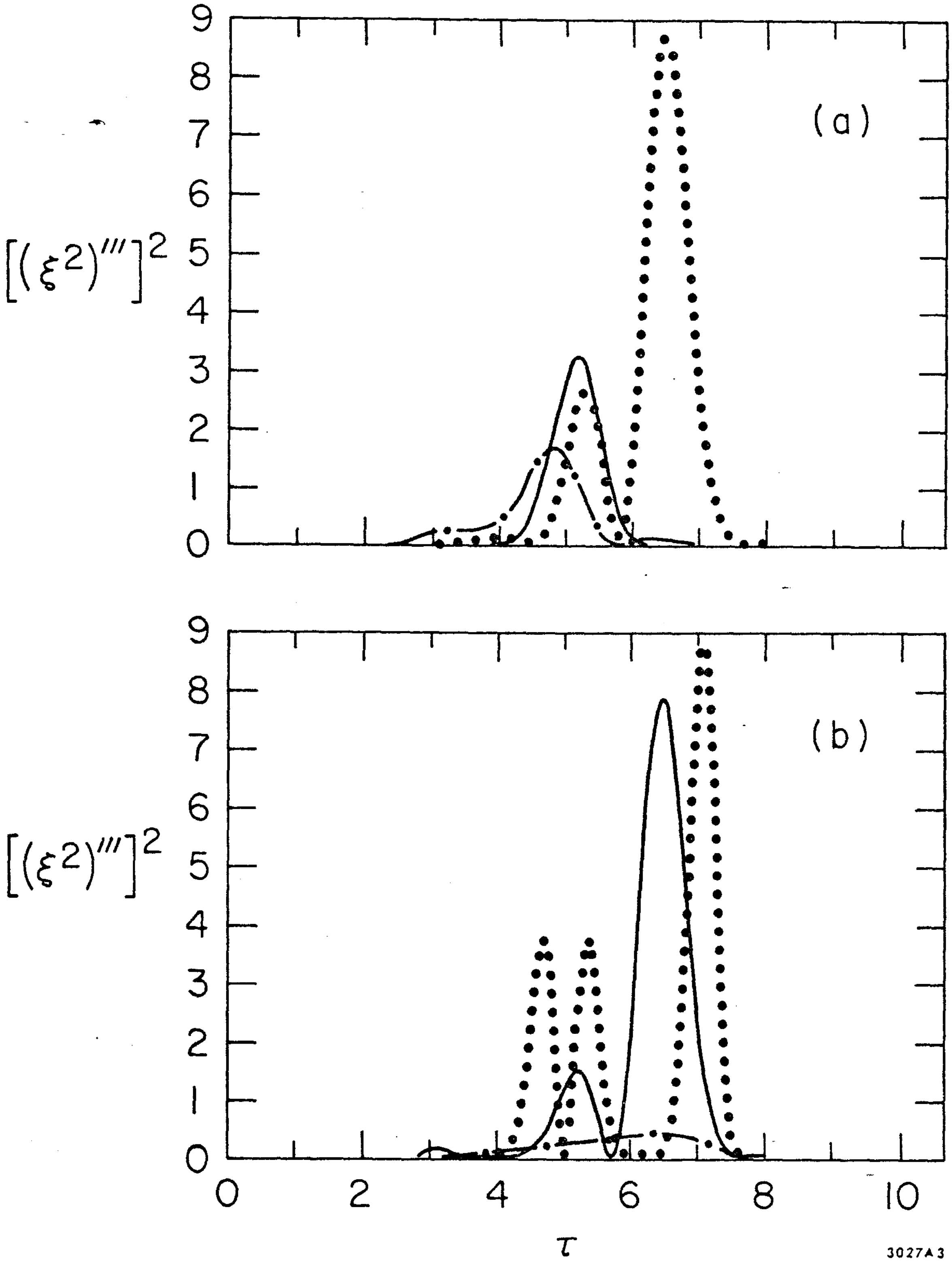


Fig. 2



(a)
(b)
$\left[(\xi^2)'''\right]^2$
$\left[(\xi^2)'''\right]^2$
0
1
2
3
4
5
6
7
8
9
0
2
4
6
8
10
τ
3027A3

Fig. 3

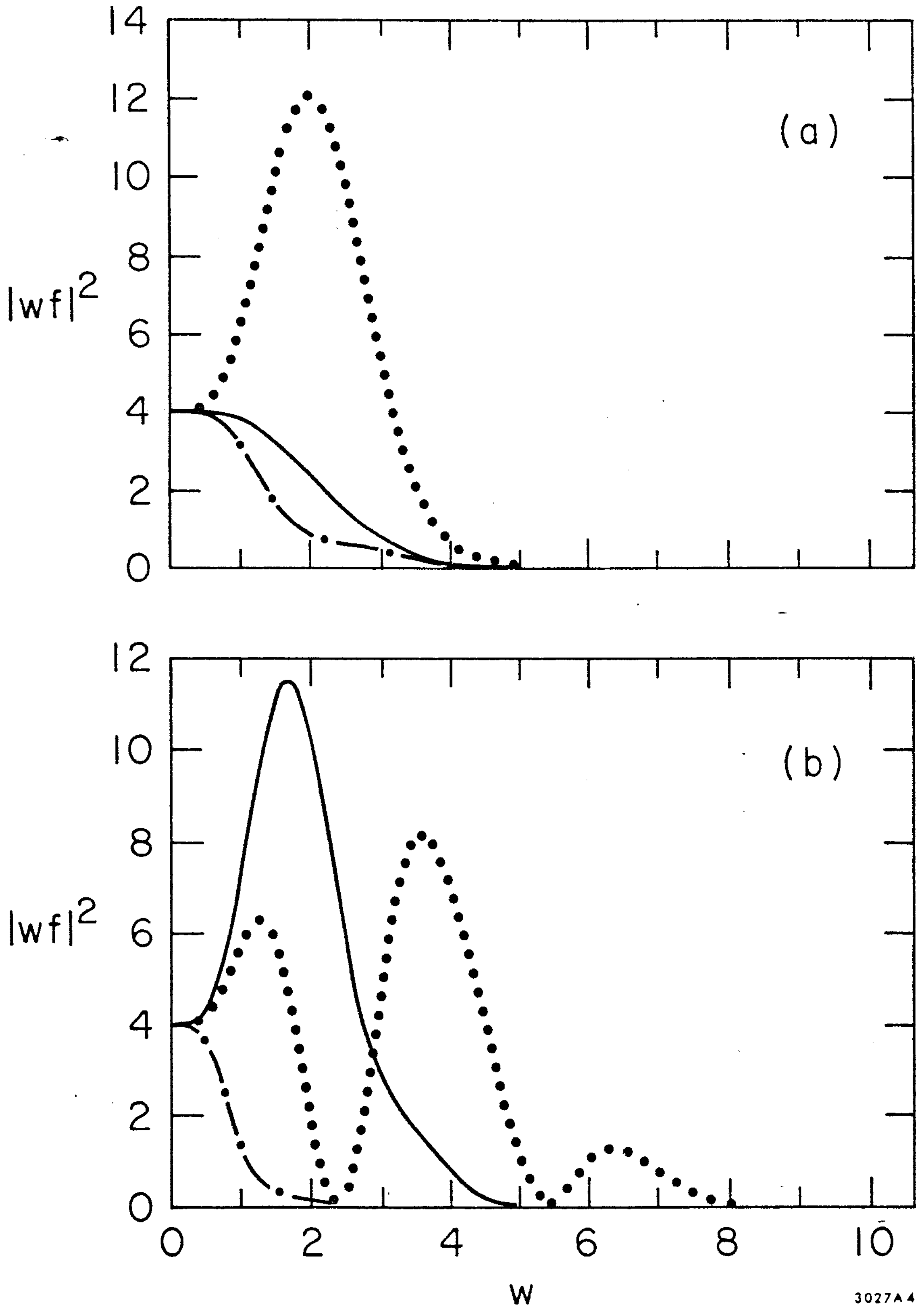


Fig. 4