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ON CONFINEMENT IN THE MASSIVE SCHWINGER MODEL*

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ABSTRACT

It is shown that the massive Schwinger model always confines, independent of the value of θ, the vacuum periodicity parameter. There are no half asymptotic particles for any value of θ, contrary to a recent claim.

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In a recent very interesting paper, S. Coleman[1] continued his investigation of the confinement (or charge screening) problem in the massive Schwinger model,[2] which he started in an earlier paper.[3] The most interesting of his results is that the physics of the model is a periodic function of an arbitrary angle θ, with the period 2π. It turns out that the appearance of this angle is very natural because of the degeneracy of the vacuum, and it takes care of the boundary condition at infinity.[4] The most surprising result of Coleman is that bound state physics explicitly depend on this vacuum periodicity parameter which is to be compared with the contrasting results related to the vacuum periodicity problems in 4 dimensions.[5]

In the following simple exercise we demonstrate that the appearance of θ explicitly in the bound state problems is due to his incorrect choice of the Hamiltonian to start with. Without trying to imitate Coleman's elegant style, let's review some general features of the Schwinger model. The model is defined by the Lagrangian density[2,3]

$$\mathscr{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\not{\partial} - e\not{A} - m)\psi \tag{1}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ .$$

To define the Hamiltonian we have to impose a gauge condition first. Following Coleman we choose the axial gauge:

$$A_1 = 0 \ . \tag{2}$$

The equation of motion for the other component A_0 becomes a constraint equation. Therefore, there are no true dynamical degrees of freedom associated with the electromagnetic (or gluon) field. This means that in one spatial dimension there are no photons, i.e., "lineland is a land of blind creatures":[6]

$$\partial_1^2 A_0 = -ej_0 \tag{3}$$

The general solution of Eq. (3), up to an arbitrary gauge parameter, is

$$A_0(x) = -e\partial_1^{-2} j_0 - Fx = -\frac{e}{2}\int dx' |x-x'| j_0(x') - Fx \tag{4}$$

and the corresponding electric field is

$$F_{01}(x) \equiv -\partial_1 A_0 = e\partial_1^{-1} j_0 + F = \frac{e}{2}\int dx' \epsilon(x-x') j_0(x') \quad . \tag{5}$$

Here F represents a constant c-number background field, and it will be clear from the discussions below that a nonzero F points out the existence of charges at the boundaries (customarily a dimensionless parameter θ is defined in terms of F as $\frac{F}{e} = \frac{\theta}{2\pi}$).

Hamiltonian density is easily found to be

$$\mathscr{H} = \bar{\psi}(i\gamma_1\partial_1 + m)\psi + [ej_0 A_0 - \tfrac{1}{2}(F_{01})^2] \quad . \tag{6}$$

If we do not pay much attention to the subtleties related to the surface terms, the interaction term in (6) can be reduced to that of Coleman by a simple integration by parts:

$$H_I = \int dx\, \tfrac{1}{2}(F_{01})^2 + (\text{S. T.}) \quad . \tag{7}$$

We shall show below that the surface term, $\text{S. T.} = \lim_{L\to\infty} [A_0 F_{01}]_{-L}^{L}$, which is cavalierly dropped by Coleman, plays a central role in his getting an incorrect result. Therefore we will try to avoid the integration by parts, and compute each term in H_I separately in a general and a special example. To check that our results are the correct ones, we shall then demonstrate that adding the surface terms explicitly in (7) eliminates all the F-dependent terms, and gives back the ordinary confining Coulomb potential. We shall substitute (4) and (5) in (6), and compute each term in (6) separately:

$$H_1 = \int dx (e j_0 A_0) = -eF \int dx\, x j_0(x) - \frac{e^2}{2} \iint dx\, dx' |x-x'| j_0(x) j_0(x') \tag{8}$$

$$H_2 = \int dx\, \frac{1}{2}(F_{01})^2 = \frac{eF}{2} \iint dx\, dx'\, \epsilon(x-x') j_0(x') - \frac{\epsilon^2}{4} \iint dx\, dx' |x-x'| j_0(x) j_0(x') + \frac{1}{2} F^2 L\,, \tag{9}$$

where in the second term we have used the identity

$$\int dx\, \epsilon(x-x')\epsilon(x-x'') = -2|x'-x''|\,.$$

Combining (8) and (9), we obtain for the full interaction term

$$H_I = -\frac{e^2}{4} \iint dx\, dx' |x-x'| j_0(x) j_0(x') - eF \left\{ \int dx\, x j_0(x) + \frac{1}{2} \iint dx\, dx' \epsilon(x-x') j_0(x') \right\} - \frac{1}{2} F^2 L\,. \tag{10}$$

For a box of length L (which will be taken to $L \to \infty$ in the end) we have the identity:

$$\int_{-L/2}^{L/2} dx\, \epsilon(x-x') = -2x' + \frac{L}{2}\left[\epsilon\left(\frac{L}{2}-x'\right) - \epsilon\left(\frac{L}{2}+x'\right)\right] \tag{11}$$

Using this identity we compute the second term in the curly bracket in (10) to be

$$-2\int dx'\, x' j_0(x') + \frac{L}{2}\int dx' j_0(x') \left[\epsilon\left(\frac{L}{2}-x'\right) - \epsilon\left(\frac{L}{2}+x'\right)\right].$$

For $L \to \infty$, the second term vanishes. This result holds even stronger for the systems with zero total charge, $Q \equiv \int dx\, j_0(x) = 0$, which we assume to be the case for the processes under consideration. So we see that the curly bracket in (10) is zero and, up to an "irrelevant" infinite constant, H_I is

$$H_I = -\frac{e^2}{4} \iint dx\, dx' |x-x'| j_0(x)\, j_0(x')\,, \tag{12}$$

a pure Coulomb interaction.

Now let us show that (7), supplemented with the surface term, gives just

this result also. First note that

$$
\begin{aligned}
A_0(+L/2) &= -\frac{FL}{2} + \frac{e}{2}\int dx'(x' - \frac{L}{2})j_0(x') = -\frac{1}{2}FL + \frac{1}{2}eD - \frac{1}{4}eLQ \\
A_0(-L/2) &= \frac{FL}{2} - \frac{e}{2}\int dx'(x' + \frac{L}{2})j_0(x') = \frac{1}{2}FL - \frac{1}{2}eD - \frac{1}{4}eLQ \\
F_{01}(L/2) &= F + \frac{e}{2}\int dx'\, \epsilon(\frac{L}{2} - x')j_0(x') = F + \frac{1}{2}eQ \\
F_{01}(-L/2) &= F + \frac{e}{2}\int dx'\, \epsilon(-\frac{L}{2} - x')j_0(x') = F - \frac{1}{2}eQ
\end{aligned}
\tag{13}
$$

where Q is the total charge, and D is the electric dipole moment of the system:

$$
Q = \int dx\, j_0(x)\,, \qquad D = \int dx\, x j_0(x) \quad . \tag{14}
$$

Using (13), we calculate the surface term to be

$$
A_0(L/2)\, F_{01}(L/2) - A_0(-L/2)\, F_{01}(-L/2) = eFD - \frac{1}{4}e^2LQ^2 - F^2L\ . \tag{15}
$$

Adding this to $\int dx\, \frac{1}{2}(F_{01})^2$, we get

$$
H_I = \frac{1}{2}F^2L - \frac{e^2}{4}\iint dx\, dx'\, |x-x'|j_0(x)j_0(x') + \frac{eF}{2}\iint dx\, dx'\, \epsilon(x-x')j_0(x')
$$

$$
+ eFD - \frac{1}{4}e^2LQ^2 - F^2L \tag{16}
$$

If we restrict ourselves to states of charge zero, that is to say, states for which

$$
\int dx\, j_0(x) = 0 \tag{17}
$$

and drop an "irrelevant" infinite constant $(-\frac{1}{2}F^2L)$ we get

$$
H_I = -\frac{e^2}{4}\iint dx\, dx'\, |x-x'|j_0(x)j_0(x') + eF\left[D + \frac{1}{2}\iint dx\, dx'\, \epsilon(x-x')j_0(x')\right]. \tag{18}
$$

But we have already shown above, in getting (12), that this curly bracket vanishes. This proves that the surface term, carefully computed, is nonvanishing, and cancels the F-dependent term in $\int dx\, \frac{1}{2}(F_{01})^2$, giving us back only the Coulomb interaction.

To amuse ourselves let us briefly repeat the formal arguments above on a very simple example, a quark-antiquark (or electron-positron) pair materialized from the vacuum, with a separation x between them. For the configuration quark being to the left of antiquark, the charge distribution is explicitly given by

$$j_0(x') = \delta(x' + \tfrac{x}{2}) - \delta(x' - \tfrac{x}{2}) \tag{19}$$

A_0 and F_{01} are easily found from (4) and (5) as

$$\begin{aligned} A_0(x';x) &= -Fx' - \tfrac{e}{2}(|x' + \tfrac{x}{2}| - |x' - \tfrac{x}{2}|) \\ F_{01}(x';x) &= F + \tfrac{e}{2}(\epsilon(x' + \tfrac{x}{2}) - \epsilon(x' - \tfrac{x}{2})) \end{aligned} \tag{20}$$

It is clear from (20) that when we put charges in the vacuum only the background field between the charges gets distorted. As the separation of the charges, $x \to \infty$,

$$\begin{aligned} A_0(x';x\to\infty) &\to -Fx' \\ F_{01}(x';x\to\infty) &\to F \end{aligned} \tag{21}$$

This proves the claim that a nonzero F means that there exist charges of magnitude F, at the boundaries (for this reason F is called the background field). Now a simple calculation gives

$$\begin{aligned} H_1 &= e^2|x| + eFx \\ H_2 &= \frac{e^2}{2}|x| + eFx + \tfrac{1}{2}F^2L \end{aligned} \tag{22}$$

and

$$H_I = H_1 - H_2 = \frac{e^2}{2}|x| \tag{23}$$

which is the pure Coulomb potential, between two oppositely charged particles. Again, at the risk of boring the expert, let us quickly check that $\int dx\, \tfrac{1}{2}(F_{01})^2$,

supplemented with the surface term, gives exactly (23):

$$S.T. = [A_0 F_{01}]_{-L}^{L}$$

First note from (20) that

$$\begin{aligned} A_0(L/2;x) &= -\tfrac{1}{2}FL - \frac{e}{2}x \\ A_0(-L/2;x) &= \tfrac{1}{2}FL + \frac{e}{2}x \\ F_{01}(\pm L/2) &= F \end{aligned} \tag{24}$$

Therefore

$$A_0(L/2)F_{01}(L/2) - A_0(-L/2)F_{01}(-L/2) = -F^2L - eFx \tag{25}$$

and finally

$$H_I = \int dx\, \tfrac{1}{2}(F_{01})^2 + (S.T.) = \left[\frac{e^2}{2}|x| + eFx + \tfrac{1}{2}F^2L\right] - [F^2L + eFx] = \frac{e^2}{2}|x|$$

Again, we dropped the irrelevant constant in the last step.

It is trivial that our results do not depend on the fact that quark is to the left of antiquark. The same arguments carry through for the other configuration also.

We have seen that a careful analysis shows no background field dependence in the $q\bar{q}$ potential. This is gratifying because the F-dependent term breaks translation invariance and P and C symmetries. Because stable bound states should be eigenstates of parity and charge conjugation, to circumvent this problem Coleman restricts himself to $\theta = 0$, i.e., he explicitly determines θ. Our result ((12) and (23)) also shows that the quark-antiquark potential is always binding, independent of the value of F, again to be compared with Coleman's result, which claims that for the particular value $F = e/2$ and for the -+ configuration (quark being to the right of antiquark) there is no long-range binding force between them.

The above investigation raises some doubts about the periodicity arguments in Ref. 1 also. We shall show below that using the full Hamiltonian doesn't change the fact that the physics of the massive Schwinger model is periodic; but it modifies the value of the period.

The interaction energy of the quark-antiquark pair is (for $x = L$) $\bar{H}=\frac{1}{2}e^2|L|$, and the vacuum energy is $H_V = \frac{1}{2}LF^2$ (since charges distort only the background field between themselves, we only look at the energy in this region). This process of spontaneous pair creation from vacuum is <u>not</u> favorable if

$$\Delta H = \tfrac{1}{2}e^2|L| - \tfrac{1}{2}F^2L > 0$$

or (26)

$$F^2 < e^2$$

So, if $|F| < e$, vacuum cannot create a $q\bar{q}$ pair. When $|F| > e$, spontaneous pair creation takes place, and this brings F down to $|F| < e$ again. Therefore, the physics of the model is periodic again, but this time with period $(-e,e)$ for F, or with period $(-\pi,\pi)$ for the variable $\theta = 4\pi\frac{F}{e}$.

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