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A SIMPLE MODEL FOR THE QUARK FRAGMENTATION FUNCTIONS SEEN IN DEEP INELASTIC PROCESSES*

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ABSTRACT

Using some simple assumptions, we present in this paper a calculation of the quark fragmentation functions seen in deep inelastic processes. The results provide evidence for the conjecture that both jets seen in high $p_\perp$ reactions at the ISR stem from similar parents. The calculated functions are in excellent agreement with the inclusive distributions seen in νp and $\bar{\nu} p$ scattering, and in particular explain the puzzling presence of leading π^-'s from the fragmentation of a leading up quark. The results are also in agreement with the charge ratios extracted from deep inelastic electron scattering.

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Introduction

Recent experiments at the ISR[1,2] on the production of high transverse momentum secondaries in proton-proton collisions point strongly in the direction of an event structure characterized by two jets whose source might be two-body scattering of proton constituents.[3] Because of the triggers involved in the experiments to date, the type of information on the two high $p_\perp$ jets in the final state is quite different. For the triggering jet, considerable information on particle composition exists. For the opposite side jet, data consist principally of inclusive distributions. In the scaling variable

$$x_e = \frac{p_\perp^{\text{hadron}}}{p_\perp^{\text{trigger}}} \quad (1) \quad ,$$

the inclusive distribution looks very similar to the distributions seen in various deep inelastic scattering reactions such as ep, μp, e^+e^-, νp and $\bar{\nu}$p scattering. In this paper, we examine within the framework of a simple model whether the particle composition seen on the trigger side might be compatible with the inclusive distribution seen on the opposite side. If so, the primaries produced could both be similar, for example quarks. In what follows we will assume they are in fact quarks and use the measured particle ratios as an ingredient in calculating meson final states in deep inelastic processes.

To do the calculation in the most clear-cut case, we will calculate the inclusive pion distributions for νp and $\bar{\nu}$p scattering in the quark fragmentation region.[4] These are described in terms of the fractional energy in the lab carried by the outgoing hadrons, $z = E_{\text{hadron}}/\nu$, where ν is the laboratory energy loss of the lepton. The distributions in the other deep inelastic processes can be described in terms of the neutrino distributions,[4,5,6] except for slight corrections due to strange-quark scattering. We will calculate the two quark fragmentation functions $D_u^{\pi^+}(z)$ and $D_u^{\pi^-}(z)$

describing particle production in $\nu p \rightarrow \mu^- + \pi^+$ (or π^-) + X. The distribution in x_e has been found to look very similar to the sum of these two functions.(1)

Details of The Model

1) We assume that the up quark fragments into a meson of type i and a quark of type j via a scale-invariant process:

$$u \rightarrow m_i + q_j ,$$

with the meson fractional energy spectrum given by $C_{m_i} f(z)$, where the C_{m_i} are constants. Only mesons containing the u quark in their wave function are produced. The quark structure of m_i is given by $m_i = (u\bar{q}_j)$. The function $f(z)$ is assumed to be independent of m_i. For simplicity we assume $f(z) = 1$. The C_{m_i} are then normalized so that:

$$\sum_{m_i} C_{m_i} = 1 .$$

The quark q_j then fragments similarly, and the process continues until the nth quark finally finds itself in the central region, where it recombines with the left over quarks from the target particle. It can be shown that summing over many fragmentation steps gives an inclusive meson spectrum

$$f_{inc}(z) = \frac{1}{z} ,$$

thus providing a specific model for the bremsstrahlung spectrum conjectured by Feynman.(4

2) For the meson spectrum, we choose the lightest pseudoscalar and vector mesons in the spirit of the discussion in Reference 7. Rather than assume SU(6) invariance, as used in that work, we take instead the particle ratios measured in inclusive high-transverse-momentum pp scattering wherever they are known.(1,8,9) Isospin invariance is then used to fill in the remaining numbers. The results for u quark fragmentation are:

Pseudoscalar	C_{m_i}	Vector	C_{m_i}
π^+	.20	ρ^+	.20[1]
π^o	.10	ρ^o	.10
η^o	.05[8]	ω^o	.10
η'	.05	K^{*+}	.10
K^+	.10[9]		

For the d quark, change $\pi^+ \to \pi^-$, $\rho^+ \to \rho^-$, $K^+ \to K^o$ and $K^{*+} \to K^{*o}$ in the Table. For the s quark we assume $C_{\bar{K}^o} = C_{K^-} = C_{\bar{K}^{o*}} = C_{K^{*-}} = .2$, $C_{\eta^o} = C_{\eta'} = .05$, $C_\phi = .1$.

The numbers for the neutral mesons relative to the charged mesons come from the wave function overlap with the u quark. The ideal quark mixing for η^o, η' would give:

$$\frac{C_{\eta^o}}{C_{\pi^o}} = 1/3 \; , \; \frac{C_{\eta'}}{C_{\pi^o}} = 2/3 \; ;$$

instead, we use the measured $\frac{\eta^o}{\pi^o}$ ratio.[8] If the s and u quarks behaved identically, the expected K^+/π^+ ratio would be 1; the departure from this value is assumed to be due to the violation of SU(3) invariance. The vector to pseudoscalar ratio may be an indication that only one helicity state of the ρ is dynamically allowed for a given quark helicity.[13]

The advantage of using particle ratios measured at high $p_\perp$ is that the very steep behavior of the cross section guarantees that the trigger is sampling the meson population near $z = 1$. In this region, meson production via resonance decay does not mask direct production ratios,[1] allowing us to extract the C_{m_i} in the table above. In the model described above, assuming an invariant high $p_\perp$ π^o production cross section $\sim 1/p_\perp^9$, we get a ratio of the high $p_\perp$ π^o to jet cross section $\simeq C_{\pi^o}/8 = 1/80$.

3) The quark mixture after the first fragmentation is given by the left over dissociated quark, leading to a quark population ratio of u:d:s = 1:1:.5. The unit u to d ratio then persists till the n^{th} quark recombines into a baryon. Thus the whole difference between $D_u^{\pi^+}$ and $D_u^{\pi^-}$ comes from the first meson generation. This gives:

$$\int (D_u^{\pi^+} - D_u^{\pi^-}) dz = C_{\pi^+} + C_{\rho^+} + \frac{2}{3} C_{K^{*+}} = .47 \ .$$

The z dependence of the π^+/π^- ratio thus depends critically on the sequential nature of the process. The inclusive distribution summed over π^+ and π^- depends only on the bremsstrahlung nature of this distribution.

At energies for which recently presented data exist, $\sqrt{s} \simeq 4$ GeV, only $\sim$ two meson generations are produced before scale invariance is violated (that is quark energy $\sim$ typical meson mass). These two meson generations, however, totally determine the large z distribution which is being studied here. The small z region ($\lesssim .3$) is further easily contaminated by spill-over from the central and target fragmentation regions at low energies.

Results

We have generated mesons by Monte Carlo methods according to the distributions outlined in the previous section. The energy ν is assumed to be asymptotically large. Resonances are allowed to decay, contributing to the number of pions and kaons seen. The contributions to $D_u^{\pi^+}(z)$ and $D_u^{\pi^-}(z)$ of the first, second, and n^{th} meson generations are calculated. The contributions of the first two generations are shown in Figure 1. It is seen that very leading π^-'s come half from first generation ρ^o decay products and half from direct production as a second generation meson. The calculations give a $\rho^o/(\pi^+ + \pi^-)$ inclusive ratio $\simeq .10$; similar to the value seen in pion-proton scattering.[10]

Figure 2 shows the comparison of the model with neutrino data.[11] The agreement with the data is excellent for $z \gtrsim .4$, which is the region where scale invariant fragmentation might hold at present energies. In particular, it gives good agreement with the π^- distribution, an unfavored product of the u quark. The fact that the absolute normalizations of the curves agree with the data is evidence that high mass resonances are not copiously produced, since they would reduce the C_{m_i} for the mesons considered here, while populating mainly small z with their multi-pion decay products. Figure 3 shows the comparison of $z\left[D_u^{\pi^+}(z) + D_u^{\pi^-}(z)\right]$ with the distributions seen in other deep inelastic processes.[6] The agreement is again good for

$z > .4$. We note that deviations are expected at the 20% level due to the presence of misidentified kaons and protons in the data samples.[6]

For comparison with other experiments, we can calculate particle ratios over the region: $0.4 \leq z \leq 0.9$. Defining:

$$R\left(\frac{\pi^+}{m_i}\right) = \frac{\int_{.4}^{.9} D_u^{\pi^+}(z)\,dz}{\int_{.4}^{.9} D_u^{m_i}(z)\,dz}$$

we find, $R\left(\frac{\pi^+}{\pi^-}\right) = 3.3$, $R\left(\frac{\pi^+}{K^+}\right) = 3.0$, and $R\left(\frac{\pi^+}{K^-}\right) = 15.6$. Note the leading K^-'s are second-generation products, coming from left-over strange quarks. The value of $R\left(\frac{\pi^+}{\pi^-}\right)$ above is in excellent agreement with the result of Reference 5. The other ratios have yet to be measured.

Concluding Remarks

In terms of a simple sequential quark-dressing model, we get a good fit to the quark fragmentation functions seen in deep inelastic processes. The model uses as input the high $p_\perp$ final state particle ratios observed in proton-proton scattering. While our model predicts meson distributions using a spectral distribution $f_{inc}(z) = \frac{1}{z}$, an alternative explanation of the fragmentation functions in terms of quark counting rules has led to a result:[12]

$$f_{inc}(z) \sim \frac{(1-z)^2}{z} .$$

If this function is applied to the directly produced mesons, and if meson production in deep inelastic processes has as rich a spectrum as indicated in the high $p_\perp$ data, then this model is unlikely to give a good description of the data over any substantial range in z. For example, we show in Fig. 3 the result of generating mesons according to $f_{m_i} = 3C_{m_i}\frac{(1-z)^2}{z}$, with the C_{m_i} given in the table earlier. Whether such a rich spectrum is indeed present, remains to be proven experimentally.

Finally, since the vector to pseudoscalar ratio is not accurately known, we have repeated the calculations outlined above using a ratio of three instead of one. The resulting quark fragmentation functions agree equally well with the neutrino data, differing by $\sim$20% from the curves shown in Figure 2 over the range $0.3 \leq z \leq 0.8$. The value of $R(\frac{\pi^+}{\pi^-})$ is changed from 3.3 to 2.7. The principal change is that $D_u^{\pi^+}(z)$ approaches .1 instead of .2 as $z \rightarrow 1$. The agreement of the present model with presently available data is therefore not very sensitive to the poorly known vector to pseudoscalar ratio.

Acknowledgement

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References

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10) F. Winkelmann et al., Phys. Lett. 56B, 101 (1975); M. Deutschmann, et al., Nucl. Phys. B103, 426 (1976).

11) B.P. Roe, "Bubble Chamber Neutrino Results from Fermilab", Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California.

12) See S. D. Ellis, M. Jacob, and P. V. Landshoff, Nucl. Phys. B108, 93 (1976) for an alternate calculation of the high-$p_\perp$ inclusive spectrum and further references on this approach.

13) A recent calculation of G. R. Farrar, to be published, in fact predicts a ρ^0/π^0 ratio of one, with the ρ^0 always longitudinally polarized. An alternative explanation might be that $q + \bar{q} \to$ meson always produces very preferentially the lightest mass mesons possible. When the q, $\bar{q}$ helicities are opposite, the lightest particles are pseudoscalars; when they are aligned, the lightest are the vector particles. This gives a unit vector-to-pseudoscalar ratio. In this paper, in the absence of more information, we have assumed that all vector particles are produced unpolarized.

Figure Captions

1) First generation meson contribution to $zD_u^{\pi^+}$ and $zD_u^{\pi^-}$. For $zD_u^{\pi^-}$, the part of the distribution due to ρ^o and ω^o decay products is separately shown. Contribution of the second meson generation, which is equal for both $zD_u^{\pi^+}$ and $zD_u^{\pi^-}$, is also shown.

2) Comparison of the calculated quark fragmentation functions with neutrino and anti-neutrino data. In the anti-neutrino case, the equality of $D_d^{\pi^-}(z)$ and $D_u^{\pi^+}(z)$ is used so that data on negatively charged particles can be used in both cases.

3) Comparison of calculated value of $z(D_u^{\pi^+} + D_u^{\pi^-})$ with data from other deep inelastic reactions. The muon scattering data cover three bins in the Bloom-Gilman scaling variable ω'. These data contain a contribution from the production of elastic ρ^o mesons, shown as the dashed curve, which should be subtracted before comparing to the predicted curve. The dashed-dotted curve, described in text, is calculated using a value $f_{m_i} = 3C_{m_i}\frac{(1-z)^2}{z}$.

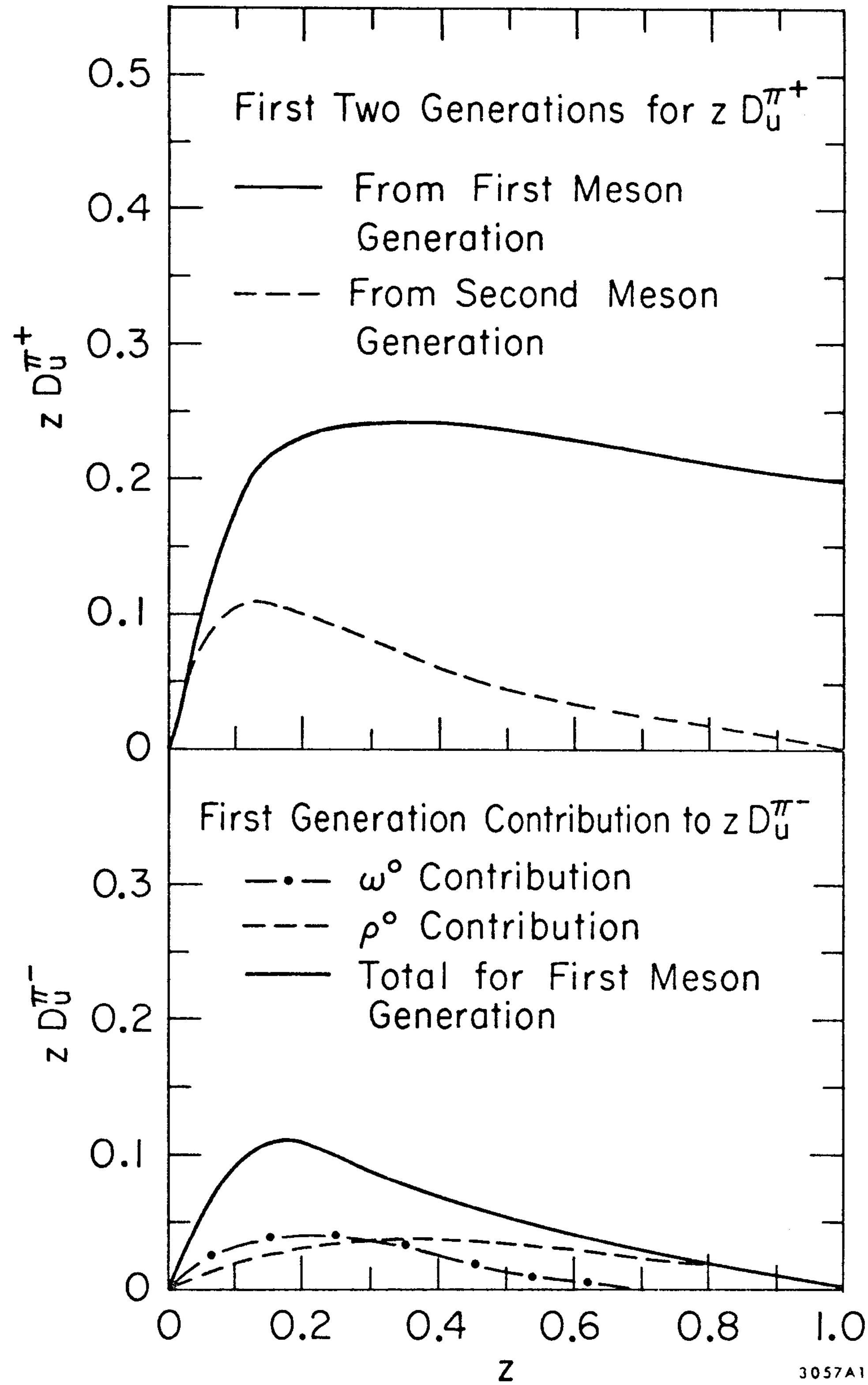

First Two Generations for $z\,D_u^{\pi^+}$
From First Meson Generation
From Second Meson Generation
$z\,D_u^{\pi^+}$
0.5
0.4
0.3
0.2
0.1
0
First Generation Contribution to $z\,D_u^{\pi^-}$
ω° Contribution
ρ° Contribution
Total for First Meson Generation
$z\,D_u^{\pi^-}$
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1.0
z
3057A1

Fig. 1

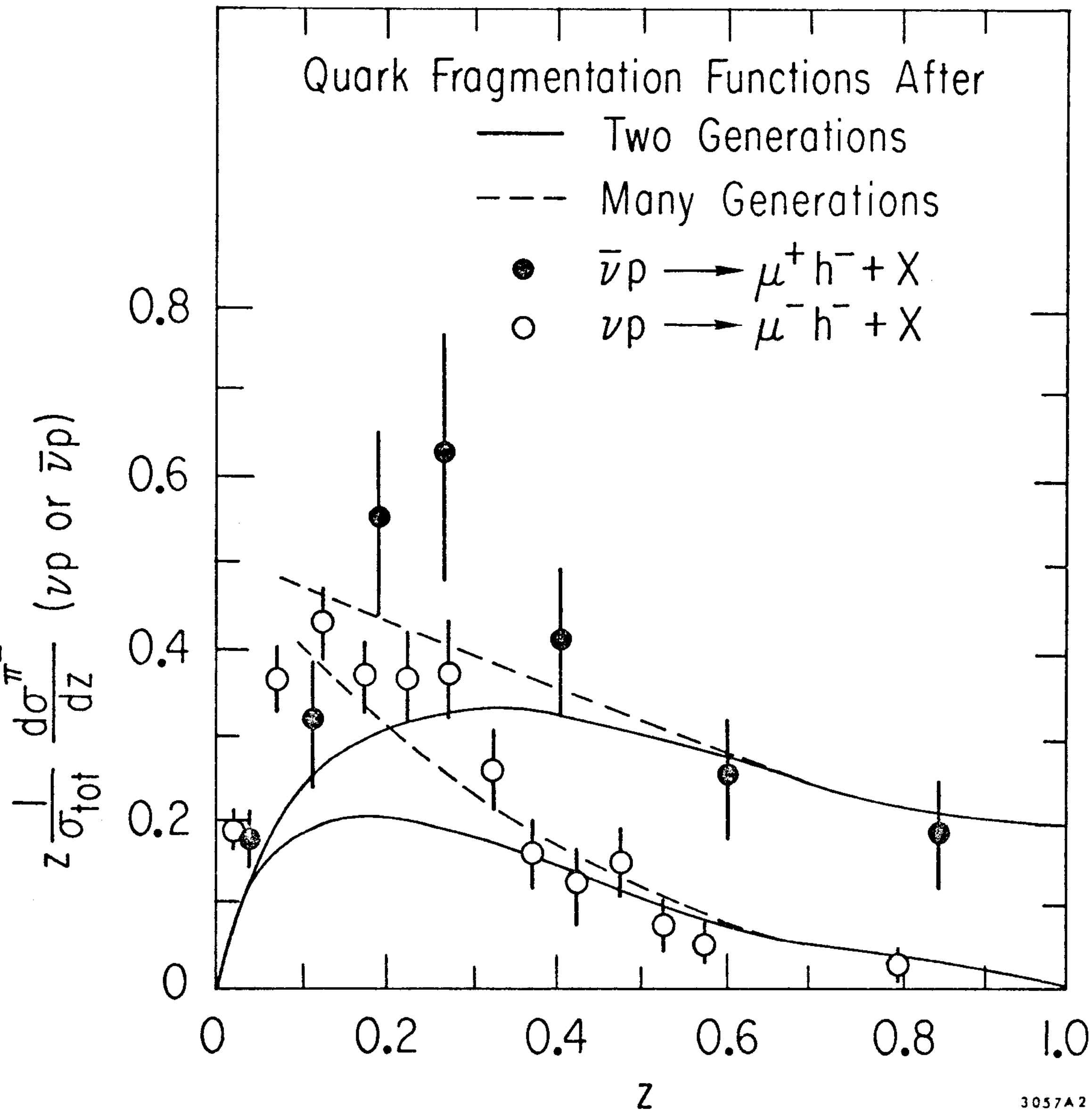
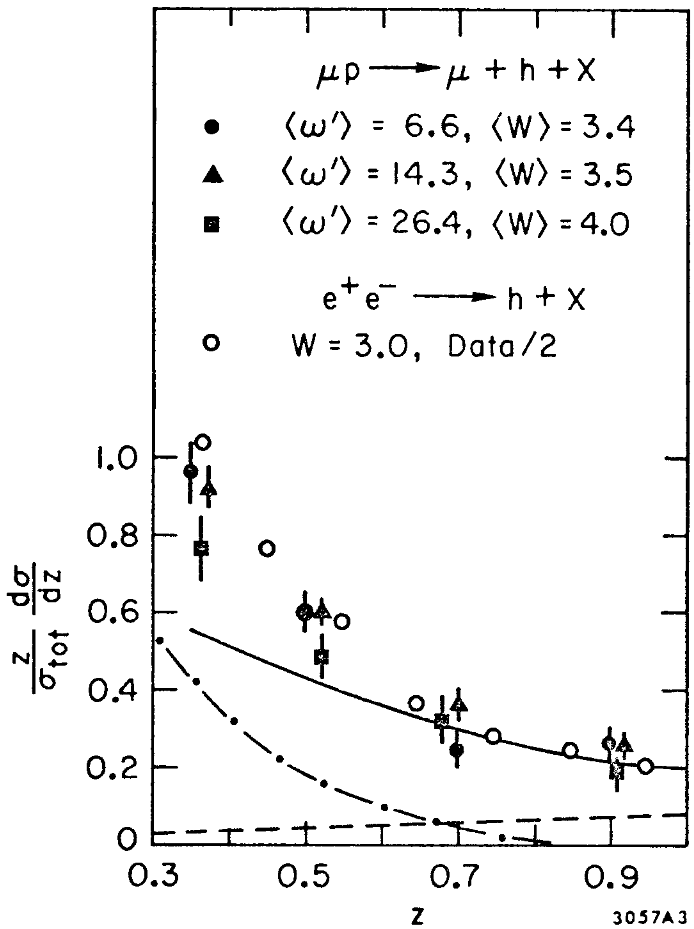

Quark Fragmentation Functions After
Two Generations
Many Generations
$\bar{\nu}p \longrightarrow \mu^+ h^- + X$
$\nu p \longrightarrow \mu^- h^- + X$
0.8
0.6
0.4
0.2
0
$\frac{1}{z\,\sigma_{tot}} \frac{d\sigma^{\pi^-}}{dz}$ (νp or $\bar{\nu}p$)
0
0.2
0.4
0.6
0.8
1.0
z
3057A2

Fig. 2



$\mu p \longrightarrow \mu + h + X$
$\langle\omega'\rangle$ = 6.6, $\langle W\rangle$ = 3.4
$\langle\omega'\rangle$ = 14.3, $\langle W\rangle$ = 3.5
$\langle\omega'\rangle$ = 26.4, $\langle W\rangle$ = 4.0
$e^+e^- \longrightarrow h + X$
W = 3.0, Data/2
$\frac{z}{\sigma_{tot}} \frac{d\sigma}{dz}$
1.0
0.8
0.6
0.4
0.2
0
0.3
0.5
0.7
0.9
z
3057A3

Fig. 3