# PHOTOPRODUCTION OF CHARMONIUM* 

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#### Abstract

Photoproduction of ortho and para charm-anticharm bound states is considered in the framework of the Cheng-Wu picture. NonAbelian gauge gluons mediate the interaction between the ce-pair and the nucleon. The angular distributions of $\psi_{c}$ and $\eta_{c}$ are determined. The influence of multigluon exchanges and quark mass variation is studied.


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[^0]Photoproduction of particles which are bound states of heavy quarks [1] permits the study of several theoretical assumptions in strong interaction dynamics and gives the possibility of investigating the dynamical implications of the large charmed quark mass [2]. The binding of the quarks into physical particles is supposed to be due to a linearly growing confinement potential [3]; this assumption was extensively used in charmonium calculations [4] and there have been many attempts to explore its deeper foundation in field theory [3]. The interaction between the quarks is commonly thought to be due to the exchange of colored gauge gluons whose interaction strength decreases with increasing gluon mass [5]; the attractive features of a gauge theory as a basic concept [6], as well as its successful application in phenomenology [7], give strong arguments in favour of such a point of view. The large charmed quark mass has dynamical implications which have been pursued in deep inelastic processes [8] but are little understood in photoproduction reactions [9]. On the phenomenological side one wonders why $\psi$-photoproduction is suppressed in comparison to photoproduction of the lighter vector mesons and why its angular distribution turns out to be less peaked in the forvard direction [1].

In this note we assume that strong interaction dynamics is correctly described by field theories of the non-Abelian type with colored gauge gluons mediating between the quarks. We therefore study the interaction of a bound constituent pair with a nucleon by gluon exchange. Within the framework of quantum electrodynamics such a problem has been studied previously by Cheng and Wu [10] and a number of other authors [11].

In the following we present the results of a simple model, which, we believe, already shows many of the characteristics resulting from our stated framework.

We consider the scattering process of a pair of charmed quarks in a scalar $1 / \mathrm{r}$ (long range) potential. The influence of the bound state nature of the quark-pair is indicated in the formal presentation of the model; however, it is dropped in its numerical evaluation since we are mostly concerned here with the consequences of gluon exchange. We first present the form of the scattering amplitude as given by Cheng and Wu [12]. Subsequently, we give the angular distribution of the ortho and para $c \bar{c}-$-states; and, thirdly, we numerically determine the dependence of the scattering amplitude on the quark mass and study the influence and behaviour of the multigluon exchange contributions.

In the present approach the scattering process shown in fig。 1 occurs in three steps: first, the incoming physical photon fluctuates into a system of freely moving constituents (c-quarks), the partons in the DLY approach [13]。 Second, each individual constituent undergoes instantaneous, elastic multiscattering processes in the gluon potential of the nucleon. There is no interaction between the quarks during this process. However, they finally interact to form the observed bound state. Within the gluon exchange framework, this three-step picture is expected to be valid at high energies where the fluctuation life time is much larger than the time needed for the interaction with the external gluon potential.

This picture has been elegantly formulated by Bjorken, Kogut, and Soper [14], using the infinite momentum frame calculus [15]. The incoming photon state is expanded in terms of the bare photon and the parton states $\mid \bar{i}>$ as:

$$
\begin{equation*}
|\gamma\rangle=\sqrt{\mathrm{z}_{\gamma}}\left[|\bar{\gamma}\rangle+\int \mathrm{d} \Gamma_{12} \cdot \mathrm{M}_{12}^{\gamma}|\overline{\mathrm{I}}\rangle|\overline{2}\rangle+\ldots\right] \tag{1}
\end{equation*}
$$

where $d \Gamma$ represents the phase space factor for the parton states and $M^{\gamma}$ is the matrix element describing the fluctuation of the photon into these states; both
quantities are determined in the infinite momentum frame．
The＂fluctuation wave function＂$M^{\gamma}$ is determined in the infinite momentum frame［14］and depends in a simple way on a longitudinal momentum $\eta$ and a transverse momentum $\overrightarrow{\mathrm{p}}$ formed by the photon and the two constituents＇mo－ menta。

The same reasoning can be applied on the final state $|\psi\rangle$（where we however exclude the existence of a bare state $|\psi\rangle\rangle$ leading to the bound state fluctuation wave function $\mathrm{M}^{\psi}\left(\overrightarrow{\mathrm{p}}^{\boldsymbol{\prime}}, \beta\right)$ ．It is related to the ordinary Schrœdinger wave function $\phi_{\mathrm{B}}(\overrightarrow{\mathrm{p}})$（which describes the bound $\mathrm{c} \overline{\mathrm{c}}$－pair），via the arguments of Cheng and Wu ［12］：

$$
\begin{equation*}
\mathrm{M}^{\psi}\left(\overrightarrow{\mathrm{p}}^{\prime}, \beta^{\prime}\right)=\sqrt{2 \mathrm{M}_{\mathrm{B}}} \cdot \phi_{\mathrm{B}}\left(\overrightarrow{\mathrm{p}}^{\prime}, \beta^{\prime} \mathrm{M}_{\mathrm{B}}\right) \cdot \mathrm{C}\left(\frac{1}{2}, \lambda_{1}, \frac{1}{2}, \lambda_{2} \mid s^{\prime}, \lambda\right) . \tag{2}
\end{equation*}
$$

$\overrightarrow{\mathrm{p}}^{\top}$ and $\beta^{\prime}$ are transverse and longitudinal momenta of the $\mathrm{c} \overline{\mathrm{c}}$－bound state system and $\mathrm{M}_{\mathrm{B}}$ is its mass．The Clebsch－Gordan coefficient $\mathrm{C}(\ldots$ ．describes the spin coupling of two fermions with helicities $\lambda_{i}$ into the ortho and para charmonium state of spin $s$ and helicity $\lambda_{\text {。 }}$ ．This spin－coupling approximation is legitimate since the constituents＇internal motion is small in the infinite momentum frame。 The overall amplitude is constructed by sandwiching the scattering operator $R \equiv S-1$ between the above initial and final states $|\psi\rangle$ and $|\gamma\rangle$ leading to the amplitude

$$
\begin{equation*}
T_{\lambda^{\prime} \lambda}=\int \frac{d \overrightarrow{\mathrm{q}}}{(2 \pi)^{2}}\left[\mathrm{~F}_{-}(\overrightarrow{\mathrm{u}}+\overrightarrow{\mathrm{q}}) \mathrm{F}_{+}(\overrightarrow{\mathrm{u}}-\overrightarrow{\mathrm{q}})-(2 \pi)^{4} \delta^{2}(\overrightarrow{\mathrm{u}}+\overrightarrow{\mathrm{q}}) \delta^{2}(\overrightarrow{\mathrm{u}}-\overrightarrow{\mathrm{q}})\right] \cdot J_{\left.\lambda^{\prime} \lambda^{(\mathrm{u}}, \overrightarrow{\mathrm{q}}\right)}, \tag{3}
\end{equation*}
$$

where $\vec{\Delta} \equiv\left(\vec{p}_{i}-\vec{p}_{\mathrm{f}}\right) \equiv 2 \overrightarrow{\mathrm{u}}$ and $\overrightarrow{\mathrm{p}_{i}}\left(\vec{p}_{\mathrm{f}}\right)$ represents the transverse momentum of the initial $\gamma$－state（final $\psi$－state）．The＂impact factor＂$J_{\lambda^{\prime} \lambda}$ contains all information on the creation process and final state binding of the constituent system through the fluctuation wave functions introduced above：

$$
\begin{equation*}
J_{\lambda^{\prime} \lambda}(\overrightarrow{\mathrm{u}}, \overrightarrow{\mathrm{q}})=\int_{-\frac{1}{2}}^{+\frac{1}{2}} \mathrm{~d} \beta \int_{-\infty}^{+\infty} \frac{\mathrm{d} \vec{\ell}}{(2 \pi)^{2}} \sum \mathrm{M}_{\lambda^{\prime}}^{\psi^{*}}(\vec{l}+\vec{m}, \beta) \cdot \mathrm{M}_{\lambda}^{\gamma}(\vec{l}-\overrightarrow{\mathrm{m}}, \beta) \tag{4}
\end{equation*}
$$

where $\vec{m}=\frac{1}{2} \vec{q}-\beta \vec{\beta}$ and the sum extends over the fermion helicities which we have omitted. The differential cross section is

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{~d} \Delta^{2}}=\frac{1}{(4 \pi)^{3}} \bar{\sum}\left|\mathrm{~T} \lambda_{\lambda^{\prime} \lambda}\right|^{2} \tag{5}
\end{equation*}
$$

The S-matrix amplitude describing the interaction of each constituent with the gluon potential is parametrized by the eikonal form

$$
\begin{equation*}
F_{ \pm}(\vec{q})=\int_{-\infty}^{+\infty} d \vec{b} \cdot e^{-i \vec{q} \vec{b}} \cdot e^{ \pm i \chi(\vec{b})} \tag{6}
\end{equation*}
$$

such that each constituent acquires an eikonal phase shift whereas their longitudinal momenta and helicities remain unchanged. Assuming a Coulomb-like gluon potential one finds

$$
\begin{equation*}
F_{ \pm}(\vec{q})= \pm i \frac{4 \pi \alpha_{S}}{{\left(\vec{q}^{2}\right)}^{2\left(1 \mp i \alpha_{s}\right)}} \cdot e^{\mp i \Phi\left(\alpha_{S}, \mu\right)} \tag{7}
\end{equation*}
$$

with the phase factor $\Phi_{S}\left(\alpha_{S}{ }^{\mu}\right)$ depending logarithmically on the small photon mass $\mu$ which was introduced in order to prevent infrared divergence。 $\alpha_{\mathrm{S}} \equiv$ $\mathrm{e}_{\mathrm{S}}^{2} / 4 \pi$ stands for the strong coupling constant with the strong interaction "charge" $e_{s}$.

We now have assembled the necessary ingredients of the $T$-matrix for the description of photoproduction of a bound quark-pair. In the following we ignore the influence of the bound state and replace the bound state wave function by a $\delta$-function. Defining the amplitudes for ortho and para charmonium production

These results reveal the following properties:

1. Keeping only $\mathrm{L}_{1}$ in $\mathrm{R}^{\mathrm{O}}$, we find the Born amplitude of single gluon exchange (forbidden by color conservation!) which reveals an angular distribution with a sharp spike near the forward direction and which then falls to zero.
2. The amplitude $R^{1}$ reveals a zero at $|\vec{\Delta}|=2 \cdot m_{c}$ due to the $\ln$-term in $\mathrm{L}_{2}(\sigma)$; in $\mathrm{L}_{2}(\sigma)$ there appears also a pole at $\sigma=1$ which is cancelled by the $\epsilon^{3 / 2}$ term in $R^{1}$.
3. Both amplitudes depend on the variable $\sigma=|\vec{\Delta}| / 2 \mathrm{~m}_{\mathrm{c}}$ and therefore scale in the c-quark mass (apart from the normalization).
4. As we go to larger $\mathrm{m}_{\mathrm{c}}$-values, the amplitudes decrease like $\sim \mathrm{m}_{\mathrm{c}}^{-7 / 2}$ and the shape of the angular distribution is shifted towards the origin.
5. Since we are working in the infinite momentum frame, the dependence on the initial energy $E_{C M}$ has completely dropped out; our formalism is therefore only valid in the asymptotic region where diffraction dominates.
6. The above results show no dependence on the target (nucleon) size since we have used an infinitely extended $1 / r-p o t e n t i a l$.
7. Our formulas are easily extended to photoproduction of electromagnetic bound-state systems as for instance "heavy leptonium" [17]; the bound-state wave function at the origin reads:

$$
\begin{equation*}
\psi_{\mathrm{B}}(0)=\frac{1}{\sqrt{\pi \mathrm{R}_{0}^{3}}}, \quad \mathrm{R}_{0}=\frac{2}{\mathrm{~m}_{\mathrm{c}^{\alpha}}}, \quad \alpha=\frac{\mathrm{e}^{2}}{4 \pi} \tag{18}
\end{equation*}
$$

and the replacement $\alpha_{S} \rightarrow(Z \alpha)$ is used. $Z(\sim 100)$ stands for the electromagnetic charge of the target atom.

We have numerically evaluated the shape of the differential cross section for $\psi_{c}$-photoproduction adjusting $\psi_{\mathrm{B}}{ }^{(0)}$ in eq。(15) such that its size agrees with
the data at $E_{C M} \sim 120 \mathrm{GeV}$ 。 In fig. 2a (and fig. 2 b ) we show its shape for $\mathrm{m}_{\mathrm{c}}$ $=1.5 \mathrm{GeV}$ (and $\mathrm{m}_{\mathrm{q}}=0.3 \mathrm{GeV}$ ) and $\alpha_{\mathrm{S}}=0.5$. The dashed lines (2-gluon exchange) represent the lowest order contribution. The solid lines (2, 4, $6 \ldots$ gluon exchanges) take multigluon corrections into account and the dashed-dotted lines ( $4,6, \ldots$ gluon exchanges) have the 2 -gluon exchange subtracted. One notices that the 2 -gluon exchange approximation is damped down by the higher order multigluon exchanges which however interfere such that their contribution is about one order of magnitude smaller. An exponential fit in the region $0.1 \leq-\mathrm{t} \leq 0.6(\mathrm{GeV} / \mathrm{c})^{2}$ gives a slope parameter $\mathrm{b} \sim 2-4 \mathrm{GeV}^{-2}$; it is less for $4,6, \ldots$ gluon exchange. Mass extrapolation to $m_{q}=0.3 \mathrm{GeV}$ (fig. 2b) brings the zero-point in the amplitude $\mathrm{R}^{1}$ (see eq. (11)) to $-\mathrm{t}=0.36(\mathrm{GeV} / \mathrm{c})^{2}$ 。This diffraction minimum is not observed in $\rho$-photoproduction [18] and it might well disappear in our model if the relativistic bound state nature of the $\rho$-meson is taken into account.

In figs. 3 a and 3 b we show the analogous curves for photoproduction of the para states $\eta_{c}$ and $\eta_{q^{\circ}}$. For illustrative purposes we have drawn the Bornapproximation (which however is forbidden by color conservation); it is strongly peaked for small $|t|$-values. $3,5, \ldots$ gluon exchange is flat over a long t-range and bends off towards zero in the extreme forward direction. The same calculation with $\mathrm{m}_{\mathrm{c}}=0.3 \mathrm{GeV}$ shows a rising curve towards smaller $|t|$-values with $\mathrm{b} \sim 5 \mathrm{GeV}^{-2}$ and a falloff to zero in the extreme forward direction.

Increasing $\alpha_{s}$ leads to a stronger influence of the higher order gluon terms besides rapidly increasing the amplitudes. The global features as presented in figs. 2 and 3 are however not substantially changed.

In this note an attempt at the descriptions of $\psi$-photoproduction in the gauge theory framework is sketched by assuming that gluons are responsible for the
interaction between the quarks. This picture leads to characteristic consequences in the shape and size of the angular distributions of $\psi_{c}$ and $\eta_{c}$. In particular, the differential cross section for $\eta_{c}$-production is $1-2$ orders of magnitude smaller in comparison to $\psi_{c}$-photoproduction and remains constant at large energies. The above presented results are distinct from other approaches and permit experimental tests.

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[19] Note that three... (four...) gluon exchanges could also contribute to ortho (para) charmonium production by taking tree-like gluon exchanges into account which have been ignored here. A simple estimate shows that their contribution is small and that no substantial changes may be expected. In addition, we have neglected higher symmetry factors due to color.

## FIGURE CAPTIONS

1. Three step picture of $\psi(c \bar{c})$ photoproduction in the gluon potential of the nucleon.

2a. Photoproduction of ortho-charmonium $\left(\psi_{c}\right)$. The solid line represents $2,4,6, \ldots$ gluon exchange, the dashed line indicates the importance of 2-gluon exchange alone, whereas the dash-dotted line shows the cross section size of $4,6, \ldots$ gluon exchange. The parameters are: $m_{c}=1.5$ GeV and $\alpha_{s}=0.5$.
23. Photoproduction of an or tho $q \bar{q}-$ state $\left(\psi_{q}\right)$. The solid line, dashed line and dash-dotted line represent $2,4,6, \ldots$ gluon exchange. The parameters are: $\mathrm{m}_{\mathrm{q}}=0.3 \mathrm{GeV}$ and $\alpha_{\mathrm{S}}=0.5$.
3a. Photoproduction of para-charmonium $\left(\eta_{c}\right)$. The solid line represents $3,5,7 \ldots$ gluon exchange, the dashed line indicates single gluon exchange (which is forbidden by color conservation?) and the dotted line indicates the size of the 3-gluon exchange near the forward direction. The parameters are: $m_{c}=1.5 \mathrm{GeV}$ and $\alpha_{s}=0.5$.
3b. Photoproduction of a para $q \bar{q}-$ state $\left(\eta_{q}\right)$. The solid line, dashed line and dotted line represent $3,5,7, \ldots$ gluon exchange, single-gluon exchange and 3-gluon exchange. The parameters are: $\mathrm{m}_{\mathrm{q}}=0.3 \mathrm{GeV}$ and $\alpha_{\mathrm{s}}=0.5$.


Fig. 1


Fig. 2


Fig. 3


Fig. 4


Fig. 5


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