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THE TRANSVERSE MOMENTUM DISTRIBUTION OF THE DILEPTONS IN HADRONIC PROCESSES*

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ABSTRACT

The dependence of $< p_{\perp}^2 >$ on the masses of the Drell-Yan dileptons are studied using the quark transverse momentum distribution in hadrons.

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The impulse approximation as applied to elementary particle interactions has been shown to be rather successful in deep inelastic scattering. Theoretically, it should be valid only when the partons can be treated as instantaneously free and when the energy is approximately conserved along with the momentum across the interaction vertex of the parton.[1] For these reasons, the fraction $x = Q^2/2p\cdot q$ of the longitudinal momentum must be finite while Q^2 and $2p\cdot q$ must be large enough in order to apply this approximation. Recently two dilepton very high energy experiments[2] were performed with $Q^2 > 25\ \mathrm{GeV}^2$. The measured dilepton cross section $\left.\frac{d\sigma}{dQdy}\right|_{y=0}$ agrees very well with the calculation of the Drell-Yan process.[4] Due to good statistics of the measured data, one can rule out several assumptions for the quark distribution functions. The mass dependence of these cross sections clearly indicates that the distributions based on power counting rules[4,5] are valid. Also, it was observed in these experiments and others[3] at low Q^2 that the averaged transverse momenta of the dileptons increases with dilepton masses. This phenomenon can be explained by the quark transverse momentum distribution obeying these simple power counting rules.

Consider a quark distribution function u. From the interchange model, one has[6]:

$$u(x, k^2, \bar{\sigma}) \propto \frac{1}{1-x} \left| \psi\left(\frac{k_\perp^2 + \bar{\sigma}}{1-x}, \bar{\sigma}\right) \right|^2 \tag{1}$$

where ψ is the wave function describing the breakup of the hadron into quarks and a core $\bar{\sigma}$. For example, the natural variable k^2 reduces to a correlated combination of the quark momenta $(1-x)$ and $k_\perp^2$. The contribution of this component of the quark wave function to the proton's spin averaged form factor at a momentum transfer $t = -q_\perp^2$ may be written as[7]:

$$F(q_\perp^2) \propto \int \frac{dx}{1-x}\, dk_\perp^2\, \psi\left(\frac{k_\perp^2 + \bar{\sigma}}{1-x}, \bar{\sigma}\right) \psi\left(\frac{[k_\perp - (1-x)\, q_\perp]^2 + \bar{\sigma}}{1-x}, \bar{\sigma}\right) \tag{2}$$

Taking

$$\psi \propto \left(\frac{1-x}{k_\perp^2 + \bar{\sigma}}\right)^n \tag{3}$$

for large value of k^2, one finds that near $x = 1$

$$u(x, k_\perp^2) \propto \frac{(1-x)^{2n-1}}{(k_\perp^2 + \bar{\sigma})^{2n}} \tag{4}$$

and

$$F(q_\perp^2) \propto (q_\perp^2)^{-n} \ell n\, q_\perp^2 \, . \tag{5}$$

According to the power counting rules[4, 7], for 5-quark minimal sea state, $n = 4$, and for the leading quarks, $n = 2$. The distribution function for the sea quarks is[4]

$$f_{sea}(x, k_\perp^2) \propto \frac{(1-x)^7/x}{(k_\perp^2 + \bar{\sigma})^8} \tag{6}$$

and for the leading quarks

$$f_{lead}(x, k_\perp^2) \propto \frac{(1-x)^3}{(k_\perp^2 + \bar{\sigma})^4} \, . \tag{7}$$

In pp scattering, the low mass pairs are formed by convoluting sea-$\overline{\text{sea}}$ distributions:

$$\int \frac{dk_{\perp 1}}{(k_{\perp 1}^2 + \bar{\sigma}_1)^8} \frac{dk_{\perp 2}}{(k_{\perp 2}^2 + \bar{\sigma}_2)^8} \tag{8}$$

while the high mass pairs are formed by convoluting $\overline{\text{sea}}$-leading distributions:

$$\int \frac{dk_{\perp 1}}{(k_{\perp 1}^2 + \bar{\sigma}_1)^8} \frac{dk_{\perp 2}}{(k_{\perp 2}^2 + \bar{\sigma}_2)^4} \tag{9}$$

where 1 and 2 stand for the incident and target proton.

The $< p_\perp^2 >$ of the dileptons can be calculated in terms of $k_{\perp 1}^2$ and $k_{\perp 2}^2$ and for any arbitrary powers n_1 and n_2:

$$p_\perp^2 = k_{\perp 1}^2 + k_{\perp 2}^2 + 2k_{\perp 1} k_{\perp 2} \cos\theta_{12}$$

and

$$< p_\perp^2 > = \frac{\displaystyle\int \frac{(k_{\perp 1}^2 + k_{\perp 2}^2 + 2k_{\perp 1} k_{\perp 2} \cos\theta_{12}) k_{\perp 1} dk_{\perp 1} d\theta_1 k_{\perp 2} dk_{\perp 2} d\theta_2}{(k_{\perp 1}^2 + \bar{\sigma}_1)^{n_1} (k_{\perp 2}^2 + \bar{\sigma}_2)^{n_2}}}{\displaystyle\int \frac{k_{\perp 1} dk_{\perp 1} d\theta_1 \, k_{\perp 2} dk_{\perp 2} d\theta_2}{(k_{\perp 1}^2 + \bar{\sigma}_1)^{n_1} (k_{\perp 1}^2 + \bar{\sigma}_2)^{n_2}}} \; . \tag{10}$$

Performing the integral, taking the limits of $k_\perp$ from 0 to ∞ (a very crude approximation):

$$< p_\perp^2 > \; = \bar{\sigma} \left[\frac{1}{n_1 - 2} + \frac{1}{n_2 - 2} \right] \quad . \tag{11}$$

A more complicated value of $< p_\perp^2 >$ is obtained if we take the proper kinematical limits for $k_\perp$. For low mass dileptons, $n_1 = n_2 = 8$, and for the high mass, $n_1 = 8$ and $n_2 = 4$. Define R the ratio of the averaged transverse momentum squared of the high mass and the low mass dileptons, replacing appropriate values of n_1 and n_2 in Eq. (11); one gets:

$$R \equiv \frac{< p_\perp^2 >_{high}}{< p_\perp^2 >_{low}} = 2 \quad . \tag{12}$$

We wish to stress that from this model, due to the nature of the power laws, the averaged transverse momentum does not grow indefinitely, but remains constant after attaining the limiting value dictated by $\bar{\sigma}$ and the powers n_1 and n_2. As seen from the nature of the argument presented, this crude calculation should give only qualitative predictions of the transverse momentum behaviors. The quantitative dependence of $< p_\perp^2 >$ on the dilepton masses and on its x_F distribution is under investigation, and this dependence is much more complicated.[8]

From Eq. (6) and (7), one can intuitively see that the $< p_\perp^2 >$ should increase with mass and also with x_F, a phenomenon commonly observed in inclusive hadron production (the "seagull" effect). In this Drell-Yan process Q^2 is not a scaling variable and we suggest that for future presentation of the data, $< p_\perp^2 >$ should be plotted versus Q^2/s for data taken at different energies. Since R differs for processes involving different incident particles and targets, the observed universal $< p_\perp^2 >$ for different energies and projectiles is considered coincidental.

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References

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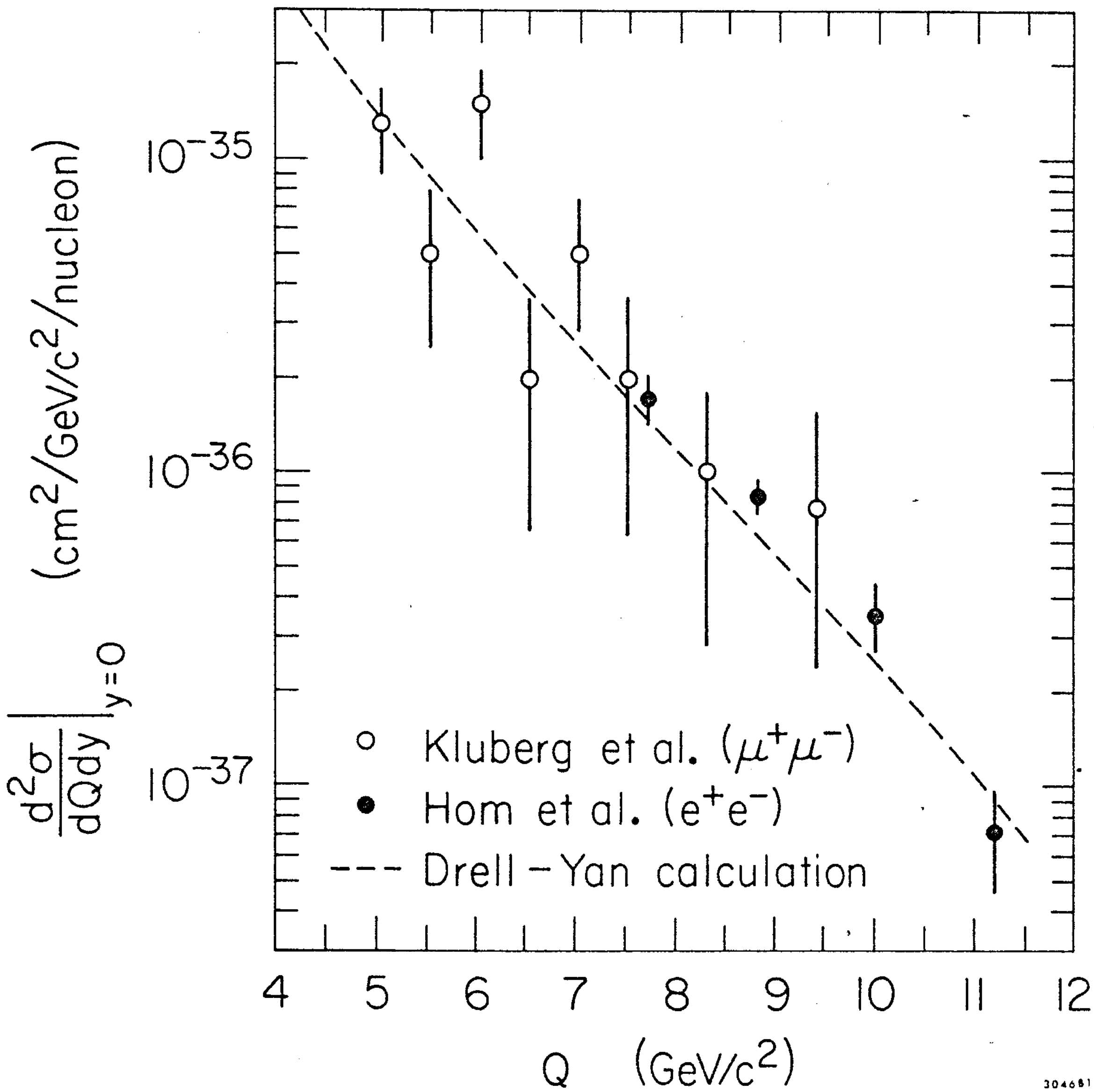
8. We are currently investigating this problem in detail.

Figure Caption

Fig. 1: The dilepton mass distribution measured at the rapidity $y = 0$ from the two high Q^2 experiments[2] at FNAL. The dotted curve is the prediction based on the calculation and quark distribution in Ref. 4.



$\left.\frac{d^2\sigma}{dQdy}\right|_{y=0}$ (cm^2/GeV/c^2/nucleon)
10^{-35}
10^{-36}
10^{-37}
Kluberg et al. ($\mu^+\mu^-$)
Hom et al. (e^+e^-)
Drell – Yan calculation
4
5
6
7
8
9
10
11
12
Q (GeV/c^2)
304681

Fig. 1